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Research Paper

Benchmarking loss given default discount rates

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ABSTRACT

This paper provides a theoretical and empirical analysis of alternative discount rate concepts for computing loss given default (LGD) rates using historical bank workout data. It benchmarks five discount rate concepts for workout recovery cashflows in order to derive observed LGDs in terms of economic robustness and empirical implications: contract rate at origination, loan-weighted average cost of capital, return on equity (ROE), market return on defaulted debt and market equilibrium return. The paper develops guiding principles for LGD discount rates and argues that the weighted average cost of capital and market equilibrium return dominate the popular contract rate method. The empirical analysis of data provided by Global Credit Data (GCD) shows that declining risk-free rates are in part offset by increasing market risk premiums. Common empirical discount rates lie between the risk-free rate and the ROE. The variation in empirical LGDs is moderate for the various discount rate approaches. Further, a simple correction technique for resolution bias is developed and increases observed LGDs for all periods, particularly recent periods.

Keywords: default; discount rates; global credit data; loss given default (LGD); recovery; resolution; systematic risk.

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1 INTRODUCTION

The loss given default (LGD) is a central modeling parameter in banks' internal credit risk models for credit risk exposures that measures the degree of shortfall of net recoveries relative to the outstanding loan amount due to, and attributable to, the default event. Credit risk exposures may result from assets, derivatives, credit lines and guarantees.

LGD modeling comprises two stages. In the first stage, observed LGDs are inferred from the relative differences between the outstanding loan amounts and the sums of discounted observed recovery cashflows at default. In the second stage, observed LGDs are explained by risk factors that are observed prior to default. Here, a large number of econometric techniques, including linear and nonlinear techniques, fractional response, beta regressions and more advanced machine learning techniques, have been proposed (for details see Loterman *et al* (2012), Baesens *et al* (2016) and Rösch and Scheule (2020)).[1]

LGD discount rates are therefore a key input parameter for the calculation of observed LGDs in the first stage. The methodology for calculating LGD discount rates is controversial for a number of reasons. First, much of the literature on discount rates focuses on unconditional asset returns. Recovery cashflows are observed post default, and this conditionality has been reflected in only a very small number of papers. Second, recovery cashflows are observed during a resolution time of multiple periods with different economic states, and this mixing effect results in a limited linkage between bank loan LGDs and economic cycles (cf Qi and Yang 2009; Yao *et al* 2017). As a result, links between systematic risk and discount rates have not been analyzed. Third, most commercial banks are unable to observe market prices for recoveries. Some studies have analyzed post-default bond prices and find large discount rate ranges and hence uncertainty due to low observation counts.

This paper analyzes five LGD discount rate concepts. First, the contract rate at origination ("contract") is the contractual interest rate of a loan as the combination of a base rate plus a risk premium at loan origination. Second, the loan's weighted average cost of capital (WACC), being a bank funding cost, is a combination of the capital-ratio-weighted cost of equity funding and the debt-ratio-weighted cost of debt funding. The capital ratio may be based on the loan-specific regulatory capital requirements. Third, the return on equity (ROE) is the cost of equity funding. Fourth,

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[1] Note the concept of Stage I and Stage II modeling is common in the literature for workout cashflows (see, for example, Do *et al* 2018). Different terms may exist. Some literature is based only on the (Stage II) modeling of observed LGDs (eg, using one minus the observed bond prices over par value at or shortly after default as a proxy for observed LGDs) and does not require a two-stage approach. However, most LGD calculations performed by commercial banks do require both stages.

the market return on defaulted debt is the return on the bond price at resolution (ie, the realized recovery at resolution) relative to the bond price at default (ie, the expected recovery at default). Fifth, the market equilibrium return is the combination of a base rate and a premium for systematic risk. The risk premium is based on the product of the systematic risk sensitivity and the equity risk premium.

The approaches we analyzed are based on a survey of Global Credit Data (GCD) member banks regarding approaches they have implemented or were considered during implementation, plus a review of the academic literature and guidance notes by regulators and accounting standards boards (see Section 3 for further details). These approaches may be separated into contract-specific, comparable and equilibrium approaches and reflect a broad spectrum.

This paper makes the following four specific contributions to the literature.

(1) It develops guiding principles for calculating LGD discount rates. Discount rates should be based on the opportunity costs of comparable financial instruments and include the risk-free rate and a premium for nondiversifiable risk. They should be based on information that is available at the time of default and exclude premiums for realized risk.

(2) The properties for five alternative discount rates are benchmarked in terms of simplicity, data availability and the avoidance of negative LGD values. As a theoretical contribution, the WACC is extended to a loan-specific WACC in order to be applicable to the data requirements of commercial banks.

(3) This paper tests the practical implementation of the approaches and implications on discount rates as well as LGDs using a large set of historical workout data provided by GCD.

(4) This paper provides a simple methodology for correcting the resolution bias in LGDs. This bias is due to the fact that short resolution times result in low LGDs and the most recent loss observations are dominated by short resolution times.

The paper finds that WACC and market equilibrium return dominate the popular contract rate method, and that this method has two main shortcomings. First, contract rates are based on the origination time: this is a violation of the principle that discount rates should relate to the default time. Second, contract rates include the expected loss in relation to default risk: this is a violation of the principle that discount rates should not compensate for realized risk. A trade-off effect is noted, as WACC and market equilibrium return are model based and are therefore somewhat more complicated to derive and validate.

The empirical analysis shows that declining risk-free rates are in part offset by increasing market risk premiums in the period 2000–13. This interaction implies limited variability in discount rates and observed LGDs in the empirical analysis. Common empirical discount rates are found to be between the risk-free rate and the ROE. The variation of empirical LGDs is moderate for the various discount rate approaches. The correction for resolution bias increases the observed LGDs for all periods.

The data set was provided by a consortium of large commercial banks and is one of the largest data sets for recovery cashflows available to researchers. GCD oversees the data collection process and ensures the homogeneity of the data collected by the various contributing banks. This paper summarizes work undertaken by a GCD working group (WG) on LGD discount rates.

The paper is structured as follows. Section 2 provides a background on LGD modeling with regard to risk, expectation and realization of LGDs. Section 3 develops guiding principles and analyzes the five different discount rate concepts. Section 4 provides the empirical analysis for GCD data. Section 5 concludes and provides an outlook for future work.

2 BACKGROUND ON LOSS GIVEN DEFAULT MODELING: RISK, EXPECTATION AND REALIZATION

Figure 1 shows the stylized evolution of credit losses from the beginning of the observation period (time 0) to the default event (time T_D), and to the resolution event (time T_R).[2] We consider two sequential risk processes: default risk with two possible outcomes (default and nondefault) and resolution risk with an infinite number of possible outcome states S. These outcomes may include a random number of cashflows at random times between the default time and the resolution time. Examples of resolution categories are workouts, modifications and cures.[3]

We follow the GCD terminology of "probability of default" being the forward estimate and "observed default frequency" or "observed default rate" being the empirical values. In this spirit, the WG has established a clear and simple terminology for LGDs, which is summarized in Table 1.

A number of different LGD definitions for our discount rates are required to solve circular references in the empirical execution and to derive regulatory capital for the WACC approach. This paper distinguishes between the random (and hence unknown) LGD and the expectation in an actuarial sense at the time of default; the realizations of LGDs do not consider the discounting of cashflows and are applied

[2] Reference values for the average time to resolution may be found in Araten *et al* (2004) (0.8 years) and Jacobs (2012) (1.7 years). These values are consistent with the GCD experience.

[3] GCD has collected a large number of observations of default outcomes (resolutions).

FIGURE 1 Evolution of credit losses.

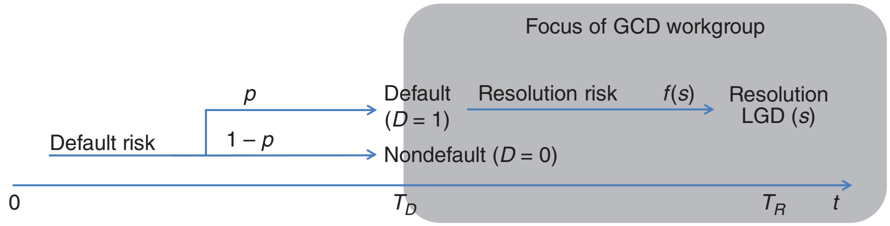

TABLE 1 LGD terminology.

Abbrev.	Description	Character	Discount	Application
LGD	LGD	Risk (RV)	Yes	Model framework
NLGD	Nominal LGD	Risk (RV)	No	Model framework
ELGD	Expected LGD	Expectation	Yes	Stage II LGD modeling
ENLGD	Expected nominal LGD	Expectation	No	Avoid circular references
OLGD	Observed LGD	Realization	Yes	Stage I LGD modeling
ONLGD	Observed nominal LGD	Realization	No	Avoid circular references
DLGD*	Basel downturn LGD	Stress	N/A	Regulatory capital

*Downturn LGD is subject to regulatory definition and is not the focus of this study. RV, random variable. N/A, not applicable.

to resolve circular references. Further, Basel III is based on the concept of downturn LGDs, which are the LGDs expected during an economic downturn. This is generally derived from expected LGDs and does not require the application of a discount rate, which is why an “N/A” was assigned. Note, however, that the model input “expected LGD” is based on a discount rate.

Cures are often considered in Stage II LGD modeling (see, for example, Do *et al* 2018); these are not analyzed separately in the derivation of discount rates and assume a value of zero.

2.1 Default risk

Default risk is assumed to have two possible realizations $D \in \{0, 1\}$.[4] The associated random loss rate given the random LGD at time of default is

$$L_{T_D} = \begin{cases} 0 & \text{if } D = 0, \\ \text{LGD} & \text{if } D = 1. \end{cases} \tag{2.1}$$

The variable "LGD" will be considered in detail in the next section. The probability distribution of the default indicator D is Bernoulli:

$$p(D) = \begin{cases} 1 - p & \text{if } D = 0, \\ p & \text{if } D = 1. \end{cases} \tag{2.2}$$

The outcome of default risk is random (hence unknown) and the expected loss rate (EL) at default may be computed by summing the probability-weighted discrete loss outcomes:

$$\text{EL}_{T_D} = ((1 - p) \times 0) + (p \times E(\text{LGD}_{T_D})) = p \times E(\text{LGD}_{T_D}). \tag{2.3}$$

The present value at origination of future loan losses results from discounting the outcomes to the origination (time T_0):

$$\begin{aligned} \text{EL}_0 &= ((1 - p) \times 0) + \left(p \times E\left(\frac{\text{LGD}_{T_D}}{(1 + \text{rf}_{[0,T_D]} + \delta_{[0,T_D]})^{(T_D - 0)}}\right)\right) \\ &= \frac{p \times E(\text{LGD}_{T_D})}{(1 + \text{rf}_{[0,T_D]} + \delta_{[0,T_D]})^{(T_D - 0)}}. \end{aligned} \tag{2.4}$$

The discount time and discount rate are equal to the risk-free rate $\text{rf}_{[0,T_D]}$ and a risk premium $\delta_{[0,T_D]}$, respectively, for all resolution outcomes, and can hence be written outside the expectation operator, as all resolution outcomes relate to the same time of default.

The discount rate for default risk includes a risk premium based on the systematic risk of the default process and (state) independent of the (unknown) outcome of default risk, as all risk premiums are (cf Damodaran (2007) and the guiding principles for risk-based discount rates in Section 3.2). The discount rate may be period specific ($[0, T_D]$) to reflect the term structure of discount rates.

