From Particles to Constrictions: Scientific Evolution of Enhanced Criteria for Internal Stability Assessment of Soils

Buddhima Indraratna[1], Jahanzaib Israr[1,2], and late Peter R Vaughan[3]
[1]*Distinguished Professor of Civil Engineering, Director, Transport Research Centre, University of Technology Sydney, 2007 NSW, Australia.*
[2]*Department of Civil Engineering, University of Engineering and Technology Lahore, Lahore 54890, Punjab, Pakistan.*
[3]*Formerly, Professor, Department of Civil and Environmental Engineering, Imperial College London, UK.*
E-mail: Buddhima.Indraratna@uts.edu.au

ABSTRACT: Internal instability occurs when steady seepage forces erode the finer fractions from non-uniform soils along pre-existing openings such as cracks in cohesive soils and voids in non-cohesive soil to induce permanent changes in the original particle size distribution. Given that the drainage characteristics of soils are significantly influenced by the shape, packing arrangement, compaction, and size distribution of their particles, even limited erosion can markedly alter their drainage characteristics. The geometrical assessment of internal instability potential is normally conducted using classical filter retention criterion based on mere particle size distribution and without giving due consideration to the above factors. These methods would determine the risk of instability by approximating the soil's constrictions based on its particle size distribution; these constrictions are pore channels connecting neighbouring void spaces that would control both permeability and retention phenomena. However, recent advances in mathematical computations have facilitated the exact delineation of constriction sizes and the introduction of more accurate constriction based methods. This study purports to shed light on the scientific evolution of particle and constriction based methods over the past four decades, including the enhanced accuracy, reduced bias, and robustness associated with the latter. An interesting case study from our experience of using these approaches for a permeable barrier design at Bomaderry, NSW (Australia) for subsurface flow treatment is presented, and recommendations for their use by practicing engineers are made to conclude this study.

KEYWORDS: Internal instability, Granular soils, Particles sizes, Constriction sizes, Relative density

1. INTRODUCTION

Naturally abundant non-uniform granular soils are commonly used as protective filters in hydraulic and transport infrastructure where combinations of complex loading, physical disturbance, and excess pore pressure may present problems such as seepage induced internal instability (Vaughan et al. 1975; Bishop and Vaughan 1962). This is a phenomenon whereby filtrates wash through the finer fraction from the coarser fabric of non-uniform soils (e.g. broadly and gap graded) and induce permanent changes in their original geo-mechanical characteristics such as altered soil gradation, volumetric strain, and permeability, etc. As Figure 1 shows, instability may be reflected by segregation piping, suffusion, internal erosion, external erosion, backward and forward erosion, and mud-pumping and lateral ejection, etc. These processes are reported to be the major causes of the failure of hydraulic structures worldwide, contributing up to 50% of all reported failures (Israr et al. 2016; Richards and Reddy 2007) as well as significant damage to transport infrastructure (Indraratna et al. 2018). For example, the occurrence of sand boiling, embankment breaching, the formation of sinkholes in hydraulic dams, as well as ballast fouling and mud-pumping in railway tracks (Indraratna et al. 2015; Wan and Fell 2008; Alobaidi and Hoare 1996; Selig and Waters 1994; Skempton and Brogan 1994; Vaughan and Soares 1982).

1.1 Critical Review of Existing Approaches

USACE (1953) pioneered an experimental evaluation of the potential for internal instability of sand-gravel mixtures, thus recommending the optimum mixtures needed to avoid the occurrence of instability for practical purposes. Kezdi (1979) and Sherard (1979) divided the particle size distribution (PSD) curve at an arbitrary point on curve to idealize an arbitrary base-filter system. This process requires the determination of a division point that corresponds to the maximum value of (D'_{15}/ d'_{85}) on a PSD curve by the trial and error method, where D'_{15} and d'_{85} are the representative particle sizes for the filter and base soils, respectively. A soil that satisfies Terzaghi's (1922) retention criterion, i.e. ($D'_{15}/d'_{85} = 4$), is considered to be internally stable, for which a more relaxed boundary of $D'_{15}/d'_{85} = 5$ was proposed by Sherard (1979).

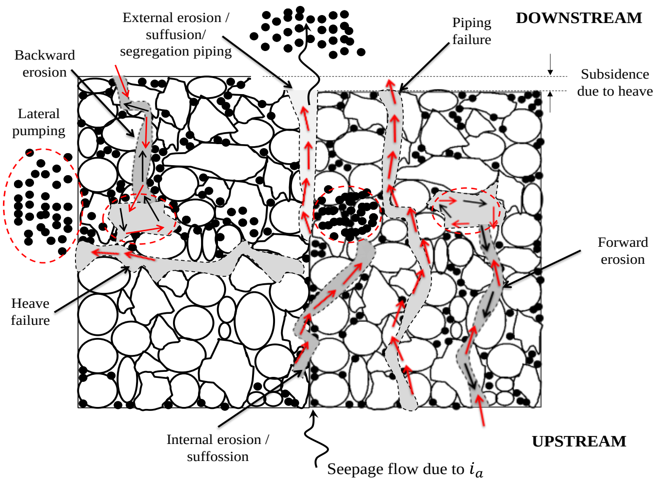

Figure 1 Illustrations of various seepage triggered instabilities in granular soils (after Israr et al. 2016, with permission from ASTM).

Kenney and Lau (1985) assessed the role of PSD in greater depth, under extreme hydraulic flow conditions accompanied by vibration, and then related the PSD to the constriction size distribution (CSD). It was proposed that an arbitrary soil particle with size D on the PSD can escape through a constriction formed by particles larger than or equal to $4D$, it may be contained by the presence of intermediate sizes between D and $4D$. The ratio $(H/F)_{min}$ was presented to assess the potential of internal instability of soils, where H represents percentages finer by mass that correspond to particle sizes between D and $4D$, where F represents the erodible fraction that corresponds to size D. The fines that erode through one constriction may be captured by finer constriction, thus experiencing local self-filtering. Furthermore, the percentage finer by mass of the erodible fraction (F) controls the potential of instability of a soil, for which Kenney and Lau (1985) assumed that erodible fines exist in the loosest state in the mix; they therefore proposed the upper limits of F subject to erosion or uniform having $C_u \leq 3$ and non-uniform soils having $C_u > 3$ as 30% and 20%, respectively.

Burenkova (1993), and lately Wan and Fell (2008), proposed identical criteria which involved different particle sizes obtained from PSD (D_5, D_{15}, D_{20}, D_{60} and D_{90}). Different zones were proposed based on the ratios D_{90}/D_{60} and D_{90}/D_{15} (Burenkova 1993) and D_{20}/D_5 and D_{90}/D_{60} (Wan and Fell 2008) to demarcate boundaries between non-suffusive (stable), suffusive and transition zones. Interestingly, Chapius (1992) comprehensively demonstrated the obvious similarity between Kenney, Kezdi and Sherard's methods, and expressed all three of them with the secant slope of PSD curve, as illustrated in Figure 2. Lately, Indraratna et al. (2015) established that Kenney and Lau's (1985) method is more accurate and conservative than the other PSD based methods.

