Title: Duo-stage decision: A framework for filling missing values, consistency check, and repair of decision matrices in multi-criteria group decision-making

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Duo-stage decision: A framework for filling missing values, consistency check, and repair of decision matrices in multi-criteria group decision-making

Abstract

With high uncertainty and vagueness in the decision-making process, maintaining consistency in the decision matrix is an open challenge. Previous studies on intuitionistic fuzzy (IF) theory focused on the consistency of preference relation but ignored consistency of the decision matrix. In this paper, efforts are made to propose a new duo-stage decision framework in the context of IFS to better circumvent the challenge. Often, decision makers (DMs) hesitate to provide certain values in the decision matrix that are filled randomly, resulting in inaccuracies in the decision-making process. To alleviate this issue, a new systematic procedure is developed that sensibly fills the missing data in the first stage. Following the first stage, consistency of the decision matrix is determined by extending Cronbach's alpha coefficient to IF context. Further, efforts are made to repair inconsistent decision matrix iteratively. In the second stage, a new aggregation operator is presented for aggregation of DMs' preferences. Also, a new mathematical model is proposed for criteria weight estimation, and a procedure is developed for ranking objects. The practical use of the proposed framework is demonstrated using a numerical example, and the strengths and weaknesses of the framework are investigated.

Keywords: Cronbach's alpha coefficient; Group decision making; Intuitionistic fuzzy set; Maclaurin symmetric mean; Missing data and COPRAS method.

1. Introduction

Group decision making (GDM) is a widely explored topic in engineering management [1], [2] and it is popular within the intuitionistic fuzzy set (IFS) context[3]. In this process, a set of DMs provides their intuitionistic fuzzy [4] preference information on each object with respect to a criterion, and these preference values are aggregated into a single decision matrix for evaluation. In general, DMs hesitate to provide certain values in the decision matrix, and random entry of these values leads to inaccuracies in the decision-making process. Motivated by this issue, Xu [5] presented a procedure for filling the missing values in intuitionistic fuzzy preference relation (IFPR). Later, Jiang and Xu [6] developed two new methods for handling incomplete IFPRsthat are more sensible and rational.

Although these methods alleviate the issue of incompleteness, they only handle the incompleteness in IFPR but ignore the incompleteness in IFS-based decision matrix. Motivated and attracted by this challenge, this paper presents a new method for handling incomplete information in IFS-based decision matrix. Additionally, scholars explored the consistency and repair process in IFPR based on additive and multiplicative measures [7]–[9]. However, the consistency of the IFS-based decision matrix was ignored. Motivated by this challenge, efforts are made in this paper to extend the Cronbach's alpha coefficient to IFS context for determining the consistency of the decision matrix. Also, a new procedure is developed for repairing an inconsistent decision matrix.

Since different DMs provide their preference information over each object, the idea of aggregation of this information continues to be vigorously explored. Many scholars have widely explored the concept of aggregation of IFS information [10]–[18]. Although these operators aggregate the IFS-based preference information of different DMs, the issue of managing the inter-relationship between criteria remains an open challenge. To address this challenge, operators such as Bonferroni mean (BM) [19], Frank [20], and Maclaurin symmetric mean (MSM) [21] were extended to IFS context. Although these operators mitigated the challenge to a certain extent, the idea of generalization was ignored. Motivated by this claim, this paper attempts to extend the generalized MSM operator to IFS context (IFGMSM). Using IFGMSM operator, all the operators mentioned above can be easily realized.

The next step in the decision framework is the estimation of criteria weights and ranking of objects. Scholars have proposed several methods for criteria weight estimation viz., analytic hierarchy process (AHP) [22], [23], which is a technique for order preference by similarity to ideal solution (TOPSIS) [24], and entropy measurements [25],which are useful when the criteria weights are entirely unknown. In contrast, when criteria weights are partially known or DMs want to explicitly give their inequality expression on each criterion, mathematical programming models [26], [27] are used. Although these models utilize the partial information provided by the DM, the idea of identifying preferences that are close to the positive ideal solution or are further away from the negative ideal solution helps DMs to properly plan the weights (relative importance) of each criterion. Also, the nature of each criterion must be considered when determining weights[28]. Motivated by these claims, this paper proposes a new mathematical model under IFS context that considers an ideal solution measurement. Further, a suitable object is selected from the set of objects by proposing a newly extended IFS-based COPRAS (complex proportion assessment) method. Although there are many ranking methods [24], [29]–[31]under IFS context, we were motivated to extend COPRAS to IFS context because of the ability of COPRAS to handle preference information from different angles and to provide ranking order with a degree of preference values.

Before identifying some significant challenges, the literature in the field of engineering and technology management that utilize MCDM methods for better decision-making are reviewed. Sun et al., [32] presented a decision support method for evaluating DMs who provide preferences to rate R&D projects and support in their selection. Later, Mogre et al., [33] proposed a new decision support system for selecting a suitable risk mitigation strategy and risk mitigation tactic from the set of available choices using a two-stage method in the context of supply chain risk management. Shahin et al., [34] integrated Kano model with fuzzy AHP to rationally prioritize critical factors for radical innovation. Similarly, Abbasianjahroni et al., [35] proposed a decision framework by integrating the Kano model and AHP method for prioritization of subcontractors in projects. Silva et al., [36] proposed an integrated method by using cognitive maps and Choquet integral to evaluate SMEs' propensity for open innovation. Raziei et al., [37] assessed the service quality in healthcare by using a fuzzy-based hybrid model. Recently, Duman et al., [38] integrated three methods viz., DEMATEL (decision-making trial and evaluation laboratory), ANP (analytical network process), and ANN (artificial neural network) for performance evaluation by considering both qualitative and quantitative factors with historical data. Dahooie et al., [39] proposed a new decision framework for prioritizing software for cloud computing systems under interval-valued IFS (IVIFS) context by extending the CODAS (combinative distance-based assessment) method.

Based on the literature analysis made above with respect to engineering/technology management and IFS, the following significant challenges can be encountered:

(1) Missing values in the decision matrix lead to inaccuracies in the decision-making process. Often, a random filling is performed to compensate for the missing values, which leads to errors in the decision-making process.

(2) The consistency of the decision matrix is ignored during the process of decision-making, which affects the final decision. In general, previous attempts by researchers to resolve the consistency issue in preference relations are not suitable for decision matrices.

(3) Considering the relationship between criteria during the aggregation process is an interesting challenge in which significant contributions are needed.

(4) Considering the nature of criteria during weight estimation is another interesting challenge that needs to be effectively addressed.

(5) Finally, ranking objects by considering information from various angles and also providing a degree of preference values is an open challenge that requires better contributions.

These challenges motivated us to find ways of circumventing them. This paper makes novel contributions in the field of decision-making under the engineering and technology management context as follows:

(1) To address the challenge (1), a new systematic procedure is presented that sensibly fills missing values.

(2) To address the challenge (2), the popular Cronbach's alpha coefficient method is extended to IFS context and an iterative method is proposed for repairing an inconsistent decision matrix.

(3) Challenge (3) is handled by proposing an IFGMSM operator for aggregating preference information that considers the inter-relationship among criteria.

