

# **Three-Dimensional Discrete Element Simulation of Cavity Expansion from Zero Initial Radius in Sand**

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Thesis submitted in fulfilment of the requirements for  
the degree of

**Doctor of Philosophy**

under the supervision of A/Prof. Behzad Fatahi and A/Prof. Hadi  
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January 2021

## **Certificate of Original Authorship**

I, *Yang Dong* declare that this thesis, is submitted in fulfilment of the requirements for the award of *the degree of Doctor of Philosophy*, in the *School of Civil and Environmental Engineering, Faculty of Engineering and Information Technology* at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This research is supported by the Australian Government Research Training Program.

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## **Acknowledgement**

I, Yang Dong, would like to express my most cordial gratitude and appreciation to those who helped me, encouraged me, guided me and accompanied me during this four-year Ph.D. study.

Firstly, I would like to particularly thank my principal supervisor, Associate Professor Behzad Fatahi for his continuous support, understanding, patience throughout this long journey, and I am particularly grateful for his encouragement during the toughest time of my life. I also appreciate my co-supervisor, Associate Professor Hadi Khabbaz for his guidance and willingness to help whenever possible.

I am appreciative to my friends, colleagues, administration staffs in the School of Civil and Environmental Engineering. I would also like to express my gratitude to Matt Gaston and Anselm Motha for their assistance in providing IT support and the facility that makes such research possible. Also, it was my honour to walk through these four years memorable time with my friends, Sienna Zhang, Navid Yeganeh and Piyush Punetha.

At this very moment, I would like to extend my heartfelt appreciation to my dearest parents, my partner Ruby who are my most powerful backing, and constantly support me spiritually at any time. Without their selfless support, I would never be able to achieve this key milestone in my life.

-Yang Dong

## List of Publications

1. Dong, Y., & Fatahi, B. (2020). Discrete element simulation of cavity expansion in lightly cemented sands considering cementation degradation. *Computers and Geotechnics*, 124, 103628.
2. Dong, Y., Fatahi, B., & Khabbaz, H. (2020). Three-dimensional discrete element simulation of cylindrical cavity expansion from zero initial radius in sand. *Computers and Geotechnics*, 117, 103230.
3. Dong, Y., Fatahi, B., Khabbaz, H., & Zhang, H. (2018). Influence of particle contact models on soil response of poorly graded sand during cavity expansion in discrete element simulation. *Journal of Rock Mechanics and Geotechnical Engineering*, 10(6), 1154-1170.
4. Dong, Y, Fatahi, B, Khabbaz, H & Hsi, J 2018, 'Impact of Initial In-Situ Stress Field on Soil Response During Cavity Expansion Using Discrete Element Simulation', *New Developments in Materials for Infrastructure Sustainability and the Contemporary Issues in Geo-environmental Engineering, GeoChina International Conference*, Springer, Hang Zhou, China, pp. 1-10.
5. Dong, Y., Fatahi, B., Khabbaz, H. & Kamruzzaman, A. 2018, 'Investigating Effects of Particle Scaling for Cavity Expansion Simulation Using Discrete Element Method', *GeoShanghai International Conference*, Springer, pp. 938-4.

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## List of Symbols

The follow symbols are used in this thesis:

- $A$ : contact area of spheres in DEM (m<sup>2</sup>);
- $A_{cf}$ : curve fitting parameter in the cavity expansion solution proposed by Collins et al. (1992);
- $a$ : cavity radius (m);
- $a_0$ : initial cavity radius (m);
- $a_f$ : final cavity radius (m);
- $b$ : radius of the external boundary in a cavity expansion problem (m);
- $c$ : radius of the plastic zone in a cavity expansion problem (kPa);
- $c_c$ : cohesion (kPa);
- $c^p$ : cohesion of the bond element in DEM (MPa);
- $D$ : pile diameter (m);
- $D_m$ : size of the prediction sphere (m);
- $d$ : distance between the centres of particles 1 and 2 (m);
- $d_{50}$ : median particle size (m);
- $d_n^p$ : normal dashpot;
- $d_s^p$ : shear dashpot;
- $d_c$ : damping constant at the contact;
- $E$ : Young's modulus (MPa);
- $E_{mod}^*$ : effective modulus of the particles in DEM (MPa);
- $E_*^p$ : bond effective modulus (MPa);