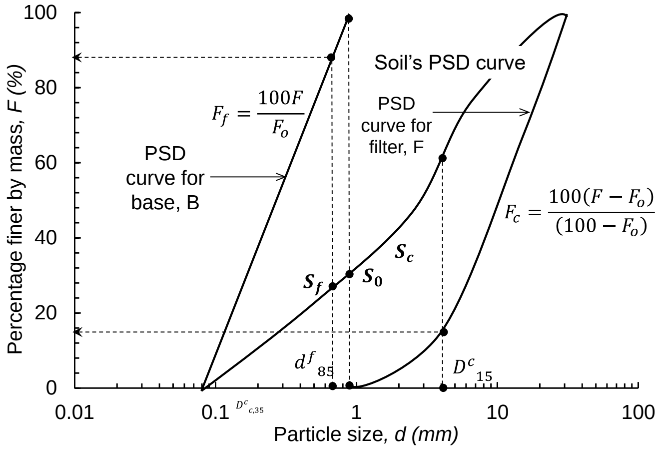

Figure 2 Similarity of PSD based criteria of internal stability assessment.

It is noteworthy that all of the above methods are based on the PSD of soils alone, and none of them are sensitive to the level of compaction of soil that could control internal stability (Israr and Indraratna 2019; Indraratna et al. 2015; Skempton and Brogan 1994). This may result in an incorrect and unsafe assessment of instability potential of some naturally abundant non-uniform soils which tend to be unstable at lower levels of compaction. For instance, Israr and Irfan (2018) recently revised the original stability boundaries of Kenney and Lau (1985) based on the level of compaction (i.e. relative density, R_d) and proposed to examine soils with $R_d \leq 70\%$ up to a percentage finer by mass F = 30%, regardless of their uniformly graded or broadly graded PSD curves (see Figure 3).

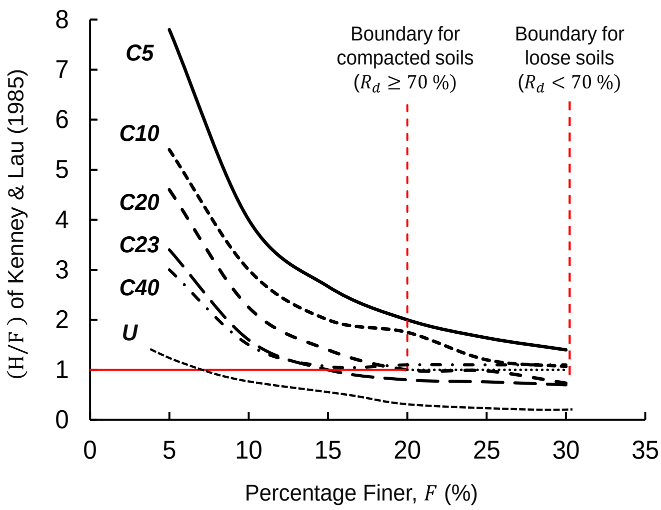

Figure 3 Original and revised stability boundaries for the method of Kenney and Lau (modified after Israr and Irfan 2018).

As a result, the rate at which this method can correctly predict the instability potential of soils could be markedly improved compared to the original method for a large published database with 108 samples. This would clearly establish that the particle size distribution and relative density govern internal stability in tandem and therefore R_d must be integrated to achieve a more accurate and robust assessment of internal instability potential. Similarly, some of the geometrically stable soils suffered from permanent changes in their PSD curves when tested under cyclic loading in the laboratory and the currently available PSD based methods proved to be unsafe in correctly capturing their instability potential (e.g. Israr and Indraratna 2018a; Israr et al. 2016; Trani and Indraratna 2010). In this study, the R_d-values were computed as the ratio between the difference of maximum void ratio e_{max} and the actual void ratio e of tested soil samples and that of e_{max} and minimum void ratio e_{min} given in percentage.

Locke et al. (2001) demonstrated that the combined effect of PSD and R_d can be captured by plotting the constriction size distribution (CSD) of a soil. The application of CSD based criteria for typical base-filter systems is well understood, whereby the CSD of a granular filter is plotted using the probabilistic approach and a well-accepted retention criterion is applied to assess whether the constrictions are fine enough to check the erosion of a protected base soil (e.g. Indraratna et al. 2007). However, evaluating the internal instability potential based on CSD requires a proper understanding of the stable coarser particles and erodible finer fraction in a given soil. For brevity, this involves the complex demarcation of a PSD curve to realize the stable coarser fraction (i.e. idealized filter) and erodible fines (i.e. base) in the subject soil, and then a well-accepted CSD based retention criterion is applied to assess whether the filter could protect the base fraction (Israr and Indraratna 2018a; Indraratna et al. 2015; Indraratna et al. 2011).

The following sections present the results from a series of internal erosion tests on various soils that conform to the typical range of protective granular filters plotted in Figure 4. Based on this analysis, a robust mechanism that will examine the correct potential of internal instability accurately is proposed, and its performance is compared to the well-known PSD based criterion of Kenney and Lau (1985) for a large published dataset. Additionally, an interesting case study from our experience with its use with a permeable barrier design at Bomaderry, NSW (Australia) for subsurface flow treatment is presented, and recommendations regarding their use by practicing engineers are made to conclude this study.

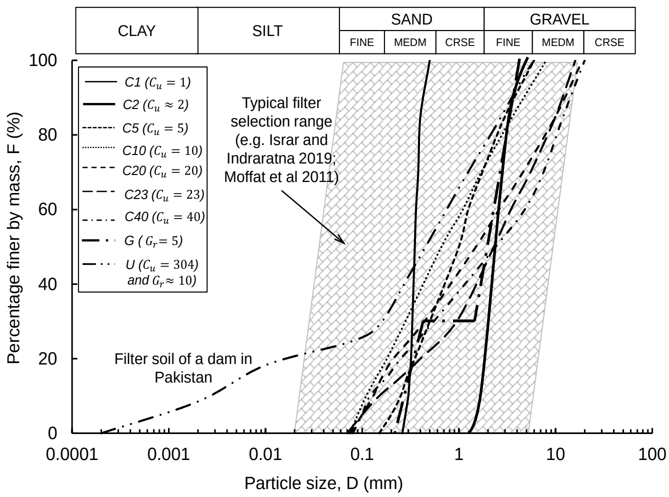

Figure 4 PSD curves of currently tested soils.

