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Keywords

Phillips curve, bargaining power, secondary workers

JEL Classification

C60, E31

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Dual Labor Market and the "Phillips Curve Puzzle"

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Abstract

Low inflation was once a welcome to both policy makers and the public. However, Japan's experience during the 1990's changed the consensus view on price of economists and central banks around the world. Facing deflation and zero interest bound at the same time, Bank of Japan had difficulty in conducting effective monetary policy. It made Japan's stagnation unusually prolonged. Too low inflation which annoys central banks today is translated into the "Phillips curve puzzle". In the US and Japan, in the course of recovery from the Great Recession after the 2008 global financial crisis, the unemployment rate had steadily declined to the level which was commonly regarded as lower than the natural rate or NAIRU. And yet, inflation stayed low. In this paper, we consider a minimal model of dual labor market to jointly investigate how blue the different factors affecting the structural evolution of the labor market have contributed to the observed flattening of the Phillips curve. We find that the the joint evolution of the level of bargaining power of workers, of the elasticity of the supply of labor to wage in the secondary market, and of the composition of the workforce is the main factor in explaining the evidence for Japan.

Keywords: Phillips curve; bargaining power; secondary workers.
JEL codes: C60; E31.

1 Introduction

Low inflation was once a welcome to both policy makers and the public. However, Japan's experience during the 1990's changed the consensus view on price of economists and central banks around the world; After the financial bubble burst at the beginning of the 1990's, Japan lapsed into deflation. During the course, the Bank of Japan (BOJ) kept cutting the nominal interest rate down to zero. Facing deflation and zero interest bound at the same time, BOJ had difficulty in conducting effective monetary policy. It made Japan's stagnation unusually prolonged.

The "Japan problem" made economists aware of long-forgotten danger of deflation. In the prewar period, deflation was a menace to the economy, and its danger was emphasized by famous economists such as Keynes (1931) and Fisher (1933). To prevent deflation, central bank must seek low inflation rather than zero inflation or stable price level. Today, following this idea, many central banks including The US Federal Reserve, BOJ, and European Central Bank target at two percent inflation of consumer price index. However, few central banks have been successful in achieving this goal in any satisfactory way.

Too low inflation which annoys central banks today is translated into the Phillips curve puzzle. The benchmark Phillips curve is as follows (Phillips, 1958; Friedman, 1968):

$$\pi_t = a\,(u - u^*) + b\,\pi_t^*, \tag{1}$$

where π and u are inflation and the unemployment rate, respectively. π^* is either inflationary expectation or inertia of past inflation. u^* is the natural rate of unemployment or the NAIRU (Non-Accelerating Inflation Rate of Unemployment). According to conventional macroeconomics, Eq. (1) or the Phillips curve determines inflation.

In the US and Japan, in the course of recovery from the Great Recession after the 2008 global financial crisis, the unemployment rate had steadily declined to the level which was commonly regarded as lower than NAIRU, u^*. And yet, inflation stayed low: 0.5% for Japan and 1.5% for the US.

The Phillips curve puzzle is acute particularly in Japan because the core of notorious Japan's deflation is the stagnation of nominal wages. For example, in 2019, the U.S. PCE of manufactured product declined by 0.5% whereas Japan's counterpart actually rose by 1.0%. However the consumer price of labor-intensive services rose by 2.4% in U.S. whereas it rose only by 0.5% in Japan. This difference reflects the stagnation of nominal wages in Japan.

In terms of the Phillips curve, Eq. (1), two things have been pointed out. First, coefficient b for inflationary expectations or lagged inflation declined significantly almost to zero in recent years. Blanchard (2018, Figure 7), for example, found that b which was almost zero in the early 1960's, rose sharply to one in the late 1960's, had stayed there for thirty years, and then declined suddenly to zero at the beginning the 2000's. While the decline in inflation from the '90s had been often attributed to better policy management, and even the tern "Great Moderation" was coined (Clarida et al., 2000), more recent analyses identify the anchoring of inflation expectations for the change in the trade-off between inflation and unemployment (Barnichon and Mesters, 2021; Blanchard, 2016). Greenspan (2001) left the following remark: "Price stability is best thought of as an environment in which inflation is so low and stable over time that it does not materially enter into the decisions of households and firms."

The second factor is a change of coefficient a for the unemployment rate in Eq. (1). A decline of a entails lower inflation than otherwise when the unemployment rate declines. Some researchers even argue that unemployment no longer has an effect on inflation, at least over some unemployment and inflation range.

Figure 1 (a) displays Japan's Phillips curve, namely, the quarterly relation between the unemployment rate and nominal wage growth for 1980.II-2019.II. We can indeed observe that the Phillips curve has flattened in recent years.

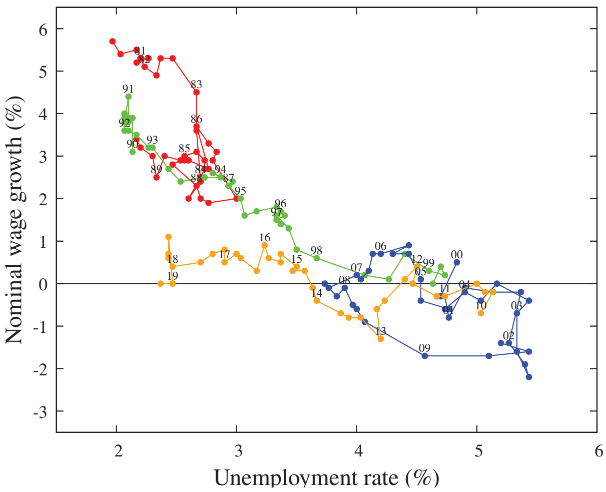
These define the "Phillips curve puzzle". In this paper, we leave the first problem untouched: we simply take b as zero, and focus on the second problem, namely why a gets smaller in Eq. (1).

