UNIVERSITY
OF TECHNOLOGY
SYDNEY

Development of Mixed Hardening Hyper-Viscoplastic Constitutive Models for Soils Incorporating Creep & Fabric Effects

by YE AUNG
BEng (1st Class Hons with University Medal, UTS)

Thesis submitted in fulfilment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

under the supervision of
A/Prof. Hadi Khabbaz & A/Prof. Behzad Fatahi

University of Technology Sydney
Faculty of Engineering and Information Technology

December 2019

Certificate of Original Authorship

Graduate research students are required to make a declaration of original authorship when they submit the thesis for examination and in the final bound copies. Please note, the Research Training Program (RTP) statement is for all students. The Certificate of Original Authorship must be placed within the thesis, immediately after the thesis title page.

Required wording for the certificate of original authorship

CERTIFICATE OF ORIGINAL AUTHORSHIP

I, *Ye Aung*, declare that this thesis, is submitted in fulfilment of the requirements for the award of *Doctor of Philosophy*, in the *Faculty of Engineering and Information Technology* at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.
**If applicable, the above statement must be replaced with the collaborative doctoral degree statement (see below).*

**If applicable, the Indigenous Cultural and Intellectual Property (ICIP) statement must be added (see below).*

This research is supported by the Australian Government Research Training Program.

Signature: Production Note:
Signature removed prior to publication.

Date: 06/12/2019

ABSTRACT

During the past several decades, the constitutive modelling for the prediction of time-dependent behaviour of soft soils has attracted an increasing attention within the geotechnical research society due to the scarcity of appropriate types of soil for construction as the regions around the globe have struggled to keep up with the meteoric rise in the infrastructure developments to cater for the substantial growth in population. Therefore, the consideration of time- and rate-dependent behaviour of geomaterials, particularly soft soils, such as creep, strain-rate dependent effects and stress relaxation behaviour, becomes a fundamental concern towards the long-term settlement deformation behaviour.

In this study, a mixed hardening hyper-viscoplastic constitutive model and its extended model are developed for describing the time-dependent stress-strain evolution of soil deformation, with the additional consideration of the arrangement of particles and the interparticle bonding, prominent in deformation of natural soils. The proposed model is intended to capture the loading-rate or strain-rate dependent behaviour of soils, accounting for the variations in the fundamental shapes of the yield loci along with the kinematic hardening and non-associated flow behaviour, with the extended model supplementing the proposed one with a β-line defining the inclination of the non-symmetrical elliptical yield locus in the p'-q plane, along with the auxiliary rotational hardening effects to the kinematic hardening behaviour. The proposed models are formulated within the context of hyperplasticity framework, mainly due to the fact that the hyperplastic constitutive models obey the fundamental laws of thermodynamics, and the resulting approach provides a well-established structure and reduces the need for 'ad hoc' assumptions. The distinctive departure from the existing viscoplasticity

models is the application of thermodynamics, based upon the use of internal variables, to postulate free-energy and dissipation potential functions, from which the elasticity law, the yield condition and corresponding flow behaviour, the isotropic and kinematic hardening laws, are derived based on a standardised systematic procedure. Firstly, the proposed model is presented, in which the free-energy function is decomposed into the elastic and the viscoplastic components, incorporating the dependence on both volumetric and deviatoric viscoplastic strains, and the viscoplastic dissipation potential function accounting for both the instantaneous energy dissipation and the additional energy dissipation due to delayed deformation. The additional viscoplastic component of the free-energy function results in the modified shift stress, to describe the kinematic hardening behaviour of the yield locus. Besides, a non-linear creep formulation is postulated to address the limitation of over-estimating long-term settlement, which is incorporated into the proposed model. Being introduced as a rational and logical extension towards the proposed model, the extended model enhances the free-energy and dissipation potential functions, in which not only the additional viscoplastic free-energy function depends on both volumetric and deviatoric viscoplastic strains, but also the fabric coupling parameter is incorporated into the free-energy and dissipation potential functions. Accordingly, the constitutive relations of the solid soil skeleton are expressed from the perspective of hyperplasticity framework in order to capture a wide variety of viscous behaviour of soils, with the emphasis on the strain-softening or hardening behaviour during the time-dependent delayed deformation in soils. The proposed model and the extended model only require minimal number of material parameters, which can readily be determined using standard laboratory testing equipment.

