TOPSIS and Modified TOPSIS: A Comparative Analysis

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Abstract:

Selection of an appropriate Multiple Attribute Decision Making (MADM) method for providing a solution to a given MADM problem is always challenging endeavour. The challenge is even greater for situations where for a specific MADM problem there exist multiple MADM methods with similar degree of suitability. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and its dominant variant the Modified TOPSIS methods are two very similar methods applicable to the same type of MADM problems. This study provides extensive simulation-based comparisons and mathematical analysis of these two popular methods in order to clarify the confusion regarding their selection for solving MADM problems.

Keywords: TOPSIS, Modified TOPSIS, Euclidean distance, Simulation comparison, MADM, Method selection.

1. Introduction:

Selecting appropriate MADM methods is always a challenging task [1]. The need for comparative comparison for methods during selection has been highlighted in studies[2-4]. Within the Multiple Attribute Decision Making (MADM) domain, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [5, 6] is highly regarded, applied and adopted MADM method. A variant of TOPSIS named modified TOPSIS was developed with the argument about how the attribute weight should be applied while solving MADM problems [7]. Both TOPSIS and modified TOPSIS have been applied for problem solving by various researchers.

TOPSIS has been used extensively with over 13000 citations [8] for practical MADM problem due to its sound mathematical foundation, simplicity, ease of applicability [9]. TOPSIS has inspired scores of new methods and comparative analysis based on it [10, 11]. TOPSIS has been widely used in areas such as purchase decisions and outsource provider selection [12, 13], manufacturing decision making [14, 15], financial performance analysis [16], service quality assessment [17], educational selection applications [18], strategy evaluation [19], and critical mission planning [20].

With close to 1000 citations [21] Modified TOPSIS has been used for comparative studies in attribute weight estimations [22]. The study is also applied in resource management [23], software selection [24], environmental assessment [25], sustainability assessment [26], material selection [27], other method development [28-30].

TOPSIS [5, 6] is one of the fundamental methods in MADM domain and has been immensely popular in applications and as foundation to numerous method development. The modified TOPSIS [7] which is also built upon the TOPSIS method, gained application popularity due to its novel use of the objective weight elicitation process based on Shannon's [31] entropy theory. Both these methods use the same Euclidean distance measure except for ***when*** the attribute weight is to be incorporated with the solution. It is very challenging for the decision makers to choose between these two methods due to their extreme similarity between them in their mathematical structures and their applicability to solve the same kind of MADM problems such as water management application [23, 32], airline assessment [33, 34], supplier selection [35]. Thus, there is a need to evaluate and compare these two methods to justify their suitability and applications.

In following sections, we first present the TOPSIS and the Modified TOPSIS methods. Case study-based comparisons are then presented followed by mathematical analysis.

2. Background of TOPSIS and Modified TOPSIS

We first present the generic form of the MADM problem before the TOPSIS and the Modified TOPSIS methods are explained by considering how these methods are applied to solve a given MADM problem.

2.1 MADM Problem Formulation

The generic MADM problem has the objective of assessing and ranking alternatives A_i $(i = 1, 2, ..., I)$ on the basis of certain attributes C_j $(j = 1, 2, ..., J)$. The set of alternatives A_i represents the available options for the decision maker which requires to be ranked. The set of attributes C_j represents the factors influencing te decision maker's choice while ranking the alternatives A_i. W_j $(j = 1, 2, ..., J)$ represents the weights indicating the relative significance of the attributes C_j. Attribute weights can be presented as a vector as shown in Eq.1 [4, 6, 36].

$$W = \{W_j\} \quad (1)$$

The decision maker's preference for every alternative A_i against every attribute C_j are known as the performance rating x_{ij} $(i = 1, 2, ..., I; j = 1, 2, ..., J)$. As shown in Eq. 2, the performance ratings for each alternative against each attribute can be displayed in the form of a decision matrix [4-6, 9, 36, 37].

