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Exponential Synchronization of Switched Neural Networks With Mixed Time-Varying Delays via Static/Dynamic Event-Triggering Rules

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ABSTRACT This paper is devoted to the exponential synchronization of switched neural networks with mixed time-varying delays via static/dynamic event-based rules. At first, by introducing an indicator function, the switched neural networks are transformed into neural networks with general form. Then, sufficient conditions are deduced to achieve exponential synchronization for drive-response systems by two different types of event-triggering rules, i.e., static and dynamic event-triggering rules. Meanwhile, we can ensure that the Zeno phenomenon does not occur by proving that the time interval between two successive trigger events has a positive lower bound. Finally, two illustrative examples are elaborated to substantiate the theoretical results.

INDEX TERMS Switched neural network, exponential synchronization, event-based control, time-varying delay, distributed delay.

I. INTRODUCTION

Neural network in modern sense usually refers to the mathematical model, which can preliminary simulate the animal brain on the basis of fully understanding of the structure and operation mechanism of biological neural network and modern network theory. This model is distributed, highly fault-tolerant and intellectually capable of processing information. It is widely used in biology [1], [2], physics [3]–[5], computer science [6]–[14], and cognitive science [15]–[20]. With the continuous exploration of the neural network system, the diversity of the dynamic behavior has been extensively studied for various typical neural networks. For example, the stability [21], passivity [21]–[24] and finite-time cluster synchronization [25]; The exponential stabilization [26], lag synchronization [27], dissipativity [28] and attractivity [29] of memristive neural networks; The Mittag-Leffler stability [30], projective synchronization [31], bifurcation [32] of fractional neural network and so on.

With the development of intelligent control, switched system has been widely used [33], [34]. The switched neural network is obtained by introducing the idea of switching into neural networks. Because highly interconnected switched neural networks provide a framework for designing large-scale parallel processors, they have potential applications in image processing, pattern recognition, associative memory, optimization of combinations as well as other fields, and attracted much attention [35], [36]. Switched neural network is a very important hybrid dynamic system as its dynamic behavior is very complex for the existence of switching. In addition, the unavoidable delay often makes the switched neural network unstable or oscillating, and makes the analysis of the switched system become more complex [37]–[39]. Therefore, the research of delayed switched neural network is of great significance.

Synchronization reflects the way in which individuals achieve certain goals through information exchange. Researchers in various fields have revealed the mechanism of synchronization from different perspectives and applied it to engineering [40]–[45]. The theory and practice prove that the synchronizable neural network system is helpful to the design of secure communication based on chaos [46]. However, for individual independent dynamic systems, even if the dynamic equations of each node are the same, different

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initial values will cause them to change asynchronously. Therefore, it becomes an indispensable topic to synchronize neural networks by appropriate measures.

To reduce the update frequency of the control signal, the event-triggering algorithm was proposed to reduce the network load and improve the operation efficiency of the whole system [47], [48]. The basic idea is that when the preset trigger condition was established, the trigger control task is executed. In recent years, researchers have introduced event-triggering mechanism into different systems [49]–[53]. Based on this, the modeling, stability analysis and design of event-triggering conditions of event-triggering systems are discussed. Although the event-triggering mechanism has the advantages mentioned above, the constant trigger parameter can not dynamically adjust the sampling interval according to the change of state error. A new problem has been raised, that is, how to improve the event-triggering mechanism so that the waste of network bandwidth can be reduced. At present, there is little research on this issue. Therefore, this paper considers using adaptive event-triggering mechanism to meet these requirements.

Inspired by the discussions above, the exponential synchronization is investigated for switched neural networks with mixed time-varying delays via event-triggering rules. The main contributions are as follows:

1) Mixed time-varying delays are taken into account in switched neural networks.
2) Sufficient conditions are studied for delayed switched drive-response neural network systems to achieve exponential synchronization via static and dynamic event-triggering algorithms.
3) Positive lower bounds can be achieved for the inter event time of both the two different event-triggering control schemes, which ensures that Zeno behavior does not occur.

This paper proceeds as follows. In Section II, the delayed switched neural network models are introduced and some necessary definitions and lemmas are presented. In Section III, the event-based controller is developed and the main results on static and dynamic event-triggering control are derived, respectively. Then, illustrative numerical examples are elaborated in Section IV to substantiate the validity of the obtained results. Finally, conclusions and future directions of research are given in Section V.

II. PRELIMINARIES

In this section, some notations used in this paper are introduced at first. Then, the drive-response switched neural network models are described. Finally, some necessary definitions and lemmas are presented.

A. NOTATIONS

$\mathbb{N}^+$ represents the positive integer set. Let $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ be the space of $n \times m$ real matrices. $P \in \mathbb{R}^{n\times n} > 0$ (< 0) means that matrix P is symmetric and positive (negative) definite. B^T corresponds to the transpose of vector or matrix B. I_n denotes the $n \times n$ real identity matrix. The 1-norm and 2-norm of vector or matrix z are denoted as $\|z\|_1$ and $\|z\|$, respectively.