For this purpose, we consider a minimal model of labor market. We believe that it is useful and important to to set up a minimal model to organize various issues concerning the Phillips curve puzzle. In a simple model of dual labor market (as in McDonald and Solow, 1981, 1985; Gordon, 2017), we explore what kind of change in the economy makes the Phillips curve flat. The analysis of the model reveals that the level of bargaining power of workers, the elasticity of the supply of labor to wage in the secondary market, and the composition of the workforce are the main factors in explaining the flattening of the Philips curve.

Our main contribution is to provide a compact model to consider the joint evolution of structural factors that have been so far investigated separately in the literature. The main finding is that the change in shape in the relationship between the level of economic activity and inflation can only be explained by the joint effects of these four factors. More specifically, none of these factors by itself appears to be able to generate the shift in the Phillips curve observed in the data. This conclusion introduces a new and original perspective since, to the best of our knowledge, the theoretical literature has so far either focused on a single factor at the time, or provided a microeconomic explanation based on changes in the behavior of employers and workers. Our study suggests that any attempt to explain the dramatic shift in the relationship between inflation and level of economic activity observed for Japan should focus on the deep structural modifications in the labor market that occurred over the last three decades.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model. Sections 3 and 4 presents the analytical and numerical results, respectively. Finally, section 5 provides some concluding remarks.

(a)



(b)

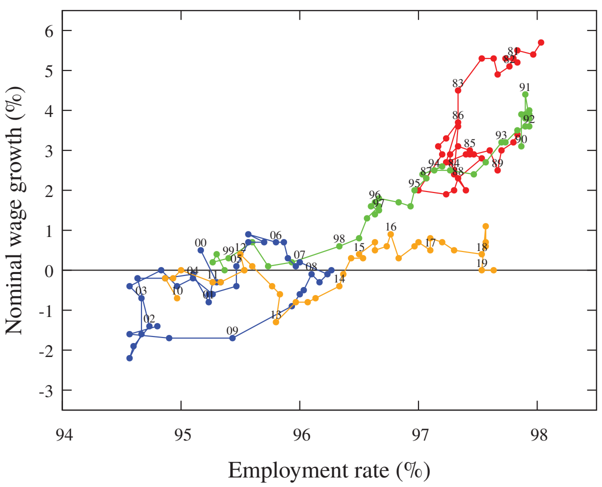


Figure 1: **Japan's Phillips curve: Unemployment rate and nominal wage growth of 157 quarters from 1980.Π to 2019.Π.** (a) The abscissa is the unemployment rate in %. (b) The abscissa is reversed and is the employment rate (=100-unemployment rate (%)). Colors: 1980s (red), 1990s (green), 2000s (blue), 2010s (orange). The last several years' data (in yellow) show definite deviation from those of the earlier years, with nominal wage growth is low in spite of the low unemployment. Sources are Ministry of Health, Labor and Welfare, Japan (2020) and Statistics Bureau of Japan (2020).

2 The Model

The model assumes a dual labor market consisting of primary labor and secondary workers (McDonald and Solow, 1981, 1985; Gordon, 2017). As in Di Guilmi and Fujiwara (2020), we identify as secondary workers all the employees without a permanent contract (agency, temporary, and part-time).

Firms are heterogeneous in size and efficiency, but adopt the same production function. Each firm produces a homogeneous goods by employing only labor, composed by primary and secondary workers. Specifically, the firm j ($j = 1, \cdots, N$) employs $L_{1,j}$ primary workers with wage $w_{1,j}$ and $L_{2,j}$ secondary workers with wage w_2. There are $L_1 = \sum_{j=1}^{N} L_{1,j}$ primary workers and $L_2 = \sum_{j=1}^{N} L_{2,j}$ secondary workers employed.

Primary workers are a fixed endowment for each firm.[1] Accordingly, $L_{1,j}$ is a given constant. In contrast, firm freely changes the level of secondary workers. Following the empirical findings of Munakata and Higashi (2016), the wage of secondary workers is determined by the market and is uniform across firms.

The output of firm j is determined as follows:

$$Y_j = A_j (L_{1,j} + c\, L_{2,j})^{\alpha}, \tag{2}$$

where $\alpha \in (0, 1)$. A_j is firm-specific total factor productivity (TFP). The parameter $c \in (0, 1)$ quantifies the productivity of secondary labor relative to that of primary labor.

The profit is given by

$$\Pi_j = Y_j - L_{1,j} w_{1,j} - L_{2,j} w_2. \tag{3}$$

In order to mimic the firm-level bargaining process that is prevailing in Japan, the primary workers' wage is set in a two-step process for each employer. Assuming a fixed endowment of primary workers (or *insiders*) $L_{1,j}$ for each firm, in the first stage firm and primary workers determine the number of secondary workers (or *outsiders*) $L_{2,j}$ to be hired. Assuming a perfectly competitive market for secondary workers, firms take the secondary wage w_2 as given. Once the number of secondary workers is determined, firms and insiders share the surplus defined by revenue less the wages paid to secondary workers through a Nash bargaining (Mortensen and Pissarides, 1994).

First stage maximization: profit

Firm maximizes its profit (3) by choosing the number of secondary workers or outsiders:

$$\max_{L_{2,j}} \left[\Pi_j\right]. \tag{4}$$

[1] The Japanese firm rarely lays off its primary workers. In 2020, for example, in the mid of Covid–19 recession, real GDP fell by unprecedented 28.1%, and yet, the unemployment rate rose slightly only to 2.8%.