$e$ : void ratio;  
 $f$ : friction force acting parallel to the slipping plane (kN);  
 $F_n$ : normal force at the contact interface between contacting particles (kN);  
 $F_s$ : shear force at the contact interface between contacting particles (kN);  
 $F_n^0$ : normal force between two contacting particles at the beginning of the timestep (kN);  
 $F_s^0$ : shear force between two contacting particles at the beginning of the timestep (kN);  
 $F_x$ : resultant force in the x direction (kN);  
 $F_y$ : resultant force in the y direction (kN);  
 $F_n^H$ : Hertz normal force (kN);  
 $F_s^H$ : Hertz shear force (kN);  
 $F_{s_0}^H$ : Hertz shear force at the beginning of the timestep (kN);  
 $F_s^{max}$ : shear strength at the contact interface (kPa);  
 $F_{s_{max}}^l$ : maximum shear strength of the linear frictional component (kN);  
 $F_{s_{max}}^p$ : maximum shear strength of the parallel cementation bond component (kN);  
 $F(e)$ : void ratio function;  
 $G$ : shear modulus (MPa);  
 $G_{ref}$ : modulus number;  
 $g_s$ : surface gap between contacting particles (m);  
 $g_{max}$ : maximum gap between contacting particle (m);  
 $H$ : horizontal force applied to the particle (kN);  
 $K_G^e$ : elastic shear modulus exponent;



$k_n$ :	normal stiffness of linear springs (N/m);
$k_s$ :	shear stiffness of linear springs (N/m);
$k_r$ :	rolling stiffness of linear springs (N/m);
$k^*$ :	stiffness ratio between normal stiffness and shear stiffness (N/m);
$k_*^l$ :	linear component spring stiffness ratio (normal stiffness/shear stiffness);
$k_n^l$ :	normal stiffness of linear springs (N/m);
$k_s^l$ :	shear stiffness of linear springs (N/m);
$k_n^p$ :	normal stiffness of bond element (N/m);
$k_s^p$ :	shear stiffness of bond element (N/m);
$L$ :	the distance between the centres of contacting particles (m);
$L_p$ :	the embedded length of the pile in Strain Path Method;
$l$ :	linear frictional components;
$M$ :	the slope of critical state line;
$M^p$ :	resultant moment (kNm);
$M_t^p$ :	twisting moment (kNm);
$M_b^p$ :	bending moment (kNm);
$M_{RR}$ :	rolling resistance torque (kNm);
$M_{RR}^{max}$ :	limiting torque of rolling resistance torque (kNm);
$N$ :	the normal force acting perpendicular to the slipping plane (kN);
$N_b$ :	total number of broken cementation bonds;
$N_c$ :	ratio between the total number of cementation bonds;
$ne$ :	modulus exponent;

- $O_1$ : the centre of the contact interface between balls A and B;
- $O_2$ : the centre of the contact interface between balls A and B';
- $P_a$ : cavity pressure (kPa);
- $P_{e0}$ : external radial stress (kPa);
- $P_i$ : internal radial stress/cavity pressure in cavity expansion theory (kPa);
- $P_{a0}$ : internal cavity pressure (kPa);
- $P_r$ : radial pressure (kPa);
- $P_h$ : hoop stress (kPa);
- $P_{max}$ : maximum cavity pressure (kPa);
- $P\%$ : the progress of the cavity expansion;
- $p$ : parallel cementation bond components;
- $p_{ref}$ : reference pressure equal to 100 kPa;
- $p_r$ : radial stress (kPa);
- $p'$ : initial mean effective stress (kPa);
- $p_{lim}$ : constant limiting pressure causing a continuous cavity expansion (kPa);
- $q$ : deviatoric stress (kPa);
- $R$ : particle size (m);
- $R_1$ : radius of particle 1 (m);
- $R_2$ : radius of particle 2 (m);
- $R_b$ : number of broken cementation bonds;
- $R_c$ : radius of the column to be installed (m);
- $R_p$ : radial distances measured from the corresponding initial cavities (m);