2.2 Resolution risk

Resolution risk is assumed to have an infinite number of possible realizations. The random net recovery cashflows for a given realization path s may occur any time

[4] Note that probabilities of default are considered in the contract rate concept.

between the default time and the resolution time: $c(s) = (c_{t_1}(s), \dots, c_{T_R}(s))$ and the probability density function is $f(s)$, with $\int f(s) = 1$.

The random absolute (eg, in US dollar terms) nominal LGD (ANLGD) is then

$$\text{ANLGD}(s) = \text{EAD} - \sum_{t=T_D}^{T_R} c_t(s). \tag{2.5}$$

The random relative nominal LGD (NLGD) per scenario is computed by relating ANLGD to the exposure at default (EAD), which comprises the outstanding principal and accrued interest and is assumed to be deterministic, rather than an ex ante bank estimate:[5]

$$\text{NLGD}(s) = \frac{\text{ANLGD}(s)}{\text{EAD}} = 1 - \frac{1}{\text{EAD}} \sum_{t=T_D}^{T_R} c_t(s). \tag{2.6}$$

The random LGD at the default time follows by means of discounting by a risk-adjusted rate:

$$\text{LGD}(s) = 1 - \frac{1}{\text{EAD}} \sum_{t=t_1(s)}^{T_R(s)} \frac{c_t(s)}{(1 + \text{rf}_{[T_D,t]} + \delta_{[T_D,t]})^{(t-T_D)}}. \tag{2.7}$$

The discount rate is the same for all loss observations and is (state) independent of the (unknown) outcome of resolution risk, as are all risk premiums (cf the guiding principles for risk-based discount rates in Section 3.2). The objective of the WG was the computation of the observed LGDs based on the time value of money, here $\text{rf}_{[T_D,t]} + \delta_{[T_D,t]}$. The discount rate may be period specific ($[T_D, t]$) to reflect the term structure of discount rates.

The outcome of resolution risk is random (hence unknown ex ante) and the expected nominal LGD (ENLGD), which is unconditional on any risk factor, may be computed by integrating over the density-weighted random nominal LGD, NLGD(s):

$$\text{ENLGD} = E(\text{NLGD}) = \int f(s)\, \text{NLGD}(s)\, \mathrm{d}s. \tag{2.8}$$

Likewise, the present value of the expected LGD is

$$\text{ELGD} = E(\text{LGD}) = \int f(s)\, \text{LGD}(s)\, \mathrm{d}s. \tag{2.9}$$

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[5] Corporate loans often have deterministic amortization schedules rather than embedded prepayment options and liquidity facilities.

2.3 Estimation of LGD

The observations of the random variables NLGD (LGD) may be called observed NLGD (observed LGD), realized NLGD (realized LGD), ultimate NLGD (ultimate LGD) or post-resolution NLGD (post-resolution LGD). We will call them observed NLGD (ONLGD) and observed LGD (OLGD); these can be estimated based on (2.6) and (2.7). This is often referred to as Stage I LGD modeling.

The observed LGDs may be linked with risk factors (via the estimation of regression models), and LGD may be estimated conditional on risk factors. The resulting estimated LGDs may be applied to compute banks' internal and regulatory (Basel III) capital requirements. This is often referred to as Stage II LGD modeling.

Further, the expected NLGD (expected LGD) may be estimated based on assumptions; this paper applies the average over the ONLGDs (OLGDs) for given risk segments as the simplest approach.

3 DISCOUNT RATES FOR COMPUTING EMPIRICAL LOSS GIVEN DEFAULT RATES

3.1 Regulatory guidance

Basel Committee on Banking Supervision (2005) mandates that the discount rate includes the time value of money and a risk premium for undiversifiable risk:

> When recovery streams are uncertain and involve risk that cannot be diversified away, net present value calculations must reflect the time value of money and a risk premium appropriate to the undiversifiable risk. In establishing appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions, the bank should focus on the uncertainties in recovery cash flows associated with defaults that arise during the economic downturn conditions identified under Principle 1. When there is no uncertainty in recovery streams (eg, recoveries derived from cash collateral), net present value calculations need only reflect the time value of money, and a risk free discount rate is appropriate.

Variations of these regulations and guidance notes are included in national guidance notes from prudential regulators. Examples are Australia (Australian Prudential Regulation Authority 2005), Hong Kong (Hong Kong Monetary Authority 2006), the United Kingdom (Financial Services Authority 2003; Bank of England 2013) and the United States (OCC–FRB–FDIC–OTS 2003, 2007).

The Prudential Regulation Authority (PRA) of the Bank of England expects banks to "ensure that no discount rate used to estimate LGD is less than 9%" (Bank of England 2013). The publication further found that "there was no widely-accepted industry approach to determining appropriate discount rates, insufficient evidence of the appropriateness of rates; and a tendency to reduce discount rates over time". The

PRA accepts lower discount rates if conservative cashflows (eg, certainty equivalent cashflows) are applied. Further, the PRA is willing to consider alternative industry approaches subject to review.

More recently, the European Banking Authority (EBA) has suggested a discount rate equal to the primary interbank offered rate applicable at the moment of default plus 5% (European Banking Authority 2017).[6]

Accounting standards differ from this interpretation, as they prescribe the use of the effective rate, ie, the rate that exactly discounts expected future cash payments or receipts through the expected life of the financial instrument to the initial investment. This is generally the contract rate for fixed-rate loans and the current interest rate for floating-rate loans. US banks have been subject to Statement of Financial Accounting Standard 114 ("Accounting by Creditors for Impairment of a Loan"), which prescribes the effective interest rate to be the discount rate. Internationally, International Accounting Standard 39 prescribes the effective interest rate of financial assets. In the future, the larger banks will be subject to the International Financial Reporting Standards (IFRS) established by the International Accounting Standards Board (IASB). In essence, International Financial Reporting Standard 9 (IFRS 9) continues to prescribe the discount rate to be the effective interest rate.

More recently, IFRS 9 (International Accounting Standards Board 2014) and Current Expected Credit Loss (CECL; Financial Accounting Standards Board 2016) require the computation of lifetime expected losses as a basis for loan loss provisioning. IFRS 9 has been implemented since 2018 outside the United States. It applies to all instruments measured at amortized cost and instruments measured at fair value through other comprehensive income, but lifetime expected losses are only computed for assets that have increased significantly in credit risk. CECL is implemented in the United States and applies to all assets.[7] There are a number of approaches to implementing the new regulations, and discounted cashflow methods are popular. Financial Accounting Standards Board (2016) specifies that "if an entity estimates expected credit losses using methods that project future principal and interest cashflows, the entity shall discount expected cashflows at the financial assets' effective interest rate". Note that IFRS 9 and CECL are different applications to the computation of observed LGDs, as IFRS 9 and CECL generally require predefault calculations while the latter generally requires post-default calculations. However, this is another example of the popularity of the contract rate/effective interest rate.

In summary, prudential regulators are open to various approaches for LGD discount rates on economic substance, provided they include the time value of money

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[6] The approaches suggested by the EBA and the PRA are not analyzed further, as these do not take the individual loan and bank characteristics into account.

[7] For further details, see Bellini (2019).

and a premium for nondiversifiable (systematic) risk. Contrary to this, accounting standards prescribe the use of the effective rate (ie, the contract rate in most instances).

3.2 Guiding principles for LGD discount rates

In this paper, we focus on discounted cashflow (DCF) methods, for consistency with the data environment in the banking industry. For example, GCD collects observed cashflow data during workout on mostly nontraded instruments. In this technique, expected cashflows are discounted by risk-adjusted discount rates. Observed LGDs are computed by discounting the observed cashflows by the risk-adjusted discount rate. The WG proposes the following guiding principles.

(1) Discount rates should be based on the opportunity costs of financial instruments with comparable price-relevant factors to avoid arbitrage opportunities for nonregulated market participants, and hence market inefficiencies.

(2) Key price factors, per regulatory guidance, include the risk-free rate and premium for nondiversifiable risk. We interpret nondiversifiable risk as systematic risk that cannot be diversified in financial markets.

(3) Discount rates should be based on information available at the time of default. This includes parameter estimates based on historical information.

(4) Discount rates should exclude premiums for realized risk and reflect only resolution risk, and thus not include a premium for default risk nor exclude the premium for resolution risk.

This paper does not consider, but encourages a more detailed analysis of, the following aspects.

(1) Recoveries may be decomposed into assets (determined ex ante) of different degrees of systematic risk. These assets may be appraised using discount rates that reflect the systematic risk levels.

(2) Banks may apply conservative estimates for recovery realizations and a certainty equivalent method with the risk-free rate as discount rate, as appropriate.

(3) Basel Committee on Banking Supervision (2005) mandates that “appropriate risk premiums for the estimation of LGDs consistent with economic downturn conditions” are applied. There are a number of approaches available to model downturn LGD. European Banking Authority (2019) distinguishes three general categories: model building approaches for banks that have sufficient loss data for the identified downturn period; haircut or extrapolation approaches

for banks that do not have sufficient loss data; and an addition of fifteen percentage points to ELGD estimates if banks cannot use the approaches in the first two categories. Banks should carefully consider any interaction between the discount rate method, the downturn method and the data on which these methods are based.

(4) Impact of taxation on cashflows and discount rates.

(5) Term structure of interest rates.

3.3 Review of discount rate approaches

The approaches analyzed in the following are based on a survey of GCD member banks and either have been implemented or were considered during implementation. We also draw on a review of the academic literature, as well as regulatory guidance notes by regulators and accounting standards boards (see Section 3.1 for further details). The approaches may be separated into loan contract-specific, comparable and equilibrium approaches.

3.3.1 Loan contract rate

Asarnow and Edwards (1995) propose utilizing the loan contract rate of the defaulted loan as the discount rate.

Default and resolution risk are priced in the contract rate. The annual return given no default is the contract rate k:

$$r_{D=0} = (1 + k) - 1 = k. \tag{3.1}$$

With reference to Figure 1, we focus on default risk and set the outcome after default to ELGD. We assume a loan with a maturity of one year and default at maturity. Assuming that interest (based on the contract rate) has accrued to a full period, the annual return given a default is

$$\begin{aligned} r_{D=1} &= (1 + k)(1 - \text{ELGD}) - 1 \\ &= k(1 - \text{ELGD}) - \text{ELGD}. \end{aligned} \tag{3.2}$$

Therefore, the expected return with regard to default risk is

$$\begin{aligned} E(R_{\text{L}}) &= (1 - p)r_{D=0} + p r_{D=1} \\ &= (1 - p)k + p(k(1 - \text{ELGD}) - \text{ELGD}), \end{aligned} \tag{3.3}$$

with probability of default p and random return on the loan R_{L}. Note that the expected return for resolution risk is embedded in ELGD in the above equations.

In contrast to the expected return with regard to resolution risk, the contract rate at origination includes the expected loss and the LGD-adjusted likelihood of nondefault (see also Chalupka and Kopecsni 2008). This becomes apparent after solving (3.3) for k:

$$k = (E(R_{\mathrm{L}}) + \underbrace{p\mathrm{ELGD}}_{\text{expected loss}})\left(\frac{1}{1 - p\mathrm{ELGD}}\right). \tag{3.4}$$

The contract rate has two shortcomings:

(i) it relates to the expected return with regard to default risk and resolution risk, which is set at origination,[8] and

(ii) it exceeds the expected return for the loan as it includes the expected loss.

As a result, the contract rate contradicts the guiding principles presented in Section 3.2. Alternatively, the expected return implied in the contract rate from (3.3) may be used, as this is more closely aligned with the guiding principles.

However, estimated expected returns imply a circular reference to the estimated ELGD assumption, which requires knowledge of the discount rate. To avoid the circular reference, we may apply ENLGD for the realized annual return given a default:

$$r_{D=1} = ((1 + k)(1 - \mathrm{ENLGD}))^{1/((T_D - 0) + (T_R - T_D))} - 1. \tag{3.5}$$

The formula assumes that all cashflows have been received at the time of resolution and computes the internal rate from the resolution time to the loan origination.