This determines demand for secondary workers $L_{2,j}^{(\mathrm{d})}$ of firm j as follows:

$$L_{2,j}^{(\mathrm{d})} = \frac{1}{c}\left[-L_{1,j} + \left(\frac{\alpha c A_j}{w_2}\right)^{1/(1-\alpha)}\right]. \tag{5}$$

The total demand for secondary workers in the economy as a whole is then:

$$L_2^{(\mathrm{d})} = \frac{1}{c}\left[-L_1 + \left(\frac{\alpha\, c\, A}{w_2}\right)^{1/(1-\alpha)}\right], \tag{6}$$

where A is the following nonlinear sum of A_j:

$$A = \left(\sum_{j=1}^{N} A_j^{1/(1-\alpha)}\right)^{(1-\alpha)}. \tag{7}$$

The level of output of firm j is

$$Y_j = A_j \left(\frac{\alpha c A_j}{w_2}\right)^{\alpha/(1-\alpha)}. \tag{8}$$

We assume the following supply function of secondary workers L_2:

$$L_2^{(\mathrm{s})} = B\, w_2^{\beta}. \tag{9}$$

where β is the Frisch elasticity. Matching demand for and supply of secondary workers, $L_2^{(\mathrm{s})} = L_2^{(\mathrm{d})}$, we obtain the following nonlinear equation for w_2:

$$B\, w_2^{\beta} = \frac{1}{c}\left[-L_1 + \left(\frac{\alpha\, c\, A}{w_2}\right)^{1/(1-\alpha)}\right], \tag{10}$$

Demand for and supply of secondary labor as functions of w_2 are shown on (L_2, w_2) plane in Fig. 2. It can be shown that the solution of Eq. (10) always exists.

Second stage maximization: primary workers' wage

Firm and primary workers (insiders) determine the wage of primary workers $w_{1,j}$ through a Nash bargaining according to:

$$\max_{w_{1,j}} \left[(L_{1,j} w_{1,j})^{\gamma} (\Pi_j)^{(1-\gamma)}\right], \tag{11}$$

where $\gamma \in (0,1)$ indicates bargaining power of primary workers.

The Nash bargaining determines $w_{1,j}$ as follows:

$$w_{1,j} = \gamma \frac{Y_j - L_{2,j} w_2}{L_{1,j}}. \tag{12}$$

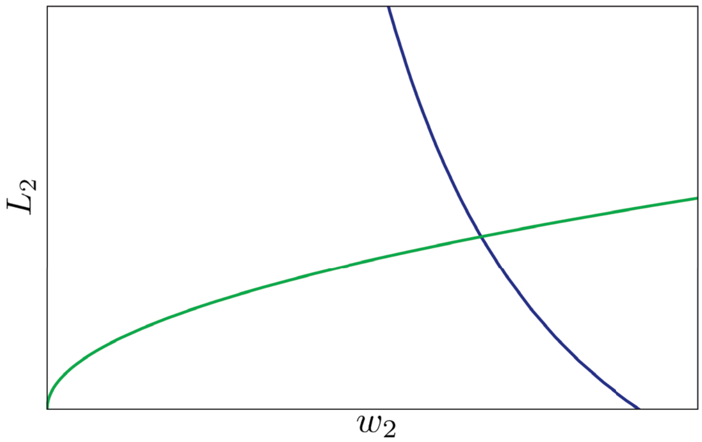


Figure 2: **Demand function $L_2^{(\mathrm{d})}$ (Eq.(6)) in blue and supply function $L_2^{(\mathrm{s})}$ (Eq.(9)) in green.**

By combining Eqs.(5) and (12), we find that

$$\Pi_j = \frac{1-\gamma}{\gamma} L_{1,j} w_{1,j}. \tag{13}$$

In the following, we study the relationship between the total employment of workers,

$$L = L_1 + L_2 = L_1 + B w_2^{\beta}, \tag{14}$$

and the average wage,

$$\bar{w} = \frac{W}{L}. \tag{15}$$

In Eq.(15), W is the total earnings of all the workers:

$$\begin{aligned} W &= \sum_{j=1}^{N} (L_{1,j} w_{1,j}) + L_2 w_2 \\ &= \gamma \sum_{j=1}^{N} Y_j + (1-\gamma) L_2 w_2 \\ &= \gamma A \left(\frac{\alpha c A}{w_2} \right)^{\alpha/(1-\alpha)} + (1-\gamma) B w_2^{1+\beta}. \end{aligned} \tag{16}$$

We use the result of the second maximization, Eq.(12). Because L and $\bar{w}$ are functions of A, we obtain functional relationship between L and $\bar{w}$ by eliminating A, while keeping other parameters $\{L_1, c,\ \alpha,\ \ B,\ \beta,\ \gamma\}$ fixed. The curve $w(L)$ is our Phillips curve, which models the relationship shown in Fig.1(b).

Our Phillips curve is expressed in level of wage (rather than wage inflation) since, as we show below, the rate of change in wage is implied by the wage level.

The parameters and variables of this model are listed in Table 1. The model is extremely parsimonious, with only seven free parameters: $A, L_1, c, \alpha, B, \beta, \gamma$. However, because of nonlinearities, its solution is not trivial and able to generate a set of interesting results.

Output	
A	Nonlinear sum of the total factor productivity A_j (Eq.(7))
L_1	Total number of primary workers
L_2	Total number of secondary workers employed (Eq.(5))
c	Secondary workers' productivity coefficient
α	Output exponent
Supply of secondary workers	
B	Coefficient of labor supply for secondary workers
β	Elasticity to wage for secondary workers
Nash Bargaining	
$w_{1,j}$	The wage of the primary workers at firm j (Eq. (12))
w_2	The wage of secondary workers (Eq. (10))
γ	Bargaining power of primary workers

Table 1: **List of the parameters and variables of the model.** Variables are determined by the equation referred in parentheses.