(4) Challenge (4) is handled by proposing a new mathematical programming model under IFS context by considering the ideal solution measurement.

(5) To handle the challenge (5), the COPRAS method is extended under IFS context to effectively handle information from various angles and to provide a degree of preference information along with the ranking order.

The remainder of the paper is constructed as follows. Section 2 presents the fundamentals of IFS, and Section 3 provides the core concept of this paper. Section 3 begins with a workflow that is followed by procedures for filling missing values, consistency check and repair, aggregation operator, weight calculation method for attributes, and ranking method. To validate the practical use of the proposed framework, a numerical example of green supplier selection is presented in Section 4. Section 5 provides a comparative analysis of the proposed framework with other methods and conclusions, and future scope are presented in section 6.

2. Preliminaries

In this section, the fundamentals of the IFS concept are presented.

Definition 1[4]**:** Consider a set B defined over a fixed set Z such that $B \subset Z$ is also fixed. The IFS $\bar{B}$in Z is an object given by,

$$\bar{B} = \left(z, \mu_{\bar{B}}(z), \upsilon_{\bar{B}}(z)\right) \tag{1}$$

where$\mu_{\bar{B}}(z) = \mu(z) \in [0,1]$ is the degree of membership, $\upsilon_{\bar{B}}(z) = \upsilon(z) \in [0,1]$ is the degree of non-membership, and $\pi_{\bar{B}}(z) = \pi(z) \in [0,1]$ is the degree of indeterminacy or hesitation with $\pi_{\bar{B}}(z) = \pi(z) = 1 - \left(\mu_{\bar{B}}(z) + \upsilon_{\bar{B}}(z)\right)$ and $\mu_{\bar{B}}(z) + \upsilon_{\bar{B}}(z) \leq 1$.

Remark 1: For ease of representation, $\tau_i = \left(\mu_i(z), \upsilon_i(z)\right)$ is called intuitionistic fuzzy value (IFV).

Definition 2[5]**:** Let τ_1, τ_2 be two IFVs. Some operational laws that they follow are:

$$\tau_1 \oplus \tau_2 = \left(\mu_1(z) + \mu_2(z) - \mu_1(z)\mu_2(z), \upsilon_1(z)\upsilon_2(z)\right) \tag{2}$$

$$\tau_1 \otimes \tau_2 = \left(\mu_1(z)\mu_2(z), \upsilon_1(z) + \upsilon_2(z) - \upsilon_1(z)\upsilon_2(z)\right) \tag{3}$$

$$\tau_1^{\lambda} = \left((\mu_1(z))^{\lambda}, 1 - \left(1 - \upsilon_1(z)\right)^{\lambda}\right) 0 \leq \lambda \leq 1 \tag{4}$$

$$\lambda\tau_1 = \left(1 - \left(1 - \mu_1(z)\right)^{\lambda}, \left(\upsilon_1(z)\right)^{\lambda}\right) 0 \leq \lambda \leq 1 \tag{5}$$

Definition 3 [40]**:** The aggregation of preference information is a mapping $U^n \longrightarrow U$that is achieved using a generalized macular in symmetric mean (GMSM) operator and is given by:

$$GMSM^{(r,\lambda_1,\lambda_2,\ldots,\lambda_r)}(\tau_1,\tau_2,\ldots,\tau_n) = \left(\frac{\sum_{1\leq j_1<j_2\ldots<j_r\leq n}\left(\prod_{k=1}^{r}\tau_i^{\lambda_k}\right)}{\binom{n}{r}}\right)^{\frac{1}{\sum_{k=1}^{r}\lambda_k}} \quad (6)$$

where $1 \leq j_1 < j_2 \ldots < j_r \leq n$ refers to r integer values taken from set of$n$integers and$\lambda_1, \lambda_2, \ldots, \lambda_r \geq 0$, r is a parameter with $r = 1,2,\ldots,n$, $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

Definition 4: The Cronbach alpha coefficient is the measure of consistency (or reliability) of a set of items given by:

$$\alpha = \frac{N\bar{c}}{\overline{\sigma^2}+(N-1)\bar{c}} \quad (7)$$

where N is the number of items, $\bar{c}$ is the average inter-correlationamong items, and $\overline{\sigma^2}$ is the average variance.

3. Proposed decision framework

This section presents the core contributions of the paper by initially depicting the working model of the decision framework. Following this, methods are proposed for filling missing values, consistency check and repair, aggregation of preferences, criteria weight calculation, and ranking of alternatives. Challenges relating to MCDM under the context of engineering/technology management are presented above after a comprehensive literature analysis of articles pertaining to MCDM under engineering/technology management context. These challenges are circumvented from the contributions presented below:

3.1. Workflow of the proposed duo-stage decision framework

The workflow consists of two stages. The first stage is dedicated to filling missing values in decision matrices, checking the consistency of decision matrices, and repairing inconsistent matrices. In the second stage, these consistent matrices are aggregated using the proposed IFGMSM operator. Criteria weights are then calculated using a newly proposed programming model, and objects are prioritized using the IFS-based COPRAS method. Fig.1 depicts the workflow of the proposed framework.

3.2 Filling of missing values in the decision matrix

This section puts forward a systematic procedure for filling missing values in decision matrices under IFS context. Previously, researchers presented methods for filling missing values in IFPRs [5], [18], [41]but, to the best of our knowledge, no work has been done with respect to filling missing values in an IFS-based decision matrix. DMs generally hesitate to provide certain preference information or are uncertain about choices face confusion during the preference elicitation process, which results in missing entries in the matrix. Random value entry or binning methods [42] fill the missing information in the dataset but does not properly reflect the context of decision matrices in group decision-making scenarios. Additionally, consideration of the expertise of DMs during the process of filling missing values is important. Motivated by these challenges and to overcome them, this paper puts forward four cases.

Case 1: Of k DMs each providing a decision matrix, one decision matrix contains the missing instance. To identify the missing instance, Eq. (8) is used.

$$\tau = \prod_{j=1}^{k^*} \mu^{\lambda_j}, \prod_{j=1}^{k^*} \upsilon^{\lambda_j} \tag{8}$$

wherek^* is the number of DMs whose decision matrix contains the preference information (for the instance under consideration) with $k^* < k$.

Case 2: A particular instance is missing in all k decision matrices. To identify the missing instance, Eq. (9) is used.

$$\tau = \prod_{j=1}^{m} \mu^{\lambda_j}, \prod_{j=1}^{m} \upsilon^{\lambda_j} \tag{9}$$

where$m$ is the number of objects studied.

Case 3: Of k matrices, only one matrix contains preference information (for a particular instance taken for consideration). To fill the missing values, repeat the preference information to all $k-1$ matrices.

Case 4: The entire row or column of all k matrices contains missing values. To fill the missing values (a) row-wise, replace the values from the adjacent row, or (b) column-wise, check if the missing criterion (column) belongs to the benefit or cost zone and replace the values from the adjacent criterion from the respective zone.

