Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 105 (2015) 160 – 166

Procedia Engineering

www.elsevier.com/locate/procedia

6th BSME International Conference on Thermal Engineering (ICTE 2014)

Thermal Diffusion Effect on Unsteady Viscous MHD Micropolar Fluid Flow through an Infinite Vertical Plate with Hall and Ion-slip Current

Nisat Nowroz Anika[a], Md. Mainul Hoque[b]*, Sheikh Imamul Hossain[c] and Md. Mahmud Alam[c]

[a]*Department of Mechanical Engineering, The University of Newcastle, Callaghan, NSW 2308, Australia*
[b*]*Department of Chemical Engineering, The University of Newcastle, Callaghan, NSW 2308, Australia*
[c]*Mathematics Discipline, Khulna University, Khulna-9208, Bangladesh*

Abstract

An analysis is carried out to study the effect of Hall and Ion-slip current and heat transfer characteristics over an infinite vertical plate for micropolar fluid in the presence of magnetic field. The governing boundary layer equation first transformed into non-dimensional form and resulting non-linear system of partial differential equations are then solved numerically by using the robust implicit finite difference technique. Also the unconditional stability and convergence test has been carried out to establish the effect of shear stresses, couple stress, Nusselt number and Sherwood number on the flow field. Finally, the effects of various parameters are separately discussed and shown graphically.

Peer-review under responsibility of organizing committee of the 6th BSME International Conference on Thermal Engineering (ICTE 2014)

*Keywords:*Micropolar fluid; Hall current; Ion-slip current ; Thermal diffusion.

* Corresponding author. Tel.: +61 406 618 193;
*E-mail address:*mohammadmainul.hoque@uon.edu.au

Peer-review under responsibility of organizing committee of the 6th BSME International Conference on Thermal Engineering (ICTE 2014)
doi:10.1016/j.proeng.2015.05.052

1. Introduction

Magneto-micropolar fluid flow with hall and ion slip currents is play an important role in the view of its wide applications in many engineering problemssuch as electric transformers and heating elements. Eringen [1] proposed the theory of micopolar fluid considering microrotation and microinertial effect. These flow characteristics cannot be described by the usual Navier-Stokes theory. Hence,the renowned NavierStokes formula is not appropriatefor modelling such type of problem. Since, the pioneering work by Eringen [1] has created a lot of interest. However, there are comprehensive works that have been carried out on viscous and incompressible fluid with the effect of Hall current by Chamhka [2], Seddeek [3], Takher et al. [4], Shateyi et al. [5,6], Salem and Abd El-aziz [7], Anika et al. [8,9], and among others.

Nomenclature

(x, y, z)	Cartesian Coordinates	(u, v, w)	Velocity components
e	Suction velocity	M	Magnetic Parameter
b_e	Hall current	t	Dimensionless time
P_r	Prandlt number	E_c	Eckert number
S_c	Schmidt number	**B**	Magnetic field
D	Microrotation parameter	T_w	Temperature at the plate
l	Spin gradient viscosity	$T_¥$	Temperature outside the boundary layer
L	Vortex viscosity	C_w	Concentration at the plate
G_r	Grashof number	$C_¥$	Concentration outside the boundary layer
G_r^*	Modified Grashof number	U	Dimensionless primary velocity
b_i	Ion-slip parameter	W	Dimensionless secondary velocity
S_o	Soret number	W	Dimensionless microrotation variable
q	Dimensionless Temperature	F	Dimensionless Concentration variable

The momentum, heat, and mass transport on vertical plate have several practical engineering applications. The heat transfer problem associated with Hall and Ion slip current under different physical conditions has been analysed by several authors. Dash et al. [10] investigated MHD free convection and mass transfer flow over a continuously moving vertical plate under the action of strong magnetic field. The hall and ion slip current in the momentum equation are considered for high speed fluid flows and the level of concentration of foreign mass have been taken very high. Anika and Hoque [11] studied the one dimensional Magnetohydrodynamics flow behaviour through an infinite vertical plate. Hall and Ion slip current with strong magnetic field and constant suction are applied perpendicular to the plate. Recently, Haque et al. [12] studied MHD free convection and mass transfer flow past a semi-infinite vertical porous plate having variable suction with constant heat and mass fluxes. The diffusion thermo, thermal diffusion terms, viscous dissipation and Joule heating terms have been considered for high speed fluid.But the boundary layer micropolar fluid flow characteristic in presence of Hall and Ion slip with magnetic field still not well understood.