2. EXPERIMENTAL PROGRAM

A total of 33 internal erosion tests were carried out on nine granular soils with C_u ranging between 1 and 304. As can be seen from Figure 4, these soils consist of sand and sand-gravel mixtures which conform to the typical selection ranges for protective filter designs for railway substructures and hydraulic structures (Israr and Indraratna 2017; Trani and Indraratna 2010; Dounias et al. 1996; Vaughan 1994; Selig and Waters 1994). All the test specimens were compacted to relative densities (R_d) between 0 and 100% to cover all the practical scenarios from the natural deposits to the engineered fills (Kenney and Lau 1985; Vaughan and Kwan 1983). In this study the hydraulic flow is applied from the bottom of the test samples in upward direction to replicate downstream sand boiling and mud-pumping in hydraulic dams and railway substructures, respectively.

Test samples were prepared by mixing the predetermined dry weight of soil and then compacting it in multiple uniform layers inside the hydraulic cell, to achieve a target compaction level for a length of 200 mm. The target relative densities between 0 and 100% are obtained by the trial and error method while considering the limiting void ratios e_{max} and e_{min} for each soil that are determined based on ASTM D-4253 and ASTM D-4254, respectively. For brevity, the method of sample preparation by Indraratna et al. (2015) could achieve $R_d \approx 50\%$, where the soil is placed in discrete layers and then compacted by a 300 mm long metal bar of 20 mm diameter and almost 0.7 kg in weight. By using the Scott et al. (2012) method, the imparted compaction energy (E_c) is estimated to be around 270 kJ/m^3 for preparing specimens at 50% relative density. Test specimens in loosest state, i.e. $R_d \approx 0$, are prepared by the method of Skempton and Brogan (1994) that involved placement by the hands and compaction under self-weight of soil. Similarly, the densest state of compaction i.e. $R_d \approx 100\%$ could be obtained by layered compaction using standard compaction test effort (after Indraratna et al. 2018). Sample saturation is done by first de-airing them under a back pressure above 100 kPa for a sufficient time, before the de-aired and filtered water is circulated for at least 24 hours. Complete saturation to a satisfactory level occurred by obtaining Skempton's B > 0.90 through multiple pressure ramps of 10 kPa difference between the cell pressure and back pressure (after Amini and Hamidi 2014).

In this current study the uniformity and repeatability of laboratory test samples with respect to particle distribution and the level of compaction is ensured by preparing additional test samples using the above sampling procedure. For example, uniformity with respect to the particle size distribution is assessed by comparing the pre-test and post-test PSD curves of the samples. No significant changes in PSD and the coefficients of uniformity (i.e. $C_u = D_{60}/D_{10}$) for the middle layer of a stable samples could clearly show excellent repeatability and uniformity with respect to particle size distribution. Given that erosion would be partially represented by a significant loss of fines that would markedly alter the original C_u of the tested soil. For instance, C_u-value of soil C20 decreased from 20 to 5 due to the erosion of fines at the particle size at the D_{10}-level. Furthermore, uniformity with respect to the level of compaction is examined by comparing the overall dry density (γ_d) of each sample with that of the small specimens retrieved from different layers of the same soil specimen. A test specimen is characterised as uniform and free of any layering effects when its local and overall dry densities are the same and there is less than 6% of standard deviation (Israr and Indraratna 2018b; Israr et al. 2016).

As Figure 5 shows, the hydraulic test chamber has a rigid wall glass cell with a smooth surface that can accommodate a 200 mm long sample (240 mm diameter). These dimensions would eliminate any boundary effects such as wall friction and particle erosion, and the development of flow channels along the cell wall (Israr and Indraratna 2018a; Zou et al., 2013; Moffat et al., 2011). Hydraulic inflow to the test sample is applied through an automated pump at predetermined pressures, while a pressure transducer installed at the outflow pipe could monitor the effluent pressure and hence the total hydraulic head loss. The hydraulic gradient applied (i_a) to induce the erosion of fines is deduced as the ratio of the differential head loss and the length of the sample, while the eroded fines are continuously captured in a sedimentation tank for post-test sieve analysis. Similarly, to minimise any possible jetting action, the saturated soil samples are subjected to controlled increments of hydraulic gradients (Δi_a) such as for geometrically assessed stable and unstable samples, where Δi_a is approximately 0.04-0.05 and 0.02-0.025, respectively (Israr 2016). The occurrence of instability is characterised by a sudden drop in i_a accompanied by a marked rise in the effluent turbidity, i.e., much higher than 60 Nephelometeric Turbidity Units (NTU), as well as visual signs of excessive washout, piping or heave failure. The i_a-values that correspond to the instability are considered to be the critical hydraulic gradient (i_{cr}). The tested samples were recovered in multiple layers for post-test PSD analysis to compare with the original PSD curve, whereas the soils with altered gradations are considered to be internally unstable.

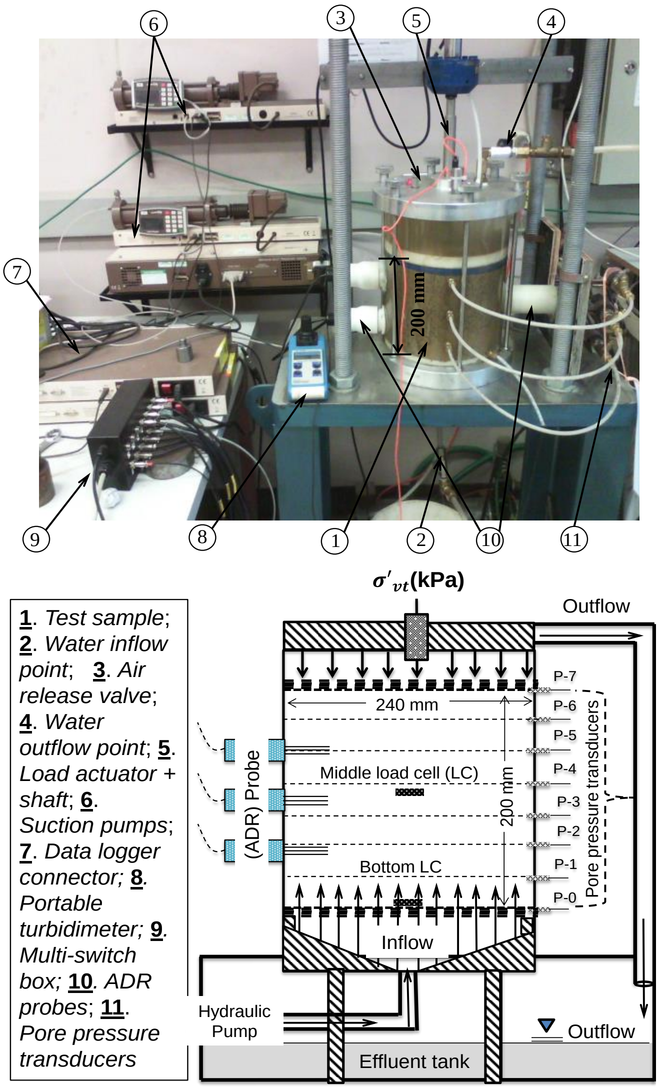

Figure 5 Details of current test setup and apparatus.