B. MODEL DESCRIPTION

Consider the following n-array switched neural network with mixed delays

$$\dot{u}(t) = -A_k u(t) + B_k f(u(t)) + C_k f(u(t-\tau(t))) + D_k \int_{t-\sigma(t)}^{t} f(u(s))ds + I \quad (1)$$

where $u(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^T \in \mathbb{R}^n$ is the state vector of neurons; $A_k = \text{diag}\{a_1^k, a_2^k, \cdots, a_n^k\} > 0$ represents the neural self-inhibitions; $B_k = [b_{ij}^k]_{n\times n}$, $C_k = [c_{ij}^k]_{n\times n}$ and $D_k = [d_{ij}^k]_{n\times n}$ are the feedback connection weight matrix, time-varying delay weight matrix and distributed delay weight matrix, respectively. Moreover, matrices (A_k, B_k, C_k, D_k) take values in the finite set $\{(A_1, B_1, C_1, D_1), \cdots, (A_N, B_N, C_N, D_N)\}$. $\tau(t)$ and $\sigma(t)$ are bounded time-varying delays satisfying $0 \le \tau(t) \le \tau$ and $0 \le \sigma(t) \le \sigma$ where τ and σ are positive real numbers. $f(u(\cdot)) = [f_1(u_1(\cdot)), f_2(u_2(\cdot)), \cdots, f_n(u_n(\cdot))]^T$ is a vector consisting of neuron activation functions; $I \in \mathbb{R}^n$ is the external constant input or bias.

The response system corresponding to drive system (1) is

$$\dot{v}(t) = -A_k v(t) + B_k f(v(t)) + C_k f(v(t-\tau(t))) + D_k \int_{t-\sigma(t)}^{t} f(v(s))ds + I + w(t) \quad (2)$$

where $w(t)$ is the external control input. Note that, although the systems (1) and (2) are switched synchronously, if their initial values are different, their state trajectories are not necessarily synchronized. Therefore, we need to apply control input $w(t)$ to the response system.

Remark 1: The switched neural network systems considered are composed of N continuous-time subsystems with some specific switching rules. Due to the existence of switching, switched systems may often have more complex dynamic behaviors than general dynamic systems [54], [55]. Thus judging the characteristics of the whole switched system should not only consider the characteristics of each subsystem, but also consider the switching rules at the same time. As discussed in [56]–[58], the memristive neural network system is a special kind of switched neural network system, in which the weight matrices of the state-dependent switched feedback connections enable the system to exhibit complex dynamics including chaos.

To facilitate discussion, we introduce the indicator functions $\mu_k(t)$ $(k = 1, 2, \cdots, N)$ as

$$\mu_k(t) = \begin{cases} 1, & \textit{when neural networks is switched to } k\textit{th state}, \\ 0, & \textit{otherwise}. \end{cases} \quad (3)$$

From which it is easy to get $\sum_{k=1}^{n}\mu_k(t) = 1$. Then, drive system (1) can be described as

$$\begin{aligned}\dot{u}(t) &= \sum_{k=1}^{n}\mu_k(t)\Big(-A_k u(t) + B_k f(u(t)) + C_k f(u(t-\tau(t)))\\ &\quad + D_k\int_{t-\sigma(t)}^{t} f(u(s))ds + I\Big)\\ &= -Au(t) + Bf(u(t)) + Cf(u(t-\tau(t)))\\ &\quad + D\int_{t-\sigma(t)}^{t} f(u(s))ds + I \end{aligned}\tag{4}$$

where $A = \sum_{k=1}^{n}\mu_k(t)A_k$, $B = \sum_{k=1}^{n}\mu_k(t)B_k$, $C = \sum_{k=1}^{n}\mu_k(t)C_k$ and $D = \sum_{k=1}^{n}\mu_k(t)D_k$.

Similarly, system (2) can be described as

$$\begin{aligned}\dot{v}(t) &= -Av(t) + Bf(v(t)) + Cf(v(t-\tau(t)))\\ &\quad + D\int_{t-\sigma(t)}^{t} f(v(s))ds + I + w(t). \end{aligned}\tag{5}$$

Define the synchronization error as $x(t) = v(t) - u(t)$, then the error dynamics system can be described as

$$\begin{aligned}\dot{x}(t) &= -Ax(t) + B\Phi(x(t)) + C\Phi(x(t-\tau(t)))\\ &\quad + D\int_{t-\sigma(t)}^{t}\Phi(x(s))ds + w(t) \end{aligned}\tag{6}$$

where $\Phi(x(\cdot)) = f(v(\cdot)) - f(u(\cdot))$.

Assumption 1: The neuron activation functions $f_i(\cdot)$ $(i = 1, 2, \cdots, n)$ are globally Lipschitz continuous and bounded. Namely, there exist positive constants F_i and $\widetilde{F}_i$ such that

$$|f_i(\xi_1) - f_i(\xi_2)| \le F_i|\xi_1 - \xi_2|$$

and

$$|f_i(\varsigma)| \le \widetilde{F}_i$$

hold for any $\xi_1, \xi_2, \varsigma \in \mathbb{R}$ and $i = 1, 2, \cdots, n$.

Assumption 2: The time-varying transmission delays $\tau(t)$ and $\sigma(t)$ are differentiable functions and satisfy $0 \le \dot{\tau}(t) \le \widetilde{\tau} < 1$ and $0 \le \dot{\sigma}(t) \le \widetilde{\sigma} < 1$, where $\widetilde{\tau}$ and $\widetilde{\sigma}$ are positive constants.

C. DEFINITIONS AND LEMMAS

Definition 1: Drive-response systems (1) and (2) are said to be exponentially synchronizable, if for any initial conditions $\psi_u(s)$, $\psi_v(s) \in \mathbb{R}$ of systems (1) and (2), respectively, there exist $\chi \ge 1$ and $\epsilon > 0$ such that

$$\|v(t) - u(t)\| \le \chi\|\psi_v(s) - \psi_u(s)\|e^{-\epsilon t}$$

holds for any $t \ge 0$. Positive real number ϵ is called the exponential convergence rate of synchronization of systems (1) and (2).

