

# **Development of Mixed Hardening Hyper-Viscoplastic Constitutive Models for Soils Incorporating Creep & Fabric Effects**

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# ABSTRACT

During the past several decades, the constitutive modelling for the prediction of time-dependent behaviour of soft soils has attracted an increasing attention within the geotechnical research society due to the scarcity of appropriate types of soil for construction as the regions around the globe have struggled to keep up with the meteoric rise in the infrastructure developments to cater for the substantial growth in population. Therefore, the consideration of time- and rate-dependent behaviour of geomaterials, particularly soft soils, such as creep, strain-rate dependent effects and stress relaxation behaviour, becomes a fundamental concern towards the long-term settlement deformation behaviour.

In this study, a mixed hardening hyper-viscoplastic constitutive model and its extended model are developed for describing the time-dependent stress-strain evolution of soil deformation, with the additional consideration of the arrangement of particles and the interparticle bonding, prominent in deformation of natural soils. The proposed model is intended to capture the loading-rate or strain-rate dependent behaviour of soils, accounting for the variations in the fundamental shapes of the yield loci along with the kinematic hardening and non-associated flow behaviour, with the extended model supplementing the proposed one with a  $\beta$ -line defining the inclination of the non-symmetrical elliptical yield locus in the  $p'$ - $q$  plane, along with the auxiliary rotational hardening effects to the kinematic hardening behaviour. The proposed models are formulated within the context of hyperplasticity framework, mainly due to the fact that the hyperplastic constitutive models obey the fundamental laws of thermodynamics, and the resulting approach provides a well-established structure and reduces the need for 'ad hoc' assumptions. The distinctive departure from the existing viscoplasticity

models is the application of thermodynamics, based upon the use of internal variables, to postulate free-energy and dissipation potential functions, from which the elasticity law, the yield condition and corresponding flow behaviour, the isotropic and kinematic hardening laws, are derived based on a standardised systematic procedure. Firstly, the proposed model is presented, in which the free-energy function is decomposed into the elastic and the viscoplastic components, incorporating the dependence on both volumetric and deviatoric viscoplastic strains, and the viscoplastic dissipation potential function accounting for both the instantaneous energy dissipation and the additional energy dissipation due to delayed deformation. The additional viscoplastic component of the free-energy function results in the modified shift stress, to describe the kinematic hardening behaviour of the yield locus. Besides, a non-linear creep formulation is postulated to address the limitation of over-estimating long-term settlement, which is incorporated into the proposed model. Being introduced as a rational and logical extension towards the proposed model, the extended model enhances the free-energy and dissipation potential functions, in which not only the additional viscoplastic free-energy function depends on both volumetric and deviatoric viscoplastic strains, but also the fabric coupling parameter is incorporated into the free-energy and dissipation potential functions. Accordingly, the constitutive relations of the solid soil skeleton are expressed from the perspective of hyperplasticity framework in order to capture a wide variety of viscous behaviour of soils, with the emphasis on the strain-softening or hardening behaviour during the time-dependent delayed deformation in soils. The proposed model and the extended model only require minimal number of material parameters, which can readily be determined using standard laboratory testing equipment.

The performance and applicability of the proposed and extended models are investigated and validated using the triaxial and oedometer experimental results available in the existing literature. Comparisons between the numerical results and the laboratory measurements are

conducted to demonstrate the versatility and capability of the proposed model in reproducing the rate-dependent behaviour of natural soft soils subjected to a variety of loading conditions. Due to the advantages of strong theoretical foundation with rigorous, yet compact and consistent procedure, with a relatively small number of required model parameters, the proposed and extended models have been signified as ideal for the numerical implementations to predict the time-dependent behaviour of soft soils, including long-term settlement behaviour in geotechnical structures.