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ x_{21} & x_{22} & \cdots & x_{2J} \\ \cdots & \cdots & \cdots & \cdots \\ x_{I1} & x_{I2} & \cdots & x_{IJ} \end{bmatrix} \quad (2)$$

With the defined decision matrix X and weight vector W in Eq. 1 & 2, the MADM problem Φ is represented in Eq. 3 [4, 36]

$$\Phi = \{X, W\} \quad (3)$$

The given MADM problem Φ can be solved by applying a range of suitable MADM methods. Generally, the MADM methods require (a) a normalisation procedure, and (b) a score aggregation technique. The normalisation transforms the performance rating x_{ij} to a comparable measurement unit. An overall weighted score V_i $(i = 1, 2, ..., I)$ for each alternative is calculated by applying the aggregation which combines the weights with the performance ratings. The final ranking of the alternatives is done using the overall score [4-6, 36].

2.2 The TOPSIS method

TOPSIS is based on the fundamental premise that the best solution has the shortest distance from the positive-ideal solution, and the longest distance from the negative-ideal one. Alternatives are ranked with the use of an overall index calculated based on the distances from the ideal solutions. [3-5, 36, 38].

The TOPSIS method can be explained as a set of stages shown below:

Stage 1: Calculate the normalised performance ratings.

Vector Normalisation is applied to obtain normalised performance ratings from Eq. 2.

In this procedure, each performance rating x_{ij} in X is divided by its norm. The normalised ratings y_{ij} $(i = 1, 2, ..., I; j = 1, 2, ..., J)$ can be calculated by Eq. 4.

$$y_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{I} x^2{}_{ij}} \tag{4}$$

Although this conversion process makes comparison across attributes easier through the use of dimensionless units, it has the challenges in performing straightforward comparison due to non-equal scale length [3, 4, 6, 36, 39].

The normalised performance ratings y_{ij} can be given as a matrix Y as shown in Eq.5.

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1J} \\ y_{21} & y_{22} & \cdots & y_{2J} \\ \cdots & \cdots & \cdots & \cdots \\ y_{I1} & y_{I2} & \cdots & y_{IJ} \end{bmatrix} \tag{5}$$

Stage 2: Integrate weigh with ratings.

The weighted and normalised performance rating v_{ij} $(i = 1, 2, ..., I; j = 1, 2, ..., J)$ is calculated from Eq. 1 & 5 as shown in Eq. 6. These weighted ratings are combined to form the weighted-normalised decision matrix V in Eq. 7 [3-5, 36].

$$v_{ij} = W_j * y_{ij} \; ; i = 1, 2, ..., I; \; j = 1, 2, ..., J. . \tag{6}$$

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1J} \\ v_{21} & v_{22} & \cdots & v_{2J} \\ \cdots & \cdots & \cdots & \cdots \\ v_{I1} & v_{I2} & \cdots & v_{IJ} \end{bmatrix} \tag{7}$$

Stage 3: Find positive and negative ideal solutions.

A^* and A^- are denoted as the positive and negative ideal solution sets respectively which can be detected from Eq. 7 as

$$A^* = [v_1^*, v_2^*, ..., v_J^*] \tag{8}$$

$$A^- = [v_1^-, v_2^-, ..., v_J^-] \tag{9}$$

Where, $v_j^* = \begin{cases} \max v_{ij}, if\ j \text{ is a benefit attribute} \\ \min v_{ij}, if\ j \text{ is a cost attribute} \end{cases}$

$v_j^- = \begin{cases} \min v_{ij}, if\ j \text{ is a benefit attribute} \\ \max v_{ij}, if\ j \text{ is a cost attribute} \end{cases}$ [4, 5, 36]

Stage 4: Obtain the separation values

The separation measure is the distance of each alternative rating from both the positive and negative ideal solutions which is obtained by applying the Euclidean distance theory. Eq. 10 & 11 show the process for positive and negative separation calculations respectively [4-6, 9, 36].

$$S_i^* = \sqrt{\sum_{j=}^{J} (v_{ij} - v_j^*)^2} \tag{10}$$

$$S_i^- = \sqrt{\sum_{j=}^{J} (v_{ij} - v_j^-)^2}\ , \tag{11}$$

Stage 5: Calculate the overall preference score

The overall preference score V_i for each alternative A_i is obtained as shown in Eq. 12.

$$V_i = \frac{S_i^-}{S_i^- + S_i^*} \tag{12}$$

Alternatives are ranked based on higher V_i values [4-6, 9, 36].