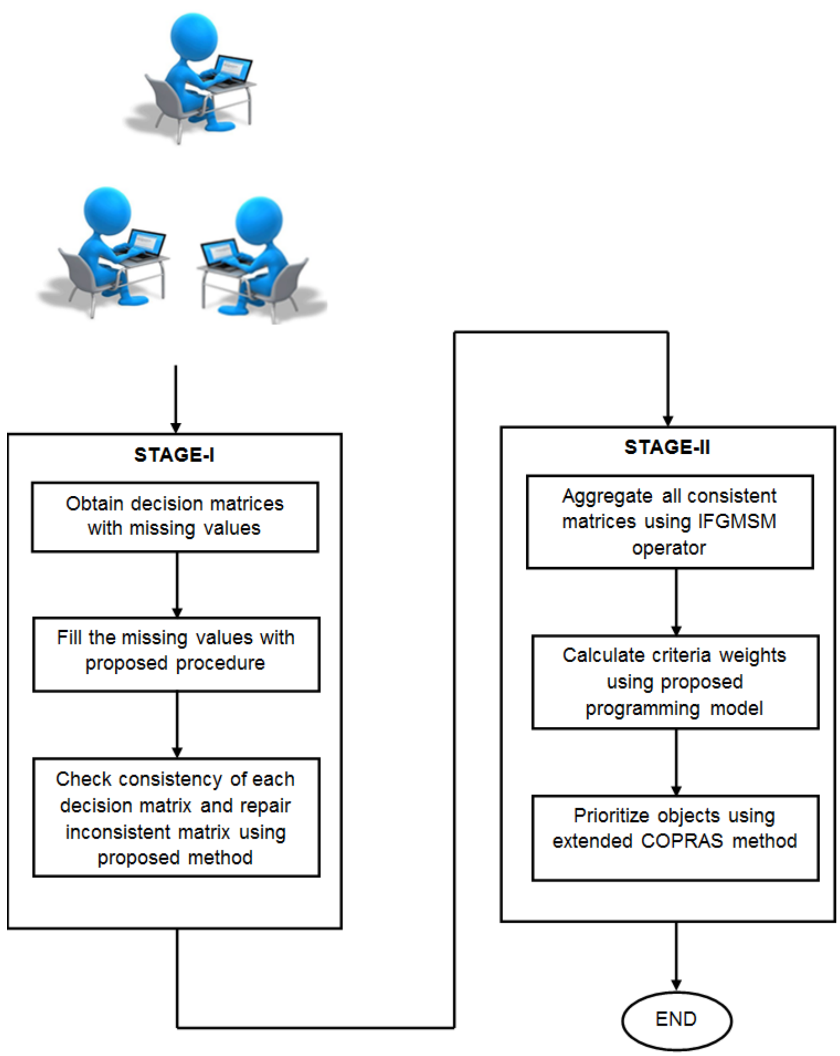

Fig.1. Workflow of the proposed duo-stage decision framework

3.3 Extension of Cronbach's alpha coefficient to IFS

This section presents a method for determining the consistency of the decision matrix. From section 1, it is clear that researchers have explored the consistency of IFPR and have ignored the consistency of IFS-based decision matrices. The Cronbach alpha coefficient is a method used for realizing the consistency (or reliability) of the data. Researchers in the field of data mining and machine learning have adopted this method for validating their input. Attracted and motivated by the power of Cronbach's alpha coefficient, this paper makes efforts to extend the method to IFS context.

This estimation helps DMs to understand the decision matrix better and also promotes rational decision making. The inter-relationship among criteria can be effectively realized using Cronbach's alpha coefficient. The coefficient is defined as follows:

Definition 5: Consider a decision matrix of order $m \times n$ where m represents the set of objects and n represents the set of criteria. IFS information is used for preference elicitation. The intuitionistic fuzzy based Cronbach alpha coefficient (IFCAC) is given by:

$$\alpha_{IFCAC} = \frac{N\overline{c(\tau_i,\tau_j)}}{1+(N-1)\overline{c(\tau_i,\tau_j)}} \tag{10}$$

Where N is the total number of elements in a decision matrix and$\overline{c(\tau_i,\tau_j)}$ is the average inter-correlation value.

The correlation between two IFVs can be determined using:

$$c(\tau_i,\tau_j) = \frac{\sum_{i,j}(\mu_i\mu_j+\upsilon_i\upsilon_j)}{2} \tag{11}$$

$$\sigma^2 = \frac{\sum_{i=1}^{m}\left(h_i(\tau)-\overline{h(\tau)}\right)^2}{m-1} \tag{12}$$

Where $h_i(\tau) = \mu + \upsilon$ is the accuracy and $\overline{h(\tau)}$ is the average accuracy value.

The consistency value is calculated for each decision matrix, and the value is compared with the consistency factor ζ, which is a parameter in the unit interval $[0,1]$. The value of ζ is determined by the DM. If $\zeta \leq \alpha_{IFCAC}$, then the decision matrix is consistent. Otherwise, efforts are made to repair consistency (see section 3.3 for details).

3.4 Repair of an inconsistent decision matrix

This section focuses on the repair of an inconsistent decision matrix and proposes an iterative procedure. In this procedure, efforts are made to transform the inconsistent matrix to a level with acceptable consistency. The matrices that fail to satisfy the consistency condition described in section 3.2 are sensibly transformed to reach an acceptable consistency level.

Definition 6: Consider a decision matrix of order $m \times n$ in which the acceptable consistencyis given by:

Let $D = (\tau_{ij})_{m\times n}$ be a decision matrix and $\overline{D} = (\overline{\tau_{ij}})_{m\times n}$ be another matrix obtained by applying Eq. (13) on D.

$$\overline{D} = \begin{cases} \bigoplus_{s=1,t=1}^{m,n}(\tau_{is} \oplus \tau_{tj}) & for\ square\ matrix \\ \bigoplus_{\substack{s=1,t=1 \\ s\neq i,t\neq j}}^{m,n}(\tau_{st}) & otherwise \end{cases} \tag{13}$$

Where τ is an IFV and $\oplus$ is an operator given in Definition 2.

Next, calculate the distance between $D$and $\overline{D}$, which is given by Eq. (14).

$$d(D,\overline{D}) = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\left(\mu_{ij} - \overline{\mu_{ij}}\right)^2 + \left(\upsilon_{ij} - \overline{\upsilon_{ij}}\right)^2 + \left(\pi_{ij} - \overline{\pi_{ij}}\right)^2\right)} \tag{14}$$

Where $\pi_{ij} = 1 - (\mu_{ij} + \upsilon_{ij})$ and $\overline{\pi_{ij}} = 1 - (\overline{\mu_{ij}} + \overline{\upsilon_{ij}})$.

$$D^{z+1} = (1-\eta)D^z \oplus \eta\overline{D^z} \tag{15}$$

Where z represents the iteration count and η is a parameter of the form $0 \leq \eta \leq 1$.

If $d(D, \overline{D}) \leq \zeta^*$, then acceptable consistency was achieved and D is considered for further evaluation. Otherwise, Eq. (15) is iteratively applied and distance is calculated using Eq. (13,14) to achieve an acceptable and consistent decision matrix. Here, ζ^* is an acceptable consistency factor.

