

# **Multi-dimensional Channel Parameter Estimation for mmWave Cylindrical Arrays**

**by Zhipeng Lin**

Thesis submitted in fulfilment of the requirements for  
the degree of

**Doctor of Philosophy**

under the supervision of Prof. Ren Ping Liu, Prof. J. Andrew  
Zhang, and Prof. Tiejun Lv

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March, 2021

# Certificate of Original Authorship

I, Zhipeng Lin, declare that this thesis is submitted in fulfilment of the requirements for the award of PhD degree, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of the requirements for a degree at any other academic institution except as fully acknowledged within the text. This thesis is the result of a Collaborative Doctoral Research Degree program with Beijing University of Posts and Telecommunications.

This research is supported by the Australian Government Research Training Program.

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## ABSTRACT

Millimeter-wave (mmWave) large-scale antenna arrays, standardized for the fifth-generation (5G) communication networks, have the potential to estimate channel parameters with unprecedented accuracy, due to their high temporal resolution and excellent directivity. However, most existing techniques have very high complexities in hardware and software, and they cannot effectively exploit the properties of mmWave large-array systems for channel estimation. As a result, their application in 5G mmWave large array systems is limited in practice.

This thesis develops new and efficient solutions to channel parameter estimation using large-scale mmWave uniform cylindrical arrays (UCyAs). The key contributions of this thesis are on the following four aspects:

We first present a channel compression-based channel estimation method, which reduces the computational complexity substantially at a negligible cost of estimation accuracy. By capitalizing on the sparsity of mmWave channel, the method effectively filters out the useless signal components. As a result, the dimension of the element space of the received signals can be reduced.

Next, we extend the channel estimation to the hybrid UCyA case, and design new hybrid beamformers. By exploiting the convergence property of the Bessel function, the designed beamformers can preserve the recurrence relationship of the received signals with a small number of radio frequency (RF) chains.

We then arrange the received signals in a tensor form and propose a new tensor-based channel estimation algorithm. By suppressing the receiver noises in all dimensions (time, frequency, and space), the algorithm can achieve substantially higher estimation accuracy than existing matrix-based techniques.

Finally, to reduce cost and power consumption while maintaining a high network access capability, we develop a novel nested hybrid UCyA and present the corresponding parameter estimation algorithm based on the second-order channel

statistics. Simulation results show that by exploiting the sparse array technique to design the RF chain connection network, the angles of a large number of devices can be accurately estimated with much fewer RF chains than antennas.

Overall, this thesis presents several applicable UCyA design schemes and proposes the efficient channel parameter estimation algorithms. The presented new UCyAs can significantly reduce the hardware cost of the system with a marginal accuracy loss, and the proposed algorithms are capable of accurately estimating the channel parameters with low computational complexities. By employing the presented UCyAs and implementing the proposed novel algorithms cohesively, the different communication and deployment requirements of a variety of mmWave communication scenarios can be met.

**KEYWORDS:** Channel parameter estimation, millimeter-wave communications, large-scale antenna array, tensor processing, hybrid beamforming.

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## Acknowledgements

I would like to extend my deep gratitude to all those who have offered me gracious help and support in the process of my thesis writing.

My deepest gratitude goes first and foremost to my supervisors, Prof. Ren Ping Liu, Prof. J. Andrew Zhang, and Prof. Tiejun Lv, for discovering my potential, constantly encouraging me to go further, and providing very valuable lessons on many different levels. Their meticulous academic attitudes and dedicated work styles have always been my model of learning. They have offered me numerous valuable suggestions with incomparable patience and encouraged me profoundly throughout my PhD study.

Also, I would like to express my sincere gratitude to Prof. Wei Ni for helping me review the thesis. The comments and the mutual discussions are very valuable for the thesis itself and for my work beyond it. Without his help, the completion of this thesis would have been impossible.

Special thanks goes to University Of Technology Sydney (UTS) and Beijing University of Posts and Telecommunications (BUPT) for providing wonderful working environments and useful study materials. I would also like to thank my former and current colleagues Xinyu Li, Jie Zeng, Yi Gong, Yuanyuan Ma, Shaoyang Wang, Yashuai Cao, Jintao Xing, Zhongyu Wang, Qixuan Zhang, Yun Zhang, Yuan Ren, Xuwei Zhang, and Qian Liu for countless technical discussions and personal talks, for sharing their knowledge and skills, for helping me out when help was dearly needed. I sincerely hope we can all stay in touch and I wish you all the best for your future.

