Engineering the Realizations of Localized Waves with Independently Addressable Pulse Driven Arrays

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***Abstract*—The various descriptions of localized wave solutions of the wave equation and Maxwell's equations are briefly reviewed. The generation of these space-time coupled fields from independently addressable pulse driven arrays is discussed. Important aspects of pulse driven array elements and their impact on the overall performance of a pulse driven array is emphasized. It is further described that many of the exotic properties associated with launched localized waves can be realized with these arrays.**

***Index Terms*—Localized waves, pencil-beams, pulse driven arrays, Rayleigh distance, ultra-wide bandwidth arrays**

I. INTRODUCTION

Localized waves (LWs) are space-time coupled fields [1]. Brittingham reported the first LW; he termed it a focused wave mode [2]. As solutions of the wave equation, LWs represent scalar (acoustic) fields. As solutions of Maxwell's equations, they represent electromagnetic fields. LWs are acoustic and electromagnetic (EM) pulses with provocative properties. They are localized in both space and time and do not disperse as they propagate. Because of their pencil-beam behavior, the LW term was introduced by the author to clearly distinguish them from commonly known plane wave solutions. Like with plane waves, more complex space-time field structures can be obtained as weighted superpositions of basic LWs. Unlike the plane wave outcomes, the LW superpositions yield space-time localized fields.

Because they are space-time fields, the LWs to be discussed here and in my presentation are fundamentally different from the frequency domain, diffraction-free Bessel beams [3]. While Bessel beams are strictly infinite energy solutions, finite energy LWs have been demonstrated [4], as was confirmed in [5]. Furthermore, many of the predicted exotic properties of LWs were confirmed experimentally when an acoustic version of the modified power spectrum (MPS) pulse [4] was launched in a water tank using an independently addressable pulse driven array (IA-PDA) [6]. The extended near field properties associated with LWs were also demonstrated with an acoustic array system formed with a pair of transmit and receive IA-PDAs [7] consisting of the same elements employed in the initial experiments [6]. The basic theory and concepts of IA-PDAs were originally summarized in [8] and [9].

The LW field is generated from an IA-PDA by driving each of its elements with the time pulse specified by the LW solution at its spatial location in the array. Consequently, multiple waveforms must be generated across the entire array. These pulses are localized in time and, hence, have ultra-wide bandwidth characteristics. While arbitrary voltage waveform synthesizers facilitated the acoustic experiments (each radiating/receiving element was an electrically small piezoelectric transducer that was stimulated by a voltage applied to it), the electromagnetic versions were not possible at the time due to the lack of programmable waveform generators in the microwave region. The non-diffracting X-wave solutions, a specific class of LWs, were also originally tested in an acoustic environment [10]. On the other hand, they have been generated in the optics domain [11]. Most recently, the Bessel beam frequency domain solutions have been launched at microwave frequencies [12]–[14] and a version of the original splash pulse [15], [5] has been generated from a metasurface convertor [16].

II. FINITE ENERGY LWS

The focus wave mode is an exact solution of the three-dimensional wave equation: $\{\nabla^2 - \partial_{ct}^2\}\,\Phi_{FWM}(x,y,z,t) = 0$ and can be represented naturally with a bi-directional decomposition [15]:

$$\Phi_{FWM,k}\,(x,y,z,t) = exp\{ik(z+ct)\}\,F_{FWM}(x,y,z-ct) \tag{1}$$

With $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$ and taking the derivative chain rule, one obtains $\{\nabla_\perp^2 + 4ik\,\partial_{z-ct}\}\,F = 0$, a Schrödinger equation in the retarded characteristic variable. Thus, the FWM solution (1) is completed with the moving, axisymmetric Gaussian pulse:

$$F_{FWM}\,(x,y,z-ct) = \frac{e^{-k(x^2+y^2)/[z_0+i(z-ct)]}}{4\pi i\,[z_0+i(z-ct)]} \tag{2}$$

