Numerical Investigation on Shear Deflection of Steel Welded I Sections with Varying Span to Depth Ratios

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Abstract

Deflection of the steel I-sections is an important phenomenon that needs to be taken into account to ensure that the serviceability limit state criteria of the Australian Standards are met. The method that is widely used to calculate the deflection of steel I-sections is by the use of existing formulae that only accommodate the bending stiffness of the beams. A numerical investigation is performed in this study to find the contribution of shear effects in the final deflection of the Welded-Beams (WB) and Welded-Columns (WC). The numerical analyses were carried out in SAP2000 and numerical model was first validated using the experimental results of welded plate girders. The model was then used to analyse simply supported WB and WC sections under uniformly distributed load (UDL) with varying span lengths. The results of the numerical analyses are reported in this study which compare the mid-span deflection values from the simply supported deflection formula with the numerical model deflection values. The data acquired from the numerical analyses were used to establish a span to depth ratio for WB and WC sections below which the shear deflection becomes significant. The analysis of the results obtained from the numerical investigation suggests that a predication error begins to emerge in the result that is acquired from flexure deflection formulae at a certain span-depth ratio.

Keywords: *Shear Deflection; Span to Depth Ratio; Welded Steel I-Sections; SAP2000; Serviceability Limit State*

1. Introduction

Accurate prediction of the deflection of beams is one of the crucial components of the structural design process. The consideration of shear deflection in calculating the final deflection of short and deep beams is essential to acquire accurate results (Esendemir 2019). The serviceability limit state (SLS) condition regarding deflection shall be satisfied by the allowance of both the flexural and shear deflections. In order to achieve this, the total deflection resulting from bending and shear shall be limited to meet the SLS deflection criteria (Ramezansefat et al. 2016).

The study of composite box girder with corrugated steel webs in South-to-North Water Diversion Bridge in China by Tang, Su and Wan (2013) discovered that with the increase of span to depth ratio of the cantilever, the effect of shear deflection became more obvious. Tang, Su and Wan (2013) made two finite element analysis models one including shear effect while the other did not. The predicted values for the deflection were then compared to the actual deflection during the construction stage. They concluded that the effects of shear deformation are very important and “should be taken into consideration in design, calculation, construction and monitoring.”

The deflection of the deep and short spanned welded beams is predominantly caused by shear effects (Saliba & Gardner 2013). Consequently, it is essential to include the shear stiffness of the I-section and the resulting shear deformations in the numerical analyses. The conventional beam theory i.e. Bernoulli-Euler beam theory neglects the effects of shear forces and the consequent deformations of shear in beams (Kitis et. Al 1994). The deep steel welded beams used in large infrastructures such as bridges and flyovers have relatively smaller span to depth ratio and therefore, it is critical to account for the shear deflection to precisely predict and the overall deflection of the beams when in service. According to Megson (2014) in beams with large depths and relatively short spans, shear deflection becomes significant enough to account for.

The next best alternative to experimentally check the deflections of steel I sections is to conduct non-linear finite element analysis to check the effect of span to depth ratio on the deflection of the simply supported beams (Lin et al. 2019). Experimental studies are relatively rare for the investigation of the steel welded beams and the widely used method of analysis for the study of the behaviour of steel beams is the numerical method. Finite element method is the most acceptable method in the parametric studies of steel beams

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(Markovic & Kovacevic 2019). However, it is extremely important that the results of numerical investigations are verified by comparing them to the experimental results (Saleh et al. 2018; Markovic & Kovacevic 2019).

Moreover, it is important that the finite element method used in the analysis does account for the shear deformation. This means that the stiffness matrices of the elements should be shear deformable. Finite element model that do take into account the shear deformation of the beams include Timoshenko Beam Theory (Stramandinoli & Rovere 2012). Timoshenko Beam Theory is developed to also take into account the effect of shear deformation in the deflection of the beam. In this theory, the cross-sections of the beam elements remain plane however not perpendicular to the neutral axis (Stramandinoli & Rovere 2012; Far et al. 2017). The experimental study conducted by Stramandinoli & Rovere (2012) for the purpose of comparing the deflections acquired from Bernoulli-beam model, Timoshenko-beam model and the experiments confirmed that the Timoshenko-beam model is more accurate in terms of predicting the actual deflections of the beam and very close to that of experimental values. It was also noted that the shear deformation was larger in beams of shorter span (Stramandinoli & Rovere 2012).

In addition to that, the significance of shear deflection can be comprehended by studying a material which has a low shear stiffness relative to its bending stiffness one good example is timber. Material properties and characteristics significantly influence the behaviour and performance of structures (Tabatabaiefar et al. 2012; Fatahi & Tabatabaiefar 2014; Tabatabaiefar & Clifton 2016). The ratio of Young's modulus of timber to its shear modulus ranges from 11 to 16 (USDA 1987). At the same time, that ratio for structural steel is 2.6 (Salmon et. Al 1990). This indicates that the effect of shear in the total deflection of timber beam is more pronounced than in a steel beam (Haydar et al. 2018). In common engineering practice, the calculation of deflections is done through the application of flexural equations derived for bending only. However, these deflection equations give satisfactory results for the span-depth ratio (L/d) of 15 and above. For the timber beams with span to depth ratio (L/d) less than 15 the predicted deflection by the formulae is significantly less than the actual deflection (Bender 1992). This signifies that the shear deflection for the steel beams will also become significant enough below a certain span-depth ratio.

2. Background

2.1 Code Recommendations

Deflection of structural members is one of the major components of serviceability limit state design (Ingkiriwang & Far 2018). Since deflection is a function of many variables including a range of material and geometric properties, therefore, a variety of strategies have been formulated to calculate the exact amount of deflection of a structural member under service loads. The American National Standard of structural steel i.e. ANSI/AISC 360-16 deals with the deflection of structural steel members thoroughly. It identifies the contributing factors in the deflection of steel members which are gravity loads, effects of temperature and construction tolerances and errors. ANSI/AISC 360-16 sets the deflection limits such as the maximum deflection for a steel beam carrying plaster ceilings to be limited to 1/360 of the span of the beam. The code provides the final deflection limits, however; it does not deal with the complexities of the calculation of the deflection. ANSI/AISC 360-16 (p. 478) states that "proper control of deflections is a complex subject requiring careful application of professional judgment". It then refers to AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings* for further discussion.

AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings, 2nd Edition* suggests taking a more sophisticated approach in the calculation of deflections when dealing with tall buildings. It includes the shear deformation of beams and columns along with P-Δ effect among the significant effects to be considered when dealing with tall structures. Yet as a method of addressing these effects, West and Fischer (2003) suggested the use of "most currently available analysis software".

In addition to that, the British Standard for the structural use of steelwork in building i.e. BS 5950.1:2000 deals with deflection of structural members in the serviceability limit state design section. Similar to ANSI/AISC 360-16, BS 5950.1:2000 also set the maximum deflection limits without referring to any specific method of calculating the total deflection in beams. It does, however, mention to consider the worst possible load combination for the calculation of deflections (Clause 2.5.2).

Moreover, the European Standard, Eurocode 3: Design of Steel Structures, briefly touches on the serviceability limit states for buildings. Clause 7.2 mentions vertical and horizontal deflections along with dynamic effects, yet it does not set any deflection limits. It leaves to the designer's and client discretion to agree on deflection limits that is best suited for the project in hand. However, it refers to EN 1990 – Annex A1.4 for the purpose of providing a guide in the load combinations and design values of action whilst specifying the deflection limits. EN 1993-1-5:2006 provides detailed design consideration in terms of shear resistance of plated structural elements. Although it does not provide an explicit procedure for the calculation of shear deformation of beams, it provides a guide for verification of ultimate and serviceability limit states using Finite Element Method of Analysis for plated structures in Annex C of EN 1993-1-5:2006. EN 1993-2: 2006 recommends that the effects of shear deformation be taken into account in the calculation of the camber of beams (Clause 7.6.2). However, it does not provide any guidance on how to account for the shear deformations.

2.2 Analysis

There have been numerous studies conducted in order to formulate a mathematical method to calculate the shear deflection of beams. Strain energy is used to derive an expression to calculate the shear deflection of a beam (Megson 2014). The strain energy due to uniform shear stress, τ is given in Equation (1).

$$U = \frac{\tau^2}{2G} \times V \qquad (1)$$

Where G is the shear modulus of the material and V is the volume of the material. Since the distribution of shear stress is not uniform in the beam cross-section Equation (1) becomes as shown in Equation (2).

$$U = \frac{\beta}{2G} \times \left(\frac{S}{A}\right)^2 \times V \qquad (2)$$

Where β is the constant form-factor, S is the applied shear force and A is the cross-sectional area of the beam. The different geometric parameters used in the calculation of form-factor, β is shown in Figure 1. Equation (3) shows the final relationship of form-factor, β with the other parameters.