3 Solving the Model

Toward the goal of solving the model, we first rewrite Eq.(10) as follows:

$$\frac{w_2}{\alpha\, c\, A} = \left(L_1 + c\, B\, w_2^{\beta}\right)^{-(1-\alpha)} . \tag{17}$$

By introducing the following scaled variable v

$$v \equiv \frac{L_1^{\beta(1-\alpha)}}{(\alpha\, c\, A)^{\beta}} w_2^{\beta}. \tag{18}$$

we can rewrite Eq.(17) as follows:

$$v = (1 + g\, v)^{-\beta(1-\alpha)} , \tag{19}$$

where

$$g \equiv c\, B\, (\alpha\, c\, A)^{\beta}\, L_1^{-1-\beta(1-\alpha)}. \tag{20}$$

Recall that solving the model amounts to finding the equilibrium w_2 which is equivalent to v. Thus, we focus on Eq.(19). We note here that both v and g are dimensionless quantities in Eq.(19) (see Appendix A). This makes the following analysis straightforward.

With variable v, Eqs.(14) and (16) are written as follows:

$$L = L_1 \left[1 + \frac{g}{c} v\right] , \tag{21}$$

$$W = \frac{g^{1/\beta} L_1^{1+1/\beta}}{\alpha c (cB)^{1/\beta}} \left[\gamma\, v^{-\alpha/(\beta(1-\alpha))} + (1-\gamma)\, \alpha\, g\, v^{1+1/\beta}\right] . \tag{22}$$

This leads to the following average wage:

$$\bar{w} = \frac{W}{L} = \left(\frac{L_1}{B}\right)^{1/\beta} Z(\alpha, c, \beta, \gamma, g, v). \tag{23}$$

The coefficient $(L_1/B)^{1/\beta}$ is the *only* factor with the same dimension as $\bar{w}$. The function $Z(\alpha, c, \beta, \gamma, g, v)$ is the following dimensionless function of the dimensionless parameters $\alpha, c, \beta, \gamma, g$ and $v = v(g)$:

$$Z(\alpha, c, \beta, \gamma, g, v) = \\ \frac{g^{1/\beta}}{\alpha\, c^{1+1/\beta}} \left[\gamma\, v^{-\alpha/(\beta(1-\alpha))} + (1-\gamma)\, \alpha\, g\, v^{1+1/\beta}\right] \Bigg/ \left[1 + \frac{g}{c} v\right] . \tag{24}$$

The Phillips curve defined as the relationship between the average wage and employment, $\bar{w}(L)$, is obtained by eliminating g (and $v = v(g)$) from Eq. (21) and Eq. (23).

Now, the second term in the parentheses of the right-hand-side of Eq.(19), gv is the ratio cL_2/L_1. Therefore, if L_1 and L_2 measured in efficiency unit are different in order, we may approximate it around the larger term. In other words, if $L_1 \gg cL_2$, namely, if the primary workers dominate in efficiency in production, we expand the right hand side around for $gv \ll 1$ or equivalently $g \ll 1$ (note that large L_1 implies small g because of Eq.(20)).

The small-g perturbative solution of Eq.(19) is the following:

$$v = 1 - \sigma g + \frac{1}{2}\sigma(1 + 3\sigma)g^2 + \cdots , \tag{25}$$

where $\sigma \equiv \beta(1 - \alpha)$. By substituting Eq.(25) into Eqs.(21), (23), and (24), we obtain the followings:

$$L = L_1 \left[1 + \frac{g}{c} - \sigma \frac{g^2}{c} + \cdots \right] \tag{26}$$

$$\bar{w} = \left(\frac{L_1}{B}\right)^{1/\beta} \frac{g^{1/\beta}}{\alpha\, c^{1+1/\beta}} \left[\gamma + \left(\alpha - \frac{\gamma}{c}\right) g + \cdots \right]. \tag{27}$$

In order to eliminate g from these two equations and obtain a relationship between L and $\bar{w}$, we first solve Eq.(26) for small g:

$$g = c\left(\frac{L}{L_1} - 1\right) + c^2\sigma\left(\frac{L}{L_1} - 1\right)^2 + \cdots . \tag{28}$$

Note that this is a perturbative series in $L/L_1 - 1 = L_2/L_1 \ll 1$. By substituting this expression into Eq.(27), we obtain the following expression of $\bar{w}$:

$$\bar{w} = \frac{\gamma}{\alpha\, c}\left(\frac{L_1}{B}\right)^{1/\beta}\left(\frac{L}{L_1} - 1\right)^{1/\beta}\left[1 + \frac{c(\alpha + \gamma - \alpha\gamma) - \gamma}{\gamma}\left(\frac{L}{L_1} - 1\right) + \cdots \right]. \tag{29}$$

Thus, the average wage $\bar{w}$ is a monotonically increasing function of total employment, L. Namely, the Phillips curve has the expected sign of slope: it is upward sloping on employment–wage plane, therefore, is downward sloping on wage–unemployment plane. We find from the leading term of this expression that the slope of the curve depends on two key factors, $\gamma/(c\,\alpha B^{1/\beta})$ and $1/\beta$.

The above analysis assumes $L_1 \gg cL_2$. We can make similar analysis in the case of $L_1 \ll cL_2$. It is given in Appendix B.

4 Comparative Statics

In order to estimate the effects of the parameter, we provide here a numerical comparative statics exercise. For the numerical calculations, the parameters are estimated as follows: $c = 0.5$ (following Di Guilmi and Fujiwara, 2020); $\gamma = 0.5$ (Carluccio and Bas, 2015); $\beta \in [0.7, 0.1]$ (Kuroda and Yamamoto, 2007), while other parameters are calibrated. The Phillips curve is plotted in Fig. 3, where the solid curve is the numerical solution of Eqs. (19), (21), (23) and the dashed and dotted curves are the analytical solutions Eq.(29) and Eq.(43), respectively.