3.5 Extension of GMSM operator to IFS

This section presents a new extension to the GMSM operator under IFS context. The operator provides flexibility to the DMs for realizing the interrelationship between criteria. The GMSM operator is a generalized operator that can realize BM, generalized BM, MSM, and HamyMean (HM) as special cases. Interesting advantages of GMSM operator include: (i) it understands the interrelationship between criteria that promotes rational decision making in a practical situation where criteria are highly interrelated and (ii) the operator is generalized and promotes high scope for DMs in the practical situation.

Motivated by the power of the GMSM operator, this paper makes efforts to extend the GMSM operator to IFS context. The IFS-based GMSM operator is defined as:

Definition 7: The aggregation of different IFVs is a mapping from $X^n \rightarrow X$ given by:

$$IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1, \tau_2, \dots, \tau_k) = \left(1 - \prod_{k=1}^{\#DM}\left(1 - \prod_{j=1}^{r} \mu_k^{\lambda_j}\right)^{w_k}\right)^{\frac{1}{\sum_{j=1}^{r}\lambda_j}}, 1 - \left(1 - \prod_{k=1}^{\#DM}\left(1 - \prod_{j=1}^{r}(1 - \upsilon_k)^{\lambda_j}\right)^{w_k}\right)^{\frac{1}{\sum_{j=1}^{r}\lambda_j}} \quad (16)$$

Where $IFGMSM$ is the IFS-based GMSM operator, r is a parameter calculated by $ceil\left(\frac{\#DM}{2}\right)$, λ_j is an additional parameter that can take values from the set $\{1,2, \dots, \#DM\}$, w_k is the weight of k^{th} DM.

Specific properties of the IFGMSM operator are presented below:

Property 1: Commutative: If τ_k^* is some possible permutation, then

$$IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1,\tau_2,\dots,\tau_k) = IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1^*,\tau_2^*,\dots,\tau_k^*).$$

Property 2: Idempotent:If $(\tau_1,\tau_2,\dots,\tau_k) = \tau$, then$IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1,\tau_2,\dots,\tau_k) = \tau$.

Property 3: Monotonicity: If τ_i^* $\forall i = 1,2,\dots,k$ is another set of IFVs with the condition $\mu_i^* \geq \mu_i$and $\upsilon_i^* \leq \upsilon_i$, then $IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1,\tau_2,\dots,\tau_k) < IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1^*,\tau_2^*,\dots,\tau_k^*)$.

Property 4:Boundedness: If τ_i $\forall i = 1,2,\dots,k$ is some set of IFVs, then $\tau^- \leq IFGMSM^{(r,\lambda_1,\lambda_2,\dots,\lambda_r)}(\tau_1,\tau_2,\dots,\tau_k) \leq \tau^+$

Where,

$\tau^- = min(\mu_i), max(\upsilon_i)$and

$\tau^+ = max(\mu_i), min(\upsilon_i)$.

Theorem 1: The aggregation of different IFS-based preference information using the IFGMSM operator also produces IFS-based preference information.

Proof: To prove the theorem, the property of IFS from Definition 1 must be satisfied. Liao and Xu [8] presented a lemma that states that $\sum_{k=1}^{\#DM} w_k x_{ij} \geq \prod_{k=1}^{\#DM}\left(x_{ij}\right)^{w_k}$. Thus, by extending the lemma, we get $0 \leq \left(1 - \prod_{k=1}^{\#DM}\left(1 - \prod_{j=1}^{r} \mu_k^{\lambda_j}\right)^{w_k}\right)^{\frac{1}{\sum_{j=1}^{r}\lambda_j}} \leq 1$ and $0 \leq 1 - \left(1 - \prod_{k=1}^{\#DM}\left(1 - \prod_{j=1}^{r}(1-\upsilon_k)^{\lambda_j}\right)^{w_k}\right)^{\frac{1}{\sum_{j=1}^{r}\lambda_j}} \leq 1$.

Next, the two inequalities are combined: $0 \leq \left(1 - \prod_{k=1}^{\#DM}\left(1 - \prod_{j=1}^{r} \mu_k^{\lambda_j}\right)^{w_k}\right)^{\frac{1}{\sum_{j=1}^{r}\lambda_j}} + \left(1 - \left(1 - \prod_{k=1}^{\#DM}\left(1 - \prod_{j=1}^{r}(1-\upsilon_k)^{\lambda_j}\right)^{w_k}\right)^{\frac{1}{\sum_{j=1}^{r}\lambda_j}}\right) \leq \sum_k w_k = 1$. Thus, the aggregation of different IFS-based preference information using an IFGMSMS operator produces IFS-based preference information.

3.6 Estimation of criteria weights with partial information

This section presents a new mathematical model for criteria weight estimation using the idea of an ideal solution. Generally, this type of weight estimation is performed when DMs have partial information about each criterion or want to express their opinion of each criterion in the form of inequality constraint. The proposed mathematical model uses partial information from the DM to calculate the weights of the criteria. Researchers previously used the AHP (analytical hierarchy process) method [23]and entropy measurements [25], [43]for criteria weight estimation and these methods work in situations where the weight information is completely unknown. Researchers have also used the mathematical model when partial weight information is known. These methods are generally complex and produce unreasonable weight values.

To circumvent this issue, we adopted the idea of an ideal solution and used a distance measure to construct the objective function. By making potential use of incomplete information, the weights of the criteria are effectively determined. The systematic procedure for determining criteria weight is presented below:

Step 1: Form a criteria weight calculation matrix of order $k \times n$ where k denotes the number of DMs and n is the number of criteria. The IFS-based preference information is used for evaluation.

Step 2: Convert these IFVs into single-valued terms using the accuracy measurement shown in Eq. (17).

$$H_{ij} = \left(\mu_{ij} + \upsilon_{ij}\right) \tag{17}$$

Where H_{ij} denotes the accuracy measurement.

Step 3: Calculate the positive and negative ideal solution (PIS, NIS) for each criterion using Eq. (18,19).

$\tau^{PIS} = max_{j\in benefit}\left(H_{ij}\right)\ or\ min_{j\in cost}\left(H_{ij}\right)$ (18)

$\tau^{NIS} = min_{j\in benefit}\left(H_{ij}\right)\ or\ max_{j\in cost}\left(H_{ij}\right)$ (19)

Where τ^{PIS} is the PIS value for each criterion and τ^{NIS} is the NIS value for each criterion. From Eq. (18,19), single valued terms are obtained for PIS and NIS. Then, select the IFVs for that corresponding term. Hence, PIS and NIS values for each criterion is an IFV.

Step 3: Use model 1, shown below, as the objective function for criteria weight estimation.

Model 1:

$$Min\ Z = \sum_{j=1}^{n} \omega_j \left(\sum_{i=1}^{k} \left(d\left(\tau_{ij}, \tau^{+}\right) - d\left(\tau_{ij}, \tau^{-}\right) \right) \right)$$

Here, $d(a,b)$ is a distance measure between two IFVs, as shown below:

$d(a,b) = 0.5\sqrt{(\mu_a - \mu_b)^2 + (v_a - v_b)^2 + (\pi_a - \pi_b)^2}$ (20)

Here, subject to constraint is given by,

$s.t. 0 \leq \omega_j \leq 1and\sum_j \omega_j = 1$.