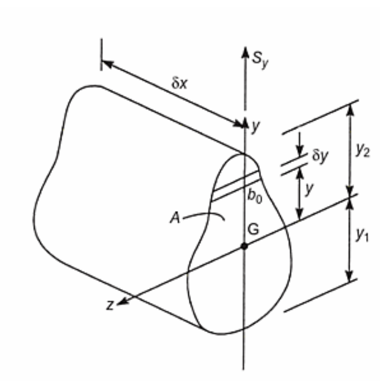


Figure 1 Geometric parameters of an arbitrary beam for the determination of Form-Factor, β (Megson 2014)

$$\beta = \frac{A}{I_Z^2} \int_{y_1}^{y_2} \frac{(A'\bar{y})^2}{b_0}\, dy \qquad (3)$$

The total deflection contributed by the shear force, υ_s can then be equated as shown in Equation (4).

$$\upsilon_s = \frac{\beta}{G} \int_L^o \left(\frac{S_y}{A}\right) dx \qquad (4)$$

Where L is the length subjected to vertical shear force S_y.

The second mathematical method that is used in calculating the shear deformation of the beam is through Timoshenko beam theory. Timoshenko beam theory is capable of taking shear deformation into account. It does not assume that the cross-section of the beam remains perpendicular to the neutral axis during deformation unlike the Euler-Bernoulli Beam Theory (Lignola et. al 2017). Timoshenko beam theory is also based on the principle of minimisation of strain energy. However, the total potential energy equation has two components first considers the flexure deformation and the second accounts for the shear deformation. The two components of the total potential energy, $\prod$, is shown in Equation (5).

$$\prod = \frac{1}{2}\int_0^L EI\left(\frac{d\theta}{dx}\right)^2 dx + \frac{1}{2}GA\int_L^0 \left(\frac{dv}{dx} - \theta\right)^2 dx \qquad (5)$$

Where $\prod$ is total potential energy, E is the Young's modulus, I is the second moment of inertia, θ is the bending rotation, G is the shear modulus and A is the cross-sectional area.

This equation is further refined and used to formulate the stiffness matrix for a beam element which then is used in the Finite Element Method of Analysis. The mathematical relationship between load vector, displacement vector and the stiffness matrix of the element can be seen in Equation (6) (Lignola et. al 2017):

$$\{f\} = [K] \times \{d\} \qquad (6)$$

Where $\{f\}$ is the load vector and $\{d\}$ is the displacement vector and $[K]$ is the stiffness matrix of the beam element. The stiffness matrix of a beam element in Timoshenko beam theory is formulated as shown by Equation (7) below:

$$[K] = \frac{EI}{L^3(1+a)}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & (4+a)L^2 & -6L & (2-a)L^2 \\ -12 & -6L & 12 & -6L \\ 6L & (2-a)L^2 & -6L & (4+a)L^2 \end{bmatrix} \qquad (7)$$

Where $a = \frac{12EI}{GAL^2}$

Due to the fact that the stiffness matrix of the beam element takes into account the shear stiffness of the beam i.e. *GA*, therefore, it enables this theory to take into account the shear deformation of the beams (Lignola et. al 2017).

3. Methodology

A comprehensive numerical analysis was conducted in this study using the non-linear finite element analysis software, SAP2000 (Computers and Structures 2018) to analyse the effect of span to depth ratio of the welded steel I-beams on its mid-span deflections. SAP2000 was chosen because of its sophisticated finite element analysis capabilities and its user friendly interface. It also has the Australian Steel Standard (AS 4100) and the Australian steel grade loaded into it. The beams were modelled using the cross-sectional dimensions of welded-beams given in Table 9 of Hot Rolled and Structural Steel Products (2018) and the material properties were also based on the same table. The beams were modelled using shell elements with uniform thickness in bending and axial deformation. Shell element has been chosen for its suitability for structural elements which have negligible third dimension as compared to the other two dimensions. Shell mesh is much easier to create with more realistic boundary conditions. Shell elements also enable a fast post-processing (Computers and Structures 2018). Each beam was modelled with the same material properties and constant cross-sectional dimensions but varying span lengths. The span lengths for each section ranged from 3m to 20m with the interval of 2m from 6m to 20m. However, below 6m every consequent meter was modelled for better clarity of the results when the span to depth ratio is significantly small. All the beams were modelled as simply supported beams and uniformly distributed load (UDL) was applied through the span of the beam. The UDL was applied to the top flange of the beam. The UDL was applied such that all the beams behaved elastically. The intensity of the load was kept constant for all the different span lengths. This was to ensure that all the parameters remained constant except for the span length of the beam.

4. The Sections Used in the Analyses

The cross-sections used in the analyses are all from Table 9 of Hot Rolled and Structural Steel Products (2018). The investigation was mainly focused on the welded-beam (WB) and welded-column (WC) sections. The dimensions of the sections and material properties used in the analyses are listed in Table 1. The cross-sectional parameters are labelled as shown in Figure 2.

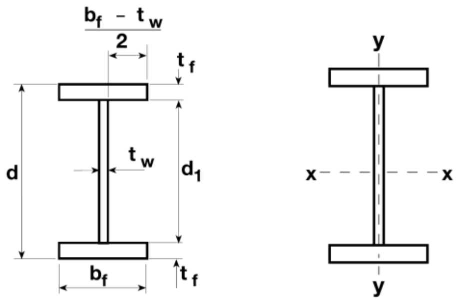


Figure 2 Cross-section parameter labels

		Flange				Yield Stress		
Designation	**Depth of Section, *d* (mm)**	**Width, b_f**	**Thickness, t_f**	**Web Thickness, t_w**	**Depth Between Flanges, d_1**	**Flange f_y (MPa)**	**Web f_y (MPa)**	**I_x (10^6) mm^4**
1200 WB 455	1200	500	40	16	1120	280	300	15300
1000 WB 322	1024	400	32	16	960	280	300	7480
900 WB 282	924	400	32	16	860	280	310	5730
800 WB 192	816	300	28	10	760	280	310	2970
700 WB 173	716	275	28	10	660	280	310	2060
500 WC 440	480	500	40	40	400	280	280	2150
400 WC 361	430	400	40	40	350	280	280	1360
350 WC 280	355	350	40	28	275	280	280	747

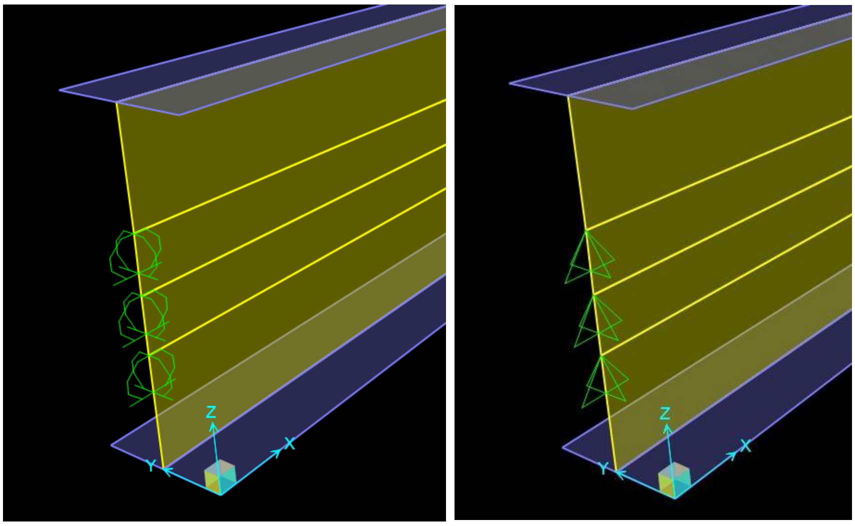
Table 1 Cross-sectional dimensions and material properties of the analysed WB and WC sections

5. Numerical Modelling Assumptions

The entire numerical modelling was performed using the finite element software, SAP2000 (Computers and Structures 2018). The following assumptions were made during the numerical analyses.

- Typical shell elements were used in the analysis.
- The WB sections were meshed using 'Automatic Area Mesh' with the size of the mesh being 50mm. This meshing size optimised the quality of the numerical analysis and the time taken for each run of the analysis.

- As illustrated in Figure 2, the simple supports were modelled with three-pin supports in the web on one end and three-roller supports on the other end. This setup reduced the local stress concentrations at the supports and simulated the real-life shear connections.
- All the intersection planes of the beams were divided into segmental areas to enhance the quality of the results. This alleviated the WB section from artificial rigidity and allowed the deflection of the beam to be as close to the actual behaviour of the beam in real life.
- The UDL was applied as uniform shell stress at the top flange such that the beam behaved elastically. This was adopted to simulate the service loads which the beams take during their design life.
- The non-linear analysis was conducted to ensure final deflections are as close as possible to the actual experimental results.
- Any manufacturing imperfections and inconsistency in cross-sectional dimensions are deemed to be negligible and hence not accommodated in the numerical modelling.



(a) (b)

Figure 3 End support configuration of the simply-supported WB & WC beams, a) Roller Support; b) Pin Support

6. Material Properties

The grade of the steel used in this analysis is hot-rolled structural steel of 300PLUS-300 grade. The minimum requirements of AS/NZS 3679.2-300 are met by the 300PLUS welded sections. The yield stress for the flange is 280 MPa whilst the yield stress for the web is 300 MPa. The elastic modulus, E is equal to 200,000 MPa with the shear modulus being equal to 82,000 MPa. The 300PLUS-300 grade has 7849 kg/m^3 mass density. It should be noted that the shear modulus of this grade of steel is approximately two and a half times less than its elastic modulus. This makes the sections less stiff in resisting shear deformations as compared to the deformations caused by bending.