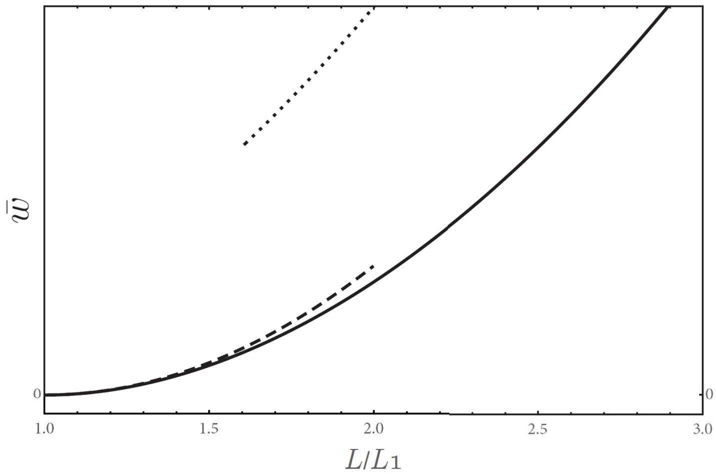


Figure 3: **The exact and approximate Phillips curve.** The solid curve: the exact solution $\bar{w}(L)$ (Eqs. (19), (21), (23)), the dashed curve: the analytical solution in case primary workers dominate (the leading term of Eq.(29)), and the dotted curve: the analytical solution in case secondary workers dominate (the leading term of Eq.(43)). The last approximation is not valid in this range of L/L_1. The parameters are chosen to be $\alpha = c = \beta = \gamma = 0.5$.

We find that the analytical solution (29) for the case when when primary workers are dominant provides a reliable approximation for it mimics well the original function in this range of L/L_1.

The wages of primary and secondary workers,

$$\bar{w}_1 \equiv \frac{1}{L_1} \sum_{j=1}^{N} L_{1,j} w_{1,j} \tag{30}$$

and w_2 as functions of total employment L are shown in Fig. 4. It is interesting to observe that the wages of primary workers determined by Nash bargaining increase more than market–determined wages of secondary workers when the total employment increases.

Our goal is to find the answer for the question why the Phillips curve flattened in recent years. For this purpose, we explore how the slope of Phillips curve depends on parameters c, B, γ, and β.

Variations of parameter β, however, require careful consideration: in the supply function Eq.(9), the coefficient B has a dimension that depends on β. Therefore, when changing the value of β, keeping the same numerical value of B does not make sense. One way of making clear the dimensional consideration is to set $B = L_1/w_0^\beta$ so that

$$L_2^{(s)} = L_1 \left(\frac{w_2}{w_0} \right)^\beta . \tag{31}$$

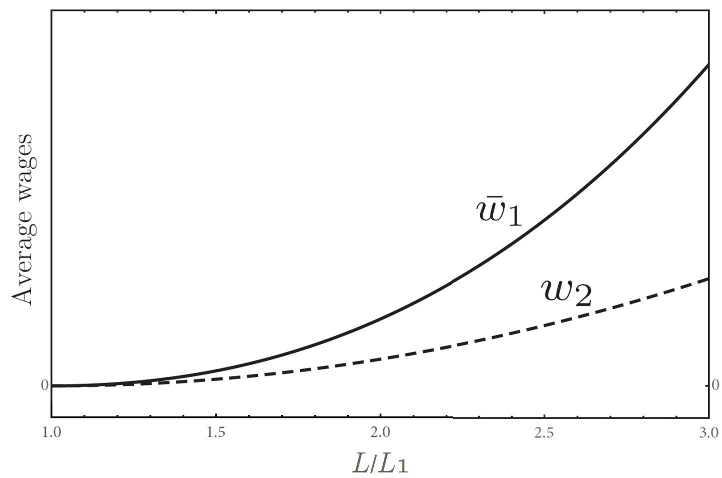


Figure 4: **The behavior of the wages $\bar{w}_1$ (solid curve) and w_2 (dashed curve).** The parameters are chosen to be the same with Fig. 3

Accordingly, the supply function is parametrized by w_0 and β instead of B and β. In this parametrization, Eq.(23) is written as follows:

$$\bar{w} = w_0 \, Z(\alpha, c, \beta, \gamma, g, v). \tag{32}$$

which makes the dimensionality trivial. Using Eq. (32), when varying β, we keep w_0 constant and vary the value of β in $Z(\alpha, c, \beta, \gamma, g, v)$ in the above equation. This situation is illustrated in Fig. 5.

Now, the Philips curve with relatively small changes of the parameters c, γ, B and β in the manner explained above are illustrated in Fig. 6 in comparison with the Phillips curve in Fig. 3.

Combining all the effects of changes of four parameters c, γ, B and β shown in Fig. 6, we obtain Fig. 7, where we can clearly observe a flattening of the Phillips curve.

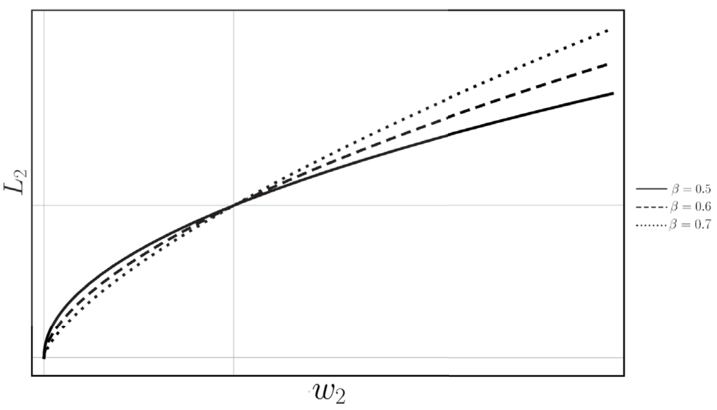


Figure 5: **The meaning of the parametrizaion of the supply function with varying β.**