Significant advantages of the proposed method are:

(1) The partial information provided by the DM is taken into consideration for criteria weight calculation. This allows the DM to express his/her view on each criterion.

(2) Criteria weight is also calculated by considering the nature (cost or benefit) of each criterion along with PIS and NIS values. This provides sensible and rational weight values.

(3) The proposed model reduces the inaccuracies in the decision-making process caused by direct weight elicitation for each criterion.

3.7 Extension of COPRAS method to IFS

In this section, the popular COPRAS ranking method is extended under IFS context for a suitable selection of the object from the set of objects. The method was originated by Zavadskas et al. [44], [45] and, attracted by the simplicity of the method, many researchers adapted it for solving interesting decision-making problems [46]–[48]. The COPRAS method considers both direct and proportional dependencies of each object over the criteria with their significance and utility degree. Further, the COPRAS method has the ability to handle preferences from different angles.

Motivated and attracted by the power of the COPRAS method, in this section efforts are made to extend COPRAS to IFS context. The proposed method is systematically formulated to preserve the power of IFS completely. The step-by-step procedure for the IFS-based COPRAS method is presented below:

Step 1: Obtain the aggregated matrix of order $m \times n$ from section 3.4. Here, m represents the number of objects and n represents the number of criteria. Also, the weight vector of the criteria is obtained from section 3.5.

Step 2: Identify the benefit and cost criteria and estimate the COPRAS parameters viz., maximizing index P and minimizing index R using Eq. (21,22).

$$P_i = \oplus_{j=1}^{benefit}\left(\omega_j \tau_{ij}\right) \tag{21}$$

$$R_i = \oplus_{j=benefit+1}^{cost}\left(\omega_j \tau_{ij}\right) \tag{22}$$

Where $\oplus$ is an operator defined in Definition 2, ω_j is the weight of the j^{th} criterion, and τ_{ij} is an IFV.

Step 3: Calculate the total index Q_i to determine a suitable object from the set of objects using Eq. (23):

$$Q_i = vH(P_i) + (1-v)\left(\frac{\sum_{i=1}^{m} H(R_i)}{H(R_i)\left(\frac{1}{\sum_{i=1}^{m} H(R_i)}\right)}\right) \quad (23)$$

Where v is the strategy of the DM that follows $0 \leq v \leq 1$and $H(R_i)$ is the accuracy measurement of IFS.

Step 4: Obtain the final ranking order using Eq. (24). The IFVs of each object are obtained from step 3.

$$Q_i^* = max_i(Q_i) \quad (24)$$

Where $Q_i$for the i^{th} object is obtained from step 3.

Arrange the Q_i values in descending order to obtain the ranking order. The object with high Q_i value is preferred.

Before demonstrating the practical use of the proposed framework, some of the key ideas of the proposed ranking method are discussed.

(1) The IFVs are retained to the maximum extent in the ranking process, which mitigates information loss and helps in rational decision-making.

(2) The parameters P_i andR_i are IFVs and are calculated for each object.

(3) The multiplication used in Eq. (20,21) uses the formula presented in Definition 2 to retain the IFVs in the evaluation process.

(4) Equations (22,23) are used to calculate the ranking order and to select a suitable object from the set of objects for the decision process.

4. Numerical example

This section presents a numerical example of the selection of a green supplier by a leading automobile company in India. India is the fourth largest country in terms of auto sales that grew over 9.5% in 2017. India was ranked seventh in terms of commercial vehicle manufacturing in

2017 and became a leading exporter of vehicles to other parts of the world. Automobile exports increased to 20.78% in 2018, thereby increasing opportunities for employment and economic growth. In mid-2018, production grew to 12.53%, with a total outreach of 21.95 million vehicle units. Recently, the Department of Industrial Policy &Promotion (DIPP) conducted a survey and identified that Indian industries attracted USD 19.29 billion as foreign direct investment (FDI) from 2000 to 2018.

Although there is a strong demand for automobile industries in India, environmental pollution and global warming cause terrible effects on the health of people within the nation and outside as well. Air pollution is a severe problem in India. It has been predicted that by 2022, the average life span of a person will have decreased by ten years. In a recent survey, India ranked the highest with respect to the number of polluted cities, with 13 cities in the top 20 and 33 cities in the top 100 from around the world. The north Indian region is adversely affected by air pollution and the advent of automobile industries pose a serious threat to living beings. Road transport serves as a major contributor to environmental pollution with approximately 94.5% CO_2. Approximately 70% of greenhouse gases are emitted from vehicles in metropolitan cities, of which two-thirds are from two-wheelers.

Motivated by this serious predicament, this paper outlines the selection of green suppliers for a leading automobile company in India. The board of directors of the company plans to go green to get themselves out of this negative situation. For this, they constituted a panel of three DMs viz., finance and audit personnel β_1, chief technical personnel β_2, and purchase and policy personnel β_3. These DMs thoroughly investigated different suppliers and identified eight green suppliers. Based on the pre-screening process and Delphi method, the panel finalized four green suppliers for evaluation $B = (b_1, b_2, b_3, b_4)$. These suppliers adopted green technologies and follow green standards ISO 14000 and 14001. Furthermore, the panel reviewed different possible criteria for evaluation and, based on brainstorming and voting, five criteria are shortlisted: $A = (a_1, a_2, a_3, a_4, a_5)$. a_1 is the total cost of raw material, a_2 is the level of adoption of green design, a_3 is resource utilization, a_4 is the quality of raw material, and$a_5$is trust relationship. The panel decides to use IFS information for rating green suppliers and the systematic procedure for prioritization of green suppliers is given below:

Step 1: Begin.

Step 2: Form three matrices of order 4×5 with four green suppliers and five criteria. The IFS information is adapted for rating suppliers.

Table 1 IFS based preference information from different DMs with missing data

Green supplier		Evaluation criteria				
		a_1	a_2	a_3	a_4	a_5
β_1	b_1	(0.11,0.82)	(0.00,0.00)	(0.54,0.20)	(0.76,0.20)	(0.00,0.00)
	b_2	(0.68,0.13)	(0.00,0.00)	(0.85,0.13)	(0.67,0.20)	(0.00,0.00)
	b_3	(0.62,0.21)	(0.00,0.00)	(0.78,0.14)	(0.44,0.52)	(0.00,0.00)
	b_4	(0.38,0.16)	(0.00,0.00)	(0.00,0.00)	(0.83,0.13)	(0.00,0.00)
β_2	b_1	(0.40,0.27)	(0.00,0.00)	(0.28,0.66)	(0.84,0.15)	(0.00,0.00)
	b_2	(0.36,0.61)	(0.00,0.00)	(0.18,0.30)	(0.91,0.01)	(0.00,0.00)
	b_3	(0.26,0.36)	(0.00,0.00)	(0.43,0.57)	(0.93,0.06)	(0.00,0.00)
	b_4	(0.48,0.26)	(0.00,0.00)	(0.78,0.13)	(0.00,0.00)	(0.00,0.00)
β_3	b_1	(0.14,0.32)	(0.00,0.00)	(0.29,0.62)	(0.50,0.17)	(0.00,0.00)
	b_2	(0.11,0.49)	(0.00,0.00)	(0.30,0.11)	(0.44,0.41)	(0.00,0.00)

b_3	(0.38,0.25)	(0.00,0.00)	(0.38,0.23)	(0.59,0.13)	(0.00,0.00)
b_4	(0.44,0.35)	(0.00,0.00)	(0.77,0.10)	(0.00,0.00)	(0.00,0.00)

Table 1 depicts the preference of information of different DMs. IFS information is used for rating. The entries $(0.00,0.00)$ are missing values that are filled in step 3.