Lemma 1 [59]: For any matrix $W \in \mathbb{R}^{n\times n} > 0$ and vectors $x, y \in \mathbb{R}^n$, the following inequality holds:

$$2x^Ty \le x^TWx + y^TW^{-1}y.$$

Lemma 2 [60]: For any matrix $Q \in \mathbb{R}^{n\times n} > 0$, scalar function $0 < \varrho(t) < \varrho$, vector-valued function $\varphi : [0, \varrho] \to \mathbb{R}^n$, the following integral inequality holds:

$$\begin{aligned}\left(\int_0^{\varrho(t)}\varphi(s)ds\right)^T Q\left(\int_0^{\varrho(t)}\varphi(s)ds\right)\\ \le \varrho(t)\int_0^{\varrho(t)}\varphi(s)^TQ\varphi(s)ds. \end{aligned}\tag{7}$$

III. MAIN RESULTS

In this section, the exponential synchronization problem of drive-response switched systems is discussed by two different event-triggering control schemes.

A. STATIC EVENT-BASED CONTROL

Define the measurement error as

$$e(t) = x(t_i) - x(t), \quad t \in [t_i, t_{i+1}) \tag{8}$$

where $x(t_i)$ is the state of $x(t)$ at trigger instant t_i which is generated by the following trigger rule

$$t_{i+1} = \inf\{t > t_i : \|e(t)\| \ge \delta\alpha\} \tag{9}$$

where $\delta \in (0, 1)$ and α is a given positive real number.

To achieve the goal of synchronizing system (1) with system (2), in any trigger interval $[t_i, t_{i+1})$, we design the following event-based controller

$$w(t) = -Gx(t_i) - H\,\mathrm{sign}x(t_i) \tag{10}$$

where gain matrix $G = \mathrm{diag}\{g_1, g_2, \cdots, g_n\} > 0$ is a constant matrix and $H = \mathrm{diag}\{h_1, h_2, \cdots, h_n\}$ is an uncertainty matrix with switching elements. Both matrices P and Q will be determined later.

Remark 2: As far as the authors know, there are many achievements on exponential synchronization of switched neural networks with mixed delays, such as [61]–[63]. Note that, the state feedback controllers in [61], [62] and the adaptive controller in [63] need to be continuously updated, while the event-triggering controller (10) only needs to be updated at discrete instants which can effectively reduce the control cost.

With event-based controller (10) and trigger rule (9), we can get the following result.

Theorem 1: Suppose that Assumptions 1 and 2 hold. Let event-based controller (10) with trigger rule (9) be applied to system (2). Systems (1) and (2) will be exponentially synchronizable, if there exist a positive real number ϵ and positive definite matrices P and Q such that

$$\lambda_{\max}(I_n - (1-\widetilde{\tau})Pe^{-\epsilon\tau}) \le 0, \tag{11}$$
$$\lambda_{\max}(I_n - (1-\widetilde{\sigma})e^{-\epsilon\sigma}\sigma^{-1}Q) \le 0, \tag{12}$$

and the control gain matrices G and H such that

$$\begin{aligned}\lambda_{\max}\big(&\epsilon I_n - 2A_k + B_kB_k^T + C_kC_k^T + D_kD_k^T\\ &- 2G + F(P + I_n + \sigma Q)F\big) \le 0, \end{aligned}\tag{13}$$

$$\begin{cases} h_j > \eta_j, & x_j(t)\mathrm{sign}(x_j(t_i)) > 0, \\ h_j \leq -\eta_j, & otherwise \end{cases} \tag{14}$$

and

$$\delta\alpha\|G\| - \eta_{\min} \leq 0, \tag{15}$$

where $F = \mathrm{diag}\{F_1, F_2, \cdots, F_n\}$, $\eta_{\min} = \min_{i=1,2,\cdots,n} \eta_i$.

Proof: Consider the Lyapunov functional as follows

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{16}$$

where

$$V_1(t) = x(t)^T x(t) e^{\epsilon t}$$
$$V_2(t) = \int_{t-\tau(t)}^{t} \Phi(x(s))^T P \Phi(x(s)) e^{\epsilon s} ds$$

and

$$V_3(t) = \int_{t-\sigma(t)}^{t} \int_{\theta}^{t} \Phi(x(s))^T Q \Phi(x(s)) e^{\epsilon s} ds d\theta.$$

Then, we have

$$\|x(t)\| e^{\epsilon t} \leq V(t). \tag{17}$$

Calculate the derivative of $V(t)$ along the solution to error system (6), we have

$$\begin{aligned} \dot{V}(t) = {} & \epsilon e^{\epsilon t} x(t)^T x(t) + 2e^{\epsilon t} x(t)^T \Big[-Ax(t) + B\Phi(x(t)) \\ & + C\Phi(x(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} \Phi(x(s))ds \\ & - Gx(t_i) - H\mathrm{sign}x(t_i) \Big] + \Phi(x(t))^T P \Phi(x(t)) e^{\epsilon t} \\ & - (1-\dot{\tau}(t))\Phi(x(t-\tau(t)))^T P \Phi(x(t-\tau(t))) e^{\epsilon(t-\tau(t))} \\ & + \sigma(t)\Phi(x(t))^T Q \Phi(x(t)) e^{\epsilon t} \\ & - (1-\dot{\sigma}(t)) \int_{t-\sigma(t)}^{t} \Phi(x(s))^T Q \Phi(x(s)) e^{\epsilon s} ds. \end{aligned} \tag{18}$$