A length of one period may be assumed for the time from loan origination to default $(T_D - 0)$ for simplicity. The resulting expected return with regard to default risk and resolution risk is

$$\begin{aligned} E(R_{\mathrm{L}}) &= (1 - p)r_{D=0} + pr_{D=1} \\ &= (1 - p)k + p(((1 + k)(1 - \mathrm{ENLGD}))^{1/(1 + T_R - T_D)} - 1), \end{aligned} \tag{3.6}$$

R_{L} is the return on the loan, which is based on the contract rate, and hence only indirectly on financial markets. Further, OCC–FRB–FDIC–OTS (2003) proposes a contractual rate of the highest risk grade.[9] This proposal is very similar to using rates

[8] The systematic risk may change post default with the realization of default risk. The WG did not discuss this in more detail but encourages further research.

[9] OCC–FRB–FDIC–OTS (2003) stipulates that "A bank must establish a discount rate that reflects the time value of money and the opportunity cost of funds to apply to recoveries and costs. The discount rate must be no less than the contract interest rate on new originations of a type similar to the transaction in question, for the lowest-quality grade, in which a bank originates such transactions. Where possible, the rate should reflect the fixed rate on newly originated exposures with term corresponding to the average resolution period of defaulting assets." This approach is no longer found in the updated guidance (OCC–FRB–FDIC–OTS 2007).

that may apply after covenant violations or, alternatively, the contract rate at default. Note that contract rates for high idiosyncratic risk exposures and prior or at default are likely to include the predefault expected loss and a similar correction may apply.

EXAMPLE 3.1 (Discount rate example) The estimated probability of default is 1%, the contract rate is 5%, ENLGD is 60% and time to resolution is two years. Therefore, by following (3.6), the estimated expected return (and hence discount rate) with regard to default risk is 4.7%:

$$E(R_{\mathrm{L}}) = (1-0.01)0.05 + 0.01(((1+0.05)(1-0.6))^{1/(1+2)} - 1) = 0.0470.$$

3.3.2 Bank weighted average cost of capital (WACC)

Witzany (2009) and Jensen (2015) proposed the application of the weighted cost of capital as a discount rate.[10] Weights are generally based on the relative proportion of equity and debt funding of a bank in market value terms.

It is often argued that the bank funding costs do not reflect the risk profile of individual credit exposures with regard to resolution risk given loan default (see, for example, Chalupka and Kopecsni 2008). Further, market funding costs for distressed/defaulted assets would be the preferred approach but are very difficult to determine. Therefore, the WG has explored the computation of loan-specific capital and debt ratios based on the capital requirements of defaulted loans relative to the post-default expected loan value. Under the assumptions that regulatory capital is a reasonable measure for systematic risk and that post-default capital and debt ratios are consistent with those of other (eg, predefault) loan instruments, bank funding costs may provide the basis for a reasonable discount rate.[11]

Default risk is realized with the default event. Hence, bank capital E_{T_D} is recomputed using a PD of unity,

$$E_{T_D} = (\mathrm{DLGD} - \mathrm{ELGD})\mathrm{EAD}, \tag{3.7}$$

and the post-default loan value V_{T_D} is assumed to equal the expected recovery:

$$V_{T_D} = (1 - \mathrm{ELGD})\mathrm{EAD}. \tag{3.8}$$

Hence, the post-default capital ratio (ie, weight) e_{T_D} is

$$e_{T_D} = \frac{E_{T_D}}{V_{T_D}} = \frac{\mathrm{DLGD} - \mathrm{ELGD}}{1 - \mathrm{ELGD}}, \tag{3.9}$$

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[10] Witzany (2009) initially presents a capital asset pricing model, which results in a WACC model for the discount rate and computes (iteratively) the spread as the capital risk charge times the regulatory capital for market risk.

[11] Prudential regulators have increased the focus on consistency between the various regulations (see Basel Committee on Banking Supervision 2014).

while the post-default equity value is assumed to be equal to the unexpected loss given default:

$$E_{T_D} = \text{DLGD} - \text{ELGD}. \tag{3.10}$$

The debt ratio (ie, weight) m_{T_D} follows as $1 - e_{T_D}$.

We use R_c to denote the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk. This expected return may be based on the combination of the expected ROE weighted by the post-default capital ratio and the expected return on debt weighted by the post-default debt ratio:

$$\begin{aligned} E(R_c) &= e_{T_D} E(R_E) + m_{T_D} E(R_M) \\ &= \frac{\text{DLGD} - \text{ELGD}}{1 - \text{ELGD}} E(R_E) + \left(1 - \frac{\text{DLGD} - \text{ELGD}}{1 - \text{ELGD}}\right) E(R_M). \end{aligned} \tag{3.11}$$

Structurally, this approach combines bank-level funding costs and loan-level funding ratios: e_{T_D} (and therefore m_{T_D}) are loan specific, as DLGD and ELGD are based on Stage II LGD models, while $E(R_E)$ is the cost of equity (eg, ROE) and $E(R_M)$ is the cost of debt (eg, average bank debt funding costs). In other words, the equity and debt weights are loan specific, while the funding costs are bank specific. In practice, $E(R_E)$ might be based on a capital asset pricing model (CAPM) or an extension thereof using the bank beta and market premium (see below for further details), while $E(R_M)$ may be derived from the weighted average cost of bank liabilities (in particular, deposit rates and wholesale funding costs).

In practice, the expected ROE and debt may be approximated by equity and debt funding costs. The formula implies a circularity issue, as ELGD and DLGD are required, which in turn are estimated in practice from OLGD, which requires the discount rate.

EXAMPLE 3.2 (Discount rate example) The estimated probability of default is 1%, estimated ELGD is 60%, DLGD is 63.2%, the post-default capital ratio is $(63.2\% - 60\%)/(1 - 0.6) = 8\%$, the debt funding costs are 4% and the ROE is 7.8%. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 4.3%:

$$E(R_c) = (0.08 \times 0.078) + (0.92 \times 0.04) = 0.0430.$$

3.3.3 Bank ROE

Eales and Bosworth (1998) discuss a range of discount rate approaches and ultimately apply the ROE as the discount rate. Their justification is that equity is debited/credited for differences between ELGD and OLGD and that the risk exposure of LGDs corresponds to the equity position. The approach may be interpreted as a special case (ie, upper boundary) of the WACC approach where the capital ratio is 100%:

$$E(R_c) = E(R_E). \tag{3.12}$$

We denote by R_c the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk. For example, the cost of equity may be computed using the CAPM,

$$E(R_E) = \mathrm{rf} + \beta E(R_M - \mathrm{rf}), \tag{3.13}$$

with the expected market excess return $E(R_M - \mathrm{rf})$ and risk-free rate rf, and

$$\beta = \frac{\mathrm{cov}(R_E, R_M)}{\mathrm{var}(R_M)},$$

where R_M is the return of the market portfolio, the use of which is somewhat controversial in the literature. However, in practice, it is often replaced by the return of the local share market index. Averages for realized market excess returns may be taken as proxies for expected values and are reported for broadly based equity market indexes in various geographies.[12] Other approaches, such as using average share returns (after adjusting for dividend payments), bank ROE target numbers or accounting ROE ratio, may be considered.

EXAMPLE 3.3 (Discount rate example) The bank equity beta to the market return is 0.8, the risk-free rate is 3% and the estimated expected market excess return is 6%. Therefore, the estimated expected ROE (and hence discount rate) is 7.8%:

$$E(R_c) = 0.03 + (0.8 \times 0.06) = 0.078.$$

3.3.4 Market return of marketable credit instrument

Brady *et al* (2006) and Jacobs (2012) compute the realized returns and average realized returns for defaulted bonds. In a similar fashion, returns on distressed debt may be applied (see, for example, Altman and Kuehne (2012), who look more broadly at returns of distressed bonds). The realized return (per annum) given a resolution outcome b of a marketable financial instrument is

$$r_{b,c} = \left(\frac{B(T_R)}{B(T_D)}\right)^{1/(T_R - T_D)} - 1, \tag{3.14}$$

with the instrument price B. Again, we use index "c" to denote an instrument that is comparable to a defaulted exposure subject to resolution risk. $B(T_R)$ is the bond

[12] Depending on the historical time period and geography, market risk premiums are between 2% and 8% per annum (see Dimson *et al* 2011).

price at resolution time T_R, and $B(T_D)$ is the bond price at default time T_D. $T_R - T_D$ is the resolution (ie, workout) period. Note that it is common to use the bond price between thirty and forty-five days past default as a proxy for the bond price at default. A positive (negative) return results if bond prices at resolution time exceed (are below) bond prices at default time. In particular, short resolution periods, in combination with bond price changes from default time to resolution time, result in large returns and large differences in mean returns for different subsamples.

The expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk may be based on the average as an estimate for the expected return:

$$E(R_{\mathrm{c}}) = \frac{1}{B}\sum_{b=1}^{B} r_{b,\mathrm{c}} = \frac{1}{B}\sum_{b=1}^{B}\left(\left(\frac{B_b(T_R)}{B_b(T_D)}\right)^{1/(T_{b,R}-T_{b,D})} - 1\right). \tag{3.15}$$

For simplicity, we have assumed that the prices of all defaulted bonds, B, are observed and that the market expectations are based on the mean of realized returns.

EXAMPLE 3.4 (Discount rate example) For two defaulted bonds, the bond price at resolution is 55 and 45 and the time to resolution is one and two years, respectively, and the bond price at default is 40 for both bonds. The realized annual return is $(0.55/0.4)^{(1/1)} - 1 = 0.375$ for the first bond and $(0.45/0.4)^{(1/2)} - 1 = 0.0607$ for the second bond. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 21.78%:

$$E(R_{\mathrm{c}}) = \frac{1}{2(0.375 + 0.0607)} = 0.2178.$$

3.3.5 Market equilibrium return

Maclachlan (2004) suggests basing discount rates on market equilibrium models, such as the CAPM. In these models, the expected return for an instrument that is comparable to a defaulted exposure subject to resolution risk may be based on the expected return of an asset, which in turn is based on the risk-free rate, the beta and the market risk premium (ie, the return of the market in excess of the risk-free rate):

$$E(R_{\mathrm{E}}) = \mathrm{rf} + \beta\,\mathrm{MP}, \tag{3.16}$$

with

$$\beta = \frac{\mathrm{cov}(R_{\mathrm{c}}, R_{\mathrm{M}})}{\mathrm{var}(R_{\mathrm{M}})} = \frac{\mathrm{Corr}(R_{\mathrm{c}}, R_{\mathrm{M}})\mathrm{SD}(C)}{\mathrm{SD}(R_{\mathrm{M}})}$$

and market risk premium $E(R_{\mathrm{M}} - \mathrm{rf})$. $\mathrm{cov}(\cdot)$ is the covariance, $\mathrm{var}(\cdot)$ is the variance, $\mathrm{Corr}(\cdot)$ is the correlation and $\mathrm{SD}(\cdot)$ is the standard deviation. R_{E} denotes that the expected return is based on an equilibrium model. The CAPM requires a number of assumptions, such as frictionless financial markets, that are controversial during economic downturns when default events are most likely to occur. Examples include the ability of market participants to borrow and lend, the absence of transaction costs and the availability of all information to all market participants.