2.3 The Modified TOPSIS method

Modified TOPSIS incorporates the attribute weights with the performance ratings in a slightly different manner compared to the TOPSIS method. Similar to TOPSIS, the overall performance score is obtained from the distance from positive and negative solutions. The distance is related with the alternative weights. The modified TOPSIS proposes the use of alternative weights with the Euclidean distances [7]. Modified TOPSIS inherits all the positive aspects of TOPSIS and supposedly rectifies the use of non-weighted Euclidean distance in TOPSIS.

The modified TOPSIS method is explained through the following stages.

Stage 1: Normalise the original decision matrix

The normalised decision matrix is calculated like the TOPSIS. The matrix can be presented as in Eq. 5 [7].

Sage 2: Identify the ideal solutions

B^* and B^- are defined as the positive and negative ideal solutions respectively and can be obtained in terms of normalised performance ratings from Eq. 5 as

$$B^* = \left[y_1^*, y_2^*, ..., y_J^*\right] \tag{13}$$

$$B^- = \left[y_1^-, y_2^-, \ldots, y_J^-\right] \quad (14)$$

Where, $y_j^* = \begin{cases} \max y_{ij} \text{ ; for benefit attribute} \\ \min y_{ij} \text{ ; for cost attribute} \end{cases}$

$$y_j^- = \begin{cases} \min y_{ij} \text{ ; for benefit attribute} \\ \max y_{ij} \text{ ; for cost attribute} \end{cases}$$ [7]

Stage 3: Obtain the weighted Euclidean distance

The weighted Euclidean distances from the positive and negative ideal solutions for each alternative A_i are obtained from Eq. 1, 5, 13, and 14 as

$$D_i^* = \sqrt{\sum_{j=1}^{J} W_j (y_{ij} - y_j^*)^2}, \quad (15)$$

$$D_i^- = \sqrt{\sum_{j=1}^{J} W_j (y_{ij} - y_j^-)^2} \quad (16)$$

Where, W_j ($j = 1, 2, \ldots, J$) is weights for attributes C_j ($j = 1, 2, \ldots, J$). [7]

Stage 4. Obtain the overall performance score

The overall score for each alternative A_i is obtained as

$$V_i = \frac{D_i^-}{(D_i^* + D_i^-)} \quad (17)$$

Performance score V_i is utilised to rank the competing alternatives. A higher score value indicates a better alternative performance [7].

3. Comparisons between TOPSIS and Modified TOPSIS

The TOPSIS and modified TOPSIS methods are compared under two different weight settings: (a) all the attributes having equal weights, and (b) the attribute weights are not equal.

3.1 A practical case study comparison

Times Higher Education (THE) ranking for universities is one of the most respected ranking for universities worldwide. The ranking of universities (alternatives in this study) is comprised of five key indicators (attributes in this study): i) Teaching, ii) Research, iii) Citations, iv) Industry Income, and v) International Outlook. The five attributes are given weights of 30%, 30%, 30%, 2.5%, and 7.5% respectively with some attributes having weighted sub attributes. Scores in each attribute is collected and adjusted using Z-scoring thus all scores are on the same scale out of 100 and no further normalisation is necessary before obtaining the overall scores [40]. Table 1 shows the scores in each attribute for top 20 universities in the rank. The overall score for each alternative (university) shown in Table 1, is obtained by using the Simple Additive Weighting (SAW) [41] where individual scores in each attribute are multiplied by the relevant weight and then added. This overall score is then used to obtain the ranking.