Hence our aim is to study the Unsteady One-dimensional Micropolar fluid flow behaviour through an infinite vertical plate with the influence of Hall and Ion-slip current having constant suction with constant heat and mass fluxes. The model has been solved by implicit finite difference technique. In general it is very complicated to evaluate the flow characteristic of this kind of problem. Therefore, it is necessary to investigate in detail the accuracy of primary velocity, secondary velocity, angular velocity, temperature and concentration across the boundary layer. The corresponding Shear stress in x and z direction, Couple stress, Nusselt number and Sherwood number have been shown graphically. In this study, all figures have not shown for brevity.

2. Mathematical Model of Flow

An unsteady one dimensional free convection flow of ionized micropolar fluid along an infinite vertical plate $y = 0$ has been expedited in this work. The flow is permeated by a non-conducting vertical plate taken along x - axis in the upward direction and y -axis is normal to the plate. A uniform magnetic field of strength B_0 is assumed to be applied along the positive y -direction normal to the plate and that induced another magnetic field on the electrically conducting fluid. The flow configuration and coordinate system are shown in Fig.1. Consequently, electrically conducting fluid is affected Hall and Ion-slip current. A force (coriolis force named microrotational force) owing to the rotating of the particle and the interaction of magnetic and electric field induces a cross flow in the z -direction. The equation of conservation of charge $\nabla . J = 0$ gives $J_y =$ constant.Since the plate is of infinite extent and the fluid motion is unsteady so all the flow variables will depend only upon y and time t . Within the framework of the above stated assumption and using the dimensionless quantities, $Y = \frac{yU_0}{\vartheta}, U = \frac{u}{U_0}, W = \frac{w}{U_0}, \tau = \frac{tU_0^2}{\vartheta}, \theta = \frac{T-T_\infty}{T_w-T_\infty}$ in the equations relevant to the problem is governed by the following coupled non-dimensional partial differential equations under the electromagnetic Boussinesq approximations as:

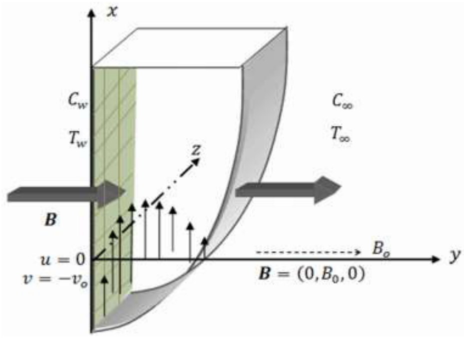

Fig.1: Boundary layer development on a vertical plate

$$\frac{\partial U}{\partial \tau} - \epsilon \frac{\partial U}{\partial Y} = (1+\Delta)\frac{\partial^2 U}{\partial Y^2} + \Delta \frac{\partial \Omega}{\partial Y} + G_r\theta + G_r^*\Phi - \frac{\mathrm{M}}{\left(\alpha_e{}^2 + \beta_e{}^2\right)}(\alpha_e U + \beta_e W) \tag{1}$$

$$\frac{\partial W}{\partial \tau} - \epsilon \frac{\partial W}{\partial Y} = (1+\Delta)\frac{\partial^2 W}{\partial Y^2} + \frac{\mathrm{M}}{\left(\alpha_e{}^2 + \beta_e{}^2\right)}(\beta_e U - \alpha_e W) \tag{2}$$