Step 3: Fill the missing values in each matrix using section 3.2. Next, the consistency of each matrix is determined by adopting the procedure from section 3.3 and the inconsistent matrices are repaired using the procedure from section 3.4.

Table 2 Decision matrix with filled IFS based preference information

Green supplier	Evaluation criteria				
	a_1	a_2	a_3	a_4	a_5
β_1					
b_1	(0.11,0.82)	(0.11,0.82)	(0.54,0.20)	(0.76,0.20)	(0.76,0.20)
b_2	(0.68,0.13)	(0.68,0.13)	(0.85,0.13)	(0.67,0.20)	(0.67,0.20)
b_3	(0.62,0.21)	(0.62,0.21)	(0.78,0.14)	(0.44,0.52)	(0.44,0.52)
b_4	(0.38,0.16)	(0.38,0.16)	(0.77,0.11)	(0.83,0.13)	(0.83,0.13)
β_2					
b_1	(0.40,0.27)	(0.40,0.27)	(0.28,0.66)	(0.84,0.15)	(0.84,0.15)
b_2	(0.36,0.61)	(0.36,0.61)	(0.18,0.30)	(0.91,0.01)	(0.91,0.01)
b_3	(0.26,0.36)	(0.26,0.36)	(0.43,0.57)	(0.93,0.06)	(0.93,0.06)
b_4	(0.48,0.26)	(0.48,0.26)	(0.78,0.13)	(0.83,0.13)	(0.83,0.13)
β_3					
b_1	(0.14,0.32)	(0.14,0.32)	(0.29,0.62)	(0.50,0.17)	(0.50,0.17)

b_2	(0.11,0.49)	(0.11,0.49)	(0.30,0.11)	(0.44,0.41)	(0.44,0.41)
b_3	(0.38,0.25)	(0.38,0.25)	(0.38,0.23)	(0.59,0.13)	(0.59,0.13)
b_4	(0.44,0.35)	(0.44,0.35)	(0.77,0.10)	(0.83,0.13)	(0.83,0.13)

Table 2 depicts the IFS-based preference information from each DM and the missing values are filled using the proposed procedure from section 3.2. As an example, case 1 is applied for entry (b_4, a_3) in β_1. Similarly, other missing values are filled with their respective cases described in section 3.1. After filling the missing values, the consistency of each decision matrix is calculated using the procedure given in section 3.3 and it is given by $\alpha_{IFCAC}^{\beta_1} = 0.88$, $\alpha_{IFCAC}^{\beta_2} = 0.86$ and $\alpha_{IFCAC}^{\beta_3} = 0.77$. Since we expect 80% to 90% consistency, the matrix from DM β_3 is inconsistent. To repair this inconsistent matrix, the procedure presented in section 3.3 is followed. The repair procedure is an iterative process.It can be observed from Fig. 2 that $\eta = 0.9$requires minimum iteration to reach acceptable consistency ($d(D, \overline{D}) \leq \zeta^*$). Here, we set $\zeta^* = 0.1$. To properly select η value, a simulation analysis was performed in which 300 matrices are taken and are used to determine the acceptable consistent matrix for different η values. The iteration count values are determined for each matrix over each η value and the average is calculated for all 300 matrices. The η value with minimum iteration count (for obtaining acceptable consistent matrix) is selected. The results of the simulation analysis are depicted in Fig.2.

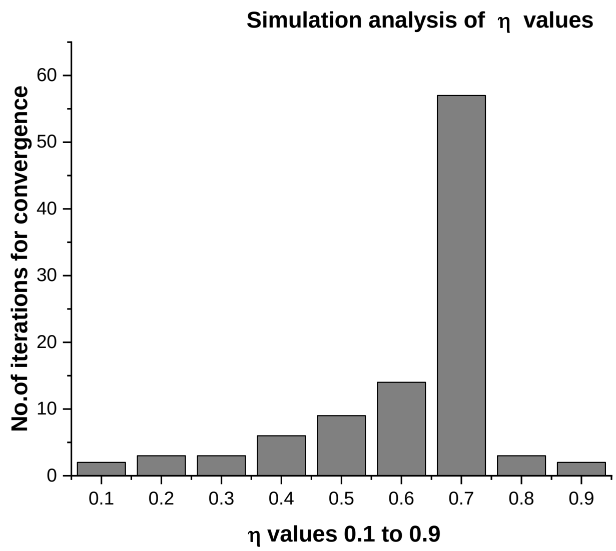

Fig.2 Selecting suitable η value using simulation analysis

Table 3 Repaired decision matrix of DM β_3

Green supplier	Evaluation criteria				
	a_1	a_2	a_3	a_4	a_5
β_3					
b_1	(0.96,0.001)	(0.96,0.0015)	(0.96,0.0015)	(0.97,0.001)	(0.97,0)
b_2	(0.96,0.001)	(0.96,0.0015)	(0.97,0)	(0.97,0.0005)	(0.97,0)
b_3	(0.97,0.001)	(0.96,0.0015)	(0.97,0.001)	(0.97,0.0007)	(0.97,0)
b_4	(0.96,0.001)	(0.95,0.0015)	(0.97,0.0007)	(0.97,0.0005)	(0.98,0)

Step 4: All consistent matrices (from step 2) are aggregated into a single matrix using the aggregation operator proposed in section 3.5.

Table 4 Aggregated matrix using IFGMSM operator

Green supplier	Evaluation criteria				
	a_1	a_2	a_3	a_4	a_5
β_{123}					
b_1	(0.82,0.0023)	(0.82,0.0032)	(0.83,0.0033)	(0.90,0.002)	(0.90,0)
b_2	(0.84,0.002)	(0.83,0.0034)	(0.87,0)	(0.91,0.001)	(0.92,0)
b_3	(0.84,0.0021)	(0.83,0.0031)	(0.85,0.0025)	(0.91,0.0014)	(0.92,0)
b_4	(0.82,0.0023)	(0.81,0.0033)	(0.89,0.0014)	(0.91,0.001)	(0.91,0)

Table 4 presents the aggregated matrix obtained using the IFGMSM operator. Here, the weights of DMs are 0.3, 0.4, and 0.3 and $\lambda_1 = \lambda_2 = 2$. Also, the decision matrix of β_1 and β_2 are considered from Table 2 and β_3 is considered from Table 3. It is observed that the aggregated information is also intuitionistic fuzzy in nature.

Step 5: A3×5evaluation matrix order is obtained for calculating the weights of each criterion using the procedure given in section 3.6.

