

# **Gaussian Process Preintegration for Inertial-Aided Navigation Systems**

**by Cedric Le Gentil**

Thesis submitted in fulfilment of the requirements for  
the degree of

**Doctor of Philosophy**

under the supervision of A/Prof. Teresa Vidal-Calleja  
and A/Prof. Shoudong Huang

University of Technology Sydney  
Faculty of Engineering and IT

February 2021



# Certificate of Original Authorship

I, Cedric Le Gentil declare that this thesis, is submitted in fulfilment of the requirements for the award of degree of Doctor of Philosophy, in the Faculty of Engineering and IT (FEIT) at the University of Technology Sydney (UTS).

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

Signature: \_\_\_\_\_  
Production Note: Signature removed prior to publication.

Date: 24/02/2021  
\_\_\_\_\_



# Gaussian Process Preintegration for Inertial-Aided Navigation Systems

by

Cedric Le Gentil

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

## *Abstract*

To perform any degree of autonomy, a system needs to localise itself, generally requiring knowledge about its environment. While satellite technologies, like GPS or Galileo, allow individuals to navigate throughout the world, the level of accuracy of such systems, and the necessity to have a direct view of the sky, do not match the precision and robustness requirements needed to deploy robots in the real world. To overcome these limitations, roboticists developed localisation and mapping algorithms traditionally based on camera images or radar/LiDAR data. Across the last two decades, Inertial Measurement Units (IMUs) became ubiquitous. Thus, LiDAR-inertial and visual-inertial pose estimation algorithms represent now the majority of the state estimation literature.

Preintegration became a standard method to aggregate inertial measurement units (IMUs) readings into pseudo-measurements for navigation systems. This thesis presents a novel preintegration theory that leverages data-driven continuous representations of the inertial data to perform analytical inference of the signal integrals. The proposed method probabilistically infers the pseudo-measurements, called *Gaussian Preintegrated Measurements* (GPMs), over any time interval, using Gaussian Process (GP) regression to model the IMU measurements and leveraging the application of linear operators to the GP covariance kernels. Thus, the GPMs do not rely on any explicit motion-model.

This thesis presents two inertial-aided systems that leverage the GPMs in offline batch-optimisation algorithms. The first one is a framework called IN2LAAMA for *INertial Lidar Localisation Autocalibration And MApping*. The proposed method addresses the issue of *motion distortion* present in most of today's LiDARs' data thoroughly by using GPMs for each of the LiDAR points.

The second GPM application is an event-based visual-inertial odometry method that uses lines to represent the environment. Event-cameras generate highly asynchronous streams of events that are individually triggered by each of the camera pixels upon illumination

changes. Our framework, called IDOL for *IMU-DVS Odometry using Lines*, estimates the system's pose as well as the position of 3D lines in the environment by considering the camera events in the framework's cost function individually (no aggregation in image-like data). The GPMs allow for the continuous characterisation of the system's trajectory, therefore accommodating the asynchronous nature of event-camera data.

Extensive benchmarking of the GPMs is performed on simulated data. The performance of IN2LAAMA is thoroughly demonstrated throughout simulated and real-world experiments, both indoor and outdoor. Evaluations on public datasets show that IDOL performs at the same order of magnitude as current frame-based state-of-the-art visual-inertial odometry frameworks.

*“Vous savez, moi je ne crois pas qu’il y ait de bonne ou de mauvaise situation. Moi, si je devais résumer ma vie aujourd’hui avec vous, je dirais que c’est d’abord des rencontres.”<sup>1</sup>*

Édouard Beaer

---

<sup>1</sup>“Well, you see... I don’t believe that there are good or bad situations. If I had to summarise my life, here, with you, I would say that it is all about encounters.”

# Acknowledgements

The first person I want to thank is someone that I forgot the name of. When people ask me how I started my PhD, I refer to him as the “Spanish dude”. During a smoko<sup>2</sup> on a building site in April 2016, I told him that I was thinking to quit my labourer job and attempt to volunteer in random software companies to try to “get back in my field”. To this, he pronounced words that profoundly changed my life:

*“Why don’t you go see a university for that?”.*

I cannot thank Shoudong Huang enough for answering my email; even if I suspect the terms “*I am ready to work on a volunteer basis*” had a non-negligible role in his decision to invite me to visit the lab and introduce me to Teresa Vidal-Calleja. After four and something years, it is always a pleasure to work with Teresa. Her passion for her job is an inexhaustible source of motivation for everyone she collaborates with. She believed in me and introduced me to the academic world. She gave me access to unique opportunities that made me grow not only as a researcher, but also as a person. Both for their technical expertise and their human qualities, Teresa and Shoudong have been wonderful supervisors that allowed me to make the best out of my PhD experience.

I want to thank every member of the Centre for Autonomous Systems that I had the chance to interact with. I especially express my gratitude to Raphaël Falque, Lakshitha Dantanarayana, and Buddhi Wijerathna for many answers to many questions, for the laughs and the road trips, more broadly for their friendship inside and outside the lab.