$$\frac{\partial \Omega}{\partial \tau} - \epsilon \frac{\partial \Omega}{\partial Y} = \Lambda \frac{\partial^2 \Omega}{\partial Y^2} - \lambda\left(2\Omega + \frac{\partial U}{\partial Y}\right) \tag{3}$$

$$\frac{\partial \theta}{\partial \tau} - \epsilon \frac{\partial \theta}{\partial Y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial Y^2} + (1+\Delta)E_c\left[\left(\frac{\partial U}{\partial Y}\right)^2 + \left(\frac{\partial W}{\partial Y}\right)^2\right] + \mathrm{M}\frac{E_c}{\left(\alpha_e{}^2 + \beta_e{}^2\right)}(U^2 + W^2) \tag{4}$$

$$\frac{\partial \Phi}{\partial \tau} - \epsilon \frac{\partial \Phi}{\partial Y} = \frac{1}{S_c}\frac{\partial^2 \Phi}{\partial Y^2} + S_0\frac{\partial^2 \theta}{\partial Y^2} \tag{5}$$

with the corresponding initial and boundary conditions

$$\tau > 0, \quad \begin{array}{ll} U = 0,\ W = 0,\ \Omega = -S\frac{\partial U}{\partial Y},\ \theta = 1,\ \Phi = 1 & \text{at } Y = 0 \\ U = 0,\ W = 0,\ \Omega = 0,\ \theta = 0,\ \Phi = 0 & \text{as } Y \to \infty \end{array} \tag{6}$$

where the symbols have their usual meaning and defined in nomenclature section.

3. Method of Solution

For simplicity the implicit finite difference method has been used to solve equations (1)-(5) subject to the initial and boundary conditions equation (6). In this case the region within the boundary layer is divided by some perpendicular lines of Y - axis, where Y - axis is normal to the medium as shown in Fig. 2.It is assumed that the maximum length of boundary layer is $Y_{\max}(= 30)$ as corresponds to $Y \to \infty$ i.e. Y vary from **0** to 30. And the number of grid spacing in Y directions is $m(= 200)$, hence the constant mesh size along Y axis becomes $\Delta Y = 0.15 (0 \le Y \le 30)$ with the smaller time step $\Delta t = 0.001$.

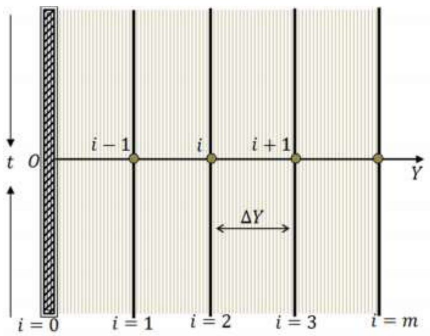

Fig 2. Implicit finite difference space grid.

Let $U_i^n, W_i^n, \Omega_i^n, \theta_i^n$ and Φ_i^n denote the values of U, W, Ω, θ and Φ at the end of a time-step respectively. Using the implicit finite difference approximation we have

$$\frac{U_i^{n+1}-U_i^n}{\Delta\tau}-\epsilon\frac{U_{i+1}^n-U_i^n}{\Delta Y}=(1+\Delta)\frac{U_{i+1}^n-2U_i^n+U_{i-1}^n}{(\Delta Y)^2}+\Delta\frac{\Omega_{i+1}^n-\Omega_i^n}{\Delta Y}+G_r\theta_i^n+G_r^*\Phi_i^n-\frac{M}{\left(\alpha_e{}^2+\beta_e{}^2\right)}\left(\alpha_eU_i^n+\beta_eW_i^n\right) \quad (7)$$

$$\frac{W_i^{n+1}-W_i^n}{\Delta\tau}-\epsilon\frac{W_{i+1}^n-W_i^n}{\Delta Y}=(1+\Delta)\frac{W_{i+1}^n-2W_i^n+W_{i-1}^n}{(\Delta Y)^2}+\frac{M}{\left(\alpha_e{}^2+\beta_e{}^2\right)}\left(\beta_eU_i^n-\alpha_eW_i^n\right) \quad (8)$$

