



# Characterization and Control of Quantum Systems using Machine Learning and Information Theory

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under the supervision of

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Akram Youssry Abdelaziz Mohamed declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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This is a THESIS BY COMPILATION. Parts of this thesis have been already published. The content have been edited to suit the formatting of the thesis and to maintain its coherence.





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# List of Abbreviations

<b>CAD</b>	Computer-Aided Design
<b>CDD</b>	Concatenated Dynamical Decoupling
<b>CPMG</b>	Carr–Purcell–Meiboom–Gill
<b>CPTP</b>	Completely-Positive Trace-Preserving
<b>CRAB</b>	Chopped Random Basis
<b>DD</b>	Dynamical Decoupling
<b>GRAPE</b>	Gradient Ascent Pulse Engineering
<b>GRU</b>	Gated-Recurrent Unit
<b>GST</b>	Gate Set Tomography
<b>LS</b>	Least-squares Estimation
<b>LSTM</b>	Long Short-Term Memory
<b>MEG</b>	Matrix Exponentiated Gradient
<b>MLE</b>	Maximum Likelihood Estimation
<b>NISQ</b>	Noisy Intermediate-Scale Quantum
<b>NN</b>	Neural Network
<b>OC</b>	Optimal Control
<b>PDD</b>	Periodic Dynamical Decoupling
<b>PGD</b>	Projected Gradient Descent
<b>PSD</b>	Power Spectral Density
<b>QCVV</b>	Quantum Characterization, Verification, and Verification

<b>QIP</b>	Quantum Information Processing
<b>QNS</b>	Quantum Noise Spectroscopy
<b>QPT</b>	Quantum Process Tomography
<b>QST</b>	Quantum State Tomography
<b>RNN</b>	Recurrent Neural Network
<b>RWA</b>	Reconfigurable Waveguide Array
<b>SPAM</b>	State Preparation and Measurement
<b>UDD</b>	Uhrig Dynamical Decoupling

# Abstract

The tasks of characterization and control of quantum systems are becoming more challenging with the advancement of quantum technology. Standard methods that were successful for simple quantum systems are becoming inadequate for more complex engineered systems. Modelling assumptions and approximations (such as Markovianity) are not justifiable anymore. As a result, the usual models fail to fit experimental measurements. In this thesis, we use state-of-the-art machine learning methods, assisted by tools from information theory as needed, to develop new frameworks that try to address these challenges. We focus on three directions. The first is developing an efficient online quantum state estimation algorithm with provable convergence properties. The second is developing a deep learning framework for characterizing and controlling closed quantum systems. The final direction is upgrading that framework to be suitable for characterization and control of open quantum systems. This thesis opens the door for a novel way of utilizing machine learning techniques for applications in quantum information specially and physics in general.