7. Validation of Numerical Model

The numerical model used in this study to analyse the relationship of the span length and mid-span deflection is validated by experimental results. Saliba & Gardener (2012), have performed a detailed numerical and experimental study of the shear response of welded plate girders. Saliba & Gardener (2012), experimentally tested nine welded plate girders. They studied the shear response of the beams under point loading including the mid-span deflections with even load increments. The numerical model used in this study is validated using one of the specimens used by Saliba & Gardener (2012), i.e. I-600 × 200 × 12 × 8-2 (section depth × flange width × flange thickness × web thickness × web panel aspect ratio) with the span length of the specimen being 2560mm. The span to depth ratio of the beam is 4.3 which makes it a deep beam. This is an ideal case for the validation of the numerical model because it tests the ability and accuracy of the numerical model to calculate the contribution of shear deformation of the beam to its overall deflection. This also aligns with the aim of this research which is to study the shear deflection of steel I-welded sections with respect to varying span to depth ratio.

The specimen was modelled in SAP2000 with the same finite element parameters used for the numerical analyses in this study. The same shell elements, non-linear analysis and mesh size were used to simulate the experimental results. The mid-span deflections were noted for each load intensity and they were compared with deflection values obtained from the experiment. In order to avoid the material non-linearity effect of lean duplex stainless steel, the validation is conducted using the load cases under which the beam behaves elastically. It was ensured that the constitutive relationship of the material is linear for the load cases used for validation. The comparison of the results of the numerical model and the experimental values is depicted in Figure 4.

After the comparison of the results obtained from the numerical analysis and from the lab experiment it has become apparent that the numerical model developed in SAP2000 for this study is valid and consistent with those noted during the experiment by Saliba & Gardener (2012). Consequently, the predictions of the numerical model can be deemed accurate and the results of this study shall be able to provide a reasonable guide in studying the shear deflection of Welded Beam (WB) and Welded Column (WC) sections.

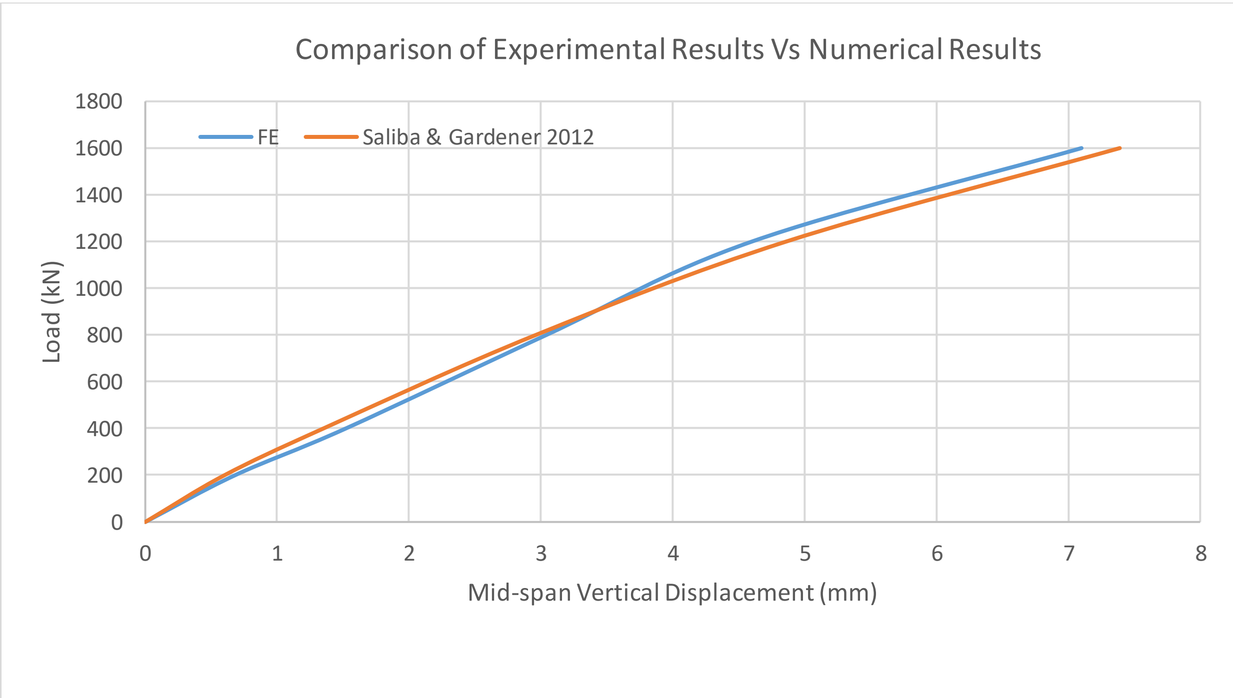


Figure 4 Comparison of the mid-span vertical deflection acquired from the numerical analysis used in this study with the experimental results reported by Saliba & Gardener (2012)

8. Numerical Study

8.1 Analysis

The numerical analyses have been conducted on the WB and WC sections that are all from Hot Rolled and Structural Steel Products. The sections have been analysed with all of the parameters constant except for the span length. It was noted that when the load was kept constant with varying span length, the mid-span vertical deflections were too small for the span-depth ratio below 10. Consequently, the applied load was increased five times to achieve noticeable mid-span deflections for the sections with span-depth ratio 10 and below.

The mid-span vertical deflections were then calculated for each section and the span length using below simply supported beam deflection formula

$$\delta_{max} = \frac{5wL^4}{384EI} \tag{8}$$

The deflection values were then compared with that acquired from the numerical model. The comparison of the deflection values revealed the difference between the two models. The results were then graphed for better visualisation. Figure 5 illustrates contour of vertical deflection of a WB simply supported beam analyised in this study.

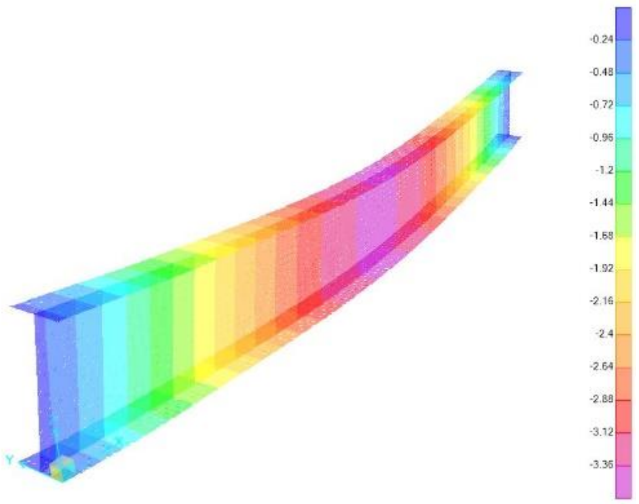


Figure 5 Contour of vertical deflection of a WB simply supported beam

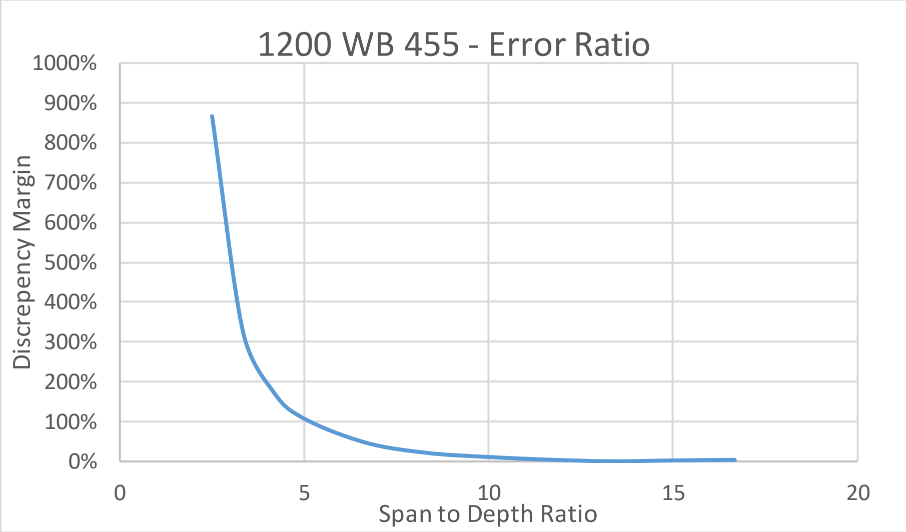
The comparison of the deflection values acquired through FE analysis and the values of the simply supported deflection formula is conducted through the calculation of **error-ratio**. The formula used for the error ratio in this study is as follows

