IOP Conference Series: Materials Science and Engineering

**PAPER • OPEN ACCESS**

# Effect of Magnetic Field on Double Diffusive Natural Convection Inside a Square Cavity with Isothermal Hollow Insert

To cite this article: Satyajit Mojumder *et al* 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **831** 012010

IOP Conf. Series: Materials Science and Engineering **831** (2020) 012010 doi:10.1088/1757-899X/831/1/012010

# Effect of Magnetic Field on Double Diffusive Natural Convection Inside a Square Cavity with Isothermal Hollow Insert

**Satyajit Mojumder[1,a], Sourav Saha[1], Sumon Saha[1,a], Rezwana Rahman[2] and Suvash C. Saha[2]**

[1]Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh
[2]School of Mechanical and Mechatronic Engineering, Faculty of Engineering and Information Technology, University of Technology Sydney, NSW, Australia

sumonsaha@me.buet.ac.bd

**Abstract.** A numerical study is carried out to analyze both heat and mass transfer phenomena in the presence of an external magnetic field inside a square cavity with an isothermal hollow circular insert located at the centre of the cavity. In the present work, the effects of Hartmann number, Lewis number and buoyancy ratio at constant Rayleigh and Prandtl numbers are investigated simultaneously to understand the importance of these parameters on the characteristics of double diffusive natural convection. Galerkin finite element method is employed for the numerical simulations. Grid sensitivity test and code validation are performed prior to confirm the numerical accuracy of the solution. Quantitative comparison is presented by showing the influence of Hartmann and Lewis numbers for different buoyancy ratios on average Nusselt and Sherwood numbers. It is found that increment of Hartmann number results in lower heat and mass transfer rates. Higher value of Lewis number produces elevated mass transfer rate. However, Lewis number has negative impact on heat transfer rate. Moreover, buoyancy ratio has significant effect on heat and mass transfer inside the cavity.

## 1. Introduction

Obstruction induced convection heat transfer is given prodigious importance because of its recurrence in many real life problems. Heat and mass transfer inside obstructed cavity has a wide range of applications, such as refrigeration systems, solar thermal heaters and coolers, attic space heating and cooling, heat exchangers, cold storages, and so on.

An extensive research has been carried out to analyse heat and mass transfer for different cavities with internal obstacle. Braga and de Lemos [1] studied laminar natural convection in cavities having circular and square rods. They reported that the obstruction had significant influence on heat transfer inside the cavity. House *et al.* [2] investigated the effect of centrally placed square heat conducting body in a vertical enclosure and concluded that heat transfer across the cavity enhanced or reduced depending on the conductivity ratio. Nasrin [3] also studied the effect of centrally placed circular obstacle in a square cavity and showed the impact of Prandtl and Hartmann numbers. Hussain and Hussein [4] investigated natural convection phenomenon for a uniformly heated circular cylinder inside a square cavity. By changing the positions of the circular cylinder, they reported that average Nusselt number behaved nonlinearly as a function of location of the cylinder. Jami *et al.* [5] applied

IOP Conf. Series: Materials Science and Engineering **831** (2020) 012010 doi:10.1088/1757-899X/831/1/012010

Lattice Boltzmann method to solve laminar natural convection problem in a square cavity having heat generating and conducting cylindrical body. De and Dalal [6] studied natural convection in a square cavity with horizontally heated square cylinder inside and reported that overall heat transfer changed by changing the aspect ratio of the heat conducting inner cylinder.

Magnetic field is an important parameter that has significant effect on heat and mass transfer for electrically conductive fluid. In fact, the magnetic field creates Lorentz force, which tries to retard the convection current and reduce the heat transfer rate. Magnetic field also has noteworthy effect on mass transfer. Venkatachalappa *et al.* [7] studied effect of magnetic field on heat and mass transfer in a vertical annulus and reported that magnetic field suppressed the double diffusive heat transfer for small buoyancy ratio, whereas for higher buoyancy ratio, magnetic field proved to be effective. They also suggested applying magnetic field perpendicular to the direction of the original flow. More related studies can be found in other literatures [8-12].