3. RESULTS AND DISCUSSION

In Figure 6 the normalised hydraulic gradient, i.e., the ratio between the observed critical hydraulic gradient i_{cr} and the classical piping theory of Terzaghi (1922) has been plotted against the relative density R_d of currently tested soil samples. The magnitude of the normalised hydraulic gradient increases proportionally with the increase in R_d, however this increase is less significant for soils with uniform C1 and the well-graded soils C5 and C10, as well as the broadly graded soil C40. Note that soils C20 and C23 exhibited marked increase in the normalised gradient from 0.47 and 0.65 at R_d = 5% to 0.9 and 0.94, respectively.

As Figure 7 shows, the mechanisms of seepage induced failure in the tested samples revealed signs of instability, such as soils C1, C2, C5 and C10 (Figure 7a) exhibited the development of heave with negligible erosion of fines, while the normalised hydraulic gradient approached unity (1.0). Similarly, the broadly graded and gap-graded soils C40 (Figure 7b), U and G experienced excessive suffusion and marked changes in their original PSD curves due to the erosion of their finer fraction. Interestingly, the seepage induced responses of soils C20 and C23 differed from the rest in that they suffered from suffusion and changes in their original gradations at $i_{cr} < 0.6$ when their R_d-values are less than 30% and 70%, respectively. However, both soils exhibited heave-piping failure at $i_{cr} > 0.8$-1.0 with no significant difference in their pre and post-test PSD curves at higher R_d-values (Fig. 7c). Based on these results; soils C1, C2, C5, C10, C20 at $R_d > 70\%$ and C23 at $R_d > 30\%$ are characterised as internally stable, while soils C20 at $R_d < 70\%$, C23 at $R_d > 30\%$, C40, G and U are characterised as unstable.

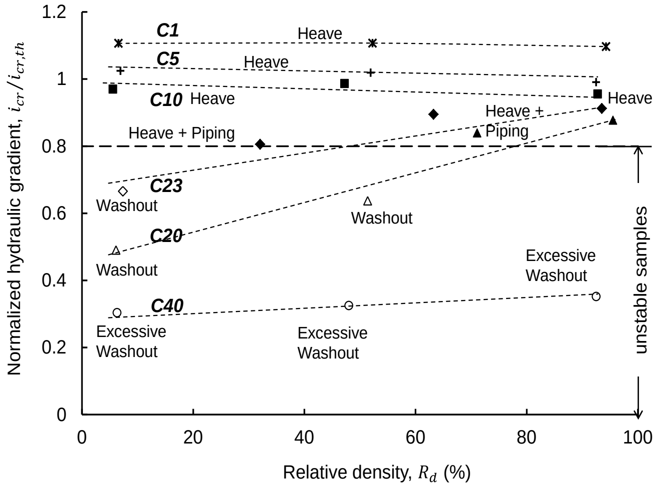

Figure 6 Compaction induced variations of normalized critical hydraulic gradients and associated seepage induced failures (adopted from Indraratna et al. 2015, with permission from ASCE).

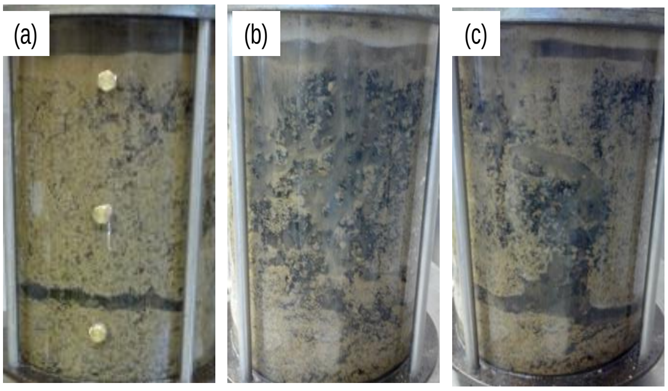

Figure 7 Illustrations of occurrence of; (a) heave in *C5* at $R_d \geq 90\%$, (b) heave in C10 at $R_d \geq 90\%$, (c) heave and piping in C20 at $R_d = 72\%$, and (d) suffusion in C40 at $R_d \geq 90\%$.

As Figure 8 shows, the largest particle sizes that were eroded and captured from the downstream sedimentation tank bear a linear correlation with the stability index, $(H/F)_{min}$ of the tested soils. This clearly indicates that all the particle sizes greater than those corresponding to $(H/F)_{min}$ on the PSD curves of tested soils would not erode and thus conform to the stable coarse fabric. Whereas the particle sizes smaller than those corresponding to the $(H/F)_{min}$ on the PSD curves represent the erodible finer fraction. Therefore, the point on the PSD curve that corresponds to the $(H/F)_{min}$ is a reasonable estimate of the demarcation point, so the portion of the curve above and below this point may be considered as an arbitrary filter and a base, respectively, for further stability analysis. Given that the erodible fines of an internally unstable soil remain freely inside the pore spaces of stable coarse fabric and acquire the loosest state of compaction with no contribution in sustainable stress transfer (Israr and Indraratna 2018b; Kenney and Lau 1985), therefore the PSD and R_d of a coarser fraction, and hence the constrictions, will control the internal stability.

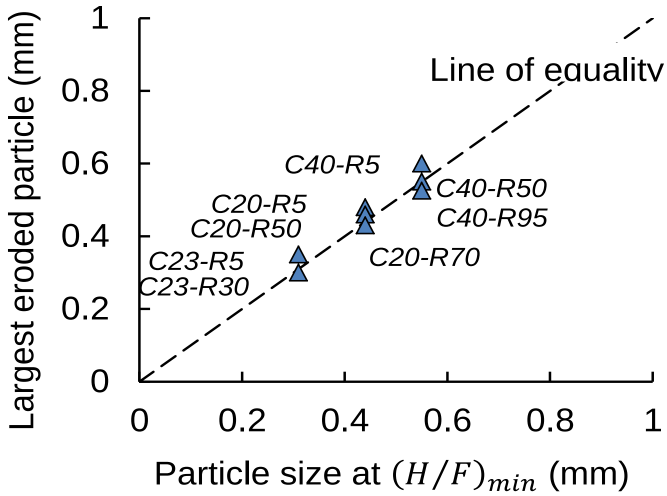

Figure 8 Correlation between sizes of the largest eroded particles from the unstable samples versus their $(H/F)_{min}$ values (modified after Indraratna et al. 2016, with permission from ASCE).

Figure 9 shows that the likelihood of the finer fraction eroding becomes maximum and the erodible distance represented by the number of penetration layers (n_l) becomes higher for fines up to 35% or finer. At 95% confidence, the steeply increasing n_l-curve for fines up to 35% finer shows that particles eroded up to a distance given by $225 \times D_m$ or beyond, will not be retained by the constrictions formed by the coarser fabric ($D_m = 0.5(D_5 + D_{15})$; where D_m and D_x represents the mean particle size and the sizes corresponding to the x percentage finer by mass). Thus, the constriction size of the coarser fraction that corresponds to 35[th] percentile finer by surface area (D^c_{c35}) would control the erosion of the finer fraction, and thus the internal stability.