EXAMPLE 3.5 (Discount rate example) The defaulted debt beta with regard to the market return is 0.5, the risk-free rate is 3% and the estimated expected market excess return is 6%. Therefore, the estimated expected return (and hence discount rate) with regard to resolution risk is 6%:

$$E(R_{\mathrm{c}}) = 0.03 + (0.5 \times 0.06) = 0.06.$$

Maclachlan (2004) estimates the defaulted debt beta based on the following.

- The correlation between the return on a defaulted bond index or defaulted bonds and the market return: the analysis results in defaulted debt betas of 36% based on the NYU Bond Index and 37.1% based on ninety defaulted US bonds. The market index was the S&P 500 index in both analyses.
- Asset correlations of 17% from Frye (2000),[13] resulting in defaulted debt betas of 30% based on an asset correlation.

The beta coefficient may be computed as

$$\beta_{i,j} = \frac{\sigma_{i,j}\sqrt{\mathrm{AC}_j}}{\sigma_{M,j}}, \tag{3.17}$$

with borrower i and risk segment j.

Alternative approaches to determine $\beta_{i,j}$ from (3.17) include the distressed bond and equity prices of defaulted borrowers. However, such approaches are limited by the number of observations and the separation of systematic and idiosyncratic risks. The WG believes that further research should be undertaken. The approach for the risk premium suggested by the WG was to regress recovery rates on equity market excess returns or Fama–French factors.

Bank loan recoveries are expected to be correlated with gross domestic product (GDP), which in turn is highly correlated with consumer consumption, and the consumption CAPM may provide a price link.[14] Rösch and Scheule (2012) model empir-

[13] Note that Basel III provides asset correlations between 12% and 24% for corporate credit exposures in the internal ratings-based approach.
[14] Note that Qi and Yang (2009) and Yao *et al* (2017) show that LGD variation is mainly explained by the collection policy, ie, idiosyncratic characteristics, and that there are limited linkages to economic cycles. Another factor may be the variation of economic states over the resolution period during which recovery cashflows are collected.

ical asset correlations AC_j by conditioning on historical average LGDs (eg, through-the-cycle models) and time-varying information (eg, point-in-time (PIT) models).[15] Assuming a normal distribution, the natural logarithms of observed recovery rates at a certain point in time ($\mathrm{ORR}_{ijt} = 1 - \mathrm{OLGD}_{ijt}$) may be described as follows:

$$\ln(\mathrm{ORR}_{ijt}) = \alpha_j + \beta_j x_{jt} + \gamma_j \varepsilon_{jt} + \delta_j \varepsilon_{ijt}, \tag{3.18}$$

with borrower i, risk segment j and default time t. The parameters may be estimated-risk-segment specific. α_j is an intercept, β_j is the sensitivity to observable (particularly macroeconomic) information, γ_j is the sensitivity to a standard normally distributed systematic random effect and δ_j is the sensitivity to an idiosyncratic standard normally distributed error term. γ_j and δ_j may also be interpreted as standard deviations. x_{jt} are time-varying systematic variables (here, the lagged average log recovery and GDP growth). Other variables, including loan-specific variables, are possible. The model may be extended to control for selection issues by the consideration of default events and cure events (see Rösch and Scheule 2012).

The asset correlation

$$\mathrm{AC}_j = \frac{\gamma_j^2}{\gamma_j^2 + \delta_j^2} \tag{3.19}$$

follows from (3.18)

3.4 Benchmarking of discount rate approaches

The WG discussed discount rate approaches in the light of a number of criteria.

(1) Guiding principles of discount rates: the degree to which an approach is supported by the principles for discount rates established by the WG. WACC and equilibrium returns satisfy these properties, while the other approaches violate aspects of these principles.

- Contract rate: this approach includes compensation for default risk (in essence, the credit spread) realized post default. Further, contract rates relate generally to the origination time, with regard to systematic risk and the time value of money.
- Return on equity: this approach assumes that the systematic risk is equal to an equity investment in the bank, which is unlikely to be reasonable given the heterogeneity of defaulted loans. The bank-specific ROE is

[15] This is comparable to default risk modeling, where asset correlations have been measured by linking time-varying defaults and default rates to historical average default rates (eg, through-the-cycle models). In extensions, frailty effects conditional on observable time-varying information (eg, PIT models) have been estimated (see, for example, Rösch and Scheule 2020).

unlikely to include an appropriate premium for the systematic resolution risk of the defaulted loan.

- Market returns of defaulted bonds: this approach is based on a small number of bond defaults, and resulting returns are unlikely to provide robust comparable discount rates.

(2) Simplicity: this approach is based on available measurable information, which does not require assumptions in the measurement process.

- Contract rates: these are generally observable but may be unclear for credit lines, derivatives and guarantees.
- WACC and ROE: these require a moderate level of assumption.
- Market returns of defaulted bonds: the comparability of defaulted bonds and defaulted loans is unclear (particularly for small and medium-sized enterprise (SME) loans).
- Equilibrium returns: these require strong assumptions for the estimation of measures for systematic risk and computation of risk premiums.

(3) Application to empirical data.

- Contract rates: these may not be observable for all loans or may be complicated by front-end, back-end, hybrid or variable features.
- WACC and ROE: data on bank funding costs may not be available; data may support the computation of the loan-level capital and debt ratios for WACC.
- Market returns on defaulted bonds: data on defaulted bonds may not be available.
- Equilibrium returns: loss data may be used to estimate the exposure to systematic risk.

(4) Negative LGDs: the realized LGD is computed based on the book value of EAD (ie, expected outstanding principal at default) and the market value of the resulting post-default realized LGD if the current interest rates and current spreads for systematic risk are applied. Low interest rate regimes (eg, as a consequence of monetary easing) in conjunction with high net recovery cashflows (eg, in the instance of a cure) may result in present values in excess of EAD, and hence negative LGDs. For example, a default followed by a loan service according to schedule and discount rates (determined at default) below the contract rate (determined at origination) would result in a recovery rate greater than 1 and hence a negative LGD. LGD values that are constrained within the

TABLE 2 Evaluation of approaches.

Criteria	Contract rate	WACC	ROE	Bond return	Equilibrium return
Guiding principles of discount rates	Disagree	Agree	Disagree	Disagree	Agree
Simplicity for:					
SMEs	Agree	Neutral	Neutral	Disagree	Disagree
large corporates	Agree	Neutral	Neutral	Neutral	Disagree
financial institutions	Agree	Neutral	Neutral	Neutral	Disagree
credit lines, derivatives and guarantees	Neutral	Neutral	Neutral	Disagree	Disagree
Application to GCD data	Neutral	Agree	Disagree	Disagree	Agree
LGD values in [0,1]	Agree	Neutral	Neutral	Neutral	Neutral

The criteria are phrased in the positive, and the evaluation categories are "agree", "disagree" or "neutral".

interval [0, 1] may support general acceptance and the input requirements of some Stage II regression models that require values between 0 and 1.

- Contract rates: LGDs are within the interval [0, 1].
- All other concepts may result in negative LGDs; however, the likelihood of negative LGDs is small.

Table 2 summarizes the WG view on these approaches. The criteria are phrased in the positive, and the evaluation categories are "agree", "disagree" or "neutral".

Two approaches are preferred by the WG, as they meet the guiding principles and can be applied in a data-rich environment that is available to GCD members:

- WACC based on the assumption that post-default capital is a reasonable reflection of systematic risk;
- market equilibrium return based on the assumption that the link between the measures for systematic risk and sensitivity to market excess returns is reasonable.

4 APPLYING DIFFERENT DISCOUNT RATES TO WORKOUT DATA

4.1 Using GCD workout data for empirical discount rate studies

Global Credit Data (GCD) is a not-for-profit initiative to help banks to measure their credit risk. It is owned by its fifty member banks across Europe, Africa, North America, Asia and Australia. GCD has collected one of the world's largest

LGD/EAD databases, with a large number of defaulted facility observations totaling over €200 billion in all Basel asset classes.

This paper analyses this database and is, to the best of our knowledge, the first to estimate discount rates from observed resolution information. The use of the GCD database for LGD discount rates has the following merits.

- Discount rates are based on systematic risk. The large number of defaulted facilities in the GCD database allows for the measurement of systematic risk, ie, the remaining nondiversifiable risk in a diversified portfolio.

- The data set is sufficiently large to form risk segments such as geographies and industries, which allows for the estimation of segment-specific systematic risk and hence the required risk premiums.

- Most discount rate approaches may be inferred from GCD data. We base the cost of equity on a bank beta of 1, and the cost of debt on the risk-free rate. The database mainly captures banking book data on nontraded bank instruments. Hence, the market return approach is not included in the empirical part of this study and we refer the reader to Jacobs (2012).

4.1.1 Screening the data and applying data filters

The empirical analysis is based on data provided by GCD in June 2015. For the purpose of this analysis, member banks agreed to a selection of filter rules; see Table 3, which shows the number of defaulted loan facilities and borrowers after the application of these various filter rules.

After the application of these filters, 29 569 defaulted facilities and 17 193 borrowers remain. We analyze LGDs at the facility level. A robustness check reveals that there is only a minor difference between facility- and borrower-based mean LGD per default year, as most borrowers relate to a single facility. Note that it is possible that some banks consolidate multiple defaulted loans by the same borrower into a single facility.

4.1.2 Risk segmentation

The following risk segments were created in terms of geographical and cultural proximity:

- Great Britain and Ireland;

TABLE 3 Number of defaulted facilities and borrowers after the application of filter rules.

Filter rules	Facilities	Borrowers
Raw data set	123 577	64 140
Facilities with FC, LC and SME as borrower	103 089	52 240
Resolved facilities	88 966	45 916
Unsecured facilities	34 498	20 395
Defaulted in 2000–13	32 521	19 294
Various robustness checks*	31 806	18 924
Borrower EAD more than €10K	29 569	17 193

FC, financial companies. LC, large companies. SME, small and medium-sized enterprises. *These checks include the following: (i) a test to see whether the amount of write-off and cashflows is reasonable; (ii) a loan reported "resolved" shows all transactions in excess of the exposure amount by less than 10%; (iii) the variable "entity asset class" must be given; (iv) if the facility asset class is "SME" or "large corporate", then the entity asset class must equal "corporate"; (v) if the facility asset class equals "Banks & Fin Co", then the entity asset class must equal "banks" or "nonbank financial company"; (vi) the entity asset class must equal "banks" or "nonbank financial company" when the primary industry code equals "finance and insurance"; (vii) the facility asset class must be given; (viii) the facility asset class must equal "Banks & Fin Co" when the primary industry code equals "finance and insurance"; (ix) the loan status must be given in the "history" table at least once for each facility ID.

- Central Europe (Austria, Belgium, France, Germany, Luxembourg, Netherlands and Switzerland);[16]
- Hispania (Portugal and Spain);
- North America (Canada and United States);
- Scandinavia (Denmark, Finland, Norway and Sweden);
- South Africa;
- others (all other countries).

Figure 2 shows on a world map the number of defaulted facilities after data filtering by country.

We have chosen the following risk segments in terms of industry segmentation:

- commerce (wholesale and retail trade);
- construction;
- finance (real estate and rental and leasing, finance and insurance);

[16] The WG is aware that Central Europe may be defined in different ways. We define Central Europe as a set of geographies that the WG believes features risk characteristics that are close to each other.

FIGURE 2 Total number of observations, by country.