Table 1. Top 20 world university ranking scores from Times Higher Education in 2020 [40]

Rank	Name	Over all	Teach ing	Resear ch	Citati ons	Industry Income	Internat ional Outlook
1	University of Oxford	95.4	90.5	99.6	98.4	65.5	96.4
2	California Institute of Technology	94.5	92.1	97.2	97.9	88	82.5
3	University of Cambridge	94.4	91.4	98.7	95.8	59.3	95
4	Stanford University	94.3	92.8	96.4	99.9	66.2	79.5
5	Massachusetts Institute of Technology	93.6	90.5	92.4	99.5	86.9	89
6	Princeton University	93.2	90.3	96.3	98.8	58.6	81.1
7	Harvard University	93	89.2	98.6	99.1	47.3	76.3
8	Yale University	91.7	92	94.8	97.3	52.4	68.7
9	University of Chicago	90.2	89.1	91.4	96.7	52.7	76
10	Imperial College London	89.8	84.5	87.6	97	69.9	97.1
11	University of Pennsylvania	89.6	87.5	90.4	98.2	74	65
12	Johns Hopkins University	89.2	81.7	91.4	98.3	91.3	73.2
13	University of California, Berkeley	88.3	83	90.6	99.2	46.1	70.4
13	ETH Zurich	88.3	81.8	92.8	90.3	56.8	98.2
15	UCL	87.1	77.8	88.7	96.1	42.7	96.2
16	Columbia University	87	85.6	82.6	98.2	44.8	79.3
17	University of California, Los Angeles	86.8	83.1	88.6	97.3	51.3	64.1
18	University of Toronto	85.5	76.6	89.5	93.6	50.5	84.7
19	Cornell University	85.1	79.7	86	96.6	37.1	73.4
20	Duke University	84	82.4	76.8	97	99.9	61.5

We have utilised these performance scores for the alternatives (universities) against the five attributes as shown in Table 1 and the attribute weights set by THE to obtain rankings. Both TOPSIS and modified TOPSIS methods were used to rank the alternatives (universities). Table 2 shows the ranking outcomes for TOPSIS and Modified TOPSIS compared with the original THE ranking. The ranking outcomes shows that different methods produce different rankings. The TOPSIS rankings were mostly similar for top 10 universities and some rank reversals for other universities. The Modified TOPSIS shows much more variations in the rank. Past studies showed that different ranking outcomes are commonly evident with different methods and have suggested techniques to choose the most suitable methods [2-4, 36]. We do not see any justification from THE ranking documents why SAW was used over other valid and equally suitable methods such as TOPSIS. However, in this study we are concerned about the fact that TOPSIS and Modified TOPSIS produced two ranking outcomes as shown in Table 2, which are vastly different despite their same mathematical origin and strikingly similar structures. This simple case study results highlights the need for through investigation and comparison between TOPSIS and Modified TOPSIS.

Table 2. Top 20 world university ranking comparisons

Name	**THE Rank**	**TOPSIS Rank**	**Modified TOPSIS Rank**
University of Oxford	1	1	2
California Institute of Technology	2	2	1
University of Cambridge	3	3	4
Stanford University	4	4	5
Massachusetts Institute of Technology	5	7	3
Princeton University	6	5	6
Harvard University	7	6	8
Yale University	8	8	11
University of Chicago	9	9	12
Imperial College London	10	12	7
University of Pennsylvania	11	10	13
Johns Hopkins University	12	11	9
University of California, Berkeley	13	14	15
ETH Zurich	13	13	10
UCL	15	16	14
Columbia University	16	17	18
University of California, Los Angeles	17	15	19
University of Toronto	18	18	16
Cornell University	19	19	20
Duke University	20	20	17

3.2 Comparison with equal weight settings

3.2.1 Simulation Results

We conducted a problem-solving simulation with more than 1,000 MADM problems (randomly generated) with equal attribute weight settings. For each problem, the TOPSIS and the modified TOPSIS methods produces the same ranking outcome. This result can be justified by the following mathematical proof.