The performance and applicability of the proposed and extended models are investigated and validated using the triaxial and oedometer experimental results available in the existing literature. Comparisons between the numerical results and the laboratory measurements are conducted to demonstrate the versatility and capability of the proposed model in reproducing the rate-dependent behaviour of natural soft soils subjected to a variety of loading conditions. Due to the advantages of strong theoretical foundation with rigorous, yet compact and consistent procedure, with a relatively small number of required model parameters, the proposed and extended models have been signified as ideal for the numerical implementations to predict the time-dependent behaviour of soft soils, including long-term settlement behaviour in geotechnical structures.

ACKNOWLEDGEMENTS

The road to the completion of my PhD journey has been mostly enjoyable and challenging, yet frustrating at times. At the jubilation end of this successful completion, I am delighted to look back over the journey and remember the support and encouragement that I have received from my family, friends, and colleagues throughout this lengthy, yet satisfying journey. I would like to take this opportunity to express my sincere gratitude towards everyone, who have helped this thesis come to fruition.

First and foremost, I would like to pay my deepest homage to my principal supervisor, Associate Professor Hadi Khabbaz, and my co-supervisor, Associate Professor Behzad Fatahi, for their continued support, and guidance on not only the research but also the other developments in my life. Under their patience and guidance, I have developed and accumulated many important skills, including technical and interpersonal, from their broad knowledge, ideas, advice and suggestions have inspired and motivated me in achieving the important objectives of my research as well as the major milestones in my life.

Secondly, my appreciation is likewise extended to Dr Lam Nguyen, along with my colleagues and other staff members in the UTS laboratory for their kind assistance and contribution at the commencement of my research project in finding the soil properties and the feasibility of conducting creep tests in the laboratory.

This research has been carried out in the School of Civil and Environmental Engineering Faculty within University of Technology, Sydney, with the support from the International Postgraduate Research Scholarship (IPRS) and the Australian Postgraduate Award (APA) by the Australian Government for three and a half years. All the support from the Faculty and University throughout my study are also gratefully appreciated and acknowledged. Moreover, I would like to thank my friends and colleagues, particularly from my geotechnical group, not only for their help but also for keeping my study life more enjoyable and pleasant.

Last, but not least, I am hugely indebted to my family for their unconditional love, moral support and encouragement throughout this arduous journey. I am deeply grateful towards my parents in always showing the faith and allowing me to study and follow my lifelong pursuit and ambition to achieve this major milestone of my life. Additionally, for my loving, caring and supportive partner, I would like to express much appreciation for her love and mental support throughout my PhD journey.

LIST OF PUBLICATIONS

- Aung, Y., Khabbaz, H. & Fatahi, B. 2019, 'Mixed Hardening Hyper-viscoplasticity Model for Soils Incorporating Nonlinear Creep Rate – H-Creep Model', *International Journal of Plasticity,* vol. 120, pp. 88-114.
- Aung, Y., Khabbaz, H. & Fatahi, B. 2019, 'Extended Mixed Hardening Hyper-viscoplasticity Model for Soft Soils Incorporating Soil Fabric', International Journal of Plasticity (Submitted).
- Aung, Y., Khabbaz, H. & Fatahi, B. 2016, 'Review on Thermo-mechanical Approach in the Modelling of Geo-materials Incorporating Non-Associated Flow Rules', 3rd International Conference on Transportation Geotechnics, Procedia Engineering, vol. 143, pp. 331-338.
- Aung, Y., Khabbaz, H. & Fatahi, B. 2016, 'Review on Thermo-mechanical Approach in the Modelling of Geo-materials Incorporating Non-Associated Flow Rules', 3rd International Conference on Transportation Geotechnics (3rd ICTG), 4-7 September, Guimarães, Portugal.
- Aung, Y., Khabbaz, H. & Fatahi, B. 2020, 'A Generalised Hyper-viscoplasticity framework for Developing Rate-dependent Plasticity Models', 4th International Conference on Transportation Geotechnics (4th ICTG), 30 August – 2 September, Chicago, Illinois (Accepted).