$R_e^H$ :	Hertz contact model particle effective radius (m);
$r_A$ :	radius of the prediction sphere A (m);
$r_B$ :	radius of the prediction sphere B (m);
$r$ :	radial distance (m);
$r_0$ :	initial radial distance of a soil element (m);
$s_u$ :	undrained shear strength (kPa);
$V_f$ :	vertical force applied to the particle (kN);
$V$ :	volume of the expansion (m <sup>3</sup> );
$Y$ :	a model parameter used in cavity expansion theory;
$\alpha$ :	a model parameter used in cavity expansion theory;
$\gamma_c$ :	a model parameter used in cavity expansion theory;
$Z$ :	a model parameter used in cavity expansion theory;
$z$ :	a cylindrical coordinate (e.g. $z$ at the ground surface is 0);
$\xi$ :	composite state parameter defined as a function of the specific volume and the mean effective stress;
$\lambda$ :	slope of the critical state line;
$\lambda^p$ :	bond radius multiplier;
$\Gamma_1$ :	the intercept on the specific volume axis when $p'/p_{ref} = 1$ ;
$\alpha_c$ :	central angle in cavity expansion simulation (degree);
$\alpha_d$ :	angle of the slipping plane (degree);
$\alpha^H$ :	Hertz contact model exponent;
$\Delta\delta_s$ :	shear displacement increment in a timestep (m);
$\delta_r$ :	radial displacement (m);

$\Delta\delta_r$ :	radial displacement increment in a timestep (m);
$\Delta\delta_n$ :	normal displacement increment in a timestep (m);
$\vartheta$ :	Poisson's ratio;
$\vartheta_s$ :	Specific volume;
$\omega_a$ :	rotational velocity of particle A (rad/s);
$\omega_b$ :	rotational velocity of particle B (rad/s);
$\Delta t$ :	increment of timestep (s);
$\mu$ :	friction coefficient/interparticle friction;
$\mu_r$ :	rolling resistance coefficient;
$\mu_{wall}$ :	wall friction coefficient;
$\mu^l$ :	friction coefficient of the linear frictional component;
$\eta$ :	a model parameter used in cavity expansion theory;
$M$ :	a model parameter used in cavity expansion theory;
$\beta$ :	a model parameter used in cavity expansion theory;
$\delta_{rSS}$ :	soil radial movement in Strain Path Method;
$\delta_{zSS}$ :	soil vertical movement in Strain Path Method;
$\sigma_1$ :	major principle stress in Mohr's circle setting (kPa);
$\sigma_3$ :	minor principle stress in Mohr's circle setting (kPa);
$\sigma_{r0}$ :	initial radial stress (kPa);
$\sigma_r$ :	cavity expansion radial stress (kPa);
$\sigma_r^e$ :	radial stress in the elastic region during the cavity expansion (kPa);
$\sigma_r^p$ :	radial stress in the plastic region during the cavity expansion (kPa);