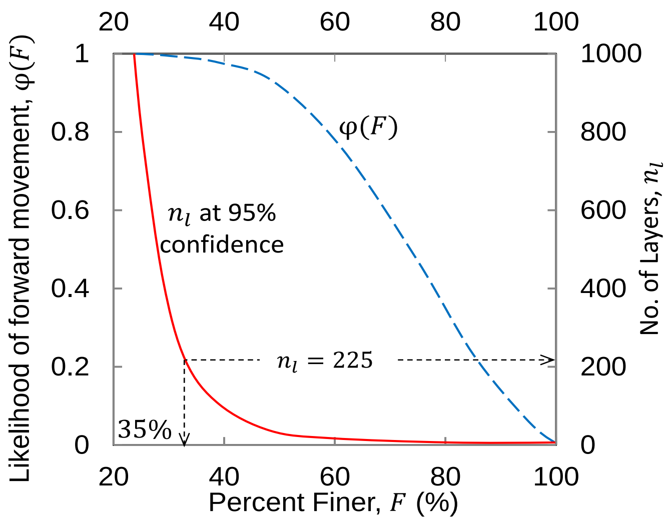

Figure 9 Determination of controlling constriction size of arbitrary coarser fraction using stochastic approach (modified after Indraratna et al. 2007, with permission from ASCE).

Figure 10 presents an interesting analysis of CSD curves plotted for the coarser fraction and the PSD curve of the finer fraction by surface area techniques for the currently tested soil C20. As shown, the constriction sizes decrease as R_d increases, and at 72% relative density and beyond, D^c_{c35} becomes increasingly finer than the controlling particle size of erodible fraction at the 85[th] percentile finer by surface area ($d^f_{85,SA}$). This shows that the soil becomes internally stable at $R_d > 70\%$, which agrees closely with the experimental results of this study.

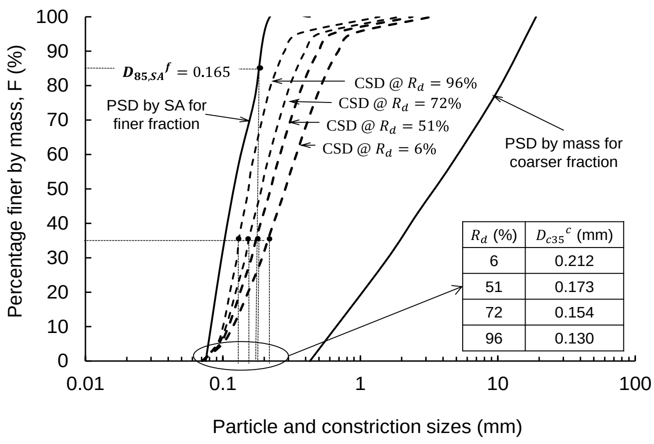

Figure 10 Variations of constriction sizes for soil C-20 with the relative density (R_d).

Table 1 Summary of calculations for hydro-mechanical assessments effectiveness and internal stability for filters F1 and F2 in protecting erodible acid sulphate soil B

Filter	Internal stability		
	$(D^c_{c35}/d^f_{85,SA})$	$i_{cr,t}$	Stable
F1	0.86	0.85	S
F2	0.97	0.85	S

Based on the above analysis, a simple but robust CSD based approach for assessing the instability potential of a granular soil is demonstrated in Figure 11.

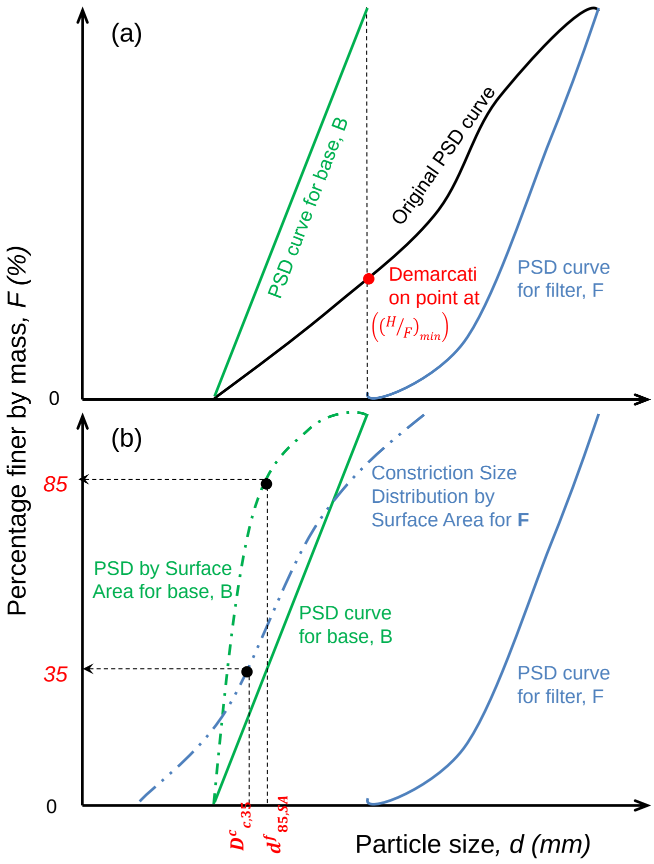

Figure 11 Proposed constriction based method for assessing internal stability of granular soils.

For brevity, it is suggested to demarcate the PSD curve of soil under examination at a point corresponding $(H/F)_{min}$ to realise stable coarser and erodible finer fractions. The PSD of the finer fraction and the CSD of the coarser fraction should be drawn using surface area techniques, and the soils meeting the following condition are considered to be internally stable:

$$D^c_{c35}/d^f_{85,\,SA} \leq 1 \quad (1)$$

To verify the proposed CSD based criterion, a large experimental database of almost 95 samples has been compiled from a number of published studies from the past four decades, as shown in Figure 12. The predictions from the current criterion are compared with those from the well-accepted and more accurate PSD based criterion of Kenney and Lau (1985).

As shown in Figure 12(a), the criterion of Kenney and Lau (1985) results in 8 incorrect assessments (6 unsafe and 2 conservative), whereas the current CSD based current criterion yields only one incorrect prediction (i.e. up to 99% success), as shown in Figure 12(b). This clearly indicates that the current approach is more rigorous and accurate than the existing PSD based criteria.