Thus, it was recognized that the FWM and more general LWs can be obtained with a bi-directional, characteristic variable solution approach, i.e., axisymmetric LW solutions can be written in the form [17]

$$\Phi_{LW}(\rho,\varphi,z,t) = \sum_{n=0}^{\infty}\int_0^{\infty}du\int_0^{\infty}dv\int_0^{\infty}d\chi\;\chi\,J_n(\chi\rho) \times e^{in\varphi}\delta(uv-\chi^2/4)\,G_n(u,v;\,\chi) \times e^{iv\,(z+ct)}\,e^{iu\,(z-ct)} \tag{3}$$

where $\rho = \sqrt{x^2+y^2}$. Choices of the spectral function G_n lead to solutions with different localization and propagation features. LW fields have been obtained with this bi-directional approach for numerous wave environments, even quantum solutions of the Klein-Gordon and Dirac equations [1], [18]. Furthermore, sub-luminal, luminal and super-luminal LWs were developed by tailoring their coupled spectra in phase space [19], [20].

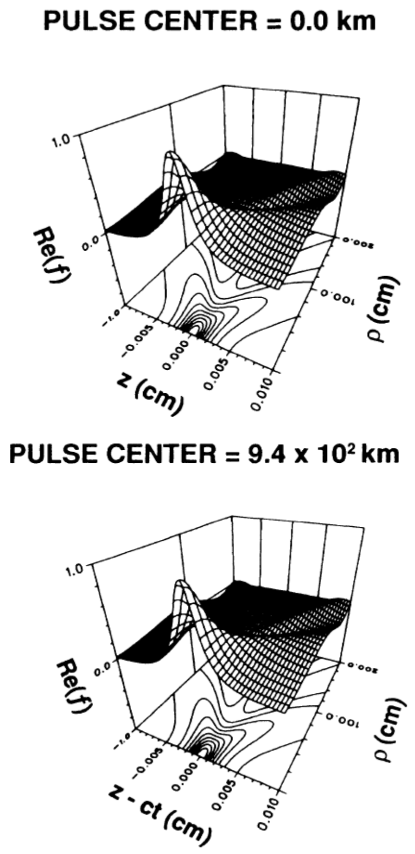

Fig. 1 Modified power spectrum pulse solution about the initial plane and about one very far from it.

It was also realized that superpositions of LWs were advantageous to develop additional solutions and to explore their properties. For instance, superpositions of the FWM solution:

$$\Phi(x,y,z,t) = \int_0^\infty dk\ A(k)\ \Phi_{FWM,k}(x,y,z,t) \qquad (4)$$

first yielded the splash pulse [15]. By properly selecting the spectrum $A(k)$, it was recognized that finite energy LWs could be obtained [4]. In particular, it was demonstrated that the MPS pulse solution of the wave equation:

$$\Phi_{MPS}(x,y,z,t) = \frac{1}{4\pi i\,[a + i\,(z - ct)]}\,\frac{e^{-b\,s\,/\,\beta}}{b + \frac{s}{\beta}} \qquad (5)$$

where

$$s = -i\,(z + ct) + \frac{\rho^2}{a + i\,(z - ct)} \qquad (6)$$

and the corresponding electromagnetic MPS solution were finite energy fields [4]. The free parameters a, b, β control the pulse shape in both space and time. The MPS solution and its pencil-beam character over very long distances is depicted in Fig. 1.

III. Generating LWs

An important aspect of the analysis of IA-PDAs and the fields they generate is the recognition that the frequency behavior of each radiating element impacts the transmitted and received signals in the time domain, e.g., an electrically small antenna acts as time derivative device. Simply, an electrically small dipole antenna is a capacitive device and the current running through it is the time derivative of the voltage pulse applied across it, i.e., $I(t) = C\,\partial_t V(t)$. Moreover, the field emitted by an antenna is directly related to the current driving it. In particular, the far field generated by any pulsed-driven finite-sized radiating element is proportional to the time derivative of the current applied to it [21], [22], [8], [9]. For instance, the electric field of a dipole antenna of length ℓ in its far-field is:

$$\vec{E}_{far\ field}\,(\vec{r},t) = -\mu\,\frac{\ell}{4\pi|\vec{r}|}\left\{\partial_t\left[I\left(t - \frac{|\vec{r}|}{c}\right)\right]\right\} \qquad (7)$$

i.e., the EM wave generated by the antenna evolves into a time derivative as it propagates from the antenna's near field to its far field. This outcome is depicted in Fig. 2.