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# Nomenclature & Abbreviations

## Latin Notations

$A, B$	functions for stress-like quantities
$C_{\alpha e}$	secondary compression coefficient
$C_{\beta}$	material constant controlling the extent of coupling
$C_r$	swelling index
$C_c$	compressive index
$D_{\beta}$	relative contribution from the volumetric and deviatoric viscoplastic strains in determining the extent of coupling
$C_v(\eta)$	volumetric target value for $\beta$
$C_q(\eta)$	deviatoric target value for $\beta$
$e_0$	initial void ratio
$e$	void ratio
$e_R$	reference void ratio
$\Delta e$	change in void ratio
$F$	overstress function
$G$	elastic shear modulus
$G_0$	initial elastic shear modulus
$g$	elastic shear modulus gradient

$J$	cross-coupling elastic modulus
$K$	elastic bulk modulus
$k$	elastic bulk modulus gradient
$M$	slope of the critical state line
$m$	power value (material constant) representing the slope of the $\log \mu - \log e$ curve
$n$	power order (dimensionless material constant)
$p$	effective stress
$\dot{p}$	change in effective stress
$p_R$	reference mean stress
$p_0$	effective stress at initial state (or reference time)
$p_c$	pre-consolidation pressure
$\dot{p}_c$	change in pre-consolidation pressure
$p_{c0}$	initial pre-consolidation pressure
$p_s$	volumetric shift stress
$p_D$	volumetric dissipative stress
$Q$	viscoplastic potential function
$q$	deviatoric stress
$\dot{q}$	change in deviatoric stress
$q_s$	deviatoric shift stress
$t_R$	reference time (or absolute equivalent time)
$V$	specific volume
$w_L$	liquid limit
$w_P$	plastic limit
$w$	flow potential function
$z$	force potential function

## Greek Notations

$\delta_{ij}$	Kronecker's delta
$\alpha$	material constant linking to the amount of deviatoric dissipation
$\beta$	cross-coupling between volumetric and deviatoric dissipation
$\varepsilon_1$	strain in axial direction
$\varepsilon_3$	strain in radial direction
$\varepsilon_{ij}$	total strain tensor
$\varepsilon_v$	total volumetric strain

$\varepsilon_q$	total shear strain
$\dot{\varepsilon}_v$	total volumetric strain increment
$\dot{\varepsilon}_q$	total shear (or deviatoric) strain increment
$\varepsilon_{ij}^e$	elastic strain tensor
$\varepsilon_v^e$	elastic volumetric strain
$\varepsilon_q^e$	elastic shear strain
$\dot{\varepsilon}_v^e$	elastic volumetric strain increment
$\dot{\varepsilon}_q^e$	elastic shear strain increment
$\varepsilon_{ij}^p$	plastic strain tensor
$\varepsilon_{ij}^{vp}$	viscoplastic strain tensor
$\varepsilon_v^{vp}$	viscoplastic volumetric strain
$\varepsilon_q^{vp}$	viscoplastic shear (or deviatoric) strain
$\dot{\varepsilon}_v^{vp}$	viscoplastic volumetric strain increment
$\dot{\varepsilon}_q^{vp}$	viscoplastic shear (or deviatoric) strain increment
$\gamma$	material constant linking to the amount of stored plastic work
$\kappa^*$	slope of swelling line in $\ln v - \ln p$ plot
$\lambda^*$	slope of normal consolidation line in $\ln v - \ln p$ plot
$\eta$	stress ratio
$\mu$	creep coefficient
$\mu_0$	initial creep coefficient
$Q$	Thermodynamics-based overstress function (in true stress space)
$Q_D$	Thermodynamics-based overstress function (in dissipative stress space)
$\sigma_0$	effective stress at a reference time
$\sigma_1$	effective stress in axial direction
$\sigma_3$	effective stress in radial direction
$\nu$	Poisson's ratio
$\delta\Phi$	dissipation increment function
$\Psi^g$	Gibbs free-energy function
$\Psi_e^g$	Elastic Component of Gibbs free-energy function
$\Psi_{vp}^g$	Viscoplastic Component of Gibbs free-energy function
$\dot{\Psi}$	the differential of the free-energy function
$\psi$	viscosity function
$\bar{\chi}'_{ij}$	generalised stress tensor
$\chi'_{ij}$	dissipative stress tensor

## Common Acronyms

CRS	Constant Rate of Strain
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CSL	Critical State Line
EVP	Elastic-viscoplastic
MCC	Modified Cam-clay
OCR	Over-consolidation Ratio
UTC	Undrained Triaxial Compression (UTC)
UTE	Undrained Triaxial Extension