It follows from Lemma 1 that

$$2x(t)^T B\Phi(x(t)) \leq x(t)^T BB^T x(t) + \Phi(x(t))^T \Phi(x(t)). \tag{19}$$

Furthermore,

$$\begin{aligned} & 2x(t)^T C\Phi(x(t-\tau(t))) \\ & \quad \leq x(t)^T CC^T x(t) + \Phi(x(t-\tau(t)))^T \Phi(x(t-\tau(t))) \end{aligned} \tag{20}$$

and

$$\begin{aligned} & 2x(t)^T D \int_{t-\sigma(t)}^{t} \Phi(x(s))ds \\ & \quad \leq x(t)^T DD^T x(t) \\ & \quad + \Big(\int_{t-\sigma(t)}^{t} \Phi(x(s))ds\Big)^T \Big(\int_{t-\sigma(t)}^{t} \Phi(x(s))ds\Big). \end{aligned} \tag{21}$$

According to the definition of measurement error $e(t)$, we have

$$\begin{aligned} -2x(t)^T Gx(t_i) &= -2x(t)^T G(e(t) + x(t)) \\ &\leq 2\|x(t)\|\|G\|\|e(t)\| - 2x(t)^T Gx(t) \\ &\leq 2\delta\alpha\|G\|\|x(t)\| - 2x(t)^T Gx(t). \end{aligned} \tag{22}$$

Combining condition (14) and $\|x(t)\|_1 \geq \|x(t)\|$, one can get

$$\begin{aligned} & -2x(t)^T H\mathrm{sign}x(t_i) \\ & \quad = -2\sum_{j=1}^{n} x_j(t) h_j \mathrm{sign}x_j(t_i) \\ & \quad \leq -2\sum_{j=1}^{n} \eta_j |x_j(t)| \leq -2\sum_{j=1}^{n} \eta_{\min} |x_j(t)| \\ & \quad = -2\eta_{\min}\|x(t)\|_1 \leq -2\eta_{\min}\|x(t)\|. \end{aligned} \tag{23}$$

Based on Assumption 2, by Lemma 2, we have

$$\begin{aligned} & -(1-\dot{\sigma}(t)) \int_{t-\sigma(t)}^{t} \Phi(x(s))^T Q \Phi(x(s)) e^{\epsilon s} ds \\ & \quad \leq -(1-\dot{\sigma}(t)) e^{\epsilon(t-\sigma(t))} \int_{t-\sigma(t)}^{t} \Phi(x(s))^T Q \Phi(x(s)) ds \\ & \quad \leq -(1-\tilde{\sigma}) e^{\epsilon(t-\sigma)} \int_{t-\sigma(t)}^{t} \Phi(x(s))^T Q \Phi(x(s)) ds \\ & \quad \leq -(1-\tilde{\sigma}) e^{\epsilon(t-\sigma)} \sigma^{-1} \Big(\int_{t-\sigma(t)}^{t} \Phi(x(s))ds\Big)^T \\ & \quad \times Q\Big(\int_{t-\sigma(t)}^{t} \Phi(x(s))ds\Big). \end{aligned} \tag{24}$$

Based the Lipschitz continuity of activation functions in Assumption 1, we have

$$\begin{aligned} & \Phi(x(t))^T (P + I_n + \sigma(t)Q)\Phi(x(t)) \\ & \qquad \leq x(t)^T F(P + I_n + \sigma Q)Fx(t). \end{aligned} \tag{25}$$

Substituting formulas (19)-(25) into (18), we have

$$\begin{aligned} \dot{V}(t) \leq {} & e^{\varepsilon t} x(t)^T \Big[\epsilon I_n - 2A + BB^T + CC^T + DD^T - 2G \\ & + F(P + I_n + \sigma Q)F\Big]x(t) + (2\delta\alpha\|G\| - 2\eta_{\min})\|x(t)\| e^{\varepsilon t} \\ & + \Phi(x(t-\tau(t)))^T (I_n - (1-\tilde{\tau})Pe^{-\epsilon\tau})\Phi(x(t-\tau(t))) e^{\varepsilon t} \\ & + \Big(\int_{t-\sigma(t)}^{t} \Phi(x(s))ds\Big)^T (I_n - (1-\tilde{\sigma})e^{-\epsilon\sigma}\sigma^{-1}Q) \\ & \times \Big(\int_{t-\sigma(t)}^{t} \Phi(x(s))ds\Big) e^{\varepsilon t} \\ \leq {} & 0. \end{aligned} \tag{26}$$

Then, we have

$$V(t) \leq V(0), \quad t \geq 0. \tag{27}$$

It follows from (17) that

$$\|u(t) - v(t)\| \leq V(t)e^{-\epsilon t} \leq V(0)e^{-\epsilon t}. \tag{28}$$

The proof is completed. ■

In the following, we show that the time sequence generated by trigger rule (9) is Zeno free.