In a typical refrigeration system, relatively warmer commodities are placed inside a relatively cooler environment of the chamber. This chamber is surrounded by the heat exchanger tubes through which refrigerants flow and control the temperature of the chamber constantly by evaporation. It is assumed that the surface of the refrigerant carrying tubes are at low temperature under steady-state operation. To model this phenomenon, the present study aims to investigate the effects of magnetic field and buoyancy ratio on a double diffusive natural convection problem inside a square cavity with isothermal hollow insert. Special attention is given to investigate the conjugate effect of Lewis and Hartmann numbers and buoyancy ratio on heat and mass transfer rate.

## 2. Problem formulation

*2.1 Physical modeling*

The details of the present model are shown in Figure 1 with appropriate boundary conditions and co-ordinate system. It consists of a square cavity of length $L$, whose left side wall is heated at temperature $T_h$ and right side wall is cooled at temperature $T_c$. The top and the bottom walls of the cavity are kept insulated. Concentration of air, with different gas mixture, is approximated to be higher near the left wall ($c_h$) and lower near the right wall ($c_l$). The tube is modelled as a hollow cylindrical insert located at the centre of the cavity, which has the constant surface temperature, $T_c$. An external magnetic field with strength $B_o$ is applied on the left side of the cavity along the positive $x$-direction. All the boundaries are assumed to be stationary. Viscous heating and radiation effect are neglected for simplifying the problem.

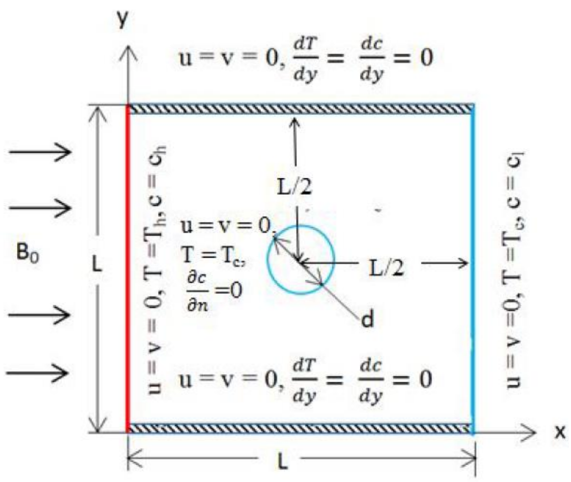



**Figure 1.** Schematic diagram of the problem with boundary conditions.

*2.2 Mathematical modeling*

The governing equations for the present problem are based on the conservation laws of mass, linear momentum, concentration and thermal energy in two-dimensions. The flow is assumed to be steady-state and laminar. Constant thermo-physical fluid properties are assumed except density variation in gravitational force term of *y*-momentum equation as related by Boussinesq approximation. Considering the above assumptions, the governing equations in non-dimensional form for the present problem can be written as,

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right) +Ra\,Pr\left(\Theta+BrC\right)-Ha^2\,Pr\,V, \tag{3}$$

$$U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}=\frac{\partial^2 \Theta}{\partial X^2}+\frac{\partial^2 \Theta}{\partial Y^2}, \tag{4}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right). \tag{5}$$

The following non-dimensional scales are used to obtain the above non-dimensional equations (1)-(5):

$$\left(X,Y\right)=\frac{\left(x,y\right)}{L},\left(U,V\right)=\frac{\left(u,v\right)}{\alpha/L},P=\frac{pL^2}{\rho\alpha^2}, \quad \Theta=\frac{T-T_c}{T_h-T_c},C=\frac{c-c_l}{c_h-c_l}. \tag{6}$$

In the aforementioned equations, *U* and *V* are the non-dimensional velocities in *X*- and *Y*-directions, *P*, Θ and *C* are the non-dimensional pressure, temperature and concentration, respectively. Dimensionless governing parameters for the present problem are Prandtl number (*Pr*), Rayleigh number (*Ra*), buoyancy ratio (*Br*), Hartmann number (*Ha*) and Lewis number (*Le*). These parameters are defined as,

$$Pr=\frac{\nu}{\alpha},Ra=\frac{g\beta_T\left(T_h-T_c\right)L^3}{\nu\alpha},Br=\frac{\beta_c\left(c_h-c_l\right)}{\beta_T\left(T_h-T_c\right)}, \quad Ha=B_oL\sqrt{\frac{\sigma}{\rho\nu}},Le=\frac{\alpha}{D}. \tag{7}$$

Here, ρ, ν, α and *D* are fluid density, kinematic viscosity, thermal and species diffusivity respectively, $\beta_T$ and $\beta_c$ are thermal and compositional expansion coefficient, *g* is gravitational acceleration constant and σ is the electrical conductivity of fluid. The boundary conditions for the present problem in the non-dimensional form are listed in Table 1.