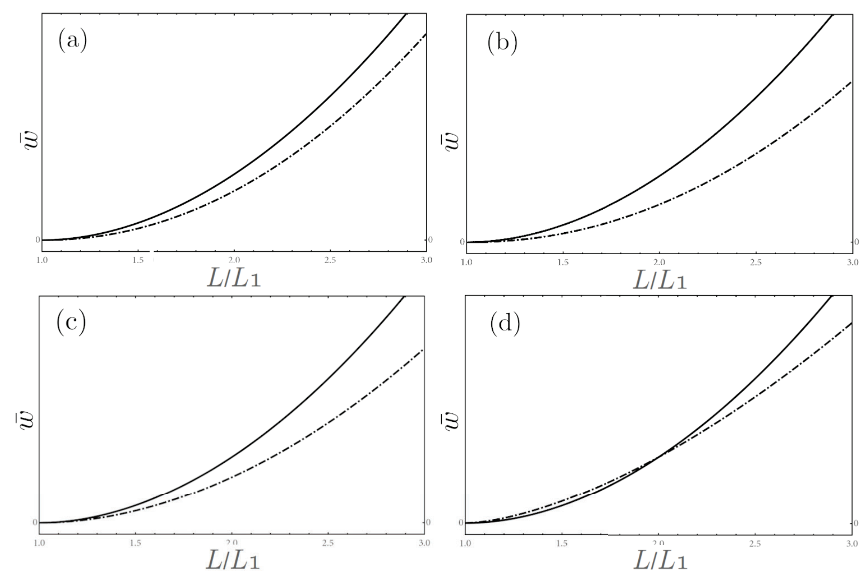


Figure 6: **The parameter dependence of the Phillips curve.** The solid curve is the same as in Fig. 3, while the dot-dashed curve is with the following parameters' change: (a) c is increased from 0.5 to 0.9; (b) γ is decreased from 0.5 to 0.2; (c) B is increased by 20%; (d) β is increased from 0.5 to 0.6;

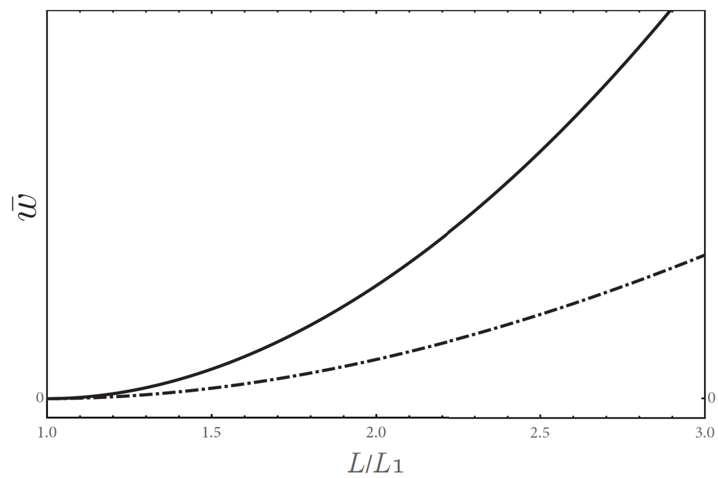


Figure 7: **Cumulative Effect of the changes of the parameters.** The parameters are $\alpha = c = \beta = \gamma = 0.5$ for the solid curve and $\alpha = 0.5,\ c = 0.9,\ \beta = 0.6,\ \gamma = 0.2$ with 20% increase in B for the dot–dashed curve. The combined effect of small parameter changes accumulate and flatten the Phillips curve.

5 Discussion

We compare the results obtained in the model with actual data. For this purpose, we focus on the first term in Eq.(29). Taking the period 2018–19 (let us denote this time-window as "period II") when the unemployment rate was about 2.5%, and the nominal wage growth was around 0.5%. The unemployment rate was about 2.5% in the late 80's (in red in the lower panel of figure 1) to the early 90's (in green in the lower panel of figure 1, identified as "period I") but the nominal wage growth was around 3%. Given 2.5% unemployment rate, the wage growth was lower in period II than in period I by 2.5%: the Philips curve had flattened. We explore how this change is generated in our model.

For this discussion we denote the values of parameters in each period with suffix I or II. For example, c in period I and II are denoted by c_I and c_{II}, respectively. Let us assume that the average wage for one period is given and take one year as the time reference. Taking period I, it increased by 3% next year. In period II, it increases only by 0.5%. Thus the next year's wage in period II is $1.005/1.03 \simeq 0.976$ times that of the next year in period I. We discuss how this difference is explained by simultaneous change in c, γ, and B, keeping β fixed in order to focus on possible structural modifications in the labor market. According to the analytical expression in Eq.(29), by taking the first term we find that

$$\frac{\gamma_{\text{ratio}}}{c_{\text{ratio}} B_{\text{ratio}}^{1/\beta}} = \frac{1.005}{1.03} = 0.976. \tag{33}$$

where we denoted the ratios of the parameters between two periods $c_{\text{ratio}} = c_{\text{II}}/c_{\text{I}}, \gamma_{\text{ratio}} = \gamma_{\text{II}}/\gamma_{\text{I}}, B_{\text{ratio}} = B_{\text{II}}/B_{\text{I}}$.

For example, let us look at the case where the three parameters change by the same ratio, $c_{\text{ratio}} = B_{\text{ratio}} = 1/\gamma_{\text{ratio}} = r$ with $\beta = 0.5$. From (33), we obtain $r = 1.00616$, that is, 0.62% increase in c and B with 0.61% reduction in γ. From this we learn that even though change in each parameters is minute, it brings the wage growth down significantly, from 3% to 0.5%. The general solutions to Eq.(33) is illustrated in Fig.8. This computational experiment suggests that the changes in the Japanese Phillips curve should be modeled as a combined effect of relatively marginal changes in the labor market.