Theorem 2: In case of Theorem 1, there is a consistent lower bound for the inter event time $T_i = t_{i+1} - t_i$ as

$$T_i \geq \frac{1}{\|A\|} \ln\Big(1 + \frac{\delta\alpha\|A\|}{\beta(t_i)}\Big) \tag{29}$$

where $\beta(t_i) = \rho\|x(t_i)\| + \varpi$, $\rho = \|A\| + \|G\|$, $\varpi = 2(\|B\| + \|C\| + \sigma\|D\|)\|\widetilde{F}\| + \|H\|$ and $\widetilde{F} = [\widetilde{F}_1, \widetilde{F}_n, \cdots, \widetilde{F}_1]^T$.

Proof: Consider the dynamics of measurement error $e(t)$ on interval $[t_i, t_{i+1})$, we have

$$\begin{aligned}\frac{d}{dt}\|e(t)\| &\leq \|\frac{d}{dt}e(t)\| = \|\dot{x}(t)\| \\ &= \| - Ax(t) + B\Phi(x(t)) + C\Phi(x(t-\tau(t))) \\ &\quad + D\int_{t-\sigma(t)}^{t}\Phi(x(s))ds - Gx(t_i) - H\text{sign}x(t_i)\| \\ &\leq \|A\|\|x(t)\| + 2(\|B\| + \|C\| + \sigma\|D\|)\|\widetilde{F}\| \\ &\quad + \|G\|\|x(t_i)\| + \|H\| \\ &\leq \|A\|\|e(t)\| + 2(\|B\| + \|C\| + \sigma\|D\|)\|\widetilde{F}\| \\ &\quad + (\|A\| + \|G\|)\|x(t_i)\| + \|H\|. \end{aligned} \tag{30}$$

Solving the above differential inequality with $\|e(t_i)\| = 0$ as the initial condition by using the comparison lemma we can obtain

$$\|e(t)\| \leq \frac{(\rho\|x(t_i)\| + \varpi)}{\|A\|}(e^{\|A\|(t-t_i)} - 1) \tag{31}$$

for all $t \in [t_i, t_{i+1})$. At the trigger instant

$$\|e(t_{i+1})\| \leq \frac{\beta(t_i)}{\|A\|}(e^{\|A\|T_i} - 1). \tag{32}$$

On the other hand, for any $t \in [t_i, t_{i+1})$, whenever events occur, we have

$$\|e(t_{i+1})\| = \delta\alpha. \tag{33}$$

It follows that

$$T_i \geq \frac{1}{\|A\|}\ln(1 + \frac{\delta\alpha\|A\|}{\beta(t_i)}). \tag{34}$$

The proof is completed. ■

Remark 3: Just like almost all the studies on the performance of dynamic systems by using event-based control schemes, this paper can also ensure that Zeno behavior does not occur. From Theorem 2, we can see the time interval is inconsistent between two consecutive trigger events. Similar problem is encountered in [58] and a unified lower bound is obtained by using the system initial values. However, the synchronization discussed in [58] is asymptotic rather than exponential. In fact, whether the positive lower bound is consistent or inconsistent, we can ensure that no countless events are triggered in a limited time.

B. DYNAMIC EVENT-BASED CONTROL

Here, we first introduce a dynamic variable $y(t) = y(t, y_0)$ which is the solution to

$$\dot{y}(t) = (-y(t) + 2\delta\alpha\|G\|\|x\| - 2\|G\|\|x(t)\|\|e(t)\|)e^{\varepsilon t} \tag{35}$$

where parameters δ, α, ϵ and matrix G are defined in Theorem 1. And the initial value of (35) is given as $y(0) = y_0 \geq 0$.

Next, we design the following dynamic trigger rule

$$t_{i+1} = \inf\{t > t_i : \|e(t)\| \geq y(t) + \delta\alpha\}. \tag{36}$$

With which the exponential synchronization problem of systems (1) and (2) is discussed in this subsection.

Theorem 3: Suppose that Assumptions 1 and 2 hold. Let event-based controller (10) with dynamic trigger rule (36) be applied to system (2). Systems (1) and (2) will be exponentially synchronizable, if there exist a positive real number ϵ and positive definite matrices P and Q which satisfy (11) and (12), and the control gain matrices G and H satisfy (13), (14) and (15).

Proof: For any $t \in [t_i, t_{i+1})$, it follows from (35) that

$$\begin{aligned}\dot{y}(t) &\geq \big[- y(t) + 2\delta\alpha\|G\|\|x\| - 2\|G\|\|x(t)\|(y(t) + \delta\alpha)\big]e^{\varepsilon t} \\ &= -(1 + 2\|G\|\|x(t)\|)y(t)e^{\varepsilon t}. \end{aligned} \tag{37}$$

Then, the solution to inequality (37) satisfy $y(t) \geq 0$.

Consider the following new Lyapunov functional

$$\overline{V}(t) = V(t) + y(t) \tag{38}$$

Calculate the derivative of $\widetilde{V}(t)$ along the solution to error system (6), we have

$$\begin{aligned}\dot{\overline{V}}(t) &= \dot{V}(t) + \dot{y}(t) \\ &\leq e^{\varepsilon t}x(t)^T\Big[\epsilon I_n - 2A + BB^T + CC^T + DD^T - 2G \\ &\quad + F(P + I_n + \sigma Q)F\Big]x(t) + 2\|x(t)\|\|G\|\|e(t)\|e^{\varepsilon t} \\ &\quad + \Phi(x(t-\tau(t)))^T(I_n - (1-\widetilde{\tau})Pe^{-\epsilon\tau})\Phi(x(t-\tau(t)))e^{\varepsilon t} \\ &\quad + \left(\int_{t-\sigma(t)}^{t}\Phi(x(s))ds\right)^T(I_n - (1-\widetilde{\sigma})e^{-\epsilon\sigma}\sigma^{-1}Q) \\ &\quad \times \left(\int_{t-\sigma(t)}^{t}\Phi(x(s))ds\right)e^{\varepsilon t} - 2\eta_{\min}\|x(t)\|e^{\varepsilon t} \\ &\quad + (-y(t) + 2\delta\alpha\|G\|\|x\| - 2\|G\|\|x(t)\|\|e(t)\|)e^{\varepsilon t} \\ &\leq 0 \end{aligned} \tag{39}$$

