

UNIVERSITY OF TECHNOLOGY SYDNEY  
Faculty of Engineering and Information Technology

## **Cooperation of Multi-agent Systems**

by

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## Certificate of Authorship/Originality

I declare that this thesis is submitted in satisfaction of the requirements for the degree of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

I certify that this thesis has been written by me. In addition, I also certify that all information sources and literature used are quoted in the thesis. This document has not been submitted for qualifications at any other academic institution.

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# ABSTRACT

## Cooperation of Multi-agent Systems

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The cooperation of multi-agent systems represents that a group of agents complete the common tasks that are difficult or impossible for an individual agent or a single system to finish.

The cooperation of multi-agent systems has received considerable attention and has been widely studied over the past few years. The consensus is the basis of the cooperation of multi-agent systems.

Researching the cooperation of multi-agent systems probably involves a large number of theories, such as graph theories and stability theories of switched systems.

In this thesis, we study consensus, tracking control, and containment control under a fixed graph. For a time-invariant network of multi-agent systems, its topological characteristic can be described by a fixed graph.

For the time-variant network, we model the multi-agent systems by switched systems in this thesis and get some new conclusions. We transform the consensus problems into stability problems first and then get some new conclusions by the aid of conventional methods.

For traditional approaches, there are many limitations. Many of the involved theories also need to be further improved. For example, the stability problems of switched systems have not yet been thoroughly resolved. Therefore, we also do much research work on stability analysis of switched systems.

For switched systems, we propose some novel theories and methods to reduce the limitations of conventional stability analysis methods. We propose some sequence-

based methods to resolve the stability problems and get some new results. It is proved that the switched systems are globally uniformly asymptotically stable when the sequence-based mode-dependence average dwell time satisfies the conditions deduced by this thesis.

We use the proposed stability theories and methods to analyse the consensus of multi-agent systems under switched systems and get some new results. A proposed model transformation can transform consensus problems into stability problems. We get some novel results.

In this thesis, we first introduce the background and then give a brief literature review on the stability analysis of switched systems and the cooperation of multi-agent systems. After that, Chapter 3 addresses multi-agent systems under a fixed topology. The research on the stability of switched systems is presented in Chapter 4. A consensus of second-order multi-agent systems under switched topologies based on the sequence is studied in Chapter 5.

The research plan, progress, and our publications on this research topic are also shown in this report. Finally, we summarize my past work and give a conclusion.

## Dedication

To my parents

To my wife

To my child

Thank you for your love and support.

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## List of Publications

### Journal Papers

- J-1. **D. Zheng**, H. Zhang and J.A. Zhang, W. Zheng, S.W. Su. “Stability of asynchronous switched systems with sequence-based average dwell time approaches,” *Journal of the Franklin Institute*, 2020, 357(4): 2149-2166.
- J-2. H. Gao, H. Zhang, **D. Zheng**, L. Zhang, Y. Li. “Finite-time event-triggered extended dissipative control for discrete time switched linear systems,” *International Journal of General Systems*, 2019, 48(5):476–491.
- J-3. **D. Zheng**, H. Zhang, J.A. Zhang, Y. Li. “Consensus of the second-order multi-agent systems under asynchronous switching with a controller fault,” *International Journal of Control, Automation and Systems*, 2019, 17(1):136–144.
- J-4. **D. Zheng**, H. Zhang, J.A. Zhang, G. Wang. “Consensus of multi-agent systems with faults and mismatches under switched topologies using a delta operator method,” *Neurocomputing*, 2018, 315:198–209.
- J-5. Y. Li, H. Zhang, **D. Zheng**, Y. Wang. “Asynchronous  $H_\infty$  control of discrete-time switched TS fuzzy systems with dwell time,” *International Journal of Fuzzy Systems*, 2018, 20(4):1098–1114.
- J-6. **D. Zheng**, H. Zhang, Q. Zheng. “Consensus analysis of multi-agent systems under switching topologies by a topology-dependent average dwell time approach,” *IET Control Theory and Applications*, 2017, 11(3):429–438.
- J-7. Q. Zheng, H. Zhang, **D. Zheng**. “Stability and asynchronous stabilization for a class of discrete-time switched nonlinear systems with stable and unstable subsystems,” *International Journal of Control, Automation and Systems*, 2017, 15(3):986–994.