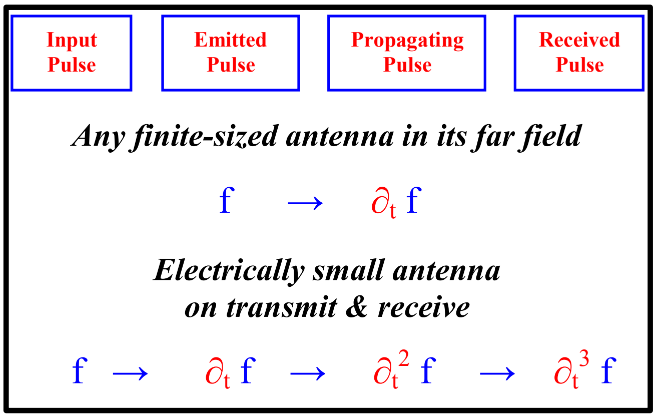

Fig. 2 Time domain characterization of the field generated by a pulse driven, finite-sized antenna.

Consequently, if the radiating element is, for example, an electrically small dipole, the far-field is proportional to two time derivatives of the input voltage pulse. This is also represented in Fig. 2. Furthermore, the voltage applied to the antenna on receive is the projection of the incident field along its electrical length. Again, if the receiving element is an electrically small dipole, the current generated in the circuit driving the load attached to it is the time derivative of that voltage. Finally, as illustrated in Fig. 2, the received time waveform is three derivatives of the input voltage signal. The prediction and verification of this outcome in [6]–[9] was reconfirmed theoretically in [22].

Note that the fact that the transmitted and received signals differ by a time derivative [21] is simply an asymmetric response of the system. The outcome still obeys reciprocity. In particular, if the same transmit and receive antennas are interchanged in the same medium, the outcome remains the same. This is simply depicted in Fig. 2 by interchanging the input and received pulse boxes.

As explained in [7]−[9], the LWs and their realization with IA-PDAs with electrically small elements give rise to extended near-to-far-field distances. Because the arrays are finite in size, the launched LWs will eventually lose their pencil-beam behavior and will begin to diffract. If an aperture has an area A and is driven with a continuous wave signal whose frequency is λ_{CW}, the near-to-far-field (Rayleigh) distance is A / λ_{CW}. When a single cycle pulse drives that aperture, it is an ultra-wide bandwidth signal. The second frequency moment of the energy of this signal defines a wavelength λ_{UWB}, and the corresponding Rayleigh distance becomes A / λ_{UWB}. Since the pulse can be designed to have a second moment that yields a shorter wavelength, the Rayleigh distance will be larger. Moreover, if the transceiver system consists of a pair of IA-PDAs with electrically small elements, then the effective wavelength associated with a LW field driving the aperture is actually the sixth moment of the energies of all the pulses associated with that LW solution [8], [9]. A MPS pulse launched and received with such a IA-PDA pair yields a Rayleigh distance much larger than the second moment of the driving waveforms as verified in [8]. It is noted that this extended near-field effect is related to, but is different from the ‘EM Missile" effect [23]. The extended near field distance is still finite because the energies of all of the waveforms are finite. On the other hand, as shown in [23], if the voltage waveform driving the aperture has a time derivative whose slope approaches infinity, then the near field of the aperture will in turn extend out to infinity. This means that the resulting electromagnetic wave will have field components that decay less than one over the distance from the aperture.

IV. Conclusions

LW solutions launched from IA-PDAs can have interesting pencil-beam and near-to-far-field characteristics. The physics of LWs and the engineering of IA-PDAs to realize them will be discussed in my presentation. While the MPS pulse will be emphasized, other LW solutions and interesting localized field structures will be discussed to highlight their essential features.

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