I want to thank Delphine and Flavien Dyièvre-Hamon who have demonstrated invaluable support when I needed it most. My PhD experience would not have been the same without rock climbing, its beautiful community, and the UTS Outdoor Adventure Club. Among the people I actually trust my life with on a weekly basis, I want to thank James Millar, who is always keen for crazy adventures, Thibaut Géry and Ronny Onggo for teaching me so much, and Marta Khomyn for changing my vision of the world and life itself. Marta deserves even more gratitude for her moral support and helping to proofread the very first version of this thesis.

Last but not least, I want to thank my parents, Valérie and Christophe Le Gentil, for their love despite the kilometres. During my whole existence, they offered me the best, even when life was not the easiest for them. The freedom and trust they gave me have been a blessing. Thank you for understanding and accepting the different choices I made throughout my life.

---

<sup>2</sup>Australian slang term designating a short break during work. Mostly used in the construction industry.



# Contents

<b>Declaration of Authorship</b>	<b>iii</b>
<b>Abstract</b>	<b>v</b>
<b>Acknowledgements</b>	<b>viii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Research topic . . . . .	1
1.2 Scope . . . . .	2
1.2.1 State estimation . . . . .	2
1.2.2 Localisation and mapping . . . . .	4
1.2.3 Extrinsic calibration . . . . .	5
1.2.4 Lidars and motion distortion . . . . .	5
1.2.5 Event-cameras . . . . .	5
1.2.6 Motivation . . . . .	6
1.3 Objectives and contributions . . . . .	7
1.4 Thesis outline . . . . .	10
1.5 List of publications . . . . .	11
1.5.1 Core contributions . . . . .	11
1.5.2 Side contributions . . . . .	12
<b>2 Review of Related Work</b>	<b>13</b>
2.1 Preintegration . . . . .	14
2.2 Continuous time state representation . . . . .	15
2.3 Lidar localisation and mapping . . . . .	16
2.4 Event-based odometry . . . . .	19
2.5 Extrinsic calibration . . . . .	21
<b>3 Gaussian Preintegration</b>	<b>25</b>
3.1 Introduction . . . . .	25
3.2 Problem statement . . . . .	26
3.2.1 System description . . . . .	26
3.2.2 IMU preintegration . . . . .	28
3.3 Definitions and background . . . . .	29

3.3.1	Gaussian Process regression . . . . .	30
3.3.2	Gaussian Process inference with linear operators . . . . .	32
3.4	Gaussian Preintegrated Measurements . . . . .	33
3.4.1	GPM - Rotation . . . . .	34
3.4.2	GPM - Velocity and position . . . . .	36
3.5	Postintegration bias and inter-sensor time-shift corrections . . . . .	37
3.5.1	Rotation GPM Jacobians . . . . .	38
3.5.1.1	Gyroscope biases . . . . .	38
3.5.1.2	Inter-sensor time-shift . . . . .	39
3.5.2	Velocity and position GPM Jacobians . . . . .	39
3.5.2.1	Accelerometer and gyroscope biases . . . . .	39
3.5.2.2	Inter-sensor time-shift . . . . .	40
3.6	Experiments and results . . . . .	40
3.6.1	Low-frequency benchmarks (0.2 - 20 Hz) . . . . .	41
3.6.1.1	Accuracy . . . . .	41
3.6.1.2	Robustness to noise . . . . .	42
3.6.1.3	Computation time . . . . .	44
3.6.2	High-frequency benchmarks (> 100 kHz) . . . . .	45
3.6.2.1	Accuracy . . . . .	46
3.6.2.2	Computation time . . . . .	47
3.7	Conclusion . . . . .	48
<b>4</b>	<b>IN2LAAMA: INertial Lidar Localisation Autocalibration And MApping</b>	<b>51</b>
4.1	Introduction . . . . .	51
4.2	Method overview . . . . .	54
4.2.1	Notation and definitions . . . . .	54
4.2.2	Cost function . . . . .	55
4.3	Back-end . . . . .	56
4.3.1	IMU factors . . . . .	56
4.3.2	IMU biases and inter-sensor time-shift . . . . .	57
4.3.3	Lidar factors . . . . .	57
4.4	Front-end . . . . .	59
4.4.1	Feature extraction . . . . .	59
4.4.2	Feature recomputation . . . . .	61
4.4.3	Data association . . . . .	63
4.4.3.1	Feature matching . . . . .	63
4.4.3.2	Outliers rejection . . . . .	65
4.4.4	Loop-closure detection . . . . .	65
4.5	On the factor graph and implementation . . . . .	68
4.5.1	Factor graph for localisation and mapping . . . . .	68
4.5.2	Factor graph for autocalibration, localisation, and mapping . . . . .	68
4.5.3	Robustness of state estimation . . . . .	70
4.5.4	Bias observability . . . . .	71
4.5.5	GPMs and memory . . . . .	71