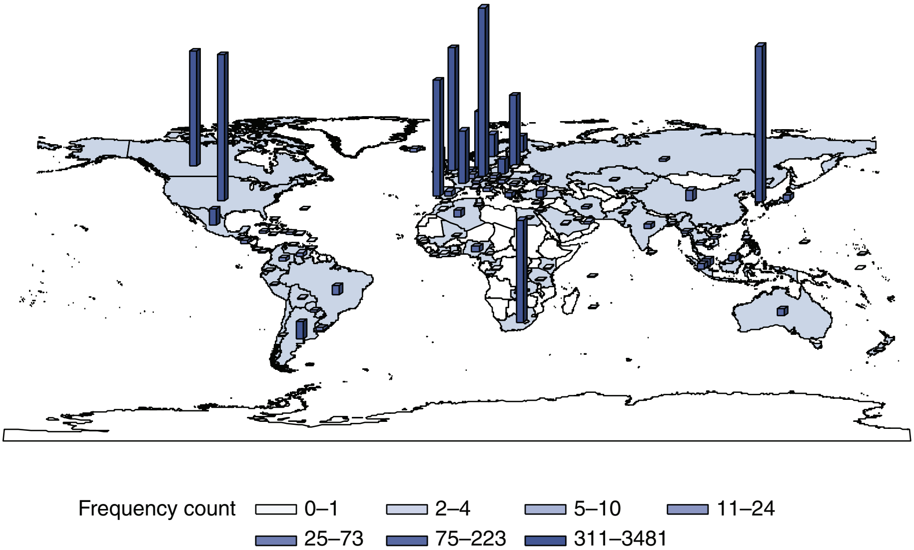
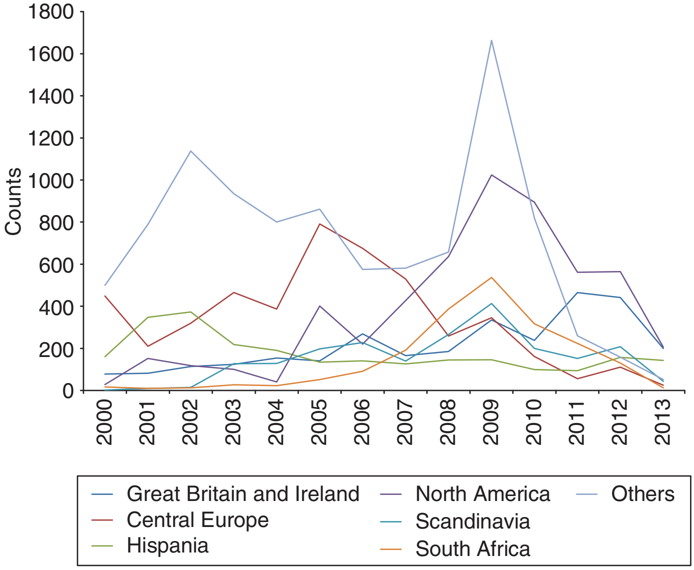


FIGURE 3 Total number of observations by geography risk segment.



- manufacturing;
- services (transportation and storage; professional, scientific and technical services; education; extra-territorial services and organizations; health and social services; hotels and restaurants; other community, social and personal services; private sector services (household));
- others (agriculture, communications, hunting and forestry, fishing and fishing products, mining, public administration and defence, utilities).

The classification by geography and industry follows the literature, as economic cycles are commonly seen as country and industry specific. The reported categories were selected based on a minimum of 100 loss event observations per default year. More granular classifications (eg, individual countries within Scandinavia) were analyzed with consistent results. Figure 3 shows the number of observations per risk segment.

The reason for this segmentation strategy is that we measure the exposure to systematic risk based on the time variation of loss rates. This requires the assumption that loss rates are not exposed to idiosyncratic risk, for which we require on average a minimum of 100 loss observations per annum.

The consequence of defining more granular risk segments[17] would be that the observed loss rate variation is based on a combination of systematic and idiosyncratic risk, and the exposure to systematic risk is overestimated.

4.1.3 Empirical strategy

The GCD collects a number of information tables. We applied the following databases to set the filter rules.[18]

Entity: general information on the borrower or guarantor.

Loan: general information on the facility.

History: dates for five events (origination, one year prior to default, default, post default and resolution).

[17] One example would be to break out and combine the industries "agriculture, hunting and forestry" and "fishing and fishing products" into single categories. This would imply a total of 904 loss observations (from 2000 to 2013) with fewer than fifty loss events per annum in economic upturns (2001 and 2002) and the remainder in economic downturns (from 2007 to 2009).

[18] We did not apply the remaining GCD data sets, which are: financial (entity, financial (sales, assets and debt)), guarantor (guarantor credit risk information) and collateral (general information on collateral).

We applied the "history" data set to obtain the timing of origination, default and resolution dates (see above), the "loan" data set to obtain the risk-free GCD discount rate and the "transaction" data set to obtain the gross resolution cashflows. In the data analysis, we carry out the following steps:

(1) application of filter rules;

(2) risk segmentation (all facilities are grouped into geographic and industry classes);

(3) computation of nominal LGD based on the LGD2 definition and a zero discount rate (LGD based on the risk-free rate is provided in the GCD database);[19]

(4) correction of risk-free and nominal LGD based on risk-free rate and zero discount rate for time to resolution bias;

(5) estimation of ENLGD as the long run average over ONLGD and of ELGD as the long run average over OLGD;

(6) merging of macroeconomic risk drivers: real GDP growth and lagged mean log recoveries (from default to resolution time);

(7) computation of systematic risk measures as dispersion of a frailty effect, which models the deviation of the log recoveries from the long run average based on observed log recoveries;

(8) computation of contract rate ("Contract") and predefault expected return ("Contract2");

(9) computation of cost of equity (ROE) and the weighted average cost of capital (WACC);

(10) computation of equilibrium returns ("Equilibrium");

(11) computation of observed LGDs based on the discount rates;

(12) correction of observed LGD for time to resolution bias;

(13) production of statistics for discount rate analysis (Section 4.2) and LGD analysis (Section 4.3).

[19] The LGD2 definition includes the sum of present values of all cashflows (excluding principal advances) and financial guarantees (except write-offs and interest accruals, which are not cashflows). For more details, see Global Credit Data (2015).

4.2 Empirical discount rate analysis

Next we analyze the risk-free rate (GCD), contract rate (Contract), adjusted contract rate (Contract2), weighted average cost of capital (WACC), return on equity (ROE) and equilibrium return (Equilibrium).

4.2.1 Assumptions

We follow the computations of Section 3.3 and make the following assumptions.

GCD: based on the three-month Euro Interbank Offered Rate (Euribor) and the short-term interest rates of the respective country. Note that Euribor is an interbank lending rate for unsecured funds and includes an average bank credit spread.

Contract: contract rates are observable for approximately 13% of all observations at facility origination. The majority (74%) of facilities that have pricing information are priced with a floating rate. We extract the spread in excess of the base rate for floating-rate loans and compute the contract rate as the sum of the spread and the risk-free rate at default. We replace missing values by median contract rates (for both the contract rate and the predefault expected return) by geography and default year.

Contract2: we consider the contract rate and the estimated predefault expected return. The discount rate is calculated by following (3.6). The mean discount rate is 3.72%, and an alternative computation using (3.3) results in similar discount rates: 3.46% if ELGD is estimated based on the risk-free rate and 3.84% if ENLGD is used to avoid the circular reference. The economic impact on LGDs is comparable.

WACC: for the downturn LGD (which is an input to the WACC concept), we assume the maximum average LGD over default years and by geography. This may be similar to conditioning on historical adverse macroeconomic states. Banks may apply alternative downturn LGD models. The discount rate is calculated by following (3.11). We avoid circular references by estimating ELGD as the mean of the observed LGD for a given risk segment based on the risk-free rate. Robustness checks using ENLGD give comparable results.

ROE: a bank beta measure of 1 is assumed; the discount rate is calculated by following (3.13).

Equilibrium: the discount rate is calculated by following (3.16). The beta measure is based on (3.17). Table 4 shows the resulting parameter estimates for the PIT regression for log recoveries, applying segmentation by geography and segmentation by industry. Here, “Int” is the intercept estimate for the regression model.

TABLE 4 Parameter estimates for a PIT regression model for log recoveries with frailty effects.

Segment	Int	GDP	Average log recovery rate	Gamma	Delta	Average cost	Beta
Great Britain and Ireland	−0.4319	−0.0518	0.3909	0.3811	1.1413	0.1003	0.5631
Central Europe	−0.3820	−0.0005	0.2437	0.1553	1.1084	0.0193	0.2467
Hispania	−0.2360	0.0327	0.2828	0.2389	0.9493	0.0596	0.4339
North America	−0.3548	0.0182	0.2389	0.2224	0.6593	0.1022	0.5682
Scandinavia	−0.5347	−0.0068	−0.0594	0.1619	0.9092	0.0307	0.3116
South Africa	−0.1793	−0.0004	0.7120	0.2099	0.9876	0.0432	0.3696
Others	−0.5961	0.0079	0.0475	0.1647	1.1726	0.0193	0.2473
Commerce	−0.2194	−0.0014	0.6112	0.1622	1.0413	0.0237	0.2737
Construction	−0.2215	−0.0073	0.3605	0.1604	0.8721	0.0327	0.3216
Finance	−0.3312	−0.0017	0.3243	0.1721	0.9375	0.0326	0.3210
Manufacturing	−0.2552	0.0027	0.4623	0.0942	0.9966	0.0089	0.1673
Services	−0.2406	−0.0030	0.5011	0.1580	0.9981	0.0245	0.2780
Others	−0.2138	−0.0032	0.7037	0.1131	1.1904	0.0089	0.1681

This table provides regression results in accordance with (3.18).

In the empirical analysis, we applied reference values $\sigma_j = 0.32$ and $\sigma_{\mathrm{M}} = 0.18$ from Maclachlan (2004) in conjunction with the AC estimates from Table 4 to estimate betas and discount rates and compute the LGDs given these discount rates. In this model, we include two systematic variables: real GDP growth and mean log recovery, which are lagged by one period. Both variables have been shown to be powerful systematic control variables for default risk (see, for example, Lee *et al* 2016). We avoid circular references by computing ORR as $1 - \mathrm{OLGD}$ based on the risk-free rate. Robustness checks using ONLGD give comparable results.

All discount rates are based on the combination of risk-free rate and a spread.

- GCD is the risk-free rate at default.
- Contract rate is the risk-free rate at default plus credit spread at origination.
- Contract2 is the risk-free rate at default plus credit spread at origination less expected loss.
- ROE is the risk-free rate at default plus the equity risk premium at default times beta (assumed to be equal to unity).
- WACC is the capital ratio times the ROE at default plus the debt ratio times the risk-free rate at default.
- Equilibrium is the risk-free rate at default plus the equity risk premium at default times beta based on the systematic variation of mean LGDs over time.

The spread for ROE, WACC and Equilibrium approaches is based on the equity risk premium.

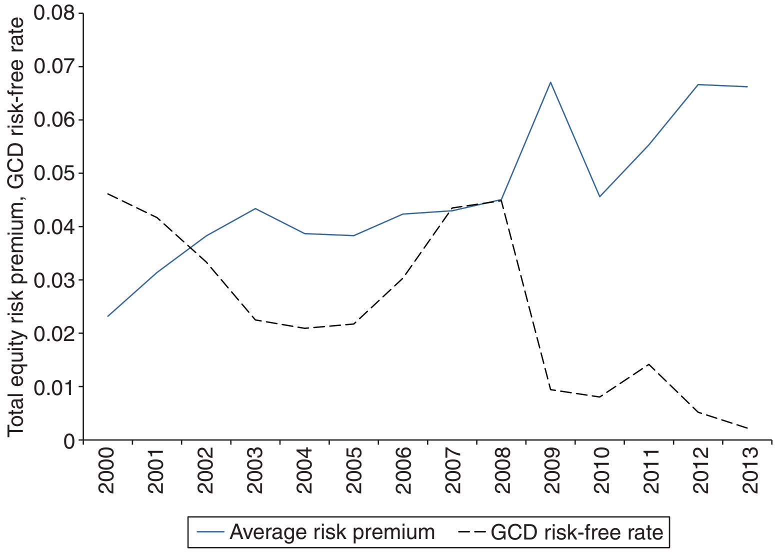
While the CAPM theory does not provide risk premiums outside the co-movement with the market return, Damodaran (2015) points out that more volatile financial markets attract a greater risk premium, which we include into our analysis. Other factors for risk premiums (eg, size, where smaller firms attract a higher risk premium) may exist, but we do not include these in our analysis, as our knowledge of the borrowers underlying the defaulted facilities is limited.