**Conference Papers**

- C-1. **D. Zheng**, H. Zhang. “ Research on the transformation of control protocols among three kinds of cooperative control for multi-agent systems,” *International Conference on Intelligent Human-Machine Systems and Cybernetics*, chengdu, 2016, 301–304



## Abbreviation

ADT: Average Dwell Time

CQLF: common quadratic Lyapunov function

DT: Dwell Time

GUES: Globally Uniformly Exponential Stability

MDADT: Mode-dependent Average Dwell Time

MLF: multiple Lyapunov functions

SBASDT: sequence-based average subsequence dwell time

SBAPDT: sequence-based average preceding dwell time

UAVs: Unmanned Air Vehicles

# Nomenclature and Notation

Capital letters denote matrices.

Lower-case alphabets denote column vectors.

$\cdot^T$  denotes the transpose operation.

$\rightarrow$  denotes going to.

$I_n$  is the identity matrix of dimension  $n \times n$ .

$0_n$  is the zero matrix of dimension  $n \times n$ .

$0_{c \times d}$  is the zero matrix of dimension  $c \times d$ .

$\mathbb{R}$ ,  $\mathbb{R}^+$  denote the field of real numbers, and the set of positive reals, respectively.

$\mathbb{Z}$ ,  $\mathbb{Z}^+$  denote the field of integers, and the set of positive integers, respectively.

$\mathbb{N}$  denotes the field of natural numbers,  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ .

$t$  is time.

$x$  is the state.

$f$  stands for functions.

$\sigma(t)$  stands for a switching signal.

$\sigma(k)$  stands for a switching signal.

$m$  is the number of subsystems or topologies.

$\mathcal{M}$  is the set of subsystems or topologies.

$\mathcal{S}$  is the set of stable subsystems or of cases that agents may reach a consensus.

$\mathcal{U}$  is the set of unstable subsystems or of cases that agents cannot reach a consensus.

$C^1$  is the space of continuously differentiable functions.

$\mathbb{R}^N$  denotes N-dimensional Euclidean space.

$t_1, t_2, t_3, \dots, t_i, t_{i+1}, \dots$  or  $k_1, k_2, k_3, \dots$  are the switching times.

$t_i^-$  represents the time approaches time  $t_i$  from the left side.

$\otimes$  represents the Kronecker product of two matrices.

$T_\uparrow$  is the time set when the Lyapunov-like function increases in certain time section.

$T_\downarrow$  is the set of time which is not in  $T_\uparrow$  in certain time section.

$\mathcal{I}$  is the set of agent indexes.

$\mathcal{V}$  is the set of nodes.

$\mathcal{E}$  is the set of edges.

$\mathcal{A}$  is the corresponding adjacency matrix.

$\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$  is a directed graph describing the topology of multi-agent systems.

$\mathcal{N}_i$  stands for the set of neighboring node  $v_i$ .

$L$  stands for the Laplacian matrix.

$H$  stands for a Laplacian-like matrix.

$\mathcal{R}(u)$  denotes the real part of a complex number  $u$ .

$\mathcal{I}(u)$  denotes the imaginary part of a complex number  $u$ .

$Dist(x, \mathcal{C})$  denotes the distance from  $x \in \mathbb{R}^N$  to the set  $\mathcal{C} \subseteq \mathbb{R}^N$ .

$\mathcal{F}$  denotes the set of the followers.

$\mathcal{R}$  denotes the set of the leaders.

$\times$  represents multiplication or Cartesian product of sets.

$>$  (or  $<$ )  $0$  represents that a matrix is positive (or negative) definite and symmetric, or that the value of a scalar is more (or less) than  $0$ .

$[p|q]$  denotes the situation when the  $p^{th}$  subsystem is aroused immediately after the  $q^{th}$  subsystem.

$T_{\downarrow}(t_l, t_{l+1})$  and  $T_{\uparrow}(t_l, t_{l+1})$  denote the unions (or the length of the unions) of the decreasing and increasing time (including unchanging time) of the Lyapunov function within the interval  $[t_l, t_{l+1})$ .

$\Delta_{\sigma(t_l)}(t_l, t_{l+1})$  represents the asynchronous time between the time slots  $[t_l, t_{l+1})$ .

$\nabla_{\sigma(t_l)}(t_l, t_{l+1})$  represents the synchronous time between the time slots  $[t_l, t_{l+1})$ .

$T_p(0, t)$  denotes the running time of the  $p^{th}$  topology between the time slots  $[0, t)$ .

$\sigma'(t)$  denotes the feedback controller switching signal for asynchronous switching.

$\Delta$  denotes the time lag between  $\sigma(t)$  and  $\sigma'(t)$  for asynchronous switching.

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