4.6	Experiments and results . . . . .	72
4.6.1	Simulation - localisation and mapping . . . . .	73
4.6.1.1	Odometry . . . . .	73
4.6.1.2	Loop-closure . . . . .	75
4.6.1.3	Robustness to inaccurate sensor model . . . . .	75
4.6.1.4	No motion model . . . . .	76
4.6.2	Simulation - front-end . . . . .	76
4.6.3	Simulation - calibration . . . . .	78
4.6.4	Real-data - Localisation and mapping . . . . .	79
4.6.4.1	Indoors . . . . .	79
4.6.4.2	Outdoors . . . . .	82
4.6.5	Real-data - Calibration . . . . .	84
4.7	Conclusion . . . . .	85
<b>5</b>	<b>IDOL: IMU-DVS Odometry using Lines</b>	<b>87</b>
5.1	Introduction . . . . .	87
5.2	Method overview . . . . .	90
5.3	Back-end . . . . .	91
5.3.1	Event-to-line factors . . . . .	92
5.3.2	Line attraction and repulsion factors . . . . .	92
5.4	Front-end . . . . .	94
5.5	Experiments . . . . .	96
5.5.1	Datasets and Evaluations . . . . .	96
5.5.2	Results . . . . .	97
5.6	Conclusions . . . . .	104
<b>6</b>	<b>Conclusions and future work</b>	<b>107</b>
6.1	Conclusions . . . . .	107
6.2	Future work and associated developments . . . . .	109
6.2.1	Semantic understanding of the scene . . . . .	109
6.2.2	Loop closure detection . . . . .	110
6.2.3	Calibration trajectories . . . . .	111
6.2.4	Event-based visual-lidar-inertial localisation and mapping . . . . .	112
	<b>Appendices</b>	<b>113</b>
<b>A</b>	<b>Overview of the Upsampled Preintegration method</b>	<b>115</b>
<b>B</b>	<b>Derivation of the bias jacobians for GPM postintegration correction</b>	<b>119</b>
B.1	Accelerometer bias . . . . .	120
B.2	Gyroscope bias . . . . .	120
<b>C</b>	<b>IN2LAAMA Jacobians</b>	<b>123</b>
C.1	IMU factors . . . . .	124

---

C.2	Biases factors . . . . .	128
C.3	LiDAR factors . . . . .	129
C.3.1	Point reprojection . . . . .	129
C.3.2	Point-to-plane . . . . .	131
C.3.3	Point-to-line . . . . .	132
C.3.4	Noise propagation . . . . .	133
<b>D</b>	<b>IDOL Jacobians</b>	<b>135</b>
D.1	Event-to-line . . . . .	136
D.2	Projection from 3D to 2D . . . . .	137
D.2.1	3D transformation . . . . .	137
D.3	Splitting force . . . . .	140
D.4	Attraction force . . . . .	141
	<b>Bibliography</b>	<b>143</b>

# Acronyms & Abbreviations

<b>1D</b>	One-Dimensional
<b>2D</b>	Two-Dimensional
<b>3D</b>	Three-Dimensional
<b>CAS</b>	Centre for Autonomous Systems
<b>CPU</b>	Central Processing Unit
<b>DoF</b>	Degree-of-Freedom
<b>DVS</b>	Dynamic Vision Sensor
<b>EKF</b>	Extended Kalman filter
<b>FoV</b>	Field-of-View
<b>GNSS</b>	Global Navigation Satellite System
<b>GP</b>	Gaussian Process
<b>GPM</b>	Gaussian Preintegrated Measurement
<b>GPS</b>	Global Position System
<b>GPU</b>	Graphic Processing Unit
<b>HDR</b>	High Dynamic Range
<b>ICP</b>	Iterative Closest Point
<b>IDOL</b>	IMU-DVS Odometry using Lines

<b>IMU</b>	Inertial Measurement Unit
<b>IN2LAAMA</b>	INertial Lidar Localisation Autocalibration And MApping
<b>KF</b>	Kalman Filter
<b>LiDAR</b>	Light Detection And Ranging Sensor
<b>LPM</b>	Linear Preintegrated Measurement
<b>MAP</b>	Maximum A Posteriori
<b>MLE</b>	Maximum Likelihood Estimation
<b>NDT</b>	Normal Distribution Transform
<b>PM</b>	Standard Preintegrated Measurement
<b>RRBT</b>	Rapidly Exploring Random Belief Trees
<b>RGB</b>	Red-Green-Blue
<b>RGBD</b>	Red-Green-Blue-Depth
<b>RMS</b>	Root Mean Squared
<b>RMSE</b>	Root Mean Squared Error
<b>SE(3)</b>	Special Euclidean group in three dimensions
<b>SLAM</b>	Simultaneous Localisation And Mapping
<b><math>\mathfrak{so}(3)</math></b>	Lie algebra of special orthonormal group in three dimensions
<b>SO(3)</b>	Special orthonormal group in three dimensions
<b>UPM</b>	Upsampled-Preintegrated-Measurement
<b>UTS</b>	University of Technology, Sydney
<b>VI</b>	Visual-Inertial
<b>VIO</b>	Visual-Inertial Odometry
<b>VO</b>	Visual Odometry