$$Formula - Deflection = x \tag{9}$$

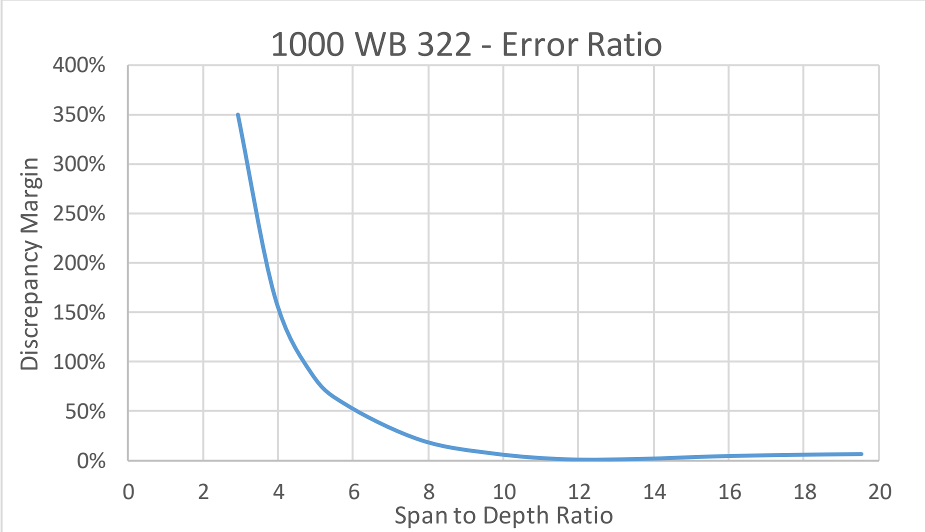
$$FE - Deflection = y \tag{10}$$

$$Error - Ratio\ \% = \left|\frac{x-y}{x}\right| \times 100 \tag{11}$$

The error-ratio is represented in the form of percentage for a better comprehension of the difference between the values acquired from two different methods. The error-ratios are then plotted against the span to depth ratio of each WB and WC section (Figure 6 - 8 and Figure 9). The plots give an elaborate visual representation of the effect of the reducing span to depth ratio and the manifestation of the shear-deflection which consequently causes error-ratio to increase. This method enables to pinpoint the range of the span to depth below which the shear-deflection becomes significant enough to account for. The results of the numerical analysis for each WB section (FE deflections) are tabulated in Appendix A along with formula deflection values (Equation 8) and beam section properties. In addition, plotted results of comparison between numercial deflections and claculted deflections from Equation 8 were illustrated in Appendix B, Figure 12-16 for WB sections and Figure 17-19 for WC sections.

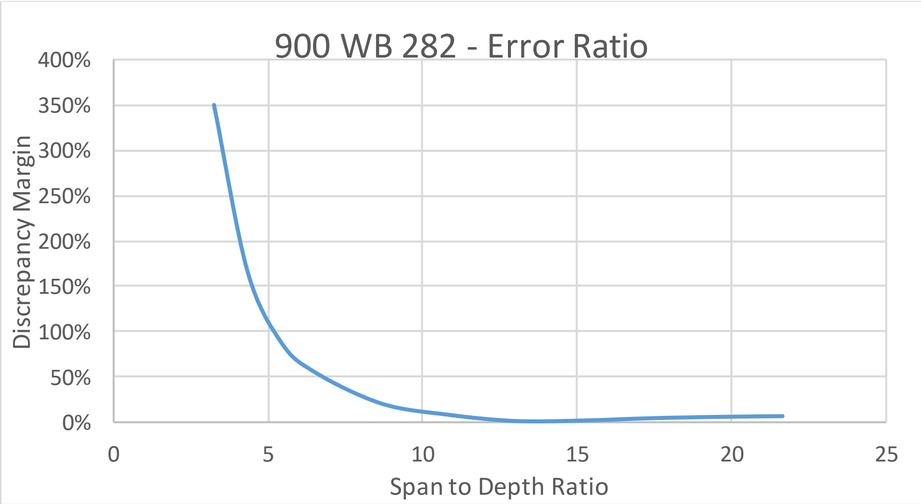


(c)

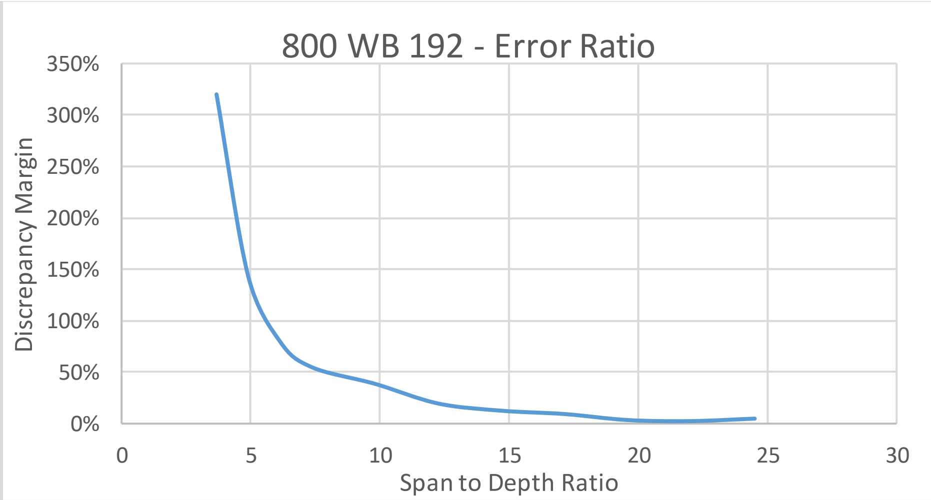


(d)

Figure 6 The Discrepancy of Deflection Values in Percentage Between FE and Formula with Reducing Span to Depth Ratio (c) 1200WB455 (d) 1000WB322



(a)



(b)

Figure 7 The Discrepancy of Deflection Values in Percentage Between FE and Formula with Reducing Span to Depth Ratio (a) 900WB282 (b) 800WB192

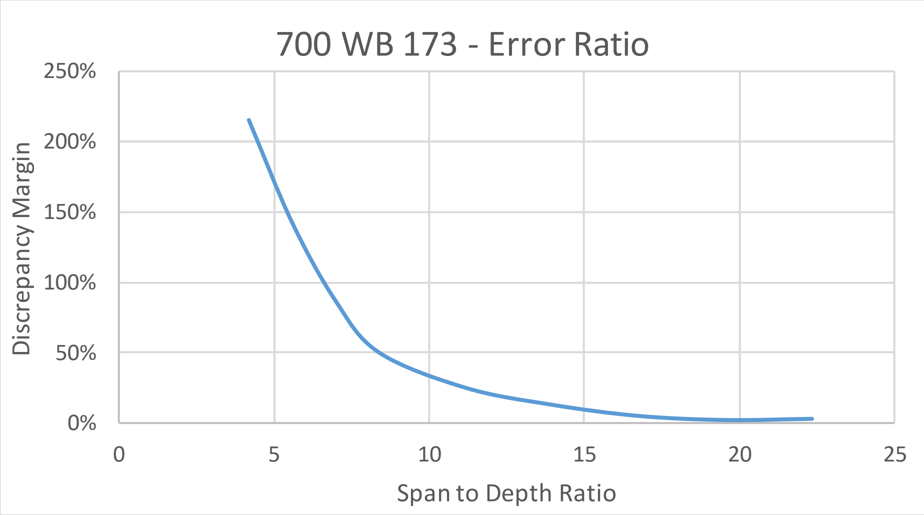


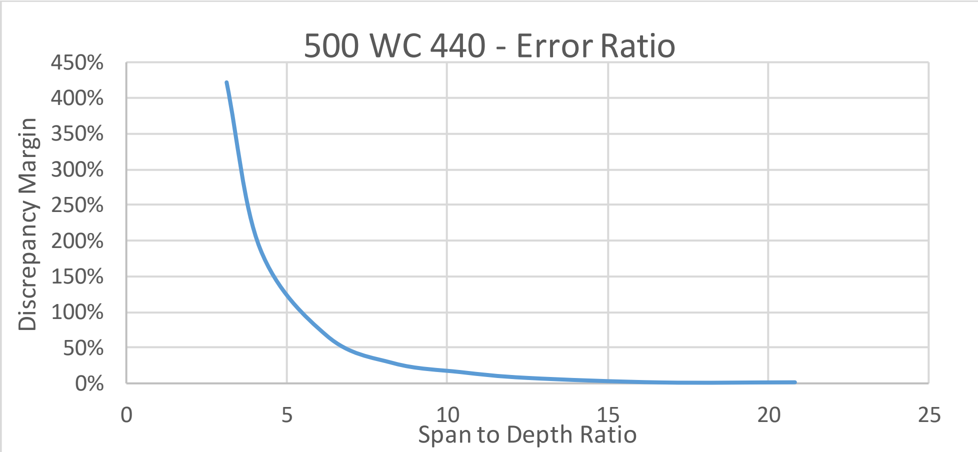
Figure 8 The Discrepancy of Deflection Values in Percentage Between FE and Formula with Reducing Span to Depth Ratio – 700WB173

Similarly, welded-column sections were analysed with varying span to depth ratio. Since welded-column sections have relatively smaller depth as compared to welded-beam sections, therefore, the span length for the analysis was adjusted to get a similar span to depth ratio to WB sections. For the sections 400WC361 and 350WC280, the loading was kept constant for all the span length during analysis. This change was made due to the relatively short span length. The results of the numerical analysis are presented in Table 2.