Table of Contents

LIST OF FIGURES

LIST OF TABLES

Nomenclature & Abbreviations

Latin Notations

A, B	functions for stress-like quantities
$C_{\alpha e}$	secondary compression coefficient
C_{β}	material constant controlling the extent of coupling
C_r	swelling index
C_c	compressive index
D_{β}	relative contribution from the volumetric and deviatoric viscoplastic strains in determining the extent of coupling
$C_v(\eta)$	volumetric target value for β
$C_q(\eta)$	deviatoric target value for β
e_0	initial void ratio
e	void ratio
e_R	reference void ratio
Δe	change in void ratio
F	overstress function
G	elastic shear modulus
G_0	initial elastic shear modulus
g	elastic shear modulus gradient

J	cross-coupling elastic modulus
K	elastic bulk modulus
k	elastic bulk modulus gradient
M	slope of the critical state line
m	power value (material constant) representing the slope of the $\log \mu - \log e$ curve
n	power order (dimensionless material constant)
p	effective stress
$\dot{p}$	change in effective stress
p_R	reference mean stress
p_0	effective stress at initial state (or reference time)
p_c	pre-consolidation pressure
$\dot{p}_c$	change in pre-consolidation pressure
p_{c0}	initial pre-consolidation pressure
p_s	volumetric shift stress
p_D	volumetric dissipative stress
Q	viscoplastic potential function
q	deviatoric stress
$\dot{q}$	change in deviatoric stress
q_s	deviatoric shift stress
t_R	reference time (or absolute equivalent time)
V	specific volume
w_L	liquid limit
w_P	plastic limit
w	flow potential function
z	force potential function

Greek Notations

δ_{ij}	Kronecker's delta
α	material constant linking to the amount of deviatoric dissipation
β	cross-coupling between volumetric and deviatoric dissipation
ε_1	strain in axial direction
ε_3	strain in radial direction
ε_{ij}	total strain tensor
ε_v	total volumetric strain

ε_q	total shear strain
$\dot{\varepsilon}_v$	total volumetric strain increment
$\dot{\varepsilon}_q$	total shear (or deviatoric) strain increment
ε_{ij}^e	elastic strain tensor
ε_v^e	elastic volumetric strain
ε_q^e	elastic shear strain
$\dot{\varepsilon}_v^e$	elastic volumetric strain increment
$\dot{\varepsilon}_q^e$	elastic shear strain increment
ε_{ij}^p	plastic strain tensor
ε_{ij}^{vp}	viscoplastic strain tensor
ε_v^{vp}	viscoplastic volumetric strain
ε_q^{vp}	viscoplastic shear (or deviatoric) strain
$\dot{\varepsilon}_v^{vp}$	viscoplastic volumetric strain increment
$\dot{\varepsilon}_q^{vp}$	viscoplastic shear (or deviatoric) strain increment
γ	material constant linking to the amount of stored plastic work
κ^*	slope of swelling line in $\ln v - \ln p$ plot
λ^*	slope of normal consolidation line in $\ln v - \ln p$ plot
η	stress ratio
μ	creep coefficient
μ_0	initial creep coefficient
ϱ	Thermodynamics-based overstress function (in true stress space)
$ϱ_D$	Thermodynamics-based overstress function (in dissipative stress space)
σ_0	effective stress at a reference time
σ_1	effective stress in axial direction
σ_3	effective stress in radial direction
ν	Poisson's ratio
$\delta\Phi$	dissipation increment function
Ψ^g	Gibbs free-energy function
Ψ_e^g	Elastic Component of Gibbs free-energy function
Ψ_{vp}^g	Viscoplastic Component of Gibbs free-energy function
$\dot{\Psi}$	the differential of the free-energy function
ψ	viscosity function
$\bar{\chi}'_{ij}$	generalised stress tensor
χ'_{ij}	dissipative stress tensor

Common Acronyms

CRS	Constant Rate of Strain

CSL	Critical State Line
EVP	Elastic-viscoplastic
MCC	Modified Cam-clay
OCR	Over-consolidation Ratio
UTC	Undrained Triaxial Compression (UTC)
UTE	Undrained Triaxial Extension