$$\frac{\Omega_i^{n+1}-\Omega_i^n}{\Delta\tau}-\epsilon\frac{\Omega_{i+1}^n-\Omega_i^n}{\Delta Y}=\Lambda\frac{\Omega_{i+1}^n-2\Omega_i^n+\Omega_{i-1}^n}{(\Delta Y)^2}-\lambda\left(2\Omega_i^n+\frac{U_{i+1}^n-U_i^n}{\Delta Y}\right) \quad (9)$$

$$\frac{\theta_i^{n+1}-\theta_i^n}{\Delta\tau}-\epsilon\frac{\theta_{i+1}^n-\theta_i^n}{\Delta Y}=\frac{1}{P_r}\frac{\theta_{i+1}^n-2\theta_i^n+\theta_{i-1}^n}{(\Delta Y)^2}+(1+\Delta)E_c\left[\left(\frac{U_{i+1}^n-U_i^n}{\Delta Y}\right)^2+\left(\frac{W_{i+1}^n-W_i^n}{\Delta Y}\right)^2\right]+\mathrm{M}\frac{E_c}{\left(\alpha_e{}^2+\beta_e{}^2\right)}\left(U_i^{n2}+W_i^{n2}\right) \quad (10)$$

$$\frac{\Phi_i^{n+1}-\Phi_i^n}{\Delta\tau}-\epsilon\frac{\Phi_{i+1}^n-\Phi_i^n}{\Delta Y}=\frac{1}{S_c}\frac{\Phi_{i+1}^n-2\Phi_i^n+\Phi_{i-1}^n}{(\Delta Y)^2}+S_o\frac{\theta_{i+1}^n-2\theta_i^n+\theta_{i-1}^n}{(\Delta Y)^2} \quad (12)$$

and the initial and boundary condition with finite difference scheme as

$$U_0^n=0,\quad W_0^n=0,\quad \Omega_0^n=-S\frac{U_1^n-U_0^n}{\Delta Y},\quad \theta_0^n=1,\quad \Phi_0^n=1,\quad \tau>0$$
$$U_L^n=0,\quad W_L^n=0,\quad \Omega_L^n=0,\quad \theta_L^n=1,\quad \Phi_L^n=1,\quad L\to\infty \quad (13)$$

Here, $i$represent the grid points while n represents a value of time, $\tau=n\Delta\tau$, where, $n=0,1,2,3,\ldots\ldots\ldots$...Also the numerical values of the shear stresses, couple stress, Nusselt number and Sherwood number are eventually by five point approximate formula for the derivatives. The continuity equation is ignored since $\mathrm{D}t$ does not appear in it. The general terms of the Fourier expansion for $U_i^n, W_i^n, \mathrm{W}_i^n, q_i^n$ and F_i^n at a time arbitrarily called $t=0$ are all e^{iaY} apart from a constant, where $i=\sqrt{-1}$. The stability conditions are not shown for brevity.

4. Results and Discussion

To investigate the physical conditions of the developed mathematical model the effects of dimensionless steady-state shear stresses (namely t_x, t_z), couple stress, Nusselt number and Sherwood number versus t has been analysed in Figs. 3 to 15 respectively. Fig. 3 has shown the effects of magnetic parameters M on the several steady-state Shear stresses. It can be observed that the main flow decreased with the magnetic parameter M. But the Shear stress in z-direction (Fig. 4) gives the opposite effect with increase of magnetic parameter.

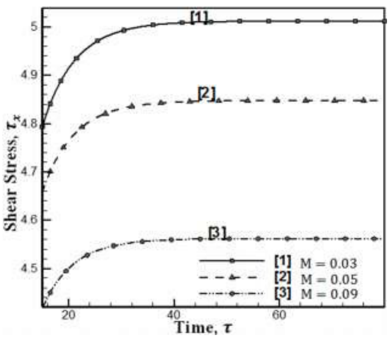

Fig. 3.Shear stressτ_x for different values of$M$.