The proof is completed. ■

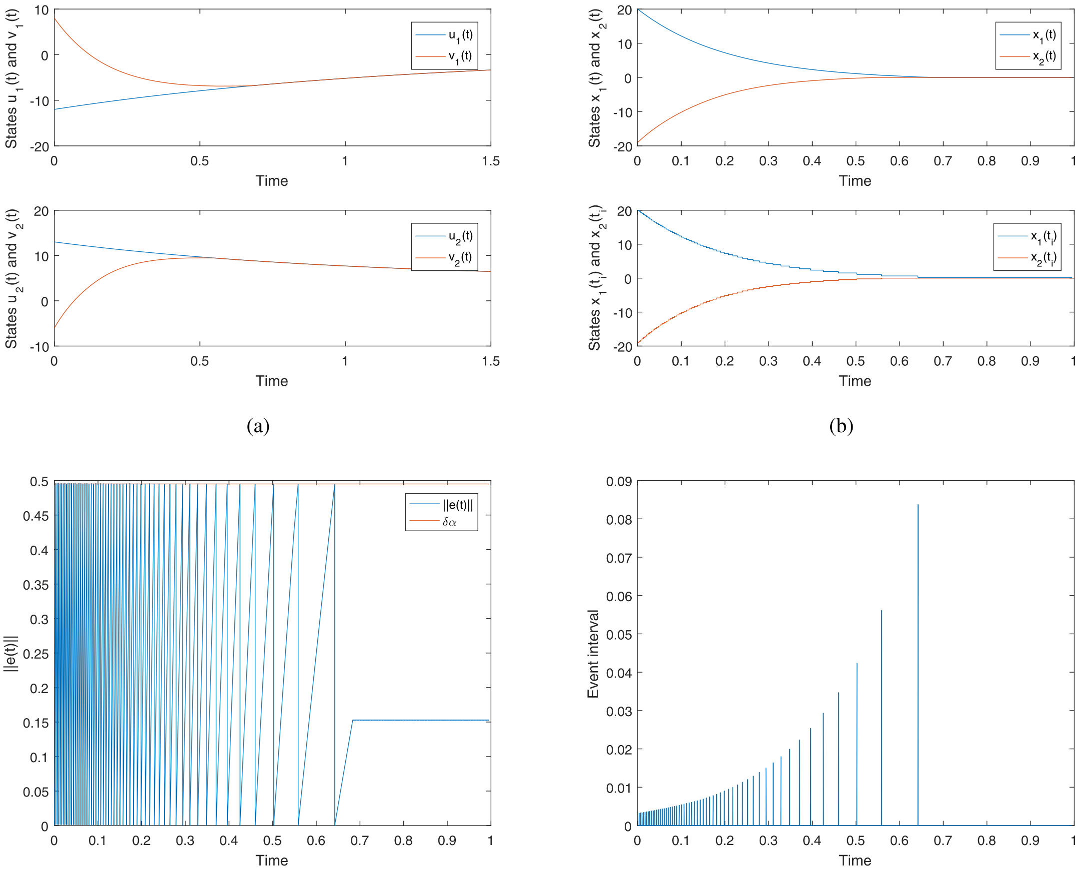
Similarly, we can also show that the time sequence generated by trigger rule (36) is Zeno free.

Theorem 4: In case of Theorem 3, there is a consistent lower bound for the inter event time T_i as

$$T_i \geq \frac{1}{\|A\|}\ln(1 + \frac{(\delta\alpha + y(t))\|A\|}{\beta(t_i)}). \tag{40}$$

Proof: Following the same procedure as the proof of Theorem 2, this conclusion can be proved. ■

Remark 4: Although the static event-triggering scheme has the advantages (reducing data transmission and power consumption) and been recognized as an effective control scheme in electronic chips with limited capacity and energy [64]–[66], it can not dynamically adjust the sampling interval according to the change of error states. This problem is solved by introducing a positive dynamic variable $y(t)$ into the trigger rule. As can be seen from (40) that the lower bound of the inter event time under dynamic event-based control mechanism is larger than that of static ones which means that the dynamic event-based control scheme is more economical and realistic.



(a) (b) (c) (d)

FIGURE 1. Systems (1) and (2) are exponential synchronized under controller (10) with static event-triggering rule (9). (a) The trajectories of driven system (1) and response system (2); (b) The trajectories of synchronization errors $x_1(t)$ and $x_2(t)$ and their sampling states $x_1(t_k)$ and $x_2(t_k)$; (c) The 2-norm of measurement error; (d) Trigger time and execution interval.

If positive definite matrices P and Q are diagonal matrices which can be described as $P = \text{diag}\{p_1, p_2, \cdots, p_n\} > 0$ and $Q = \text{diag}\{q_1, q_2, \cdots, q_n\} > 0$, then we can get the following corollary.