**Table 1.** Non-dimensional boundary conditions of the problem.

| Boundary wall | Velocity | Temperature | Concentration |
|---|---|---|---|
| Left | $U = V = 0$ | $\Theta = 1$ | $C = 1$ |
| Right | $U = V = 0$ | $\Theta = 0$ | $C = 0$ |
| Top and bottom | $U = V = 0$ | $\partial\Theta/\partial Y = 0$ | $\partial C/\partial Y = 0$ |
| Hollow surface | $U = V = 0$ | $\Theta = 0$ | $\partial C/\partial N = 0$ |

The performance of the double diffusive natural convection is evaluated by the average Nusselt number (*Nu*) and average Sherwood number (*Sh*) at the left vertical heated wall and those are defined as follows:

$$Nu=-\int_0^1\left(\frac{\partial\Theta}{\partial X}\right)_{X=0}dY,\ Sh=-\int_0^1\left(\frac{\partial C}{\partial X}\right)_{X=0}dY. \tag{8}$$

## 3. Numerical procedure and validation

Galerkin finite element method is adopted for the numerical solution of the present problem. The entire computational domain is discretised into several triangular mesh elements. The boundary wall is meshed more intensely using finer elements to ensure rapid change of physical variables. The governing differential equations (1) -(5) are transformed into a system of nonlinear algebraic equations and iteration technique is adopted to solve those equations. The convergence criterion set for the present problem is $|\gamma^{m+1} - \gamma^{m}| \leq 10^{-5}$, where γ is the general dependent variable. A grid sensitivity test is also performed to check the numerical accuracy of the present solutions. From the result of grid sensitivity test, it is found that both the average Nusselt and Sherwood numbers at the heated wall are very sensitive to grid size for smaller element number. However, when the element number is more than 4224, both *Nu* and *Sh* become insensitive to the further refinement of mesh elements. Therefore, grid having 4224 elements number is considered as optimum grid for the entire simulation process to reduce the computational effort.

To ensure numerical accuracy of the present model, a validation test has been performed. The present model is validated with the work of Teamah *et al.* [12] on double diffusive natural convection heat transfer in rectangular cavity with the presence of magnetic field. Here the effect of Rayleigh number on the variation of average Nusselt number is observed for *Ha* = 10, *Le* = 2, *Pr* = 0.7, *Br* = 1, aspect ratio of the cavity is 2 and zero inclination angle. Comparison of the model validation test is presented in Figure 2. From the figure, it is evident that the present model is quite reliable to solve a numerical problem with reasonable accuracy.

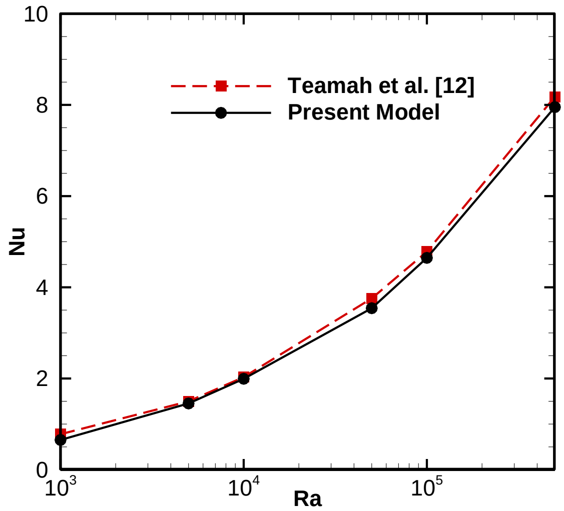



**Figure 2.** Validation of the present model in terms of average Nusselt number with the results of Teamah et al. [12].

## 4. Results and discussions

The goal of the present study is to investigate magneto-hydrodynamic heat and mass transfer phenomena inside a square cavity having a cylindrical hollow insert. For the present study, Prandtl number and Rayleigh number are fixed at *Pr* = 0.71 and *Ra* = $10^4$ respectively. To investigate mass transfer, buoyancy ratio is varied from $-5 \leq Br \leq 5$ and Lewis number is varied from $1 \leq Le \leq 20$. Hartmann number considered for the present problem is also varied from 0 to 50. The quantitative analysis is done in light of 3D plots incorporating variation of *Nu* and *Sh* against simultaneous variation of *Le* and *Ha*.