Designation	Depth of Section (mm)	Width (mm)	Thickness (mm)	Web Thickness (mm)	I (10^6mm^4)	Load (kN/m)	Span Length (m)	Span to Depth Ratio	FE - Deflection (mm)	Formula - Deflection (mm)
500 WC 440	480	500	40	40	2150	50	10	21	14.91	15.14
							8	17	6.28	6.20
							6	13	2.11	1.96
							5	10	1.10	0.95
						250	4	8	2.50	1.95
							3	6	1.00	0.60
							2	4	0.35	0.12
							1.5	3	0.20	0.04
400 WC 361	430	400	40	40	1360	50	10	23	23.06	23.94
							8	19	9.64	9.80
							6	14	3.19	3.10
							5	12	1.61	1.50
							4	9	0.72	0.61
							3	7	0.27	0.19
							2	5	0.08	0.04
							1.5	3	0.04	0.01
350 WC 280	355	350	40	28	747	50	10	28	42.70	43.58
							8	23	17.80	17.85
							6	17	5.86	5.65
							5	14	2.94	2.72

							4	11	1.30	1.12
							3	8	0.50	0.35
							2	6	0.14	0.070
							1.5	4	0.070	0.022

Table 2 Results of numerical analysis of Welded-Column sections



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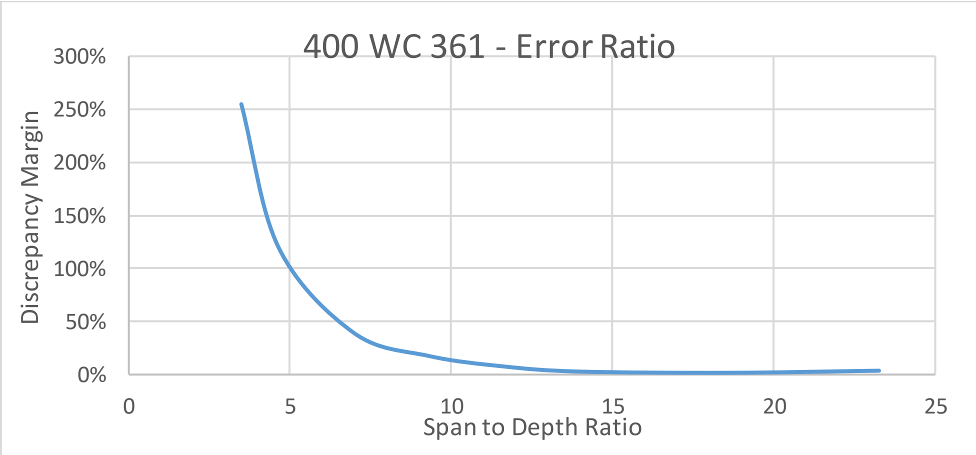

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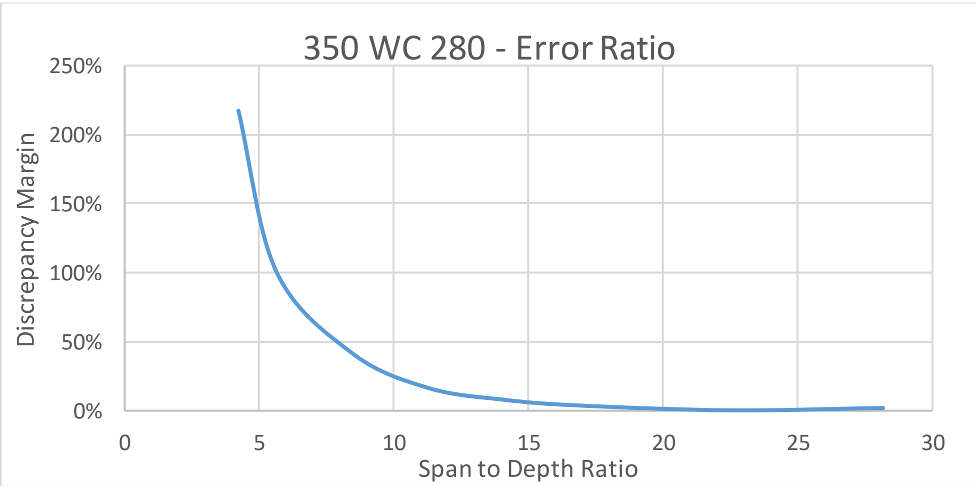

(e)

Figure 9 The Discrepancy of Deflection Values in Percentage Between FE and Formula with Reducing Span to Depth Ratio (c) 500WC440 (d) 400WC361 (e) 350WC280

8.2 Comparison and Analysis of WB and WC Error Ratios

The WB sections analysed in this numerical study follow a similar trend in regards to the difference in deflection values obtained from FE analysis and deflection formula of simply supported beams. The difference between the two deflection values at mid-span is quite small for all the WB sections when the span-depth ratio is well above 10. However, the deflection values resulted from numerical analyses diverge at an exponential rate for the span to depth ratios below 10. Figure 10 below compares the error-ratios of 1200WB455, 1000WB322, 900WB282, 800WB182 and 700WB173. The comparison of the error ratios in Figure 10 shows that all the WB sections follow a similar trend. The discrepancy between the FE deflection values and that of the formula begins to get pronounced at span to depth ratio of 8 where the discrepancy is approximately 50%.

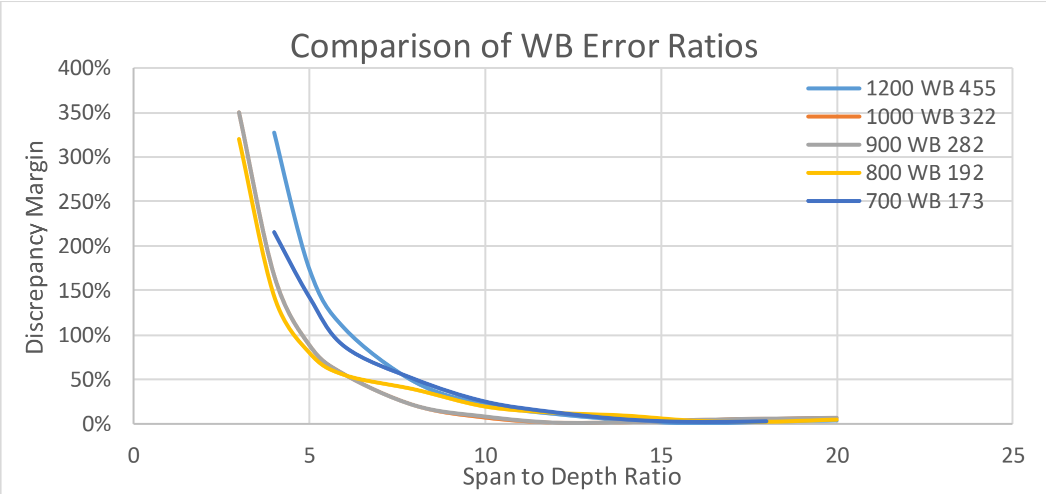

Figure 10 Comparison of the Error Ratios of All the WB Sections Analysed

Similarly, the WC sections analysed in this study show a comparable behaviour to the WB sections. The WC sections follow the same pattern as the WB section in terms of the increasing discrepancy between FE and formula deflection values. The error ratios of the 500WC440, 400WC361, 350WC280 were compiled to compare in Figure 11. Interestingly, the span to depth ratio at which the discrepancy gets to 50% is the same as WB sections i.e. 8.

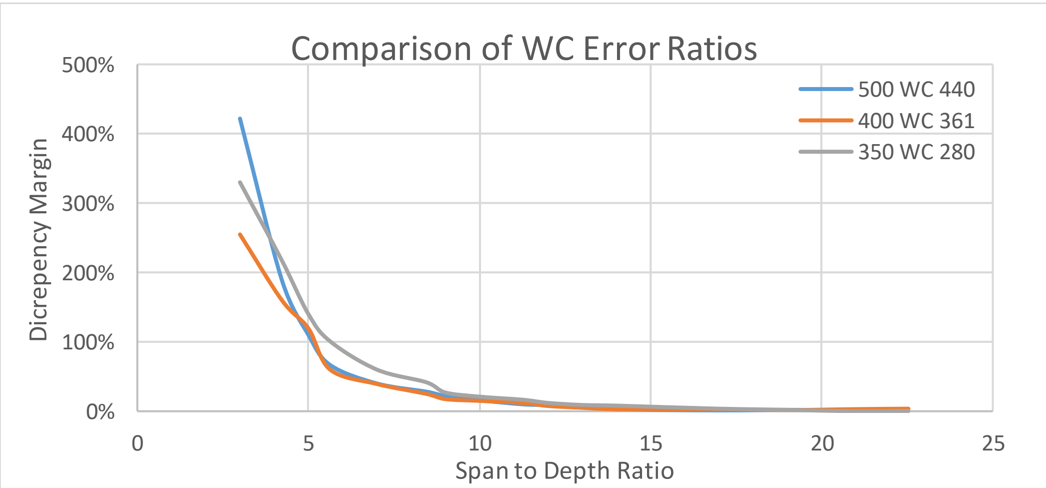

Figure 11 Comparison of the Error Ratios of All the WB Sections Analysed

The mid-span deflection values for the all three WC sections acquired from FE analysis vary very subtly from the formula values for the span to depth ratio of 10 and above. However, a noticeable split begins to emerge between the deflection values when the span to depth ratio becomes less than 10. This trend is in line with the trend depicted by the WB sections. This behaviour in the deflection of deep and short beams is expected because of the prominence of shear deformation. The deflection formula (Equation 8) is devised based on the assumption that the cross-section of the beam remains perpendicular to the neutral axis whilst the beam deflects. The conventional beam theory i.e. Bernoulli-Euler beam theory neglects the effects of shear forces and the consequent shear deformations in beams. The deflection formula based on Bernoulli-Euler beam theory only considers the bending stresses caused by the shear loads (Schramm et. al 1994). However, at low span to depth ratio the cross-section of the beam is no longer perpendicular to the neutral axis which results in a discrepancy between actual deflection values and the one predicted by the formula.