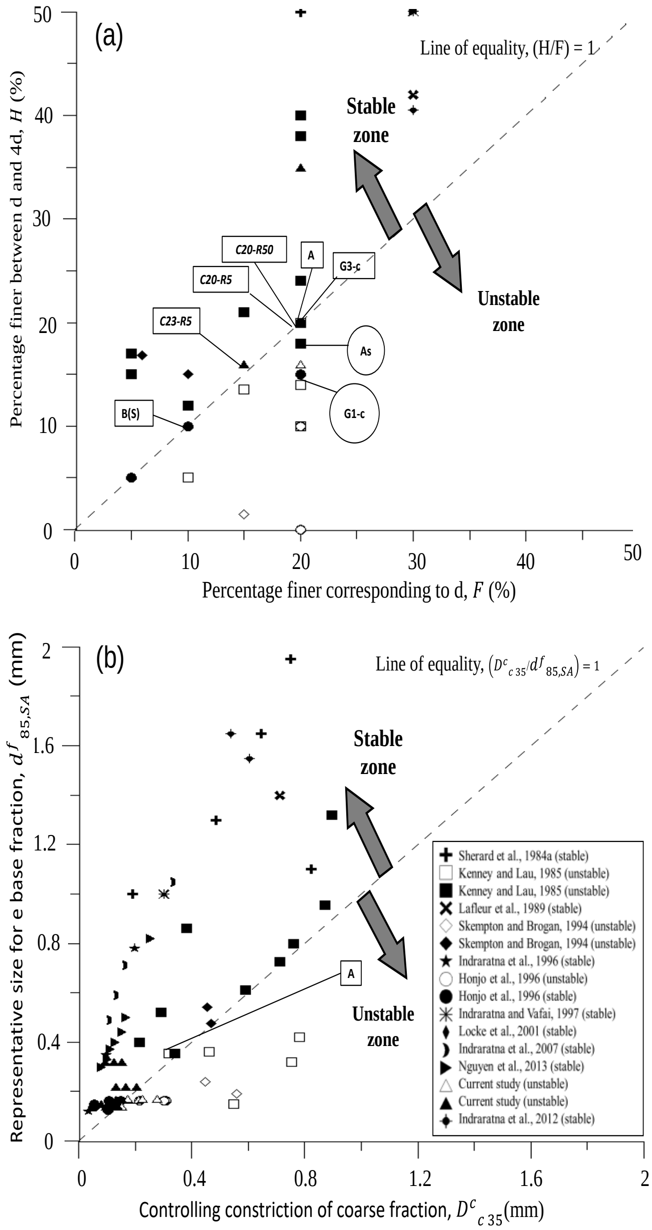

Figure 12 Comparison of predictions of internal instability potential by; (a) Kenney and Lau (1985) and (b) current constriction based method (modified after Indraratna et al. 2015, with permission from ASCE).

4. CASE STUDY: REACTIVE AND NON-REACTIVE PERMEABLE BARRIERS DESIGN AT BOMADERRY, NEW SOUTH WALES, AUSTRALIA.

As part of a joint venture between the University of Wollongong and Shoalhaven City Council in 2007, reactive permeable barriers (PRB) were installed to treat ground water contaminated with acid sulphate at a section of the Shoalhaven River at Bombaderry, NSW Australia (Figure 13). A highly dispersive clayey-silt with high sand fraction was planned to be protected by a non-reactive barrier (P1) to allow the contaminated ground water to seep through adequately, as shown in Figure 14(a).

Later, another reactive permeable barrier (P2) was installed on the downstream of P1 with a twofold objective of protecting P1 from erosion and treating the acid sulphate ground water before being disposed into the river. This would effectively prevent the P2 from clogging and hence guarantee the longevity of the treatment system, which is still functioning to date. In this study, the internal stability of the bi-layered filtration system has been reassessed using the proposed procedure.

Figure 14(b) shows the CSD curves with D^{c}_{c35} for layers P1 and P2 and the modified PSD curves with regraded representative particle sizes (d^{*}_{85}) of the protected soils (B and P1), where d^{*}_{85} is the representative particle size of the regraded curve (after Raut and Indraratna 2008). Given that the retention ratio D_{c35}/d^{*}_{85} for both P1-B and P2-P1 are less than 1, the selected filters are geometrically effective (Israr 2016).

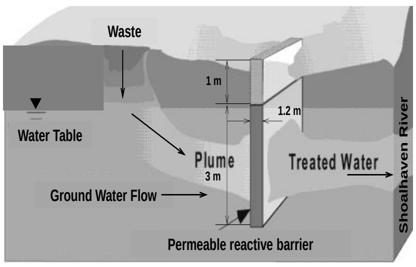

Figure 13 Schematic illustration of functioning of permeable reactive barrier (after Israr 2016).

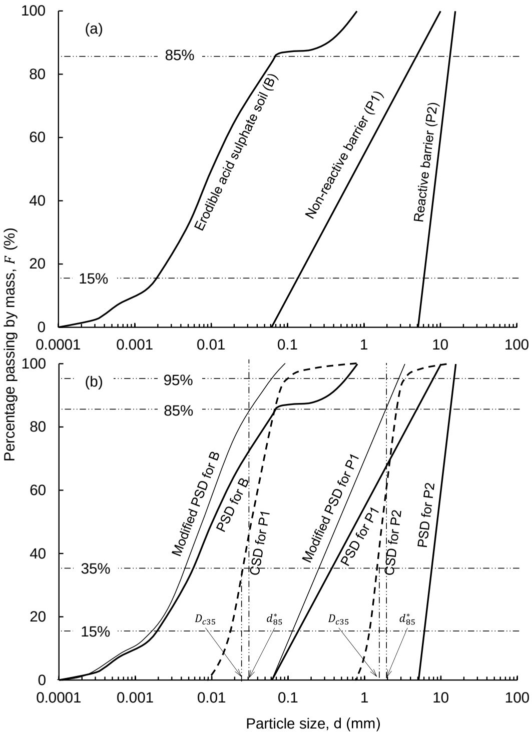

Figure 14 (a) PSDs of base (B) and filter (P1 and P2) soils, and (b) re-grading of the base soil PSDs based on dominant constriction size (D_{c95}) of filters P1 and P2.

Figure 15(a) shows the PSD curves of anticipated self-filtering layers for P1-B and P1-P2, obtained by the procedure of Indraratna and Raut (2006). The currently proposed CSD based criterion is used to examine the potential for internal instability of the self-filtering layers P1-B and P1-P2, both of whom are characterized as internally stable, as shown in Figure 15(b). In contrast, all existing criteria assess both P1-B and P1-P2 layers as internally unstable (Wan and Fell 2008; Burenkova 1993; Kenney and Lau 1985; Kezdi 1979; Sherard 1979). Note that the flow conditions are horizontal due to ground water seepage under a very mild hydraulic gradient $i_a \leq 0.01$ (Israr 2016), and the proposed method could still capture the actual behaviour accurately. Moreover, the permeable barriers are still working at full capacity, which is fully consistent with the current internal stability analysis.