Yet, none of all this would have been possible without the tremendous support of my parents. Finally, I am deeply indebted to my parent for their continuous support

and encouragement. Thank you for providing me safe and sorrow-free childhood, for discovering and supporting my abilities, and for being patient in times where work made me hard to get by. Their selfless love and silently supports give me the courage and determination to overcome all difficulties.

Zhipeng Lin  
Beijing, China, 2021.

## List of Publications

### Journal Papers

- J-1. **Z. Lin**, T. Lv, W. Ni, J. A. Zhang and R. P. Liu, "Nested Hybrid Cylindrical Array Design and DoA Estimation for Massive IoT Networks," *IEEE J. Sel. Areas Commun.*, Aug. 2020. (early access)
- J-2. **Z. Lin**, T. Lv, and P. T. Mathiopoulos, "3-D indoor positioning for millimeter-Wave massive MIMO systems," *IEEE Trans. Commun.*, vol. 66, no. 6, pp. 2472-2486, June 2018.
- J-3. **Z. Lin**, T. Lv, W. Ni, J. A. Zhang and R. P. Liu, "Tensor-Based Multi-Dimensional Wideband Channel Estimation for mmWave Hybrid Cylindrical Arrays," *IEEE Trans. Commun.*, vol. 68, no. 12, pp. 7608 - 7622, Dec. 2020.
- J-4. **Z. Lin**, T. Lv, W. Ni, J. A. Zhang, J. Zeng and R. P. Liu, "Joint Estimation of Multipath Angles and Delays for Millimeter-Wave Cylindrical Arrays with Hybrid Front-ends," *IEEE Trans. Wireless Comm.*, Mar. 2021 (early access)
- J-5. T. Lv, **Z. Lin**, P. Huang and J. Zeng, "Optimization of the Energy-Efficient Relay-Based Massive IoT Network," *IEEE Internet Things J.*, vol. 5, no. 4, pp. 3043-3058, Jun. 2018.
- J-6. Q. Liu, T. Lv and **Z. Lin**, "Energy-Efficient Transmission Design in Cooperative Relaying Systems Using NOMA," *IEEE Commun. Letters*, vol. 22, no. 3, pp. 594-597, Mar. 2018.
- J-7. J. Zeng, T. Lv, **Z. Lin**, R. Liu, J. Mei, W. Ni and Y. Guo, "Achieving Ultrareliable and Low-Latency Communications in IoT by FD-SCMA," *IEEE Internet of Things J.*, vol. 7, no. 1, pp. 363-378, Jan. 2020.
- J-8. Z. Wang, T. Lv, **Z. Lin**, J. Zeng and P. T. Mathiopoulos, "Outage Performance of URLLC NOMA Systems With Wireless Power Transfer," *IEEE Wireless*

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- J-9. Y. Cao, T. Lv, **Z. Lin**, P. Huang and F. Lin, “Complex ResNet Aided DoA Estimation for Near-Field MIMO Systems,” *IEEE Trans. Vehic. Techno.*, vol. 69, no. 10, pp. 11139-11151, Oct. 2020.
- J-10. X. Zhang, T. Lv, Y. Ren and **Z. Lin**, “Joint Content Push and Transmission in NOMA With SWIPT Caching Helper,” *IEEE Commun. Letters*, vol. 24, no. 4, pp. 922-925, Apr. 2020.
- J-11. S. Wang, T. Lv, X. Zhang, **Z. Lin**, and P. Huang, Learning-based multi-channel access in 5G and beyond networks with fast time-varying channels, *IEEE Trans. Vehic. Techno.*, vol. 69, no. 5, pp. 5203-5218, May 2020.