9. Discussion

The mid-span deflection values acquired from FE analysis when compared to the values derived from the simply supported beam deflection formula are reasonably synonymous for the span to depth ratio ranging between 10 - 20. However, when the span to depth ratio gets below 10, a split between the results emerges. The difference between the results is the function of the span to depth ratio because all the other parameters were kept constant. This divergence is expected because of the significance of shear force that becomes pronounced when welded I-sections become deeper relative to their span length. This effect is rightly supported by the Timoshenko beam theory when the cross-section of the beam does not remain perpendicular to the neutral axis after the deflection (Karttunen et. Al 2016). The smaller the span to depth ratio of the welded beams is the more prominent this effect becomes. Consequently, it manifests itself in the deflection values of the welded I-sections under loading.

As shown in Equation (7), the stiffness matrix of Timoshenko beam element accommodates the shear stiffness which in turn allows for the rotation between the cross-section and the bending line. This effect becomes more pronounced in the welded steel beams when the span to depth ratio of the beam is less than 10. Due to the fact that the cross-section of the beam does not remain perpendicular to the neutral axis in deep beams, it is important to account for the shear deformation of the beam in order to calculate the deflections accurately. The Timoshenko beam element takes into account this effect and hence provides more accurate results.

The discrepancies between the deflection values of FE analysis and the mid-span deflection formula for simply-supported beams under UDL are visualised in Figure 10. The error ratios of 1200WB455, 1000WB322, 900WB282, 800WB182 and 700WB173 show a similar trend. The shear deformation is negligible when the span length is large compared to the depth of the beam. However, as the span length ratio reduces, shear deformation increases such that below span to depth ratio of 8 it starts to increase exponentially. It is evident from the plots that the inconsistency becomes too significant to ignore when the span to depth ratio gets less than 8. The difference in the results from that point onwards continues to diverge.

The comparison of the deflection values of all the WC sections (Figure 11) reinforces the deductions from the deflection results of WB sections. The mid-span deflection values of higher span to depth ratio i.e. 10 and above are in agreement with the deflection values predicted by the formula. Shear deformation is a function of many variables such as shear modulus, form factor and shear area (Megson 2014). It is important to note the difference between shear area and gross cross-sectional area. Shear area is the effective area of the section contributing to shear deformation (Iyer 2015). It is always less than the gross cross-sectional area. Additionally, the shear stress distribution in beams with small span to depth ratio is differs to that of beams with large span to depth ratio. Consequently, the differing shear stress distribution in different beams result in varying shear area of the section. This overall has an effect on the amount of shear deformation of the beam.

Moreover, the comparison of 700WB173 to 500WC440 deflections reinforces the abovementioned effect of shear modulus in the deflection of the beam. The second moment of inertia (I_x) of 700WB173 and 500WC440 are 2060×10^6 mm^4 and 2150×10^6 mm^4, respectively. This is relatively close and a reasonable comparison can be made between the two sections. The Young's Modulus of both sections is the same because they are of the same grade of steel. The mid-span deflection values of 700WB173 and 500WC440 predicted by the formula with the UDL of 50 kN/m and span length of 8m are 6.47mm and 6.20mm. The difference between the deflection values is only 0.27mm which can be accounted for by the slight difference in I_x values. However, the deflection value acquired from numerical analysis with the same parameters is 8.13mm for 700WB173 and 6.28mm for 500WC440. The difference is approximately 2mm which is almost seven times larger than 0.27mm. This variance is too large to be associated to the change in I_x values. In order to explain this difference, another important cross-sectional property needs to be considered i.e. cross-section area (A_g). The gross cross-section area of 700WB173 is 22000 mm^2 whilst that of 500WC440 is 56000mm^2 which is more than double of the former. The direct effect of cross-sectional area is not captured in Equation 8 but it is accounted for in the numerical analysis. Shear stiffness is the product of shear modulus (G) and the cross-sectional area (A). Therefore, due to this fact that 500WC440 is stiffer overall than 700WB173 and thus the latter deflects 2mm more than the former.

10. Conclusions and Recommendations

A series of numerical analyses have been conducted in this study to investigate the significance of shear deflection in steel welded I-sections with varying span to depth ratio. The welded-beam sections analysed in this study were 1200WB455, 1000WB322, 900WB282, 800WB182 and 700WB173 and the welded-column sections analysed were 500WC440, 400WC361 and 350WC280. The numerical analysis was performed using SAP2000 (Computers and Structures 2018). The mid-span deflection values acquired from the numerical analysis were compared to the predicated deflection values by the simply supported deflection formula (Equation 8). The WB sections analysed in this numerical study follow a similar trend in regards to the difference in deflection values obtained from FE analysis and deflection formula of simply supported beams. The error-ratio, the deflection values of numerical analysis and the formula were graphed for all the analysed sections. The comparison between the theoretical and numerical results revealed that the difference between the two deflection values at mid-span is quite small for all the WB sections when the span-depth ratio is well above 10. However, the deflection values resulted from numerical analyses diverge at an exponential rate for the span to depth ratios below 10. At the span to depth ratio of 8 and below, the comparison of the values showed that the significance of shear deformation increases exponentially. As a result, it has been concluded that considering shear deformation in the calculation of deflection of still welded steel sections with span to depth ratio of 8 and below is necessary to ensure the structural design safety and integrity.

It is highly recommended to practicing engineers and engineering consultancies to not solely rely on the simply supported deflection formula when calculating the maximum deflection of simply supported WB and WC beams when the span to depth ratio is less than 8 because it takes in to account the bending stiffness of the beam only. Neglecting the shear deformation in the calculation of deflection of the beam which has small span to depth ratio will lead to unrealistic predictions and consequently unpredicted problems after construction. The beams with relatively small span to depth ratio are used in railway bridges and flyovers. Since deep beams are integral part of important infrastructure, therefore, the margin for error is very small. To ensure that the deflection serviceability criteria of AS4100 (1998) are met, a method of analysis that incorporates the shear deflection shall be implemented in the calculation of maximum deflection of beams with span to depth ratio of 8 and below.

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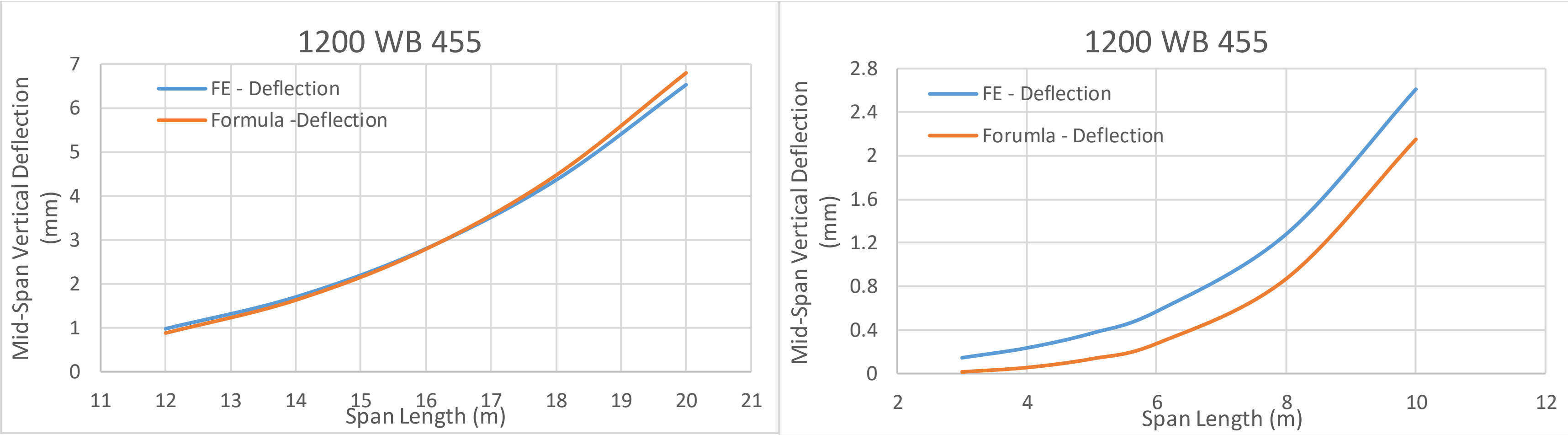
Appendix A - Results of numerical analysis of Welded-Beam sections

Designation	Depth of Section (mm)	Width (mm)	Thickness (mm)	Web Thickness (mm)	I ($10^6 mm^4$)	Load (kN/m)	Span Length (m)	Span to Depth Ratio	FE - Deflection (mm)	Formula - Deflection (mm)
1200 WB 455	1200	500	40	16	15300	10	20	17	6.53	6.8
							18	15	4.36	4.47
							16	13	2.8	2.79
							14	12	1.7	1.63
							12	10	0.978	0.88
						50	10	8	2.61	2.15
							8	7	1.28	0.87
							6	5	0.57	0.275
							5	4	0.37	0.135
							4	3	0.235	0.055
							3	3	0.145	0.015
1000 WB 322	1024	400	32	16	7480	10	20	20	12.99	13.93
							18	18	8.60	9.14
							16	16	5.45	5.70
							14	14	3.28	3.34
							12	12	1.83	1.80
						50	10	10	4.68	4.38
							8	8	2.15	1.78
							6	6	0.88	0.56
							5	5	0.52	0.28
							4	4	0.30	0.13
							3	3	0.17	0.038
900 WB 282	924	400	32	12	5730	10	20	22	10.39	11.14
							18	19	6.88	7.31
							16	17	4.36	4.56
							14	15	2.62	2.67
							12	13	1.46	1.44
						50	10	11	3.79	3.5
							8	9	1.72	1.43
							6	6	0.7	0.45
							5	5	0.42	0.22
							4	4	0.24	0.09
							3	3	0.14	0.03
	816	300	28	10	2970	10	20	25	36.80	35.10