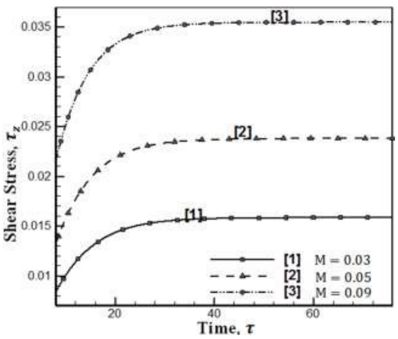

Fig. 4.Shear stress τ_z for different values of$M$.

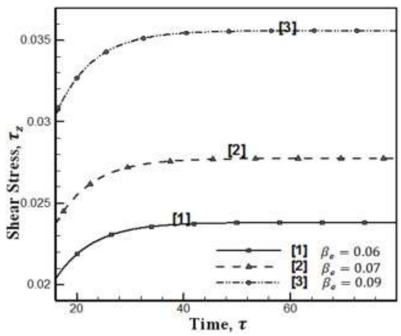

Fig. 5. Shear stress $\tau_z$for different values ofβ_e.

Also from Fig. 5 it can be concluded that shear stress in the z -direction has increased with increaseof Hall parameter b_e as compared to time t , whereas the shear stress in x -direction has no visible effect(figure is not shown for brevity).Generally, it has been seen that an electrically conduction fluid is affected by Hall currents in the presence of transverse magnetic field. So the shear stress t_z increases with increasing values of b_e , when b_e £ 1 . Fig.6 ascertain that the effect of P_r on Nusselt $(- N_u)$ number. The value of Nusselt $(- N_u)$ significantly increases with increase of P_r . Schmidt number S_c embodies the ratio of the momentum diffusivity to the mass

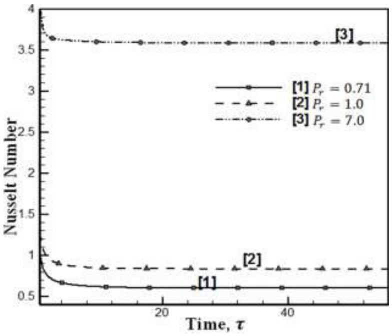

Fig. 6.Nusselt number for different values of P_r

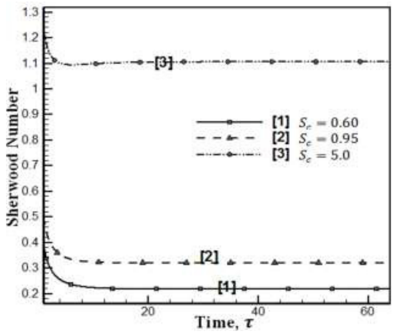

Fig. 7. Sherwood number for different values of S_c

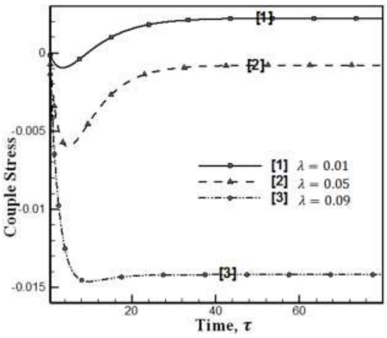

Fig. 8. Couple stressfor different values of λ.

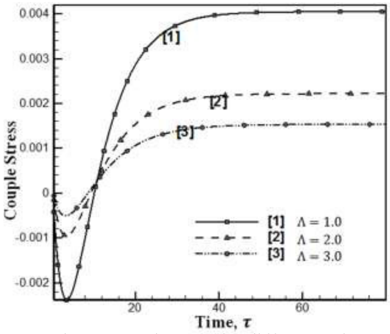

Fig. 9. Couple stressfor different values of Λ.

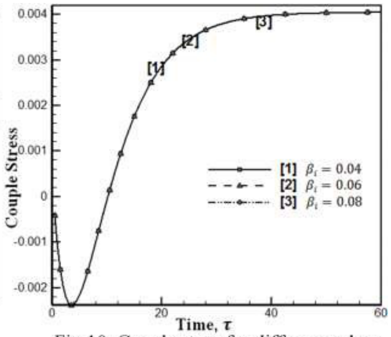

Fig.10. Couple stressfor different values of dimensionless Ion-slip parameterβ_i.