The risk-free rate is based on the GCD discount rate, and the implied equity risk premiums are from Damodaran (2015).[20] We computed an implied country risk premium on a country level by

$$\mathrm{IRP}_{kt} = \mathrm{RP}_{kt} - \mathrm{RP}_{\mathrm{US},t} + \mathrm{IRP}_{\mathrm{US},t}. \tag{4.1}$$

[20] Damodaran publishes the annual country risk premiums based on an average risk premium (RP) and the time-varying risk premium implied by future dividends (IRP) for the United States; see http://pages.stern.nyu.edu/~adamodar.

FIGURE 4 Average total equity risk premium, and the GCD risk-free rate.



For example, for Canada in 2000 we observed the following values: $\mathrm{RP}_{\mathrm{Canada},t} = 0.0611$, $\mathrm{RP}_{\mathrm{US},t} = 0.0551$ and $\mathrm{IRP}_{\mathrm{US},t} = 0.0205$. We compute the IRP for Canada as follows:

$$0.0265 = 0.0611 - 0.0551 + 0.0205.$$

We denote the index for a country by k (which is different from the broader risk segmentation j used before). The assumption behind the computation is that the US market (which is the largest in size) provides a base equity risk premium for national risk premiums. Figure 4 shows the average total equity risk premium as the sum of a global market risk premium and a country risk premium (solid lines); it also shows the risk-free GDC discount rate (dashed line). Equity risk premiums and risk-free rates offset, and may mitigate, the time variation of discount rates during the period 2000–13.

4.2.2 Euro risk-free rates

In this section, we use the euro as a pivot currency; transaction amounts and amounts at default are converted to and expressed in euro. The LGDs are then calculated using those converted amounts. Consistently with the euro-denominated cashflows, the Euribor is used as a risk-free rate for all facilities, regardless of geography.

FIGURE 5 Median discount rates over time (observations with recorded contract rate).

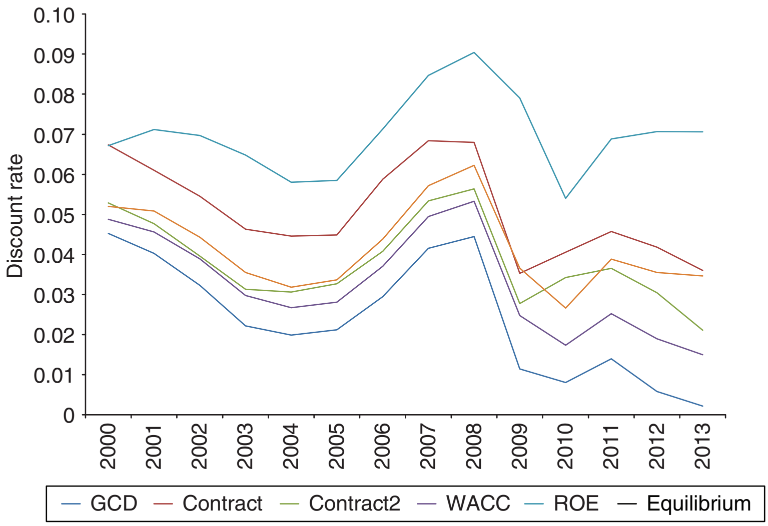


TABLE 5 Descriptive statistics: discount rates.

Measure	GCD	Contract	Contract2	WACC	ROE	Equilibrium
N	29 569	2 560	2 560	29 569	29 569	29 569
Min	0.0000	0.0000	−0.0275	0.0027	0.0205	0.0051
Max	0.0539	0.2762	0.1946	0.0647	0.1168	0.0782
Mean	0.0236	0.0491	0.0372	0.0326	0.0707	0.0413
SD	0.0147	0.0229	0.0214	0.0131	0.0136	0.0136

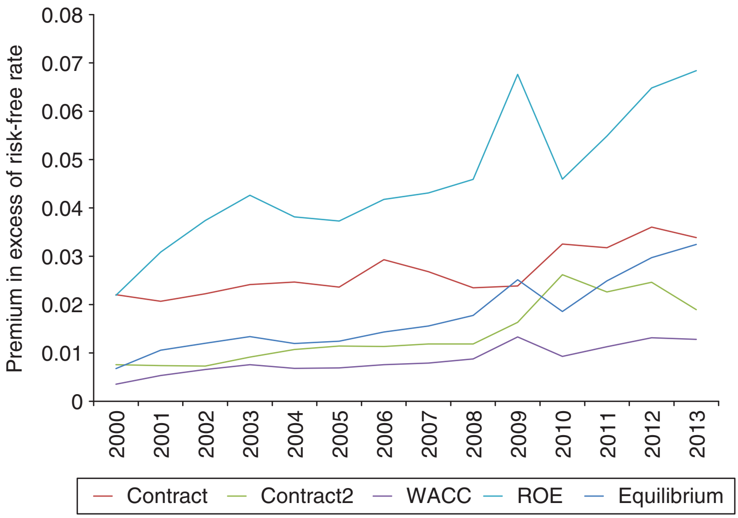
For the avoidance of doubt, in this table, all percentages are written as decimal fractions, eg, 0.0236 is to be read as 2.36%. Contract2 is computed by following (3.6), which is corrected for the expected loss. A negative value is unlikely but possible if expected losses exceed the contractual rate. In most instances, this can be attributed to low contractual rates.

Figure 5 shows the resulting discount rates. The lowest discount rate is the risk-free rate (GCD) and the highest discount rate is the ROE. Contract rates and market-implied discount rates are consistent, which is a reflection of the integration of lending and capital markets. Table 5 shows the moments for the main discount rates.

Figure 6 shows the mean premiums in excess of Euribor, which increase during the reference period.

All risk-adjusted approaches are directly (equilibrium approach) or indirectly (other approaches) based on market prices for the time value of money (risk-free

FIGURE 6 Mean premiums in excess of Euribor over time, joint cross-section, observations with recorded contract rate.



rate), systematic risk (market risk premium) and exposure to systematic risk. This implies that loan exposures with a higher risk-free rate, a higher market price for systematic risk or a greater exposure to systematic risk result in higher discount rates. High levels of default risk for a borrower, an industry or a country do not necessarily imply a high discount rate, as a high level of idiosyncratic risk may not be causal for a high level of systematic risk and a high market price.

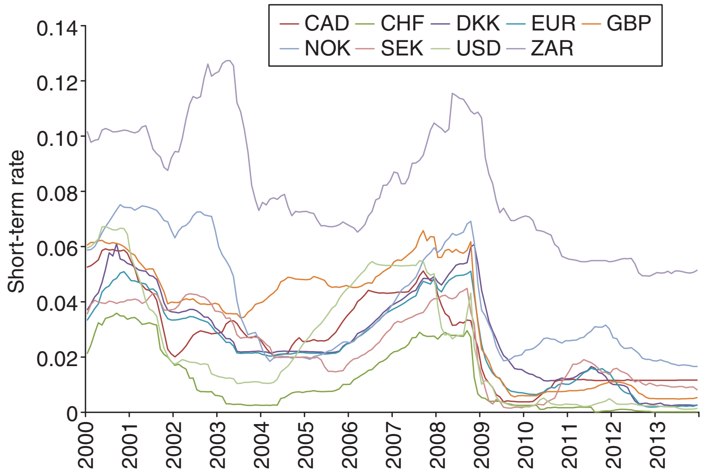
4.2.3 Currency-specific risk-free rates

The previous analysis used the euro as a pivot currency. Figure 7 shows the short-term interest rates that co-move for GCD countries, including South Africa (currency ZAR, top line), which has a higher level.

Figure 8 shows the mean discount rates by geography. We do not report the contract rate, as the number of observations is insufficient. Differences in mean currency-specific risk-free rates are co-moving over the countries analyzed. This shows the integration of the various economies. Differences may be attributed to the sovereign credit spread. Switzerland has the lowest rates and South Africa has the highest rates of the countries analyzed.

The Euribor is lowest, followed by the WACC rate, Contract2 rate, Equilibrium rate, Contract rate and ROE based on an asset beta of 1. The Contract rate is relatively

FIGURE 7 Mean discount rates by currency.



high, as it includes compensation for the expected loss. ROE is highest, as a beta of 1 is assumed, while most discount rates suggest an implied level of systematic risk, which is equivalent to a beta of less than 1.

4.3 Empirical LGD analysis

4.3.1 Euro risk-free rates

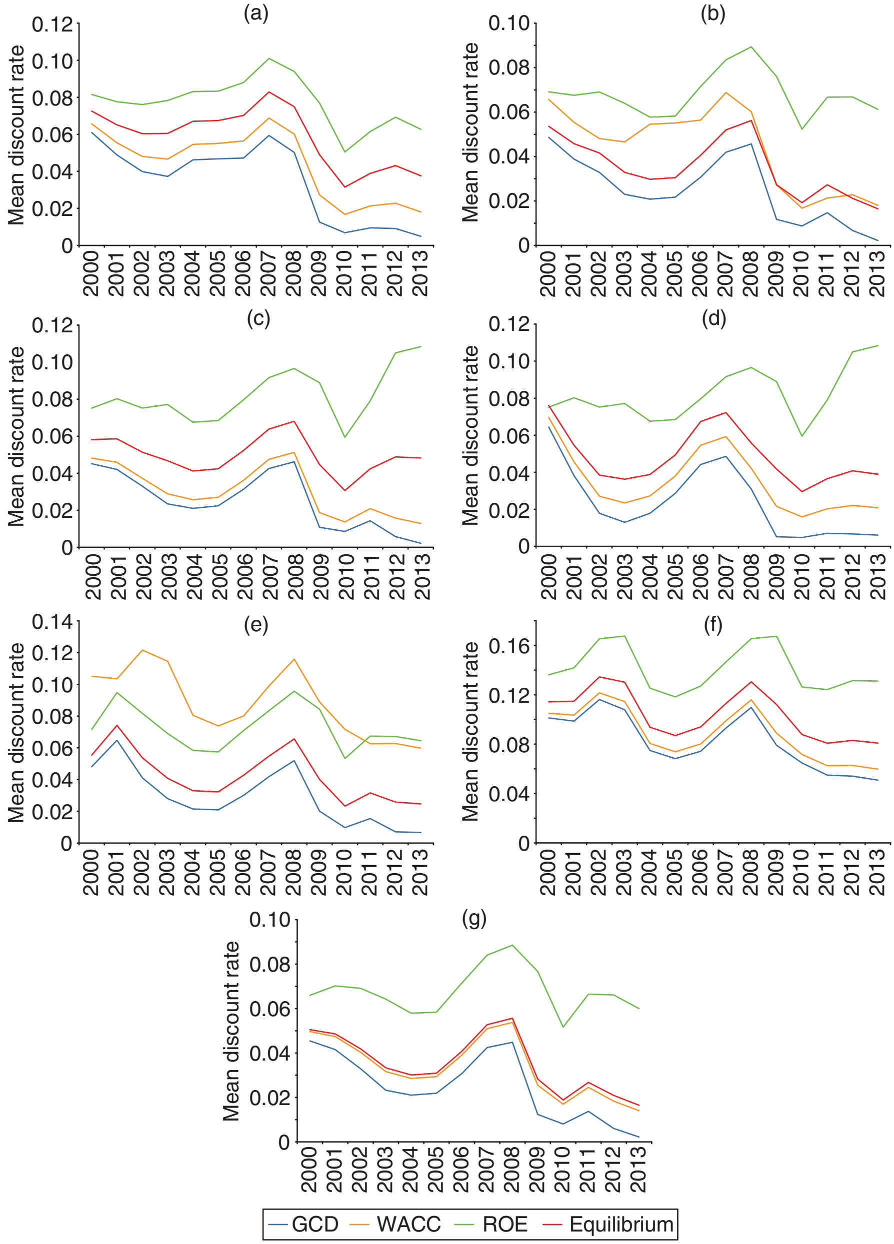
In this section, we compute and compare the LGDs that result from the various discount rate concepts without further corrections for time to resolution bias. Resolution bias will be scrutinized in Section 4.3.2.