The changes of parameters in the model which make the Phillips curve flatter are related to the structural evolution of the Japanese economy, and more specifically to: (1) an increase of productivity of secondary workers relative to primary workers, (2) weaker bargaining power of primary workers, (3) an increase of supply of secondary workers, and (4) an increase of wage elasticity of supply of secondary workers. These are indeed changes which occurred in the Japanese economy over the last thirty years.

The share of secondary or irregular workers in Japan was 15–16% during the late 1980's, but has steadily increased since then to almost 40% in 2020 (Kawaguchi and Ueno, 2013; Gordon, 2017). After the bubble busted at the beginning of the 1990's, Japanese firms facing unprecedented difficulties had attempted to cut labor cost by replacing highly-paid primary workers with low-wage secondary workers. Historically, secondary workers are considered to have

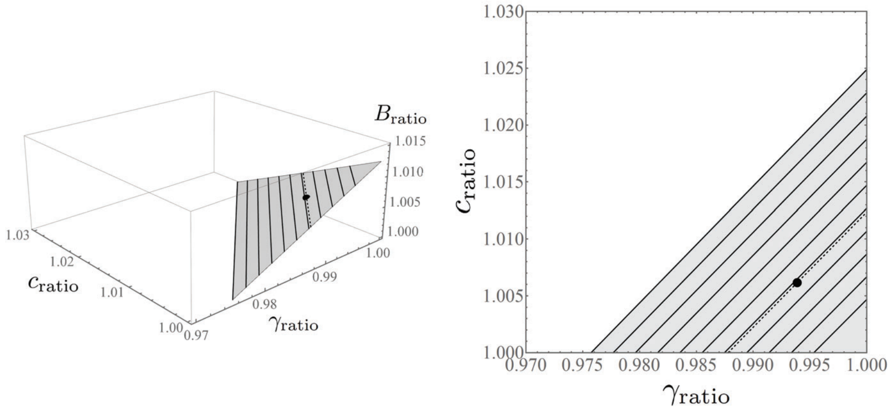


Figure 8: Change of parameters $\gamma_{\text{ratio}}, c_{\text{ratio}}$ and B_{ratio} that explain flattening of the Phillips curve from period I to II, given by Eq.(33). The left panel show the surface the solution covers, with the solid lines for $B_{\text{ratio}} = 1, 1.001, 1.002 \cdots, 1.01$, a black sphere for the case when c, B and γ are changed simultaneously by the similar level explained in the text (see (33)), and the dashed line for the required value of change in B ($B_{\text{ratio}} = 1.00616$). The right panel is the projection to the $(\gamma_{\text{ratio}}, c_{\text{ratio}})$-plane.

a lower productivity due to less education/training and lower attachment to the employer (Fukao and Ug Kwon, 2006; Shinada, 2011). However, in face of a dramatic increase in secondary workforce, overall productivity has been mostly stagnant and has not showed the decline that the variation in the proportion of secondary workers would imply. Consequently, in the absence of disaggregated data, it is possible to infer that the relative productivity of secondary workers has improved, reducing the gap with primary workers' one.

The increase in the secondary employment and workforce has also relevant implications for wage setting. Chen (2018) finds that secondary workers have typically low bargaining power, which have been exploited by employers to suppress wage growth. In contrast with the conclusions by Hirata et al. (2020) and Iwasaki et al. (2021) who emphasize downward wage rigidity, Chen (2018) attributes upward wage rigidity at the increase in the proportion of secondary workers with lower wage growth compared to permanent workers.

The bargaining power of primary workers has also been affected by the general de-unionization of capitalist economies over the last decades, which has recently been confirmed by micro-data analysis, as in the cited paper by Stansbury and Summers (2020). Furthermore, Japanese unions have faced a higher pressure to restrain wage demand, in comparison to Western economies, in order to limit the loss of jobs following the bust of the double bubble in the stock and real estate markets and the lost decade.

As for labor supply and wage elasticity of secondary workers, the Bank of Japan report of July 2018 on wages and prices provides an empirical support to our theoretical findings. The Bank of Japan explicitly point at the dual labor market with different wage-setting mechanisms as the reason behind the sluggish wage dynamics. In particular, the report argues that the increase in the labor force participation rates among seniors and women is the main cause of the weaker reaction of wages to employment. Indeed, in the last two decades, post-war baby boomers have reached the age of retirement leaving primary jobs and entering secondary labor market. At the same time, in line with what happened in other developed economies, an increase in female participation has created additional availability of workforce, in particular for part-time employment. As consequence, the supply of secondary workers has increased. The Bank of Japan estimates relatively higher wage elasticity for secondary labor such as seniors and women than primary age male workers. As a result, the increase in labor demand does not translate into high wage increase. The relatively large wage elasticity of supply of senior workers is confirmed also for the other OECD countries (Mojon and Ragot, 2019).

To summarise, our findings stress the need of an holistic approach and a broad perspective on the long-term evolution of the labor market for the identification of the causes of deflation. Moreover, the study of the microeconomic factors affecting the demand and supply of labor can be enhanced by properly framing individual behavior and choices within the structural transformations that have modified the supply of labor and the composition of the workforce. Also the discussion about the role of expectations, in our opinion, should consider the structural and institutional factors considered here, since they have possibly become embedded in the public's expectations, amplifying their effect on inflation dynamics.

In terms of policy implications, our results underlines the importance of structural changes in the labor market in order to reduce the risk of deflation. We must recognize that reforms aimed to make the job market more flexible and the progressive de-unionization observed in virtually all the developed economies actually contribute to flatten the Phillips curve, and thereby made the macroeconomy susceptible to deflation.