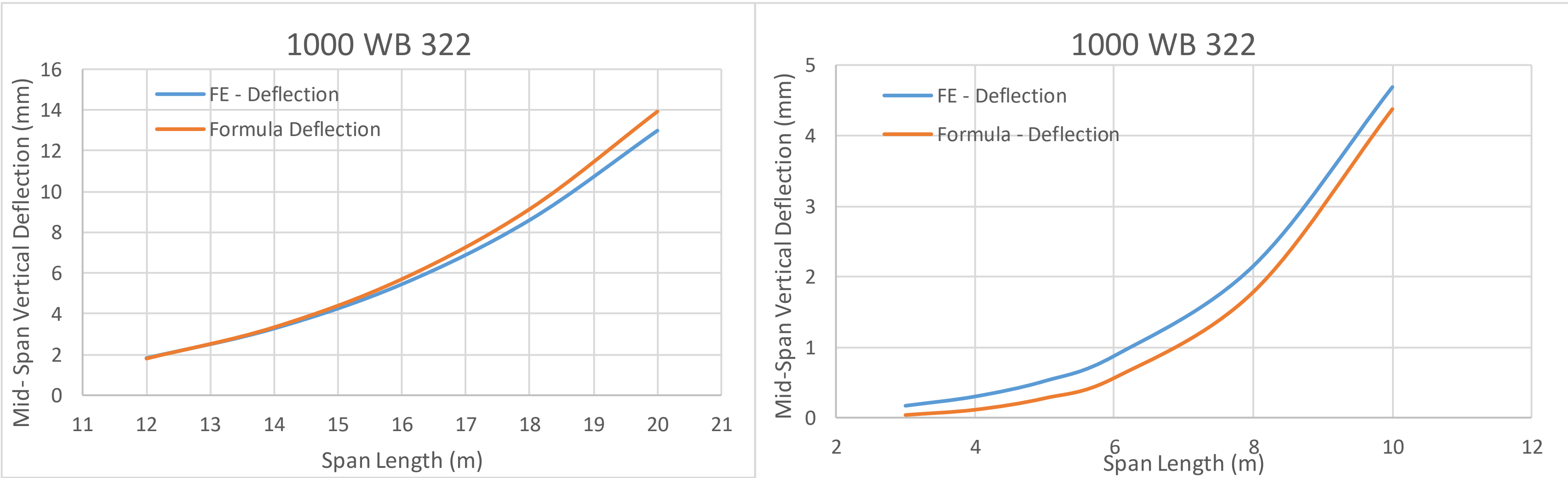
800 WB 192							18	22	22.45	23.01
							16	20	14.86	14.37
							14	17	9.20	8.42
							12	15	5.12	4.55
						50	10	12	13.10	10.96
							8	10	6.24	4.50
							6	7	2.2	1.42
							5	6	1.26	0.70
							4	5	0.73	0.30
							3	4	0.42	0.10
700 WB 173	716	275	28	10	2060	10	16	22	21.33	20.71
							14	20	12.40	12.14
							12	17	6.88	6.55
						50	10	14	17.85	15.80
							8	11	8.13	6.50
							6	8	3.02	2.01
							5	7	1.87	1.00
							4	6	0.97	0.40
							3	4	0.41	0.13

Appendix B - Comparison of FE and Formula Deflection Values



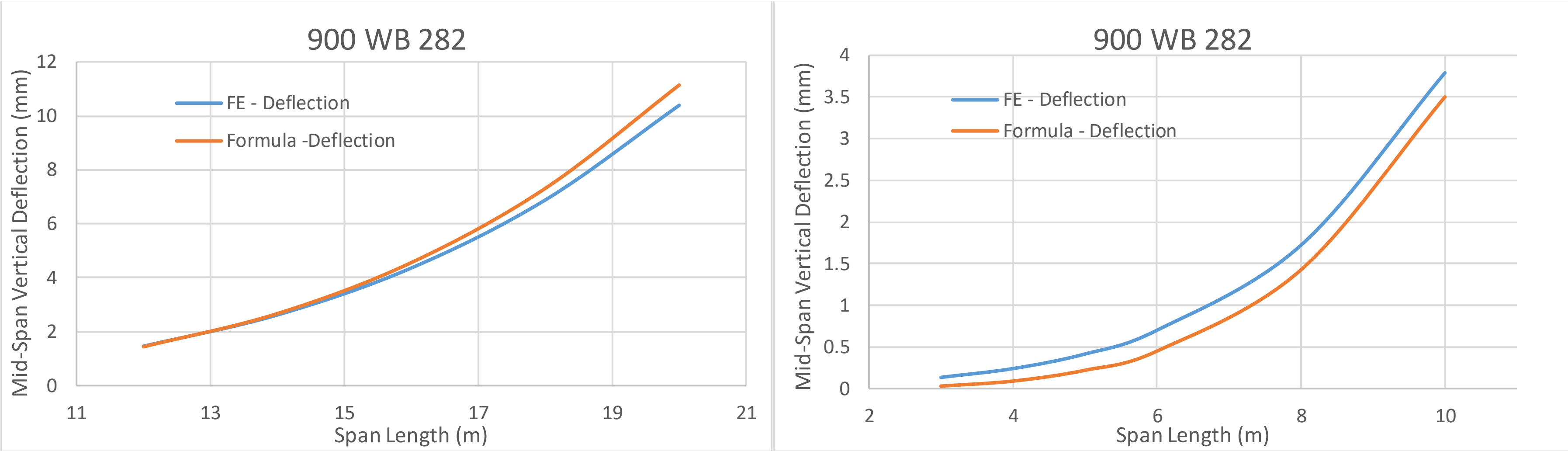
(a) *(b)*

Figure 12 Comparison of FE and Formula Deflection Values for (a)Span Length 12-20m (b)Span Length 3-10m - 1200WB455



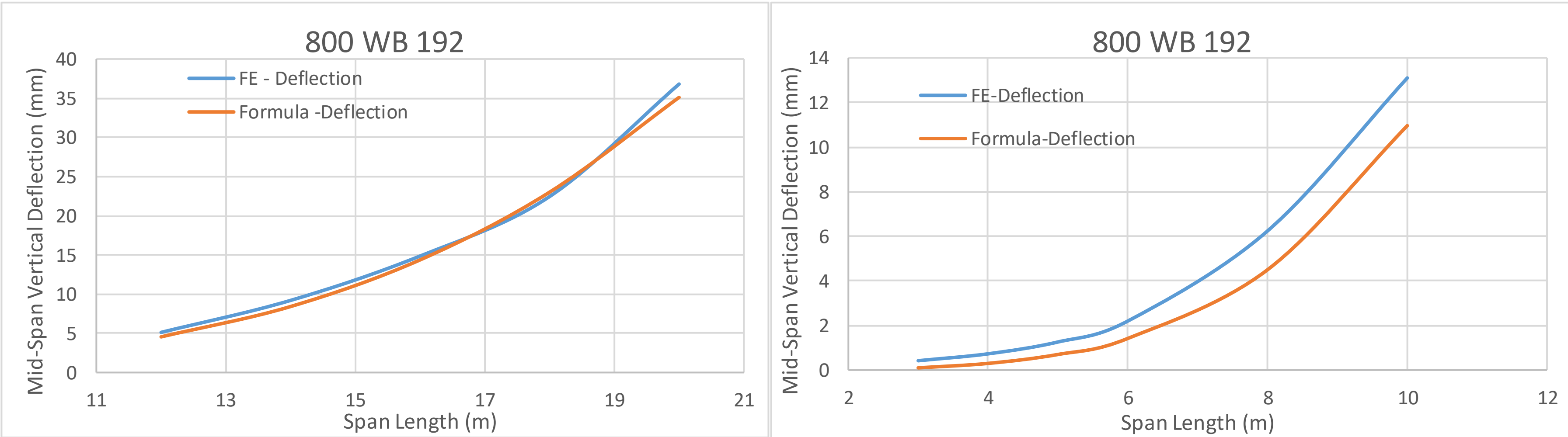
(c) *(d)*

Figure 13 Comparison of FE and Formula Deflection Values for (c)Span Length 12-20m (d)Span Length 3-10m - 1000WB322