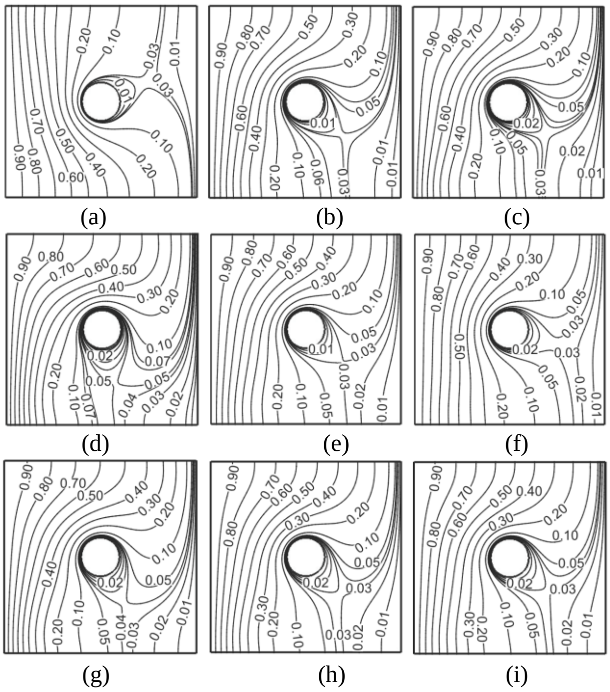



**Figure 3.** Effects of buoyancy ratio: (a) $Br = -5$, (b) $Br = 1$ and (c) $Br = 5$, Hartmann number: (d) $Ha = 10$, (e) $Ha = 25$ and (f) $Ha = 50$ and Lewis number: (g) $Le = 1$, (h) $Le = 5$ and (i) $Le = 20$ on isotherm contours for $Ra = 10^4$.