6 Concluding Remarks

The paper presents a parsimonious model of the labor market in which the labor force is composed of primary and secondary workers. A Phillips curve which includes the structural parameters of the labor market is analytically derived. We find that the slope of the Phillips curve depends on the joint effects of the composition of the workforce, the bargaining power of primary workers, and the wage elasticity to supply of secondary workers.

The solution of the model allows for a qualitative assessment of the role of a series of structural change observed in the Japanese economy on the flattening of the Phillips curve. In particular, none of these effects in isolation appears to be able to generate the shifts in the Phillips curve observed in the data. Our analysis suggests that the partial disentanglement of the dynamics of economic activity and wages is the result of deep modifications in the labor market and of its long-term structural evolution. Any quantitative analysis must therefore take a long-term perspective and consider these structural factors.

In this paper, we limit our analysis to the labor market in order to better highlight the causal nexus between its long-term evolution and the flattening of the Phillips curve. Future extensions can explicitly model the feedback effects of the sluggish dynamic of wages and aggregate demand.

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Declarations

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Conflicts of interest/Competing interests None.
Availability of data and material Data are publicly available at Ministry of Health, Labor and Welfare, Japan (2020) and Statistics Bureau of Japan (2020).
Code availability Codes are available upon request.

A Dimensional Analysis

In this appendix, we discuss the dimensions of various parameters and variables in our model.

Dimensions play important role in various fields of natural science. Basic dimensions in natural science are: Length, Weight, Time and Charge. In any equation that deals with natural quantities, the dimension of the left-hand side has to be equal to the dimension on the right-hand side. For example, "1 [in meter] = 1 [in kilogram]" does not make sense. For this reason, we often learn a lot by simply looking at the dimensions of the constants and variables. This is called "dimensional analysis".

Also, dimensionless quantities play important roles in analysis: The most famous dimensionless constant is the fine structure constant $\alpha = e^2/\hbar c = 1/137.035...$ (in cgs units), where e is the unit of electric charge, $\hbar$ is the reduced Planck's constant and c is the speed of light. As this quantity is dimensionless, α has this value, regardless of whether length is measured in meters or feet, or whether weight is measured in kilogram or pounds, and so on.

Our analysis of the model benefits greatly by the dimensional analysis. Let us examine dimensional properties of quantities in our model. We denote the dimension of the number of workers by $\mathbf{H}$, unit of value, like the dollar or yen, by $\mathbf{V}$, and time by $\mathbf{T}$.

First, the parameters c, α, β and γ are dimensionless by their definitions. Dimensions of the fundamental variables are the following:

$$\dim Y = \mathbf{V}\,\mathbf{T}^{-1}, \tag{34}$$

$$\dim L_{1,2} = \mathbf{H}, \tag{35}$$

$$\dim w_{1,2} = \mathbf{H}^{-1}\,\mathbf{V}\,\mathbf{T}^{-1}, \tag{36}$$

as Y is value created per a unit of time (yen per year, for example), $L_{1,2}$ are number of workers, and $w_{1,2}$ are value per person per time. From these, we find the following dimensions of the parameters:

$$\dim A = \mathbf{H}^{-\alpha}\,\mathbf{V}\,\mathbf{T}^{-1}, \tag{37}$$

$$\dim B = \mathbf{H}^{1+\beta}\,\mathbf{V}^{-\beta}\,\mathbf{T}^{\beta}. \tag{38}$$

The former is obtained by the requirement that dimensions of the right-hand side and the left-hand side of Eq.(2) matches, and the latter similarly from Eq.(9). From these, we find that the scaled variable v (Eq.(18)) and the parameter g (Eq.(20)) are dimensionless. For this reason, the nonlinear equation Eq.(10), which plays a central role in our model but is rather complicated, is simplified to a form much simpler and easier to analyse, Eq.(19).

B Domination of secondary workers

Large-g perturbative solution for Eq.(19) is the following:

$$v = g^{-\sigma/(1+\sigma)} \left[1 - \frac{\sigma}{1+\sigma} \, g^{-1/(1+\sigma)} + \frac{\sigma}{2(1+\sigma)^2} \, g^{-2/(1+\sigma)} + \cdots \right] \tag{39}$$

This leads to,

$$L = \frac{L_1}{c} \, g^{1/(1+\sigma)} \left[1 + \left(c - \frac{\sigma}{1+\sigma} \right) g^{-1/(1+\sigma)} + \cdots \right], \tag{40}$$

$$\bar{w} = \left(\frac{L_1}{B} \right)^{1/\beta} \frac{\alpha + \gamma - \alpha\gamma}{\alpha \, c^{1/\beta}} g^{1/(\beta(1+\sigma))} \left[1 + a_1 \, g^{-1/(1+\sigma)} + \cdots \right]. \tag{41}$$

The coefficient a_1 of the non-leading term in $\bar{w}$ is a complicated function of α, β, c and γ, which is not essential for our discussion and is not given here.Perturbative solution of Eq.(40) for g is the following:

$$g = \left(c \, \frac{L}{L_1} \right)^{1+\sigma} \left[1 - \left(1 + \sigma - \frac{\sigma}{c} \right) \frac{L_1}{L} \cdots \right]. \tag{42}$$

Note that current assumption is that $L/L_1 = 1 + L_2/L_1 \gg 1$. Therefore, we find that

$$\bar{w} = \left(\frac{L}{B} \right)^{1/\beta} \frac{\alpha + \gamma - \alpha\gamma}{\alpha} \left[1 + \cdots \right], \tag{43}$$

where the non-leading term '$\cdots$' is of order of L_1/L ($\ll 1$). Again we obtain a monotonically increasing Phillips curve, whose gradient is determined by B.

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