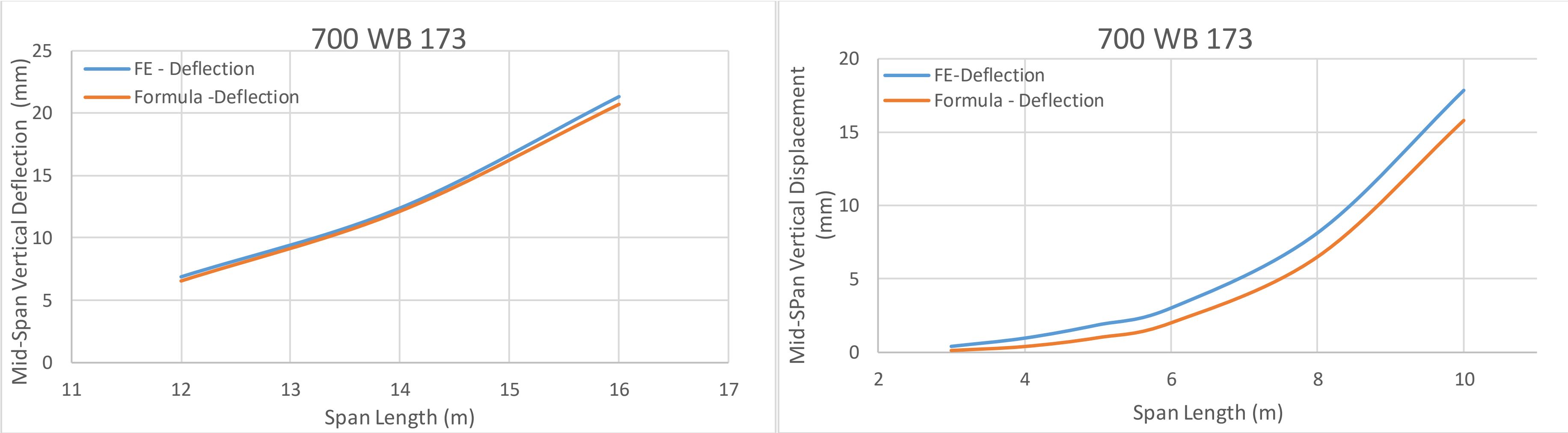
(a) (b)

Figure 14 Comparison of FE and Formula Deflection Values for (a)Span Length 12-20m (b)Span Length 3-10m – 900WB282



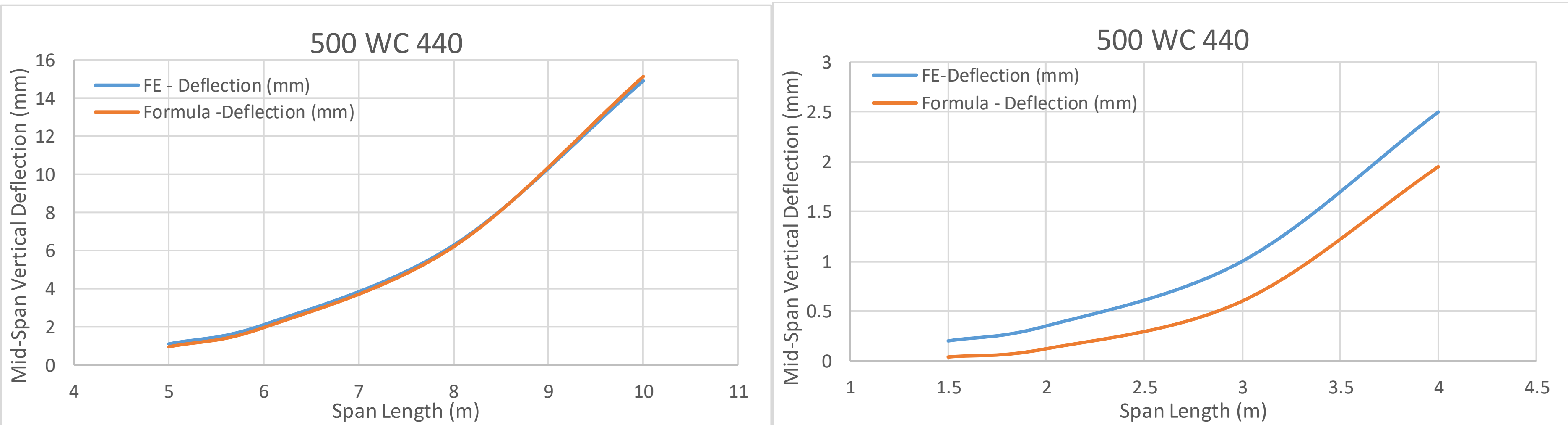
(c) *(d)*

Figure 15 Comparison of FE and Formula Deflection Values for (c)Span Length 12-20m (d)Span Length 3-10m - 800WB192



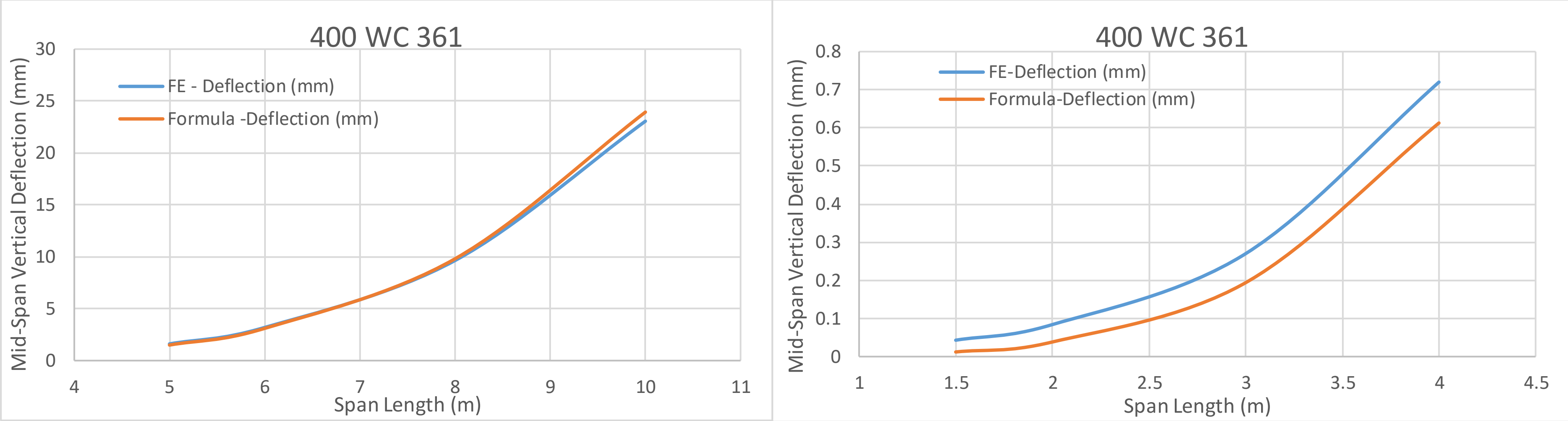
(a) *(b)*

Figure 16 Comparison of FE and Formula Deflection Values for (c)Span Length 12-20m (d)Span Length 3-10m - 700WB173



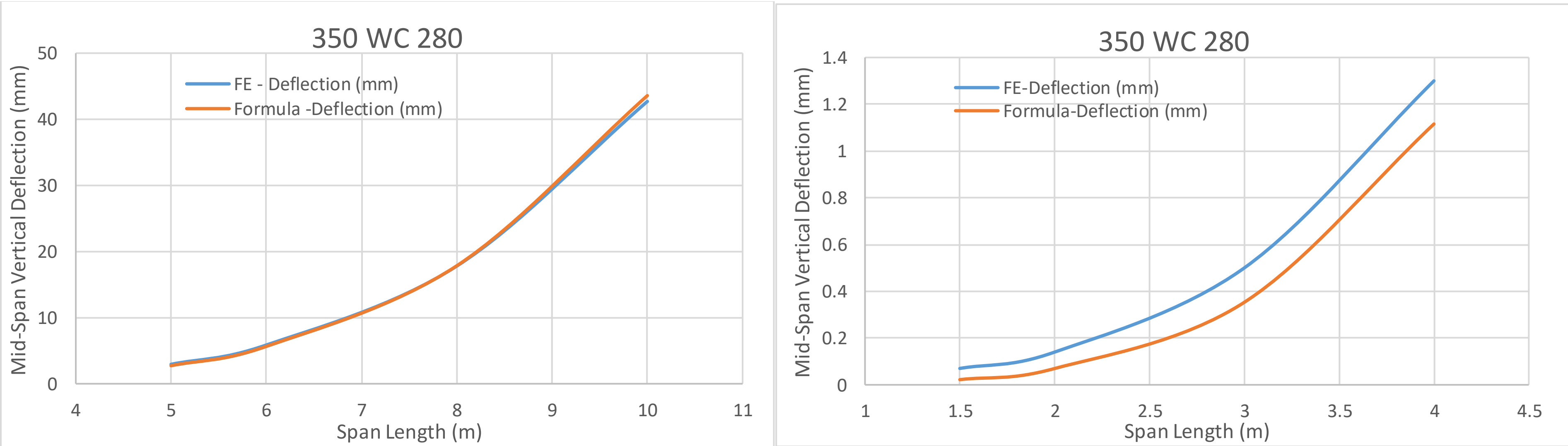
(a) *(b)*

Figure 17 Comparison of FE and Formula Deflection Values for (a)Span Length 5-10m (b)Span Length 1.5-4m – 500WC440



(c) *(d)*

Figure 18 Comparison of FE and Formula Deflection Values for (c)Span Length 5-10m (d)Span Length 1.5-4m – 400WC361



(a) *(b)*

Figure 19 Comparison of FE and Formula Deflection Values for (a)Span Length 5-10m (b)Span Length 1.5-4m – 350WC280