$\sigma_{\theta}$ : cavity expansion hoop stress (kPa);  
 $\sigma_{\theta}^e$ : hoop stress in the elastic region during the cavity expansion (kPa);  
 $\sigma_{\theta}^p$ : hoop stress in the plastic region during the cavity expansion (kPa);  
 $\sigma_x$ : initial stress in x direction (kPa);  
 $\sigma_y$ : initial stress in y direction (kPa);  
 $\sigma_f^p$ : tensile strength of a cementation bond (kPa);  
 $\sigma_n^p$ : instant normal stress of the cementation bond (kPa);  
 $\sigma_f^p$ : tensile strength of the cementation bond (kPa);  
 $\tau_f^p$ : shear strength of a cementation bond (kPa);  
 $\Delta\varphi_r$ : relative rotation increment between contacting particles in a timestep (degree);  
 $\emptyset$ : friction angle (degree);  
 $\emptyset'$ : effective stress friction angle (degree);  
 $\emptyset_m$ : mobilised friction angle (degree);  
 $\emptyset_f$ : ultimate value of the friction angle (degree);  
 $\emptyset'_{cv}$ : critical state/constant volume friction angle (degree);  
 $\psi$ : dilation angle (degree);  
 $\psi_f$ : ultimate value of the dilation angle (degree);  
 $\psi_m$ : mobilised dilation angle (degree);  
 $\tau$ : shear stress (kPa);  
 $\varepsilon_v$ : volumetric strain;  
 $\varepsilon_{\gamma}$ : shear strain;  
 $\gamma$ : deviatoric strain;

- $\Omega$  : cross section area in Strain Path Method;
- $\chi$  : a model paramater used in cavity expansion theory;
- $T$  : a model paramater used in cavity expansion theory;
- $\omega$  : a model paramater used in cavity expansion theory;
- $\theta$ : angle of the rotation of the cementation bond due to twisting (degree);
- $\beta$ : angle of the rotation of the cementation bond due to bending (degree);

## **Abstract**

Rigid inclusions, generally made of structural concrete, are widely employed to reduce the settlement and enhance the bearing capacity of the ground by transferring loads from superstructures through weak soil layers to a firm underlying stratum. However, the installations of rigid inclusions such as driven piles and controlled modulus columns can induce irreversible changes of the soil stress - strain state, and lead to excessive lateral soil movements during the auger penetration or pile driving/hammering process.

This thesis proposes a rigorous numerical modelling to investigate the installation effects of rigid inclusions on surrounding ground via cavity expansion simulation adopting discrete element method. The benefits of adopting the discrete element method is attributed to its capability in simulating large displacements and distortions, as well as incorporating the discontinuous nature of granular materials and providing a microscopic insight into the problem. True scale three-dimensional discrete element models simulating the creation of cylindrical cavities from zero initial cavity radius in dry clean and lightly cemented sands are developed. Contact constitutive models mimicking the behaviour of dry clean granular materials and lightly cemented sands are calibrated against existing laboratory experimental results. The numerical models proposed contain up to 500,000 particles with boundary conditions carefully selected to reproduce realistic scenarios. Embedded scripting is adopted to precisely record both the local and global stress and strain variations, as well as the cementation bond breakage during the cavity expansion process.

The results confirm that the selection of arbitrary initial cavity radius could significantly influence the soil response at the earlier stages of the cavity expansion. For a given expansion volume, creating a cylindrical cavity from zero initial radius induces larger stresses in the soil compared to expanding existing cavities in the same soil medium from a nonzero given initial radius. This implicates that the estimation of the pile driving force may be largely underestimated adopting the approximation method based on the existing cavity expansion theories, requiring an assumptive initial cavity radius. In addition, the soil lateral displacements, depending on the gradation and the relative density, can reach up to  $30 R_c$  ( $R_c$  is the radius of the pile) during the installation, and the loose sand in a plane strain condition can even exhibit dilation during the early stages of the cavity expansion. In the lightly cemented sands, the installation of rigid inclusions or cavity expansion can lead to significant cementation degradations. The influence zone of cementation degradation observed in cemented sand with various cement content can extend to approximately  $4R_c$ , in which the shear strength of the soil is significantly reduced due to the cementation bond breakage, which may lead to the reduction in axial capacity, adversely influencing the pile toe stability. Within this influence zone, the displacement induced by the installation is not sensitive to the level of cementations, while soils with higher cement content are expected to experience larger radial displacements beyond this influence zone. Hence, extra care must be taken by practicing engineers when assessing the required pile driving pressure and the installation effects of ground inclusions in the vicinity of existing structures such as pipelines and bridge abutments in both granular materials and lightly cemented sand.