3.2.2 Mathematical Analysis

The TOPSIS Eq. 12 is expanded using Eq. 10 & 11 as

$$V_i = \frac{\sqrt{\sum_{j=1}^{J}(v_{ij} - v_j^-)^2}}{(\sqrt{\sum_{j=1}^{J}(v_{ij} - v_j^*)^2} + \sqrt{\sum_{j=1}^{J}(v_{ij} - v_j^-)^2})}, \tag{18}$$

Eq. 18 can be further extended by applying Eq. 6 & 9 as

$$V_i = \frac{\sqrt{\sum_{j=1}^{J}(W_j y_{ij} - W_j y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J}(W_j y_{ij} - W_j y_j^*)^2} + \sqrt{\sum_{j=1}^{J}(W_j y_{ij} - W_j y_j^-)^2})} \tag{19}$$

Or,

$$V_i = \frac{\sqrt{\sum_{j=1}^{J} W_j^2 (y_{ij} - y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J} W_j^2 (y_{ij} - y_j^*)^2} + \sqrt{\sum_{j=1}^{J} W_j^2 (y_{ij} - y_j^-)^2})} \tag{20}$$

With the equal weight settings, applying $W_j = W$ to Eq. 20

$$V_i = \frac{\sqrt{\sum_{j=1}^{J} W^2 (y_{ij} - y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J} W^2 (y_{ij} - y_j^*)^2} + \sqrt{\sum_{j=1}^{J} W^2 (y_{ij} - y_j^-)^2})} \tag{21}$$

or

$$V_i = \frac{\sqrt{\sum_{j=1}^{J}(y_{ij} - y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J}(y_{ij} - y_j^*)^2} + \sqrt{\sum_{j=1}^{J}(y_{ij} - y_j^-)^2})} \tag{22}$$

Similarly, the Modified TOPSIS Eq. 17 can be expanded by using Eq. 15 & 16 as

$$V_i = \frac{\sqrt{\sum_{j=1}^{J} W_j (y_{ij} - y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J} W_j (y_{ij} - y_j^*)^2} + \sqrt{\sum_{j=1}^{J} W_j (y_{ij} - y_j^-)^2})}, \tag{23}$$

With the equal weight settings, applying $W_j = W$ to Eq. 23

$$V_i = \frac{\sqrt{\sum_{j=1}^{J} W(y_{ij} - y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J} W(y_{ij} - y_j^*)^2} + \sqrt{\sum_{j=1}^{J} W(y_{ij} - y_j^-)^2})}; \quad (24)$$

or

$$V_i = \frac{\sqrt{\sum_{j=1}^{J} (y_{ij} - y_j^-)^2}}{(\sqrt{\sum_{j=1}^{J} (y_{ij} - y_j^*)^2} + \sqrt{\sum_{j=1}^{J} (y_{ij} - y_j^-)^2})} \quad (25)$$

Comparing Eq. 22 and 25, it is observed that the two methods are the same. This mathematical explanation justifies the same ranking results obtained during the simulation study. It also highlights the extreme structural similarities between the two methods and justifies the need for further investigation under non-equal weights.

3.3 Comparison with non-equal weight settings

A simulation study and results are presented before providing a mathematical comparison of the TOPSIS and modified TOPSIS methods under non-equal weight settings.

3.3.1 Simulation Results

In this simulation study, the decision matrix from the graduate fellowship applicants ranking case presented by [6] is used. Table 3 shows the decision matrix.

Table 3. Graduate fellowship decision matrix for the simulation [6]

	Attributes				
Alternatives	*C1*	*C2*	*C3*	*C4*	*C5*
A1	690	3.1	9	7	4
A2	590	3.9	7	6	10
A3	600	3.6	8	8	7
A4	620	3.8	7	10	6
A5	700	2.8	10	4	6
A6	650	4	6	9	8

The simulation is started with equal attribute weight W = (0.2, 0.2, 0.2, 0.2, 0.2) for the five attributes. With this equal weight setting, the decision problem is solved with both the TOPSIS and modified-TOPSIS. The ranking outcomes obtained, are the same and are used as the base outcomes.