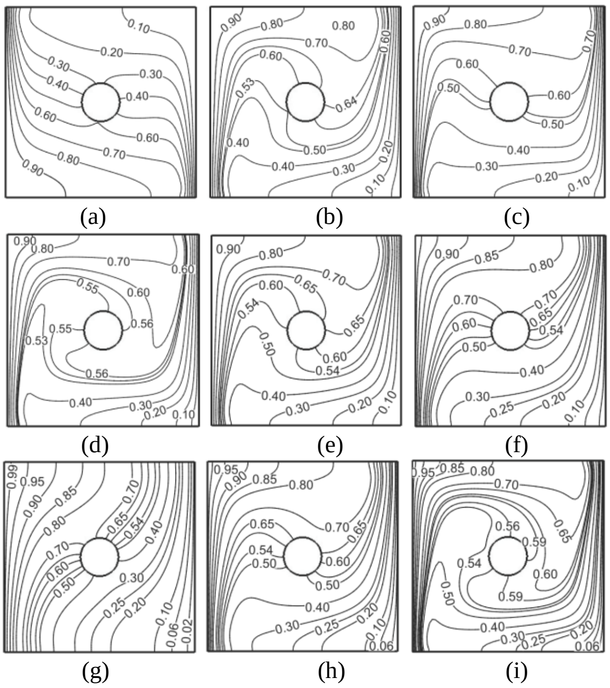


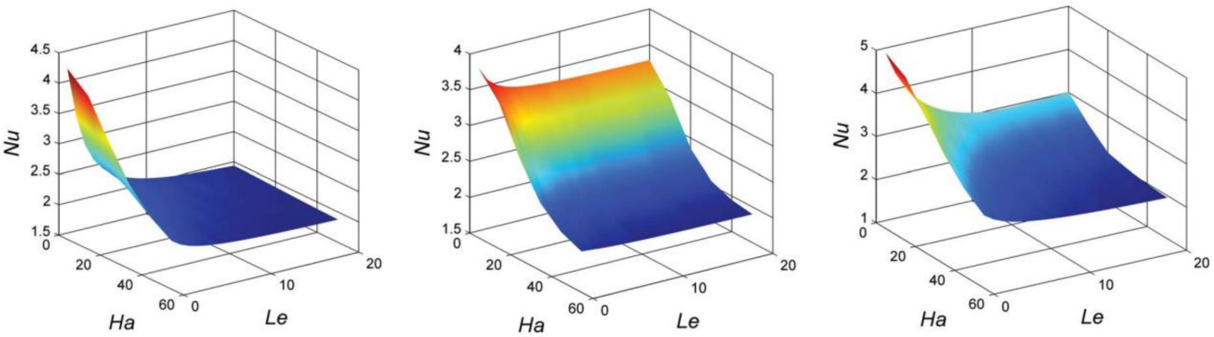

**Figure 4.** Effects of buoyancy ratio: (a) $Br = -5$, (b) $Br = 1$ and (c) $Br = 5$, Hartmann number: (d) $Ha = 10$, (e) $Ha = 25$ and (f) $Ha = 50$ and Lewis number: (g) $Le = 1$, (h) $Le = 5$ and (i) $Le = 20$ on isoconcentration contours for $Ra = 10^4$.

The effects of buoyancy ratio, Hartmann number and Lewis number on isotherm contours for $Ra = 10^4$ are visualised in Fig. 3(a)-(i) respectively. Isotherm contours for different buoyancy ratios are shown in Fig. 3(a), (b) and (c) at $Le = 10$ and $Ha = 25$. These figures show that for each value of $Br$, isotherm contours are almost parallel to the left vertical wall and insert wall. The contours are closely packed near the right vertical wall. All of these indicate a steep temperature gradient at those regions which favours conduction over convection. Isotherm contours for different *Ha* are shown in Fig. 3(d), (e) and (f). The contours are nearly parallel near the vertical walls and the cylindrical insert indicating formation of thermal boundary layers in those regions. On the other hand, the isotherm pattern does not vary significantly with the increment of Lewis number as can be seen from Fig. 3(g), (h) and (i).

Figures 4(a), (b) and (c) depict isoconcentration contours for different buoyancy ratios. From figures it can be observed that for $Br = -5$, contours with higher value reside at the bottom of the left wall. Near the top of the left wall and bottom of the right wall, contours are parallel to each other. These indicate strong mass diffusion in those regions. Isoconcentration contours' map for different *Ha* is shown in Fig. 4(d), (e) and (f). Impact of external magnetic field is not that apparent in these figures. However, it is seen that, density of isoconcentration lines becomes less with higher Hartmann number. However, isoconcentration contours undergo drastic changes with varying *Le* (Fig. 4(g), (h) and (i)). With higher values of *Le*, isoconcentration contours are shifting toward the right vertical wall.

A 3D surface plot in Figure 5 shows the combined effect of varying *Le* and *Ha* for different buoyancy ratios on average Nusselt number. From Figure 5, it is found that with the increment of both Lewis and Hartmann numbers, average Nusselt number is reduced. However, those rate and magnitude of reduction vary for different combinations of *Le* and *Ha*. For both $Br = -5$ and 5, with the increment of *Le*, the average Nusselt number decreases at a significant rate, whereas for $Br = 1$, the rate of decrement of average Nusselt number with *Le* is quite less. Again, it is evident that a combination of low Hartmann number and low Lewis number can yield the best possible heat transfer rate. However, impact of Hartmann number is more important on heat transfer rate and even for high *Le*, if low *Ha* can be ensured, quite high heat transfer rate is possible. Moreover, positive and high value of *Br* ensures better heat transfer rate.

Figure 6 shows the combined effect of varying *Le* and *Ha* for different buoyancy ratios ($Br = -5, 1, 5$) on average Sherwood number. From Figure 6, it is observed that the average Sherwood number increases with the increment of the Lewis number and decreases with increasing Hartmann number. A combination of low *Ha* and high *Le* can yield best mass transfer rate at a given value of *Br*. Moreover, change in the value of average Sherwood number is highly influenced by the change of *Le* than that of *Ha*. Hence, if a high value of *Le* is ensured, high *Ha* can also produce satisfactory mass transfer rate. Figure 6 also reveals that positive and high value of *Br* yields higher mass transfer rate.




(a) (b) (c)

**Figure 5.** Variation of average Nusselt number as a function of Hartmann and Lewis numbers for different buoyancy ratios: (a) $Br = -5$, (b) $Br = 1$ and (c) $Br = 5$.

