CONSTRAINED LOW-RANK MATRIX/TENSOR FACTORISATION

by Shuai Jiang

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Doctor of Philosophy

under the supervision of Assoc. Prof. Richard Yi Da Xu

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Shuai Jiang declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Electrical and Data Engineering, Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

I also certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of the requirements for a degree at any other academic institution except as fully acknowledged within the text. This thesis is the result of a Collaborative Doctoral Research Degree program with Beijing Institute of Technology.

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ABSTRACT

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Constrained low-rank matrix and tensor factorisation (MF/TF) have been widely used in machine learning and data analytics. Studies on the way of modelling constraints and the solution of optimisation task in general can provide theoretical supports for applications like image clustering, recommender systems and data compression. This thesis studies three algorithms of constrained low-rank MF/TF.

Imposing constraints on each feature vector of factor matrices is a common practice in many constrained low-rank MF algorithms. However, in many real scenarios, the relationships among features can influence the factorisation results as well. In order to better characterise the relationships among features, a novel MF algorithm, Relative Pairwise Relationship Constrained Non-negative Matrix Factorisation, is proposed. It places soft constraints over relative pairwise distances amongst features as regularisations to retain expected relationships after factorisation. It conforms to the so-called "multiplicative update rules" and detailed convergence proofs are provided. Experiments on both synthetic and real datasets have verified that imposing such constraints can keep most expected relationships unchanged after factorisation.

Directly adopted on tensor data, low-rank TF can effectively avoid the information loss caused by matricisation. The relationships among features of factor matrices in TF have practical meanings in many real scenarios. To describe such relative relationships in low-rank TF, this thesis proposes Relative Pairwise Relationship Constrained Non-negative Tensor Factorisation. It deals with both Camdecomp/Parafac and Tucker decomposition schemes and both squared Euclidean distance and divergence measures. The utilisation of tensor factorisation matricisation equation simplifies the update rules and greatly improves the computation efficiency. Experiments have demonstrated that the proposed algorithm can achieve higher accuracy when adopted on tensor applications.

There exists a problem of acquiring out-of-bounds and fluctuating values over predictions when applying low-rank MF on recommender systems. The commonly used solutions, truncation and imposing penalties, can cause the decrease in the number of effective predictions and affect the recommendation accuracy. This thesis creatively proposes Magnitude Bounded Matrix Factorisation to handle the above problem by imposing magnitude constraints for the first time. It first converts the original quadratically constrained quadratic programming task to an unconstrained one which is then solved by the well-known stochastic gradient descent. An acceleration approach for improving computation efficiency, an extracting method for magnitude constraints and a variant of MBMF for non-negative data are also introduced. Experiments have demonstrated that the algorithm is superior to existing bounding algorithms on both computing efficiency and recommendation performance.

Dissertation directed by Assoc. Professor Richard Yi Da Xu
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Dedication

I dedicate my dissertation work to my family without whom I would never accomplish this project. A special feeling of gratitude to my loving mother, Xueqin Jiang whose words of encouragement and push for tenacity have been helping me all throughout the way.

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List of Publications

Journal Papers

J-1. **S. Jiang**, K. Li, and R. Y. D. Xu. Relative Pairwise Relationship Constrained Non-Negative Matrix Factorisation. *IEEE Transactions on Knowledge and Data Engineering*, 31(8), pp.1595-1609, 2018.

J-2. **S. Jiang**, K. Li, and R. Y. D. Xu. Magnitude Bounded Matrix Factorisation for Recommender Systems. *IEEE Transactions on Knowledge and Data Engineering*, doi: 10.1109/TKDE.2020.2998218, 2020. Online.

J-3. **S. Jiang**, K. Li, and R. Y. D. Xu. Non-negative CP Tensor Decomposition with Relative Pairwise Relationship Regularizations. *Pattern Recognition Letters*, 2020. Under review.

J-4. L. Bai, K. Li, J. Pei, and **S. Jiang**. Main objects interaction activity recognition in real images. Neural Computing and Applications, 27(2), pp.335-348, 2016.

Conference Papers

C-1. C. Huang, **S. Jiang**, Y. Li, Z. Zhang, J. Traish, C. Deng, S. Ferguson, R. Y. D. Xu. End-to-end Dynamic Matching Network for Multi-view Multi-person 3d Pose Estimation. *In Proceedings of the European Conference on Computer Vision (ECCV)*, 1267, 2020.

C-2. Y. Li, K. Li, **S. Jiang**, Z. Zhang, C. Huang, and R. Y. D. Xu. Geometry-driven Self-supervised Method for 3D Human Pose Estimation. *AAAI Conference on Artificial Intelligence (AAAI)*, 7454, 2020.

C-3. Z. Zhang, R. Y. D. Xu, **S. Jiang**, Y. Li, C. Huang, and C. Deng. Illumination adaptive person reid based on teacher-student model and adversarial training. *IEEE International Conference on Image Processing (ICIP)*, 2020.

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Abbreviation

MF - Matrix Factorisation

NMF - Non-negative Matrix Factorisation

CNMF - Constrained Non-negative Matrix Factorisation

MUL - Multiplicative Update Rules

GNMF - Graph Regularised Non-negative Matrix Factorisation

LCNMF - Label Constrained Non-negative Matrix Factorisation

RPR-NMF - Relative Pairwise Relationship Constrained Non-negative Matrix Factorisation

BMF - Bounded Matrix Factorisation

BMC-ADMM - Bounded Matrix Completion in Alternating Direction of Multiplier Method

MBMF - Magnitude Bounded Matrix Factorisation

TF - Tensor Factorisation

NTF - Non-negative Tensor Factorisation

CNTF - Constrained Non-negative Tensor Factorisation

LRNTF - Laplacian Regularised Non-negative Tensor Factorisation

RPR-NTF - Relative Pairwise Relationship Constrained Non-negative Tensor Factorisation

SVD - Singular Value Decomposition

CP - Candecomp/Parafac

Nomenclature and Notation

Lower-case non-bold characters denote iterative variables (e.g. i, j, k, n).

Lower-case bold characters denote vectors (e.g. $\boldsymbol{u}$).

Upper-case non-bold characters denote constant scalars (e.g. N, K, I).

Upper-case bold characters denote matrices (e.g. $\boldsymbol{U}$).

Upper-case non-bold Euler characters denote functions (e.g. $\mathcal{F}$).

Upper-case bold Euler characters denote tensors (e.g. $\boldsymbol{\mathcal{X}}$).

$\boldsymbol{U}_{i:}$ denotes the i^{th} row of matrix $\boldsymbol{U}$.

$(.)^T$ denotes the transpose operation.

$\mathbb{R}$ denotes the field of real numbers.