Figure 9 shows the mean LGD without correction for resolution time bias. The discounting of recovery cashflows implies that a lower discount rate results in a lower LGD. The variation in LGDs given different discount rate concepts is limited and decreases with the time to resolution in more recent years. Table 6 shows the moments for the mean LGD without correction for resolution time bias.

4.3.2 Correction for time to resolution bias

The data sample is subject to a resolution bias, as unresolved LGDs with a longer time to resolution are not analyzed. For the same data set, Betz *et al* (2020) document a positive correlation between the time to resolution and LGD for corporate loans. Do *et al* (2018) show the same finding for mortgage loans. To correct for this bias,

FIGURE 8 Mean discount rates by geography.



(a) Great Britain and Ireland. (b) Central Europe. (c) Hispania. (d) North America. (e) Scandinavia. (f) South Africa. (g) Others.

FIGURE 9 Mean LGD without correction for resolution time bias.

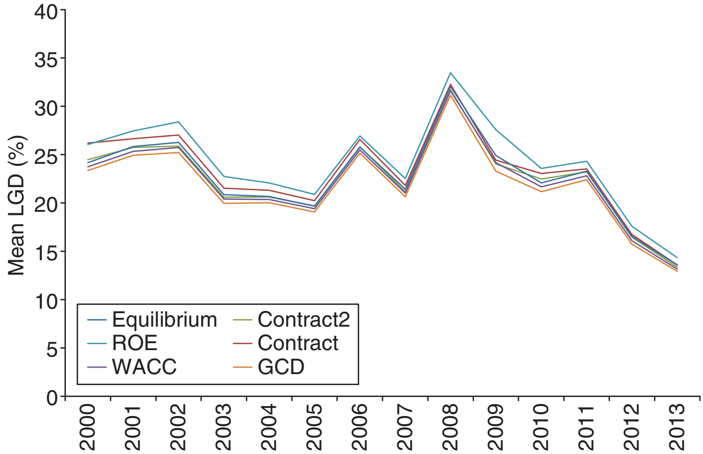


TABLE 6 Descriptive statistics: LGDs, without correction for resolution time bias.

Measure	GCD	Contract	Contract2	WACC	ROE	Equilibrium
N	29 569	29 569	29 569	29 569	29 569	29 569
Min	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Max	150.0000	150.0000	150.0000	150.0000	150.0000	150.0000
Mean	22.4122	23.8168	23.1600	22.8887	24.9517	23.3500
SD	35.9819	35.6135	35.7517	35.8369	35.3605	35.7172

For the avoidance of doubt, in this table, values for all variables except N are percentages, eg, 22.4122 is to be read as 22.4122%.

we adjust the mean LGDs ($\mathrm{LGD}_{t,\mathrm{adjusted}}$) as follows:

$$\mathrm{LGD}_{t,\mathrm{adjusted}} = (\mathrm{CR}_t \times \mathrm{LGD}_{t,\mathrm{resolved}}) + ((1 - \mathrm{CR}_t) \times \mathrm{LGD}_{t,\mathrm{unresolved}}). \qquad (4.2)$$

CR_t is the completion rate, which is the fraction of defaulted loans that have been resolved for a given default year. $\mathrm{LGD}_{t,\mathrm{resolved}}$ is the mean resolved and hence observed LGDs. We estimate the mean of unresolved (and hence unobserved) LGDs for a given default year by the mean over earlier resolved LGDs with a time to resolution (TTR) greater than the observed LGDs for a given default year:

$$\mathrm{LGD}_{t,\mathrm{unresolved}} = \frac{\sum_{i=1|\mathrm{TTR}_{it,\mathrm{resolved}} \geqslant 2015.5-t}^{N} \mathrm{LGD}_{it,\mathrm{resolved}}}{\sum_{i=1}^{N} I(\mathrm{TTR}_{it,\mathrm{resolved}} \geqslant 2015.5 - t)}. \qquad (4.3)$$

TABLE 7 Resolved LGD, completion rate, estimated unresolved LGD and adjusted LGD.

Default year	No. of obs	Resolved LGD	CR	Unresolved LGD	Adjusted LGD
2000	1234	23.3581	0.9724	70.6098	24.6613
2001	1599	24.9387	0.9423	70.6098	27.5762
2002	2089	25.2126	0.9368	70.6098	28.0830
2003	1996	19.9498	0.9541	70.6098	22.2746
2004	1722	20.0275	0.9498	70.6098	22.5664
2005	2578	19.0705	0.9457	60.5835	21.3243
2006	2199	25.1420	0.9524	47.5273	26.2085
2007	2161	20.6456	0.9134	44.3458	22.6991
2008	2536	31.1533	0.8493	42.3420	32.8395
2009	4463	23.2844	0.8780	41.8823	25.5529
2010	2728	21.1609	0.8289	40.1759	24.4139
2011	1812	22.4168	0.7451	36.9915	26.1324
2012	1771	15.7451	0.6856	35.1343	21.8404
2013	681	12.9344	0.6329	31.9048	19.8985

For the avoidance of doubt, in this table, all values are percentages, eg, 23.3581 is to be read as 23.3581%.

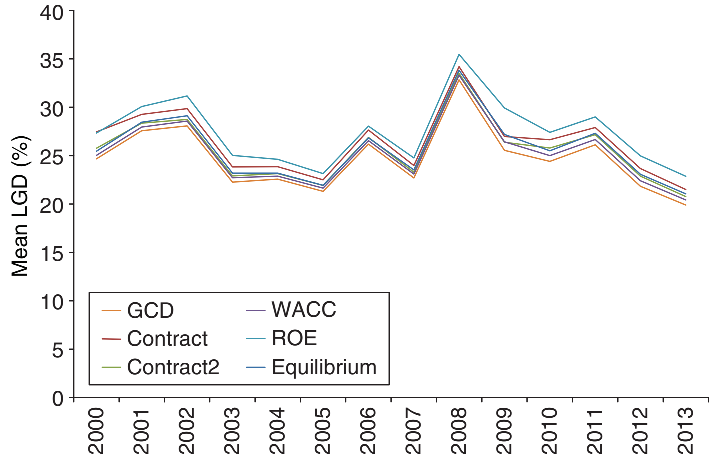
Table 7 shows the resolved LGD before correction, completion rates, estimated unresolved LGD and the adjusted LGD after correction by default year (LGDs are based on the risk-free rate, GCD). For example, $\text{LGD}_{2013,\text{resolved}} = 12.93$, CR $=$ 63%, $t = 2013$ and the end of the observation period is mid-2015. The mean LGD of unresolved LGDs (ie, defaulted loans with a time to resolution of over 2.5 years) is 31.90. The adjusted mean LGD is computed as follows:

$$\text{LGD}_{2013,\text{adjusted}} = (12.93 \times 0.63) + (31.90 \times (1 - 0.63)) = 19.90.$$

Figure 10 shows the mean LGD with correction for resolution time bias. The mean LGDs are higher than in Figure 9 over all default years, as the decreasing completion rate (from 2000 to 2013) is offset by a decreasing implied LGD (from 2000 to 2013) that is higher than the observed LGD in all instances.

The choice of discount rates has a low to moderate impact, which is a function of the average resolution time, which decreases from 3.16 in 2000 to 0.51 in 2013, and the average equity risk premium: the price for systematic risk increases from 2.19 in 2000 to 6.84% in 2013. These findings are in line with those of Gibilaro and Mattarocci (2011), who analyze the impact of the risk-free rate, contract rate and two equilibrium models (based on the beta between average recoveries and GDP as well as average recoveries and a defaulted bond index). The average LGDs vary between 0.5051 (using a risk-free discount rate) and 0.5327 (using the equilibrium model based on the defaulted bond index). Table 8 shows the descriptive statistics

FIGURE 10 Mean LGD with correction for resolution time bias.



for the main discount rates for all years, plus the start year (2000) and the end year (2013).

Mean LGDs differ in terms of levels and volatilities for geographies (Figures 11) and industries (Figure 12). North America demonstrates the greatest variability, with some linkage to the economic cycle: mean LGDs are highest during the 2001–2 economic downturn and lowest in 2005, prior to the global financial crisis.

The LGDs for industries are averaged over countries and are more aligned in terms of fluctuations but differ in terms of LGD levels. The differences in mean LGDs due to discount rates support our earlier discussion regarding the whole data set.

Figure 11 shows the mean LGDs based on various discount rates for geographies after a correction for time to resolution bias, and Figure 12 shows the mean LGDs based on various discount rates for industries after a correction for time to resolution bias.

4.3.3 Currency-specific risk-free rates

Figure 13 shows the mean LGDs based on various discount rates for geographies, using the original cashflows and the currency-specific short-term interest rate reported by the Organisation for Economic Co-operation and Development as the risk-free rate.

Contract rates are excluded from this analysis as they are independent of the choice of risk-free rate. The resulting LGD ranges are similar to those reported in Figure 11. Minor differences are visible for South Africa (particularly during the early years of

TABLE 8 Descriptive statistics: LGDs after adjustment for resolution bias.