Corollary 1: Suppose that Assumptions 1 and 2 hold. Let event-based controller (10) with trigger rule (9) or (36) be applied to system (2). Systems (1) and (2) will be exponentially synchronizable with exponential convergence rate ϵ, if there exist positive definite diagonal matrices P and Q such that

$$1 - (1 - \widetilde{\tau})p_{\min}e^{-\epsilon\tau} \leq 0, \quad (41)$$

$$1 - (1 - \widetilde{\sigma})e^{-\epsilon\sigma}\sigma^{-1}q_{\min} \leq 0, \quad (42)$$

and the control gain matrices G and H satisfy (13), (14) and (15) in Theorem 1, where $p_{\min} = \min_{i=1,2,\cdots,n} p_i$ and $q_{\min} = \min_{i=1,2,\cdots,n} q_i$.

Furthermore, if $P = \tilde{p}I_n$ and $Q = \tilde{q}I_n$ where $\tilde{p}$ and $\tilde{q}$ are positive constants, then the following corollary can be obtained.

Corollary 2: Suppose that Assumptions 1 and 2 hold. Let event-based controller (10) with trigger rule (9) or (36) be applied to system (2). Systems (1) and (2) will be exponentially synchronizable with exponential convergence rate ϵ, if there exist positive constants $\tilde{p}$ and $\tilde{q}$ such that

$$1 - (1 - \widetilde{\tau})\tilde{p}e^{-\epsilon\tau} \leq 0, \quad (43)$$

$$1 - (1 - \widetilde{\sigma})e^{-\epsilon\sigma}\sigma^{-1}\tilde{q} \leq 0, \quad (44)$$

and the control gain matrices G and H satisfy (13), (14) and (15) in Theorem 1.

Remark 5: In this paper, the time-varying delays caused by limited information processing and transmission speed of amplifiers [67] are considered. However, the communication delay is neglected. In fact, the communication delay is inevitable when employing protocols to set communication to coordinate system through remote terminals [68]. For complex neural network systems with communication delays and mixed time-varying delay, how to take control measures to study their exponential synchronization and finite/fixed-time synchronization are problems to be solved.

IV. NUMERICAL SIMULATION

Consider systems (1) and (2) with $n = 2$, $N = 2$, $f_1(\xi) = f_2(\xi) = tanh(\xi)$, $\tau(t) = 0.01$, $\sigma(t) = 0.01$, and $I = [0, 0]^T$. Then, the activation functions $f_i(i = 1, 2)$ satisfy

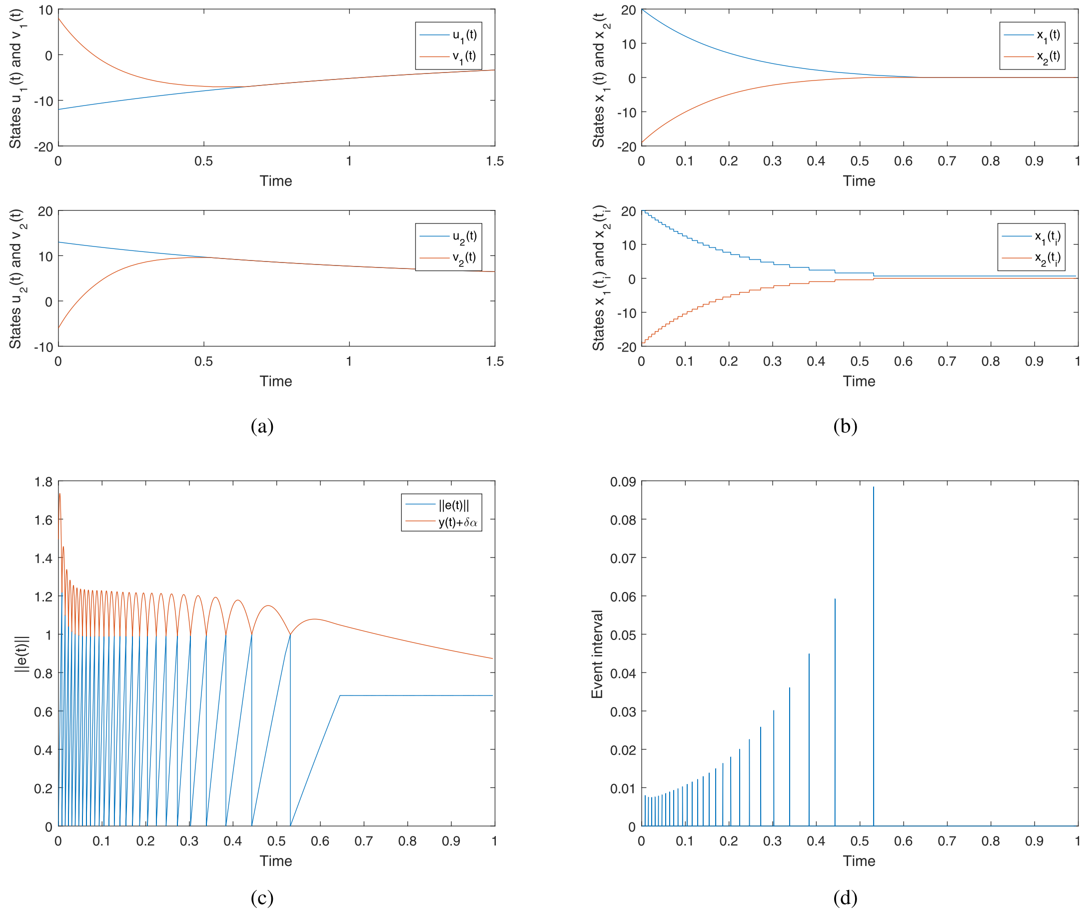


FIGURE 2. **Systems (1) and (2) are exponential synchronized under controller (10) with danamic event-triggering rule (36) where $y(0) = 1$. (a) The trajectories of driven system (1) and response system (2); (b) The trajectories of synchronization errors $x_1(t)$ and $x_2(t)$ and their sampling states $x_1(t_k)$ and $x_2(t_k)$; (c) The 2-norm of measurement error; (d) Trigger time and execution interval.**