### Conference Papers

- C-1. **Z. Lin**, T. Lv, J. A. Zhang and R. P. Liu, 3D wideband mmWave localization for 5G massive MIMO systems, in *Proc. IEEE Int. Global Commun. (GLOBECOM)*, Waikoloa, HI, USA, Dec. 2019, pp. 1-6.
- C-2. **Z. Lin**, T. Lv, J. A. Zhang and R. P. Liu, Tensor-based high-accuracy position estimation for 5G mmWave massive MIMO systems, in *Proc. IEEE Int. Conf. Commun. (ICC)*, Dublin, Ireland, Jun. 2020, pp. 1-6.
- C-3. Y. Zhang, **Z. Lin** and T. Lv, “Sum-Rate-Driven Energy Efficiency Optimization in Massive MIMO Relay Networks,” in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, Beijing, China, Aug. 2018, pp. 158-162.
- C-4. Q. Zhang, **Z. Lin** and T. Lv, “Fast Sparse Bayesian Channel Estimation for Wideband mmWave Systems,” in *Proc. IEEE Int. Symp. Pers. Indoor Mobile Radio Commun. (PIMRC)*, Bologna, Sept. 2018, pp. 1-5.
- C-5. Q. Cheng, **Z. Lin**, J. A. Zhang, D. N. Nguyen, X. Huang, A. Kekirigoda and K. Hui, “Multi-user MIMO with Jamming Suppression for Spectrum-Efficient Tactical Communications, in *Proc. Int. Con. Signal Process. Commun. Systems (ICSPCS)* (Accepted).



- C-6. Q. Zhang, T. Lv and **Z. Lin**, “Variational Bayesian Channel Estimation for Wideband Multiuser mmWave Systems,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, Shanghai, China, May 2019, pp. 1-6.
- C-7. A. Kekirigoda, K. Hui, Q. Cheng, **Z. Lin**, J. A. Zhang, D. N. Nguyen and X. Huang, “Massive MIMO for Tactical Ad-hoc Networks in RF Contested Environments, in *Proc. IEEE Military Commun. Conf. (MILCOM)*, Norfolk, VA, USA, 2019, pp. 658-663.

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# Abbreviation

1-D - One-Dimensional

2-D - Two-Dimensional

3-D - Three-Dimensional

5G - Fifth-Generation

ADC - Analog-to-Digital Converter

AoA - Angle of Arrival

AoD - Angle of Departure

AP - Access Point

AWGN - Additive White Gaussian Noise

B5G - Beyond Fifth-Generation

BS - Base Station

CMOS - Complementary Metal Oxide Semiconductor

CP - CANDECOMP/PARAFAC

CRLB - Cramér-Rao Lower Bound

CS - Compressed Sensing

CWSSM - Coherent Wideband Signal-Subspace Method

DAC - Digital-to-Analog Converter

DFT - Discrete Fourier Transformation

DoA - Direction-of-Arrival

DoF - Degree of Freedom

ESPRIT - Estimation of Signal Parameter via Rotational Invariance Technique

EVD - Eigenvalue-Decomposition

GPS - Global Position System

HOSVD - Higher-Order Singular Value Decomposition  
LoS - line-of-sight  
IoT - Internet of Things  
ISSP - Incoherent Signal-Subspace Processing  
IWSSM - Incoherent Wideband Signal-Subspace Method  
MHA - Minimum Hole Array  
mIoT - Massive Internet of Things  
ML - Maximum Likelihood  
mmWave - Millimeter-Wave  
MRA - Minimum Redundancy Array  
MS - Mobile Station  
MUSIC - Multiple Signal Classification  
NLoS - Non-Line-of-Sight  
OBA - Open Box Array  
OFDM - Orthogonal Frequency Division Multiplexing  
OMP - Orthogonal Matching Pursuit  
Q-DFT - Quasi-Discrete Fourier Transform  
RF - Radio Frequency  
RMa - Rural Macro  
RMSE - Root Mean Square Error  
RSS - Received Signal Strength  
SNR - Signal-to-Noise Ratio  
TDoA - Time Difference of Arrival  
TLS - Total-Least-Squares  
UCA - Uniform Circular Array  
UCAMI - Unitary Constrained Array Manifold Interpolation  
UCyA - Uniform Cylindrical Array

ULA - Uniform Linear Array

URA - Uniform Rectangular Array

UMa - Urban Macro

UMi - Urban Micro

WSSM - Wideband Signal-Subspace Method

## Nomenclature and Notation

$a$ ,  $\mathbf{a}$ ,  $\mathbf{A}$ , and  $\mathbb{A}$  stand for a scalar, a column vector, a matrix, and a set, respectively.