(a) All years

Measure	GCD	Contract	Contract2	WACC	ROE	Equil	Equil 2
N	29 569	29 569	29 569	29 569	29 569	29 569	29 569
Min	1.9475	1.9848	1.9582	1.9524	2.0025	1.9432	1.9648
Max	147.8104	147.8477	147.8211	147.8153	147.8654	147.8061	147.8277
Mean	25.0829	26.6813	25.9287	25.5887	27.9014	25.3255	26.1114
SD	31.7808	31.4584	31.5755	31.6553	31.2335	31.7091	31.5493

(b) 2000

Measure	GCD	Contract	Contract2	WACC	ROE	Equil	Equil 2
N	1 234	1 234	1 234	1 234	1 234	1 234	1 234
Min	1.9475	1.9848	1.9582	1.9524	2.0025	1.9432	1.9648
Max	147.8104	147.8477	147.8211	147.8153	147.8654	147.8061	147.8277
Mean	24.6613	27.3862	25.7986	25.0397	27.3447	24.5213	25.4740
SD	30.5928	30.4993	30.4002	30.5786	30.6137	30.5729	30.5627

(c) 2013

Measure	GCD	Contract	Contract2	WACC	ROE	Equil	Equil 2
N	681	681	681	681	681	681	681
Min	11.7122	12.8886	12.3366	12.0870	13.7965	11.8924	12.4638
Max	77.3427	78.5057	77.9596	77.7118	79.4042	77.5225	78.0872
Mean	19.8985	21.5108	20.8231	20.4184	22.8705	20.1990	21.0495
SD	19.9879	19.8625	19.9010	19.9433	19.7503	19.9484	19.8662

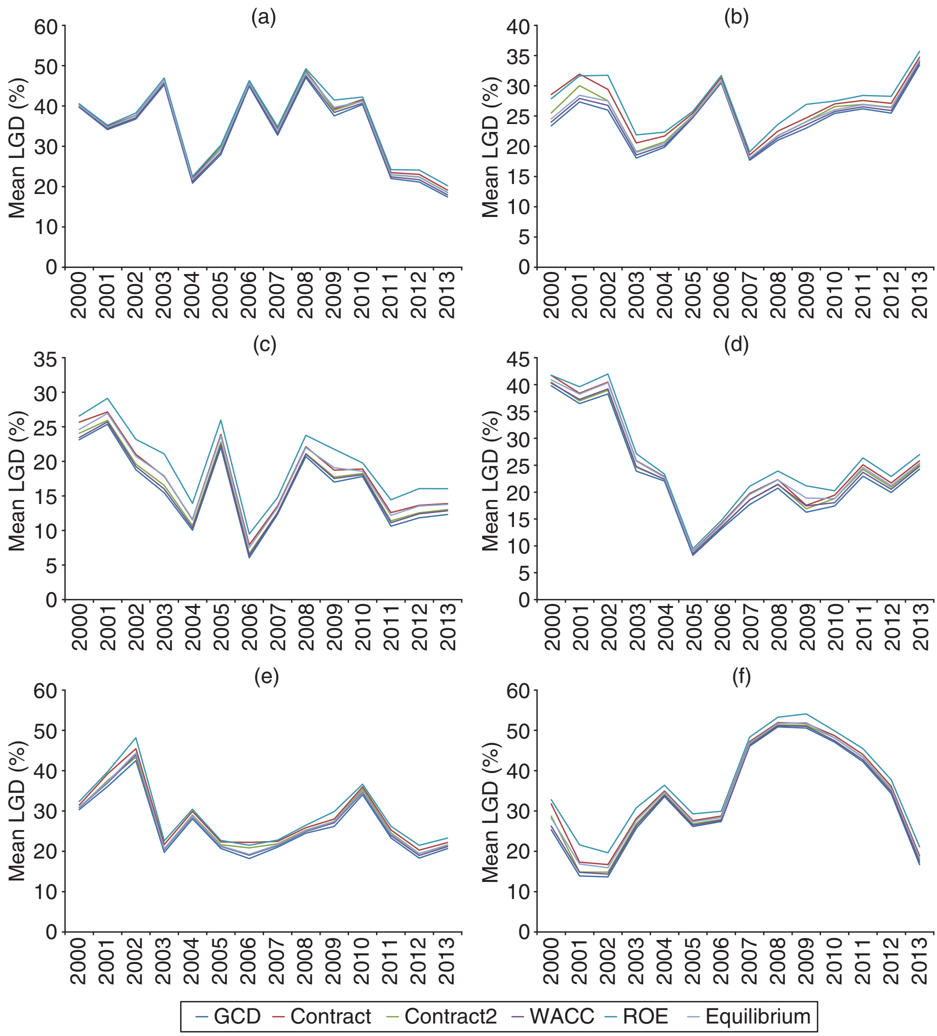
For the avoidance of doubt, in this table, the values of all variables except N are percentages, eg, 25.0829 is to be read as 25.0829%.

this period) as the country had higher than average risk-free rates translating into higher mean LGDs.

5 CONCLUSION

This paper analyzes five LGD discount rate concepts (contract rate at origination, loan-weighted average cost of capital, ROE, market return on defaulted debt and market equilibrium return) based on four guiding principles (the opportunity costs of comparable instruments, the risk-free rate and premium for systematic risk at default and the exclusion of premiums for realized risk) for simplicity, data avail-

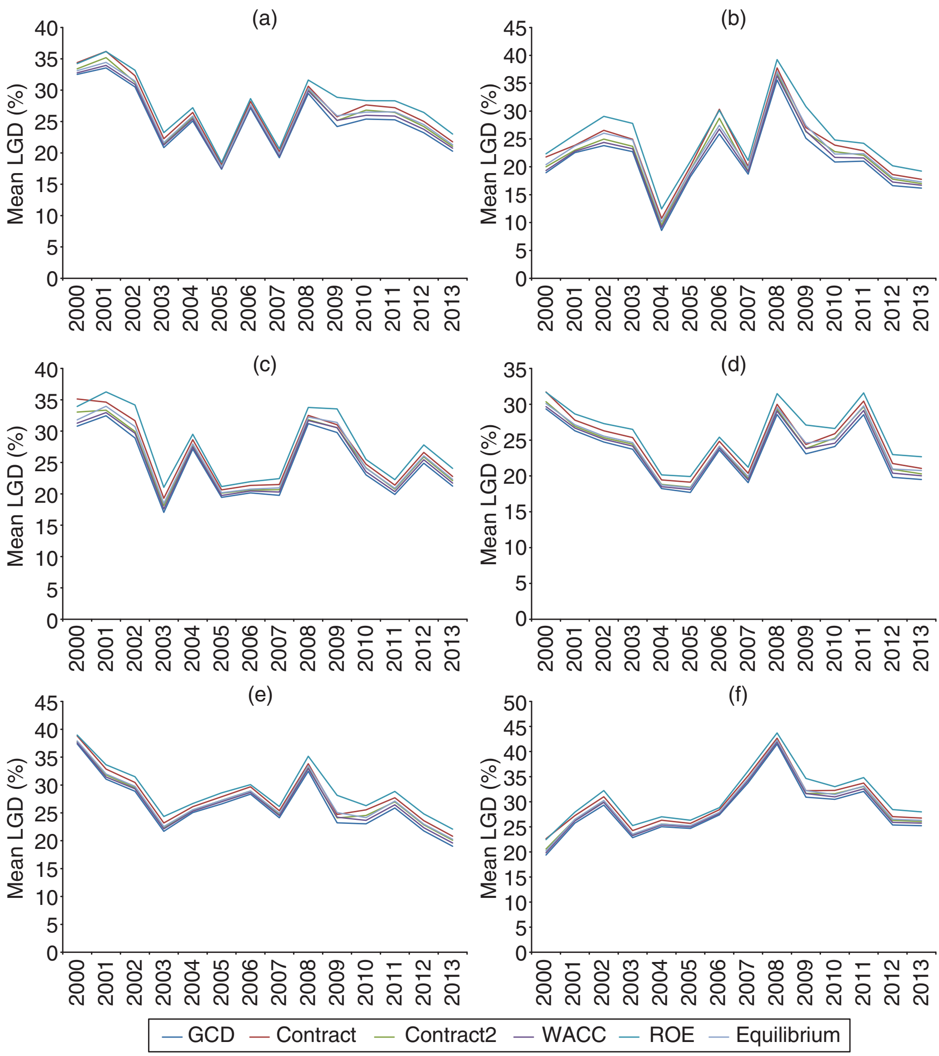
FIGURE 11 Mean LGD with correction for resolution time bias, by geography, euro discount rate.



(a) Great Britain and Ireland. (b) Central Europe. (c) Hispania. (d) North America. (e) Scandinavia. (f) South Africa. (g) Others.

ability and avoidance of negative LGDs. Further, two methodological advancements were made: the WACC approach was extended for loan-level equity ratios and debt ratios, and a correction technique was presented to correct for resolution bias due to

FIGURE 12 Mean LGD with correction for resolution time bias, by industry, euro discount rate.

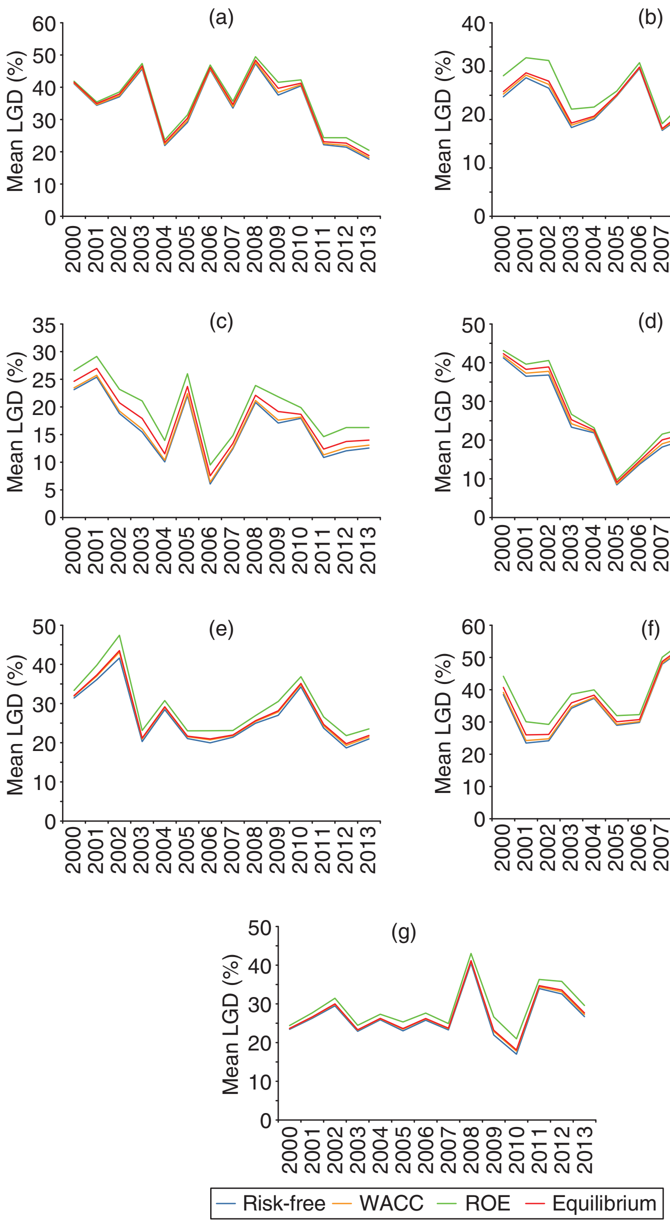


(a) Commerce. (b) Construction. (c) Finance. (d) Manufacturing. (e) Services. (f) Others.

the correlations between LGDs and resolution periods and appropriate censoring of data.

The paper identifies WACC and market equilibrium return as our preferred discount rate concepts, as these are in line with the guiding principles and require

FIGURE 13 Mean LGD with correction for resolution time bias, by geography, local short-term rate.



(a) Great Britain and Ireland. (b) Central Europe. (c) Hispania. (d) North America. (e) Scandinavia. (f) South Africa. (g) Others.

only limited additional effort relative to the readily available contract rate. Other approaches have some key disadvantages: the contract rate is based on the origination time and violates the principle that LGDs should relate to the time of default; the contract rates also include the expected loss; the ROE approach does not measure the systematic risk; bond returns are not available for the general credit risk exposures of commercial banks.

The variation of empirical LGDs is moderate for the various discount rate approaches, as changes in risk-free rates are partly offset by changes in market risk premiums, alongside limited differences in discount rates, resolution periods and, hence, durations.

There is scope for further research. First, the discipline would benefit from a more granular understanding of the systematic risk of recovery cashflows. Systematic risk measures are challenging to compute. In our case, we used geographic and industry clusters. Further research on systematic risk on other dimensions such as unconditional idiosyncratic risk (eg, credit score, loan-to-value bands) is needed. Such measures include asset correlation or variation coefficients (eg, the ratio of the standard deviation of the average LGD to the mean LGD over time). New methodologies may be developed to estimate measures for single borrower or loan exposures.

Second, discount rates are also important for other credit risk applications. For example, discount rates may be required in IFRS 9 and CECL lifetime expected loss modeling. At present, accounting boards require the effective (contract) rate, but future research may scrutinize the assumptions and start a more general discussion beyond existing regulations.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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