Table 5 Evaluation matrix for criteria weights

Green supplier	Evaluation criteria				
	a_1	a_2	a_3	a_4	a_5
β_1	(0.12,0.13)	(0.25,0.37)	(0.20,0.11)	(0.35,0.40)	(0.50,0.17)
β_2	(0.23,0.59)	(0.13,0.70)	(0.90,0.010)	(0.18,0.60)	(0.31,0.19)
β_3	(0.45,0.22)	(0.80,0.10)	(0.10,0.21)	(0.55,0.41)	(0.27,0.38)

Table 6 Positive and negative ideal solution for each criterion

Ideal solution	Evaluation criteria				
	a_1	a_2	a_3	a_4	a_5
h^+	(0.23,0.59)	(0.80,0.10)	(0.90,0.01)	(0.35,0.40)	(0.31,0.19)
h^-	(0.12,0.13)	(0.25,0.37)	(0.20,0.11)	(0.55,0.41)	(0.50,0.17)

Criteria weights are determined from Table 5 and 6. Criteriaa_1, a_2, and a_3 are benefit type and the remaining are cost type. Model 1 from section 3.5 is used to determine the weights of the criteria. Here, the objective function is determined as $(-0.043)\omega_1 + (0.246)\omega_2 + (0.439)\omega_3 + (-0.094)\omega_4 + (-0.033)\omega_5$. Constraints are given by $\omega_1 \leq 0.25$, $\omega_2 \leq 0.25$, $\omega_3 \leq 0.15$, $\omega_4 \leq 0.20$ and $\omega_5 \leq 0.20$. Using optimization toolbox of MATLAB®, weights are calculated and they are given by $\omega_1 = 0.25$, $\omega_2 = 0.25$, $\omega_3 = 0.10$, $\omega_4 = 0.20$ and $\omega_5 = 0.20$.

Step 6: Prioritize the green suppliers using the proposed method given in section 3.7.

Additionally, discuss the superiority and weakness of the proposed framework by comparing it with other methods (refer to section 5).

Table 7 COPRAS parameters for ranking suppliers

Green supplier	COPRAS parameters					
	P (ub)	R (ub)	Q (ub)	P (b)	R (b)	Q (b)
b_1	(0.66,0)	(0.60,0)	5.65	(0.66,0)	(0.60,0)	5.65
b_2	(0.69,0)	(0.62,0.052)	5.10	(0.68,0)	(0.62,0)	5.09

b_3	(0.68,0)	(0.63,0)	5.42	(0.68,0)	(0.63,0.052)	5.42
b_4	(0.69,0)	(0.62,0)	5.51	(0.67,0)	(0.62,0)	5.50

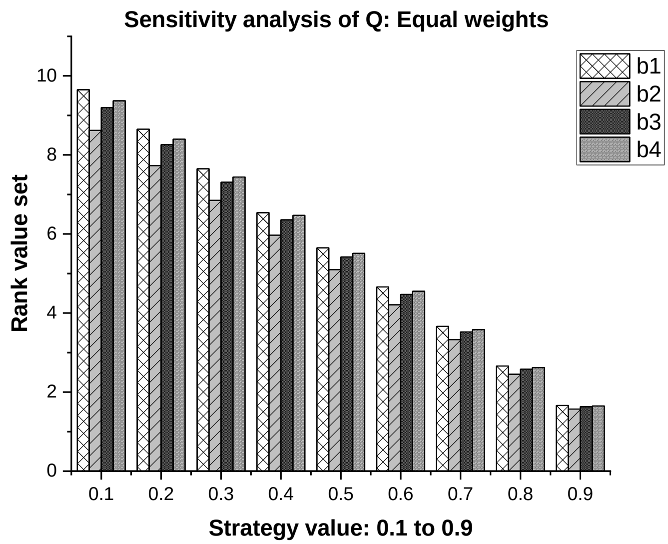

Fig.3 Sensitivity analysis: Q over strategy values with unbiased criteria weights

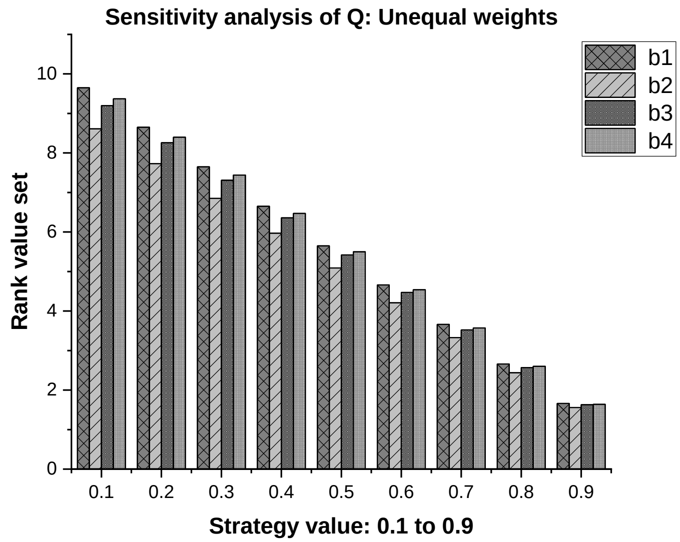

Fig.4 Sensitivity analysis: Q over strategy values with biased criteria weights

Table 7 presents the values for the proposed IFS-based COPRAS parameters for each object. In Table 7, the ranking order is given by $b_1 \succ b_4 \succ b_3 \succ b_2$for equal and unequal weight values. Furthermore, from Figs.2 and 3, we can infer that the proposed method is stable even after considerable changes are made to the strategy values.

Step 7: End.

5. Comparative analysis: proposed vs. others

A comparative analysis of the proposed decision framework with other methods is presented in this section. Three case studies [49]–[51] from the literature are used in this analysis and all these case studies use IFS information. l: These case studies are analyzed in detail:

- **Case study 1** [51]**:** A supplier selection problem is presented with three suppliers and five attributes. Three experts rate the suppliers using IFS information and weight values of each expert and attribute are provided. When suppliers are prioritized using the method discussed in [51], the result is represented as $A_2 \succ A_1 \succ A_3$ and when suppliers are prioritized using the proposed method, the result is represented as$A_2 \succ A_1 \succ A_3$ for unequal weights and $A_2 \succ A_3 \succ A_1$ for equal weights. All three matrices are consistent and hence, repair is not required. These matrices are aggregated using the proposed operator. From the work of [52], we can observe that the proposed method is *stable* even after adequate changes are made to the suppliers. Also, the proposed method is *partially stable* (rank order of highly preferred supplier is retained) even after adequate changes are made to the attributes.