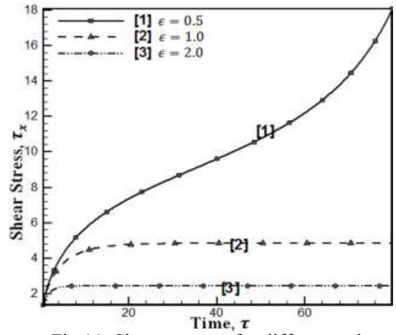

Fig.11. Shear stress $\tau_x$for different values of dimensionless Suction parameter ϵ.

(species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass transfer (concentration) boundary layer. Fig. 7 has shown the effect of Sherwood $(- S_h)$ number for different values of S_c .The time development of couple stress for the different values Vortex viscosity l has been presented in Fig.8. Shear viscosity causes a backward flow and then it rise to become linear. Therefore, the couple stress decrease as vortex viscosity l increases with dimensionless time until they reach the steady-state condition as t ® ¥ .Local spinning motion of the fluid usually decreases in viscosity at higher shear rates. The effects of spin gradient viscosity L on couple stress have been plotted in Fig.9 with respect to t . The steady state couple stress has a backward flow and increase steeply onwards as compared to t . Again the couple stress increases with the increase of gradient viscosity L . Considerable effect of dimensionless Ion-slip parameter b_i on time development has been plotted in Fig.10. Because of considering very small magnetic parameter, the increase of b_i has negligible effects on those profiles.In order to get the physical insight of suction parameter e , it has been seen from Figs. 11, 12 that shear stresses for both x and z direction has fallen for

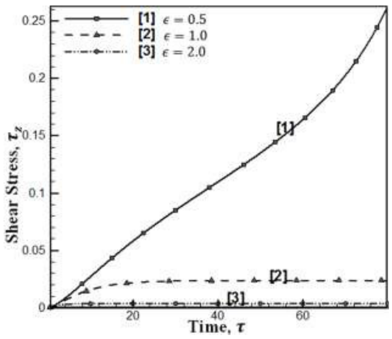

Fig.12. Shear stress $\tau_z$for different values of dimensionless Suction parameterϵ

the rise of e . So, boundary layer flow attains minimum velocity for large suction.And the Sherwood number (- S_h) has increases for low suction as illustrated in Fig.13.The effect of dimensionless Soret number S_o has been discussed in Figs.14 and 15. It is noticed that for increasing S_o , at low times, Couple stress decelerates for large S_o . And then onwards it was boosted from below for large time steps with the increase of S_o . The Sherwood number (- S_h) decreases drastically for the increase of dimensionless Soret number S_o as illustrated in Fig.15.

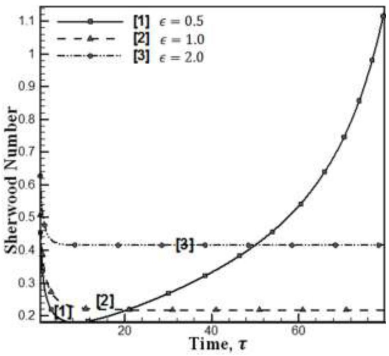

Fig.13. Sherwood numberfor different values of dimensionless Suction parameter ϵ

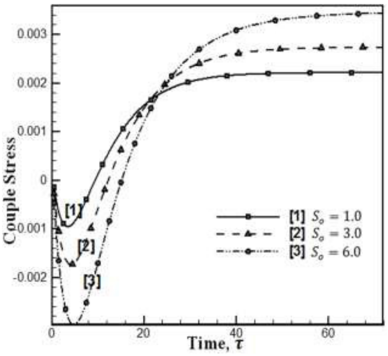

Fig.14. Couple stress for different values of dimensionless Soretnumber S_o.