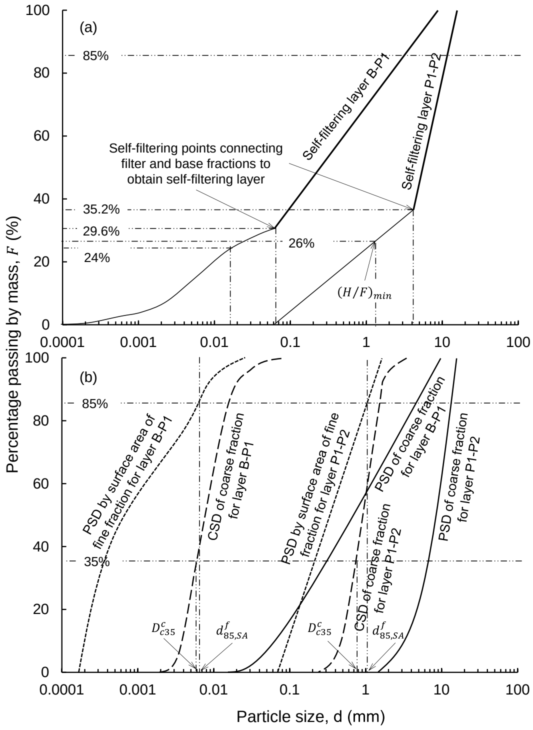

Figure 15 (a) PSDs of self-filtering layers for base filter systems B-P1 and P1-P2, and (b) PSDs of coarse and fine fractions, CSDs of coarse fraction, PSDs by surface area for fine fractions obtained after demarcating the PSDs of self-filtering layer.

5. CONCLUSIONS

The principal findings from this study are as follows:

- The particle size distribution and relative density work in tandem to govern the internal stability of soils. Soils with uniformity coefficients up to 10 showed higher internally stability and exhibited heave with no erosion at hydraulic gradients approaching unity, whereas soil with uniformity coefficients up to 23 tended to be internally stable at higher relative densities.
- The existing particle size based criteria are mostly empirically formulated, they are more prone to personal and procedural bias, and they lack a robust analytical basis. This means most of the well-accepted criteria tested in this study could not show a success rate beyond 70%, except the original criterion of Kenney and Lau (1985) which had more than 80% success rate.
- The criterion of Kenney and Lau (1985) could be improved to above 90% by revising the stability boundary based on the level of compaction. However, no existing PSD based method could still show 100% success, it may only be obtained from the constriction size based method.
- The proposed CSD based method demarcates a given PSD curve in arbitrary coarser and finer fractions at a point corresponding

to $(H/F)_{min}$ and examines the capability of controlling constriction size formed by the former in retaining the representative particle size of the latter. A large published experimental dataset from various studies could successfully verify the rigor of proposed method which showed 99% success.

- A real life practical example of permeable barrier design at the Shoalhaven River in Bomaderry, NSW Australia, was successfully used to verify the application and rigor of the proposed CSD based criterion. This also illustrated that the proposed method is applicable for horizontal flow conditions. Nevertheless, further research under dynamic loading in transport infrastructures and complex stress states in hydraulic dams is needed to extend the scope of this study for more practical cases.

6. ACKNOWLEDGEMENT

A significant amount of work reported in this paper was initially inspired by late Professor Peter Vaughan through his detailed discussions and correspondences with the first author, Prof Buddhima Indraratna. Professor Vaughan's strong belief that internal instability of a material under increasing hydraulic forces should also be a function of constrictions and the degree of particle interlock (i.e. not just particle sizes) epitomized the subsequent research on granular filtration and internal instability of geo-materials conducted at the University of Wollongong producing 6 PhDs under the guidance of the first author. In this regard, the doctoral research work of F. Vafai, M. Locke, A. Raut, D. Trani, Vo-Trong and J. Israr (second author) is acknowledged. The authors are indebted to Professor Antonio Gens for his insightful review and suggestions for improving this paper prior to publishing. The authors also appreciate both ASCE and ASTM for granting permissions to reuse some of the modified figures from previously published works of the authors. Furthermore, the second author acknowledges the financial support from University of Engineering and Technology Lahore Pakistan and the University of Wollongong (UOW) Australia in the form of Faculty Development Program Scholarship for his PhD and during his Post-doctorate, respectively. The assistance of the laboratory staff at UOW is appreciated.

7. REFERENCES

Alobaidi, I., and Hoare, D. J., (1996) "The development of pore water pressure at the subgrade-subbase interface of a highway pavement and its effect on pumping of fines." Geotextiles and Geomembranes. 14, Issue 2, pp111-135.

American Society for Testing and Materials (ASTM). (2006) "Standard test methods for maximum index density and unit weight of soils using a vibratory table," *ASTM* D4253-06, West Conshohocken, Pa.

American Society for Testing and Materials (ASTM). (2006) "Standard test methods for minimum index density and unit weight of soils and calculation of relative density." *ASTM* D4254-00, West Conshohocken, Pa.

Amini, Y., and Hamidi, A., (2014) "Triaxial shear behavior of a cementtreated sand–gravel mixture." Journal of Rock Mechanics and Geotechnical Engineering, 6, Issue 5, pp455–465 (DOI: 10.1016/j.jrmge.2014.07.006).

Bishop, A. W., and Vaughan, P. R., (1962) "Selset reservoir: design and performance of the embankment." Proceedings of the Institution of Civil Engineers, 21, Issue 2, pp305-346.

Burenkova, V. V., (1993) "Assessment of suffusion in non-cohesive and graded soils." Filters in geotechnical and hydraulic engineering, J. Brauns, M. Heibaum, and U. Schuler, eds., Bakema, Rotterdam, The Netherlands, pp357-360.

Chapius, R. P., (1992) "Similarity of internal stability criteria for granular soils." Canadian Geotechical Journal, 29, pp711-713.

Dounias, G. T., Potts, D. M., and Vaughan, P. R., (1996) "Analysis of progressive failure and cracking in old British dams." Geotechnique, 46, Issue 4, pp621–640.

Honjo, Y., Haque, M. A., and Tsai, K. A., (1996) "Self-filtration behavior of broadly and gap-graded cohesionless soils." Geofilters'96, J. Lafleur and A. Rollin, eds., Bitech Publications, Montréal, pp227–236.

Indraratna, B., and Vafai, F., (1997) "Analytical model for particle migration within base soil-filter system." Journal of Geotechnical and Geoenvironmental Engineering, 123, Issue 2, pp100-109 (DOI: 10.1061/(ASCE)1090-0241(1997)123:2(100))

Indraratna, B., Israr, J., and Li, M., (2018) "Inception of Geohydraulic failures in granular soils-An Experimental and Theoretical Treatment." Geotechnique, 68, Issue 3, pp233-248.

Indraratna, B., Israr, J., and Rujikiatkamjorn, C., (2015) "Geometrical method for evaluating the internal instability of granular filters based on constriction size distribution." Journal of Geotechnical and Geoenvironmental Engineering, 141, Issue 10, 04015045.

Indraratna, B., Nguyen, V. T., and Rujikiatkamjorn, C., (2011) "Assessing the potential of internal erosion and suffusion of granular soils." Journal of Geotechnical and Geoenvironmental Engineering, 137, Issue 5, pp550-554.

Indraratna, B., Nguyen, V. T., and Rujikiatkamjorn, C., (2012) "Hydraulic conductivity of saturated granular soils determined using a constrictionbased technique." Canadian Geotechnical Journal, 49, Issue 5, pp607–613.