$\mathbf{I}_K$  denotes a  $K \times K$  identity matrix.

$\mathbf{0}_{M \times K}$  denotes an  $M \times K$  zero matrix.

$\|\mathbf{A}\|_F$  denotes the Frobenius norm of  $\mathbf{A}$ .

$\text{invec}(\cdot)$  denotes the inverse algorithm of vectorization.

$\det(\mathbf{A})$  is the determinant of  $\mathbf{A}$ .

$\text{Tr}(\mathbf{A})$  denotes the trace of  $\mathbf{A}$ .

$\text{vec}(\mathbf{A})$  is the vectorization of  $\mathbf{A}$ .

$\mathbf{A}^*$  denotes the conjugate of  $\mathbf{A}$ .

$\mathbf{A}^T$  denotes the transpose of  $\mathbf{A}$ .

$\mathbf{A}^H$  denotes the conjugate transpose of  $\mathbf{A}$ .

$\mathbf{A}^{-1}$  denotes the inverse of  $\mathbf{A}$ .

$\mathbf{A}^\dagger$  denotes the Moore-Penrose pseudo inverse of  $\mathbf{A}$ .

$\mathbf{A} \otimes \mathbf{B}$  denotes the Kronecker products of  $\mathbf{A}$  and  $\mathbf{B}$ .

$\mathbf{A} \diamond \mathbf{B}$  is the Khatri-Rao product of  $\mathbf{A}$  and  $\mathbf{B}$ .

$\text{E}\{\cdot\}$  denotes the expectation of a random variable.

$\odot$  is the Hadamard product operator.

$\lceil \cdot \rceil$  and  $\text{mod}(\cdot)$  represent the ceiling function and the modulo operator, respectively.

$O(\cdot)$  denotes the computational complexity.

$\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times \dots \times I_N}$  denotes an order- $N$  tensor, whose elements (entries) are  $a_{i_1, i_2, \dots, i_N}$ ,  $i_n = 1, 2, \dots, I_n$ , and the index of  $\mathcal{A}$  in the  $n$ -th mode ranges from 1 to  $I_n$ .

$\mathcal{A}_{:, :, \dots, :, i_n = k, :, \dots, :}$  denotes a subtensor of  $\mathcal{A}$ , where the index of the mode- $n$  is set to  $k$  ( $0 \leq k \leq I_n$ ).

$[\mathcal{A} \sqcup_n \mathcal{B}]$  denotes the tensor concatenation of  $\mathcal{A}$  and  $\mathcal{B}$  in mode- $n$ .

$\mathbf{A}_{(n)} \in \mathbb{C}^{I_n \times (I_1 I_2 \dots I_N / I_n)}$  denotes the mode- $n$  unfolding (also known as matricization) of  $\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times \dots \times I_N}$ .

$\text{Rank}_n(\mathcal{A})$  is the rank of the mode- $n$  unfolding of tensor  $\mathcal{A}$ , i.e.,  $n$ -rank of  $\mathcal{A}$ .

$\mathcal{C} = \mathcal{A} \times_n \mathbf{B} \in \mathbb{C}^{I_1 \times \dots \times I_{n-1} \times J_n \times I_{n+1} \times \dots \times I_N}$  is the  $n$ -mode product of a tensor  $\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times \dots \times I_N}$  and a matrix  $\mathbf{B} \in \mathbb{C}^{J_n \times I_n}$ . It can be written in the form of the mode- $n$  matricized tensor:  $\mathbf{C}_{(n)} = \mathbf{B} \mathbf{A}_{(n)}$ .

$\mathcal{C} = \mathcal{A} \circ \mathcal{B} \in \mathbb{C}^{I_1 \times I_2 \times \dots \times I_N \times J_1 \times J_2 \times \dots \times J_M}$  is the outer product of two tensors  $\mathcal{A} \in \mathbb{C}^{I_1 \times I_2 \times \dots \times I_N}$  and  $\mathcal{B} \in \mathbb{C}^{J_1 \times J_2 \times \dots \times J_M}$ , whose elements are  $c_{i_1, i_2, \dots, i_N, j_1, j_2, \dots, j_M} = a_{i_1, i_2, \dots, i_N} b_{j_1, j_2, \dots, j_M}$ .

Some important properties of tensor operations used in this paper are presented in Appendix A.