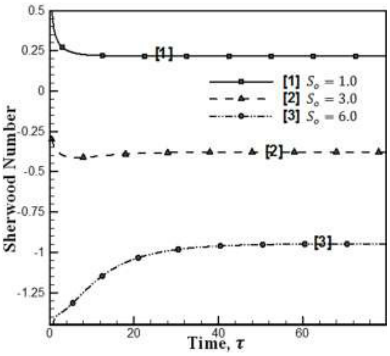

Fig.15. Sherwood number for different values of dimensionless Soretnumber S_o.

5. Conclusions

MHD mass transfer problem by free convection flow of an ionized incompressible viscous micropolar fluid across the infinite vertical plate under the action of Hall current and Ion-slip parameter has been taken into account. The physical properties are graphically discussed for different values of corresponding parameters. Some important findings of this study are given below:

1. The Shear stress in x -direction decreases with the increase of M and e .
2. The Shear stress in $\boldsymbol{z}$-direction increases for of M and b_e while it decreases with the increase of e .
3. The Couple stress increases with increase of L and decrease for l and S_0 .
4. The Sherwood number increases with the increase of S_c and e and decrease for S_o .
5. The Nusselt number increases with increase of P_r .

References

[1] A. C. Eringen, Theory of micropolar fluids,Journal of Mathematics and Mechanic. 16 (1966) 1–18.
[2] A. J. Chamkha, MHD-free convection from a vertical plate embedded in a thermally stratified porousmedium with hall effects, Applied Mathematical Modelling. 21(10) (1997) 603–609.
[3] M. A. Seddeek, Effects of hall and ion-slip currents on magneto-micropolar fluid and heat transferover a non-isothermal stretching sheet with suction and blowing,Proceedings of the Royal Society A. 457 (2001) 3039–3050.
[4] H. S. Takhar, A. J. Chamkha, and G. Nath, MHD flow over a moving plate in a rotating fluid withmagnetic field, hall currents and free stream velocity,International Journal of Engineering Science.40 (13) (2002) 1511–1527.
[5] S. Shateyi, P. Sibanda, and S. S. Motsa, Magnetohydrodynamic flow past a vertical plate withradiative heat transfer, Journal of Heat Transfer. 129 (12) (2007) 1708–1713.
[6] S. Shateyi, S. S. Motsa, and P. Sibanda, The effects of thermal radiation, hall currents, soret, anddufour on MHD flow by mixed convection over a vertical surface in porous media,MathematicalProblems in Engineering. Article ID 627475 (2010) 1-20.
[7] A. M. Salem and M. Abd El-Aziz, Effect of hall currents and chemical reaction on hydromagneticflow of a stretching vertical surface with internal heat generation/absorption, Applied Mathematical Modelling. 32 (7) (2008) 1236–1254.
[8] N. N. Anika, M. M. Hoque and M. M. Alam, Unsteady free convection flow with flow control parameter, Current Trends in Technology and Science. 2 (1) (2012) 193-201.
[9] N. N. Anika, M. M. Hoque and N Islam, Hall current effects on magnetohydrodynamics fluid over an infinite rotating vertical porous plate embedded in unsteady laminar flow, Annals of Pure and Appl. Mathematics. 3 (2) (2013) 189-200.
[10] K. Dash, M. M. Alam and M. Wahiduzzaman, MHD free convection and mass transfer flow from a vertical plate in the presence of hall and ion slip current,Advances in Mechanical Engineering. Article ID 851957 (2012) 1-20.
[11] N. N. Anika and M. M. Hoque, Thermal buoyancy force effects on developed flow considering hall and ion-slip current, Annals of Pure and Appl. Mathematics. 3 (2) (2013) 179-188.

[12] Z. Haque, M. M. Alam, M. Ferdows and A. Postelnicu, Micropolar fluid behaviour on steady MHD free convection and mass transfer flow with constant heat and mass fluxes, joule heating and viscous dissipation, J. King Saud Univ. Engineering Sciences, 24(2) (2012) 71-84.