Assumption 1 with $F_i = \widetilde{F}_i = 1$, and the time-varying delays $\tau(t)$ and $\sigma(t)$ satisfy Assumption 2 with $\dot{\tau}(t) = \widetilde{\tau} = 0$ and $\dot{\sigma}(t) = \widetilde{\sigma} = 0$. Systems (1) and (2) switch according to the following switching rule

$$\begin{cases} (A_k, B_k, C_k, D_k) = (A_1, B_1, C_1, D_1), & \|u(t)\| < 1, \\ (A_k, B_k, C_k, D_k) = (A_2, B_2, C_2, D_2), & otherwise \end{cases}$$

where

$$A_1 = \begin{bmatrix} 1.0 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 1.0 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1.6 & -0.3 \\ -1.2 & 1.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.5 & -0.2 \\ -1.1 & 1.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -1.5 & 0.2 \\ -1.3 & 0.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -1.6 & 0.4 \\ -1.5 & 0.8 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.8 & 0.4 \\ -1.1 & -1.5 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.7 & 0.5 \\ -1.0 & -1.6 \end{bmatrix}.$$

Taking

$$P = \begin{bmatrix} 1.101 & 0 \\ 0 & 1.101 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11 \end{bmatrix},$$

then inequalities (11) and (12) hold. Let controller (10) be applied to response system (2) with control gain matrices

$$G = \begin{bmatrix} 3.9411 & 0 \\ 0 & 5.5361 \end{bmatrix}, \quad H = \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix}$$

where $\eta = [\eta_1, \eta_2]^T = [2.8403, 2.8403]^T$ and

$$h_i = \begin{cases} \eta_i + 0.2, & x_j(t)\text{sign}(x_j(t_i)) > 0, \\ -\eta_i - 0.2, & otherwise. \end{cases} \tag{45}$$

Then, according to Theorems 1 and 3, systems (1) and (2) are exponential synchronization under both the static event-triggering control and the adaptive dynamic event-triggering control with trigger parameters $\delta = 0.99$, $\alpha = 0.5$ and exponential convergence rate $\epsilon = 0.1$. The simulation results are shown in FIGURE 1 and FIGURE 2, respectively. From which we can see that the trigger time sequences generated by both trigger rules are Zeno free.

Remark 6: As can be seen from FIGURE 1(c) and FIGURE 2(c) that, under the dynamic event triggering control scheme, the fluctuation frequency of measurement error is lower and the times of updating controllers is less. This

TABLE 1. The numbers of events occurring under two different event-based control schemes.

parameter \ trigger scheme	static	dynamic
$\alpha = 0.2$	141	69
$\alpha = 0.4$	71	35
$\alpha = 0.6$	47	24
$\alpha = 0.8$	36	18
$\alpha = 1.0$	29	15

TABLE 2. The time of the last trigger event under two different event-based control schemes.

parameter \ trigger scheme	static	dynamic
$\alpha = 0.2$	0.7976	1.2632
$\alpha = 0.4$	0.6812	0.6062
$\alpha = 0.6$	0.5516	0.5821
$\alpha = 0.8$	0.5612	0.4830
$\alpha = 1.0$	0.5240	0.6506

implies that the cost of the dynamic event-triggering control scheme is lower than that of the static ones. This also can be seen from TABLE 1. In addition, the datum in TABLE 1 also show that under the same event-triggering control scheme, the larger the upper bound of the trigger rule is, the more beneficial it is to reduce the number of controller updates, which is consistent with the conclusions of most related studies.

In numerical simulation, we also investigate the difference of the last trigger event time, which can indirectly reflect the synchronization time of the drive-response systems, under different trigger schemes and under the same trigger scheme with different trigger thresholds as presented in TABLE 2. We regret to find that the dynamic event-triggering scheme is not conducive to speeding up the time required for synchronizing the drive-response systems. Then the question arises, what factors will directly affect the time required for two or multiple systems to achieve synchronization? This may be a topic worth exploring.

V. CONCLUSION

This paper investigated the exponential synchronization for delayed switched neural networks via static/dynamic event-triggering rules. By using a constant bound event trigger rule and an adaptive dynamic bound event trigger rule, we obtain the sufficient conditions for exponential synchronization of drive-response system combining Jensen's inequality and Lyapunov stability theory. Experiments show that the main results in this paper are economical and feasible.

To further improve the application of adaptive event-triggering control scheme in neural networks systems, a lot of research work needs to be explored. Further investigations may aim at studying the more complex dynamic behaviors including passification and chaos of switched neural networks with mixed time-varying delays and white noise disturbance, and more practical finite/fixed time synchronization of two or multiple classical synchronous or asynchronous switched neural networks by designing appropriate adaptive event-triggering rules.

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CONFLICT OF INTEREST STATEMENT

There is no conflict of interest in this paper.

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