Indraratna, B., Raut, A. K., and Khabbaz, H., (2007) "Constriction-based retention criterion for granular filter design." Journal of Geotechnical and Geoenvironmental Engineering, 133, Issue 3, pp266-276 (DOI: 10.1061/(ASCE)1090-0241(2007)133:3(266)).

Indraratna, B., Vafai, F., and Dilema, E., (1996) "An experimental study of the filtration of a lateritic clay slurry by sand filters." Proceedings of Institution Civil Engineers, Geotechnical. Engineering Journal, 119, Issue 2, pp75–83.

Israr, J., (2016) "Internal instability of granular filters under cyclic loading." Ph.D. Thesis, Ph.D. thesis, University of Wollongong, Wollongong, Australia.

Israr, J., and Indraratna, B., (2017) Internal Stability of Granular Filters under Static and Cyclic Loading, Journal of Geotechnical and Geoenvironmental Engineering, 143, Issue 6, 04017012.

Israr, J., and Indraratna, B., (2018a) "Mechanical Response and Pore Pressure Generation in Granular Filters subjected to Uniaxial Cyclic Loading." Canadian Geotechnical Journal, 55, Issue 12, pp1756-1768 (DOI: 10.1139/cgj-2017-0393).

Israr, J., and Indraratna, B., (2018b) "Assessment of internal stability of filters under static and cyclic loading: an experimental and theoretical treatment." Australian Geomechanics Society Journal, 53, Issue 4, pp102-115.

Israr, J., and Indraratna, B., (2019). "Study of critical hydraulic gradients for seepage induced failures in granular soils" Journal of Geotechnical and Geoenvironmental Engineering, 145, Issue 7, 04019025 (DOI: 10.1061/(ASCE)GT.1943-5606.0002062).

Israr, J., and Irfan, M., (2018) "Effects of Compaction on Internal Stability of Granular Soils: An Experimental Evaluation of Particle-Based Methods." Journal of Testing and Evaluation, 41, Issue 1, pp103–116 (DOI: 10.1520/JTE20170321).

Israr, J., Indraratna, B. and Rujikiatkamjorn, C., (2016) "Laboratory Investigation of the filtration of Granular Soils under Static and Cyclic Loading." Geotechnical Testing Journal, 39, Issue 5, pp795-812 (DOI: 10.1520/GTJ20150288).

Kenney, T. C., and Lau, D., (1985) "Internal stability of granular filters." Canadian Geotechical Journal, 22, pp215–225.

Kezdi, A. (1979). Soil physics, Elsevier Scientific, Amsterdam, The Netherlands.

Lafleur, J., Mlynarek, J., and Rollin, A., (1989) "Filtration of broadly graded cohesionless soils." Journal of Geotechnical Engineering, 115, Issue 12, pp1747-1768 (DOI: 10.1061/(ASCE)0733-9410(1989)).

Locke, M., Indraratna, B., and Adikari, G., (2001) "Time-dependent particle transport through granular filters." Journal of Geotechnical and Geoenvironmental Engineering, 127, Issue 6, pp521–529.

Moffat, R., Fannin, R. J., and Garner, S. J., (2011) "Spatial and temporal progression of internal erosion in cohesionless soil." Canadian Geotechical Journal, 48, Issue 3, pp399–412.

Nguyen, V. T., Rujikiatkamjorn, C., and Indraratna, B., (2013) "Analytical solutions for filtration process based on constriction size concept." Journal of Geotechnical and Geoenvironmental Engineering, 123, Issue 2, pp1049-1061 (DOI: 10.1061/(ASCE)GT.1943-5606.0000848)

Raut, A. K., and Indraratna B., (2008) "Further advancement in filtration criteria through constriction-based techniques." Journal of Geotechnical and Geoenvironmental Engineering, 134, Issue 6, pp883-887.

Richards, K. S. and Reddy, K. R., (2007) "Critical appraisal of piping phenomena in earth dams." Bulletin of Engineering Geology and Environmental, 66, Issue 4, pp381-402.

Scott, B., Jaksa, M., and Kuo, Y. L. (2012) "Use of proctor compaction testing for deep fill construction using impact rollers." Ground Improvement and Ground Control, B. Indraratna, C. Rujikiatkamjorn, and J. S. Vinod, eds., Woloongong, Australia, pp1107-1112.

Selig, E. T. and Waters, J. M., (1994) Track geotechnology and substructure management. London: Thomas Telford.

Sherard, J. L., (1979) "Sinkholes in dams of coarse broadly graded soils." Proceedings of 13th Congress on Large Dams, New Delhi, Issue 2, pp25-35.

Sherard, J., Dunnigan, L., and Talbot, J., (1984a) "Basic properties of sand and gravel filters." Journal of Geotechnical and Geoenvironmental Engineering, 110, Issue 6, pp684-700 (DOI: 10.1061/(ASCE)0733-9410(1984)).

Skempton, A. W., and Brogan, J. M. (1994). "Experiments on piping in sandy gravels." Geotechnique, 44(3), 449–460.

Terzaghi, K., (1922). Failure of dam foundations by piping and means for preventing it (in German). Die Wasserkraft, Zeitschrift fur die gesamte Wasserwirtschaft., 17, Issue 24, pp445–449.

Trani, L. D. O. and Indraratna, B., (2010). "Assessment of Subballast Filtration under Cyclic Loading." Journal of Geotechnical and Geoenvironmental Engineering, 136, Issue 11, pp1519-1528.

Unites States Army Corps of Engineers USACE. (1953). "Investigation of filter requirements for underdrains." Tech. Memo. No. 3–360, U.S. Waterways Experiment Station, Vicksburg, Miss.

Vaughan, P. R. and Soares, H., (1982) "Design of filters for clay cores of dams." Journal of Geotechnical Engineering Division, 108, Issue 1, pp17-31.

Vaughan, P. R., (1994) "Criteria for the use of weak and weathered rock for embankment fill, and its compaction control." State-of-the-art address. Proceedings of 13th International Conference on Soil Mechanics and Foundation Engineering, Delhi, 6, pp195–206.

Vaughan, P. R., and Kwan, C.-Y., (1983) "Weathering, structure and insitu stress in residual soils." Geotechnique, 34, Issue 1, pp43-59.

Vaughan, P. R., Lovenbury, H. T., and Horswill, P., (1975) "The design, construction and performance of Cow Green embankment dam." Geotechnique, 25, Issue 3, pp555–586.

Wan, C. F., and Fell, R., (2008) "Assessing the potential of internal instability and suffusion in embankment dams and their foundations." Journal of Geotechnical and Geoenvironmental Engineering, 134, Issue 3, pp401–407.

Zou, Y., Chen, Q., Chen, X., and Cui, P., (2013) "Discrete numerical modelling of particle transport in granular filters." Computers and Geotechnics, 47, pp48–56.