

Advanced Design and Optimization Techniques for Electrical Machines

by Bo Ma

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This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abbreviations

ANN	Artificial Neural Network
ANOVA	Analysis of Variance
BLUE	Best Linear Unbiased Estimation
CAE	Computer-Aided Engineering
CI	Chebyshev Interval
CPM	Claw Pole Motor
DACE	Design and Analysis of Computer Experiments
DEA	Differential Evolution Algorithm
DFSS	Design for Six Sigma
DG	Differential Gear
DPMO	Defects Per Million Opportunities
DTC	Direct Torque Control
EA	Evolutionary Algorithms
EDAs	Estimation of Distribution Algorithm
EMF	Electromotive Force
EV	Electric Vehicle
FOC	Field-Oriented Control
GA	Genetic Algorithm
HEV	Hybrid Electric Vehicle
HM	Homogenization Method
ICE	Internal Combustion Engine
IPMSM	Interior Permanent Magnet Synchronous Motor
LSL	Lower Specification Limit
LSM	Least Square Method
MCA	Monte Carlo Approach
MDO	Multi-disciplinary Design Optimization
MLE	Maximum Likelihood Estimation
MMA	Method of Moving Asymptotes
MPC	Model Predictive Control

MPP	Most Probable Point
MTPA	Maximum Torque Per Ampere
NSGA	Non-dominated Sorting Genetic Algorithm
OC	Optimality Criteria
PCCI	Polynomial Chaos Chebyshev Interval
PCE	Polynomial Chaos Expansion
PID	Proportion Integration Differentiation
PM	Permanent Magnet
PMSM	Permanent Magnet Synchronous Motor
POF	Probability of Failure
PSO	Particle Swarm Optimization
RBDO	Reliability-Based Design Optimization
RBF	Radial Basis Function
RDO	Robust Design Optimization
RSM	Response Surface Model
SA	Sensitivity Analysis
SIMP	Simplified Isotropic Material with Penalization
SM	Space Mapping
SMCA	Scan and Monte Carlo Approach
SOM	Sequential Optimization Method
SPMSM	Surface-mounted Permanent Magnet Synchronous Motor
SS	Switching Signal
SVM	Support Vector Machine
TFM	Transverse Flux Motor
USL	Upper Specification Limit

Abstract

To investigate the design optimization techniques of electrical machines, a literature survey is conducted about the problem modeling, and techniques utilized for the effective optimization conduction with various case studies. As a comprehensive design optimization example, an in-wheel motor development for distributed direct vehicle driving is investigated in detail. The works on the application analysis, new material application (grain-oriented silicon steel), topology development, manufacturing process, experiment verification, and parametric deterministic optimization are presented.

Based on the state-of-art design optimization methods and case studies, challenges and proposals are also presented in the survey. In the design stage, the bring-up of new topology depends on the expertise of the designers. This means the expert system still plays an important role in the application-oriented design optimization process. Moreover, parametric optimization is carried out based on the specific design which means the freedom of optimization is limited. To overcome the restriction of parametric optimization for high freedom optimization topology optimization method is investigated. At the current stage, the topology optimization of the soft magnetic components of electrical machines considering various electromagnetic performances is studied. The optimization results of the design examples verified the effectiveness of the proposed method in achieving the optimal shape of the design.

Another problem is about the uncertainties in manufacturing such as tolerances which bring in reliability problems for the conventional deterministic optimal solution. Under this circumstance, the robustness optimization is important for searching optimal solution with both high objective performance and reliability. For the robust optimization of electrical machines, the additional uncertainty quantification and robustness assessment further aggravating the complexity and computation cost of the problem. Considering the problem with different types of uncertainties, high effective optimizers are proposed based on effective uncertainty quantification methods with a general framework. The numerical study results proved the effectiveness of the proposed method.

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