- **Case study 2** [50]**:** A hazardous waste carrier selection problem is presented with five waste carriers and 14 attributes. Three experts rate the carriers using IFS information and the weights of each expert and attribute are provided. All three matrices are consistent and hence the repair procedure is not needed. The matrices are aggregated using the proposed operator. Prioritization order generated by the method proposed in [50] is represented as $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$. In contrast, the proposed method generates a prioritization order of $A_5 \succ A_1 \succ A_3 \succ A_2 \succ A_4$ for both unequal and equal weight values. This clearly shows that the proposed method is *robust* and, from [52], we observe that the proposed method is *partially stable* even after adequate changes are made to the waste carriers and attributes.
- **Case study 3** [49]**:** A sustainable supplier selection problem is presented with three suppliers and nine attributes. Three experts rate the suppliers using IFS information and the weights of each expert and attribute are provided. First, two decision matrices are consistent, and the third decision matrix is inconsistent. To make the matrix consistent, the repair procedure is adopted and, with $\eta = 0.5$, acceptable consistency is obtained. These

matrices are aggregated using the proposed operator and the prioritization order is represented as $A_3 \succ A_1 \succ A_2$ (for both the weight of unequal and equal attributes). In contrast, the prioritization order obtained from meththe od proposed in [49] is represented as $A_2 \succ A_3 \succ A_1$. From [52], we can infer that the proposed method is *robust and stability* is ensured even after adequate changes are made to the suppliers. Moreover, *partial stability* is maintained when adequate changes are made to the attributes.

This analysis identified the following advantages of the proposed method:

(1) Based on the literature analysis, it is inferred that filling of missing values in decision matrices has not been performed before. Moreover, the filling of missing values in decision matrices retains the IFS information and provides reasonable values by considering different cases of occurrence of missing values.

(2) Also, from the literature analysis, we can infer that the consistency check and repair has not been performed for decision matrices. To circumvent this challenge, Cronbach's alpha coefficient is extended for IFS context and inconsistent matrices are repaired systematically using iterative procedures.

(3) Once we obtain consistent matrices, aggregation is performed by properly capturing the interrelationship between different attributes. To address this challenge, the GMSM operator is extended to IFS context.

(4) The weights of attributes are calculated using partial information from the DMs and by considering the nature of attributes. Furthermore, objects are prioritized by extending the COPRAS method to IFS. Here, the COPRAS method is formulated in such a manner that IFS information is retained to the maximum extent possible.

(5) Finally, the practical use of the proposed method is validated using a green supplier selection problem. To further understand the superiority of the proposed method, we conducted a simulation study. According to this study, 540 decision matrices were considered. First 180 matrices are of order 3×5; the next 180 matrices are of order 4×5, and the remaining are of order 4×5. These matrices use IFS information and they are completely filled matrices. Based on the literature analysis, we identified these orders as being popular for analysis. Equal weights are considered for the attributes and consistency is evaluated for each matrix. Inconsistent matrices are repaired using $\eta = 0.5$ in an arbitrary fashion and they are made acceptably consistent. All these matrices are fed as input to the proposed ranking method and a sensitivity analysis test is conducted over the weights of the attributesand strategy value. Table 8 depicts the result of the analysis.

Table 8 Sensitivity analysis on weights and strategy values: Stability test with and without consistency repair

Order	Consistency repair (y/n)	Sensitivity analysis (Stability)	
		Weights (in %)	Strategy (in %)
3×5	n	(71.43,91.43)	(64.28,88.57)
	y	(91.43,100)	(82.86,100)
4×5	n	(72.73,90.91)	(67.53,88.31)
	y	(88.31,100)	(76.62,93,51)
5×5	n	(76.56,93.75)	(68.75,85.94)
	y	(89.06,100)	(81.25,93.75)

Note: (a,b) refers to (% of stability, % of partial stability); y refers to yes, consistency repair is adopted and n refers to no, consistency repair is not adopted; weights represent the weights of attributes (both equal and unequal weight values).

Table 8 presents the sensitivity analysis of criteria weights and strategy values. Initially, from each set (3 by 5, 4 by 4, and 5 by 5), inconsistent matrices are identified using Cronbach's alpha coefficient. In the first set (matrices of order 3 by 5), 70 matrices are identified to be inconsistent; similarly, in the second set, 77 matrices are found to be inconsistent; in the third set, 64 matrices are found to be inconsistent. These matrices are repaired using the proposed repairing procedure and the proposed ranking method is applied to these matrices for prioritization of objects.

In Table 8, we depict two scenarios viz., (a) without consistency repair and (b) with consistency repair for sensitivity analysis on both criteria and strategy values. When the ranking order remains unchanged by varying criteria values (that is, equal and unequal weights) and strategy values, the method is said to be *stable*. Thus, the values shown in Table 8 convey the stability (in %). We also depict *partial stability* (in %). When the object with high preference remains unchanged even after a considerable change of values, the method is said to be partially stable. For example, in the case of matrices of order 4 by 5, when consistency repair procedure is adopted, 68 of 77 matrices are stable (that is, stability is 88.31%). A similar idea is used in Table 8, and from this analysis, the proposed method with the consistency repair procedure outperforms the method without consistency repair.

6. Conclusions

This paper presents a new decision framework under IFS context for solving MCGDM problems. The framework consists of two stages. In the first stage, the missing values in each decision matrix are filled using the proposed procedure. Subsequently, consistency of each decision matrix is determined using the IFCAC method, and the inconsistent matrices are repaired iteratively using the repair procedure to attain acceptable consistency. Further, in the second stage, these matrices are aggregated using the proposed IFGMSM operator, and the criteria weights are calculated using a mathematical programming model that effectively uses partial information from the DMs. Then, objects are prioritized using the newly extended COPRAS method under IFS context that reasonably mitigates information loss. Finally, the proposed decision framework is validated using a green supplier selection problem, and the superiority of the proposal is demonstrated by comparison with other methods. From the analysis, we can infer that the proposed framework is stable even after adequate changes of parameters such as criteria weights and strategy value.

Before discussing the implications, let us discuss some social impacts of the proposed decision framework:

(1) The duo-decision framework provides a rich and flexible platform for rational business decisions that will help organizations promote their business in the global market.

(2) The framework provides a systematic procedure for selecting a suitable choice from the set of available choices. This not only reduces the time for decision-making but also provides mathematical justification for the selection.

(3) Organizations spend a lot of money and time on different types of decisions in their day-to-day activities, such as recruitment of personnel for the project and selection of a supplier for purchasing raw materials. The DMs can select strategies for business growth that can be effectively handled by the proposed decision framework and a rational decision.

Some managerial implications are presented below:

(1) The proposed decision framework focuses on a substantial and unresolved problem in MCGDM. To the best of our knowledge, filling of missing values and consistency check and repair procedures in decision matrices are unexplored ideas in MCGDM that are addressed for the first time in this paper.

(2) Moreover, in the second phase, these matrices are used to arrive at a rational decision by the process of aggregation, criteria weight calculation, and prioritization of objects. New methods are proposed in each stage to overcome some of the limitations of the existing methods.

(3) The decision framework is a *ready-to-use* tool that helps DMs to arrive at rational decisions effectively. The framework addresses some of the real-time problems such as (a) missing values in decision matrices due to hesitation, confusion, or pressure. Also, sometimes, completely filled matrices lack consistency because of the lack of complete knowledge about the objects and criteria.

(4) The proposed decision framework clearly supplements the decision-making process by systematically providing prioritization order.

As part of future directions, plans are being made to extend the idea of filling missing values and consistency check and repair in decision matrices under hesitant fuzzy set to the linguistic term set contexts and its variants. Also, plans are being made to integrate artificial intelligence, granular computing, and recommendation systems, with MCGDM.

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