The attribute weights are then changed gradually with a step of 0.1 producing 126 distinct weight sets between the range of (0.6, 0.1, 0.1, 0.1, 0.1) and (0.1, 0.1, 0.1, 0.1, 0.6). The incremental step is decided to be 0.1 because it produces significant result variations required for this study.

For each set of weights, the MADM problem is then solved using both the TOPSIS and the modified TOPSIS methods. The simulation shows that 70% of the 126 weight sets generates distinct ranking outcomes for the two methods.

The simulation results and the previous sections for equal weight settings highlight the fact that the only difference between TOPSIS and modified TOPSIS is in how the attribute weight is incorporated during calculations. A closer inspection of expanded TOPSIS Equation (20) and expanded modified TOPSIS Equation (23) shows that the only difference between the two methods is that in TOPSIS, W_j^2 is used but in modified TOPSIS W_j is used while calculating the distances from the positive and the negative ideal solutions. Thus, further mathematical analysis under non-equal weight settings is required to establish the differences of these methods.

3.3.2 Mathematical Analysis

The modified TOPSIS method suggests that the distance between performance ratings should be weighted, rather than the performance ratings as done in TOPSIS. Considering this argument rational and valid, the equation is derived from the basic Euclidean distance theory [42].

A single dimension problem with two vectors P [x_1] and Q [x_2] shown in Fig.1.

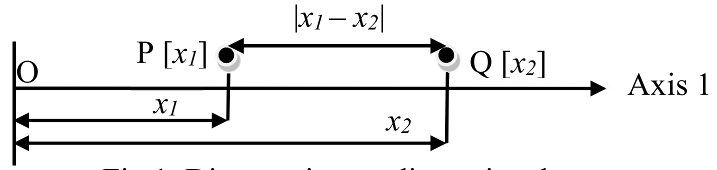

Fig 1. Distance in one dimensional space

The distance between P and Q is obtained as

$$|PQ| = d_x = |x_1 - x_2| \tag{26}$$

If the dimension has any weight associated with it, then the weighted distance can be expressed as

$$D_x = W_1 |x_1 - x_2| \tag{27}$$

Now consider the problem with two dimensions with vectors P [x_1, x_2] and Q [y_1, y_2] as shown in Fig. 2.