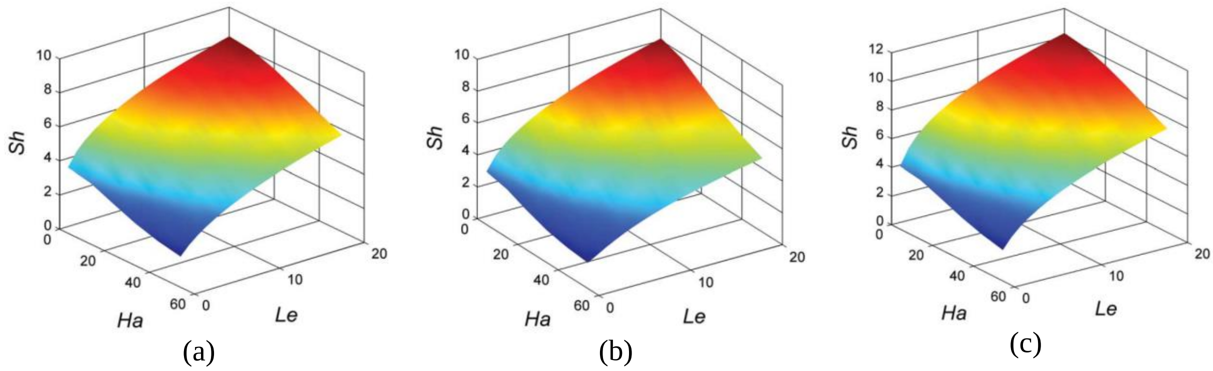



**Figure 6.** Variation of average Sherwood number as a function of Hartmann and Lewis numbers for different buoyancy ratios: (a) *Br* = −5, (b) *Br* = 1 and (c) *Br* = 5.

## 5. Conclusions

The present investigation is carried out in order to get a better insight on the influence of Lewis number, Hartmann number and buoyancy ratio on heat and mass transfer in a practical refrigeration model. The following conclusions can be made from the study. Higher values of positive and negative buoyancy ratios (*Br* = −5, 5) cause better heat transfer for low Hartmann number and low Lewis number. When *Br* = 1, the effect of Lewis number is insignificant on heat transfer rate. Increment of buoyancy ratio and Lewis number always assists the mass transfer, whereas increment of Hartmann number reduces the mass transfer. Higher value of Hartmann number retards the flow strength significantly and thus creates hindrance to both heat and mass transfer. With proper combination of governing parameters like low buoyancy ratio (*Br* = 1), low Hartmann number (*Ha* = 0 − 10) and higher Lewis number, it is possible to enhance heat and mass transfer simultaneously. This understanding can be used to design equipment efficiently in a refrigeration system.

## 6. References

[1] E.J. Braga, M.J.S. de Lemos, Int. Commun. Heat Mass Transf. **32**, 1289–1297 (2005)
[2] J.M. House, C. Beckermann, T.F. Smith, Numer. Heat Transf. A **18**, 213–225 (1990)
[3] R. Nasrin, Int. Commun. Heat Mass Transf. **38**, 625–632 (2011)
[4] S.H. Hussain, A.K. Hussein, Int. Commun. Heat Mass Transf. **37**, 115–1126 (2010)
[5] M. Jami, A. Mezrhab, M. Bouzidi, P. Lallemand, Int. J. Ther. Sci. **46**, 38–47 (2007)
[6] A.K. De, A. Dalal, Int. J. Heat Mass Transf. **49**, 4608–4623 (2006)
[7] M. Venkatachalappa, Y. Do, M. Sankar, Int. J. Eng. Sci. **49**, 262–278 (2011)
[8] M.A. Teamah, Int. J. Therm. Sci. **47**(3), 237–248 (2008)
[9] O. Laguerre, S. Benamara, D. Remy, D. Flick, Int. J. Heat Mass Transf. **52**, 5691–5700 (2009)
[10] X. Zhang, H. Huang, Int. Commun. Heat Mass Transf. **51**, 31–38 (2014)
[11] H. Yamaguchi, X.R. Zhang, X.D. Niu, J. Magn. Magn. Mat. **322**, 698–704 (2010)
[12] M.A. Teamah, A. F. Elsafty, M.Z. Elfeky, E.Z. El-Gazzar, Alex. Engg. J. **50**, 269−282 (2011)

## Acknowledgment

The authors gratefully acknowledge the support provided by the Department of Mechanical Engineering, Bangladesh University of Engineering and Technology (BUET) during this research work.