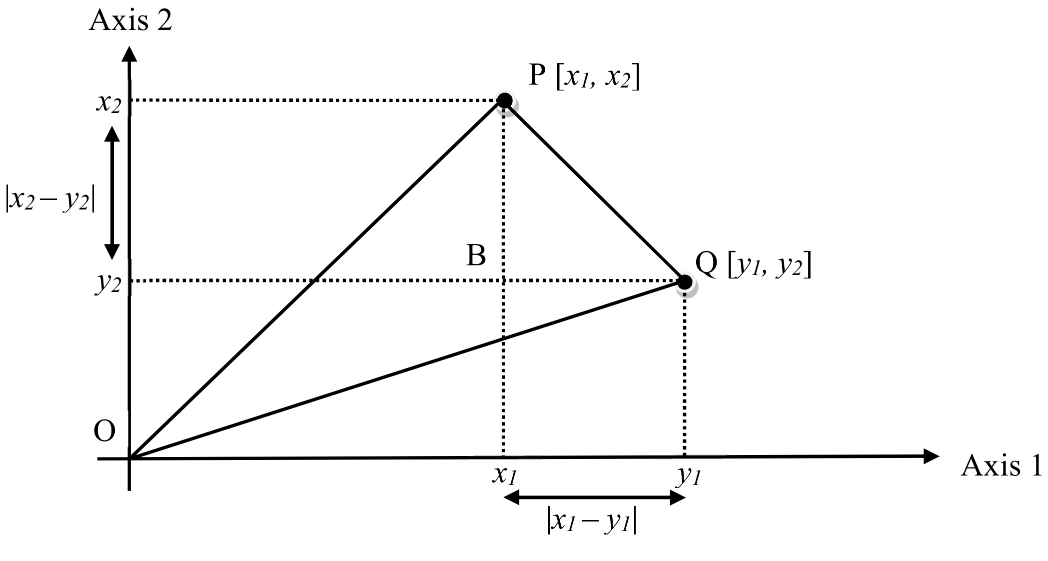

Fig. 2. Distance in two-dimensional space (Source: Adapted from Greenacre [42])

Using the Pythagoras' theorem for right-angled triangle, from Figure 2 we can write the distance between P and Q as

$$|PQ|^2 = (d_{xy})^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2 \quad (28)$$

or

$$d_{xy} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \quad (29)$$

By applying Equations (26) and (27) the two-dimensional weighted Euclidean distance can be obtained from Equation (29) as

$$D_{xy} = \sqrt{(W_1 | x_1 - y_1 |)^2 + (W_2 | x_2 - y_2 |)^2} \quad (30)$$

Similarly, the Euclidean distance and the weighted Euclidean distance can be obtained for three dimensional problems with P [x_1, x_2, x_3] and Q [y_1, y_2, y_3] as shown in Equations (31) and (32) respectively.

$$d_{xy} = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2} \quad (31)$$

$$D_{xy} = \sqrt{(W_1 | x_1 - y_1 |)^2 + (W_2 | x_2 - y_2 |)^2 + (W_3 | x_3 - y_3 |)^2} \quad (32)$$

The weighted Euclidean distance for vectors P and Q with j ($j = 1, 2, \ldots, J$) dimensions can be obtained similarly as

$$D_{xy} = \sqrt{\sum_{j=1}^{J} (W_j | x_j - y_j |)^2} \quad (33)$$

or

$$D_{xy} = \sqrt{\sum_{j=1}^{J} W_j^2 (x_j - y_j)^2} \quad (34)$$

The mathematical derivation of Equation (34) proves that while calculating the weighted Euclidean distance, squared weight should be used. The multi-dimension used in the derivation is analogous to MADM problem solving by TOPSIS and modified TOPSIS where the attributes are considered as dimensions. Comparisons between Equation (34) and the TOPSIS Equation (20) and the modified TOPSIS Equation (23) prove that the TOPSIS method applies the weight in a correct manner.

The concept of distance weighting introduced in the modified TOPSIS is valid and rational. The modified TOPSIS method derives objective weight using the entropy concept [31, 43] based on information variation in the MADM problem [7]. This objective weight shows the relative importance of the attributes in terms of their impacts on the decision outcomes. The objective weight should be treated differently from the attribute weights provided by the decision maker and should never be used in the process of solving the MADM problem. The objective weight certainly can indicate the decision maker regarding the significance of attributes so that the decision maker can be careful while solving the problem.

On the other hand, although the TOPSIS method uses the weighting of normalised performance rating and does not explicitly applies the distance weighting concept, the mathematical structure of TOPSIS is implicitly the same as that of the weighted Euclidean distance.

4. Conclusions

This study has provided extensive simulation and mathematical proof-based comparisons between two widely used MADM methods: the TOPSIS method and the modified TOPSIS method. The evaluations have shown the validity of the arguments presented for the modified TOPSIS however the application of the weight incorporation process during the aggregation is inappropriate. The objective weight elicitation concept in modified TOPSIS is valid and highly useful to understand criteria significance. However, the way weight is incorporated will produce incorrect ranking, as the weigh is squared in the aggregation process. It has been proved that the TOPSIS method provides correct weight aggregation process, and it should be used for MADM problems where both TOPSIS and modified TOPSIS could be applied. This study will benefit the decision makers who are not sure about choosing between these two methods and would eliminate the confusions among practitioners.

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