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Improving on Simple Majority Voting by Alternative Voting Mechanisms

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Summary

Majority voting is the predominant mechanism for collective decision making. However, majority voting is generally not efficient as it does not allow voters to express the intensity of their preferences. In addition, majority voting suffers from the "tyranny of the majority," i.e. the risk of repeatedly excluding minority groups from representation. A final drawback is the "winner-take-all" nature of majority voting, i.e. it offers no compensation for losing voters. Economists have recently proposed various alternative mechanisms that aim to produce more efficient and more equitable outcomes. Under *storable voting*, voters allocate a budget of votes across several issues. Under *vote trading*, voters can exchange votes for money. Under *linear voting* or *quadratic voting*, voters can buy votes at a linear or quadratic cost respectively. These alternative mechanisms hold the promise to improve on majority voting but have their own shortcomings. Additional theoretical analysis and empirical testing is needed to produce a mechanism that robustly delivers efficient and equitable outcomes.

Keywords: *electoral design, inefficient voting, storable votes, vote trading, linear voting, quadratic voting, experiments*

1. Majority voting

The most popular decision rule in groups when making a binary decision is majority voting. Majority voting is popular across the full spectrum of human groups from hunter-gatherer tribal societies (Boehm, 1996; Boyd and Richerson, 1988; Wilson, 1994) to modern industrial democracies (Mueller, 1989). It is the decision rule most frequently adopted to make formal social choices in elections, legislatures, and committees.

Many countries use majority-based referenda to decide whether to undertake public projects, implement a policy change, or to accept a new law. Most US states, for example, add bond issues for schools or roads as "yes/no" items on the ballot of a general election. European countries have issued national referenda to decide whether to join the European Union, accept European lawmaking, and a common currency. Referenda are also commonly employed on a local level, e.g. whether to accept a change in zoning restrictions, whether to build a nuclear plant in a certain municipality, whether to repurpose land (including demolishing houses) to allow for an infrastructure project, etc. This type of "direct democracy" is particularly prevalent in Switzerland where business hours, vacation duration, etc., are often settled by referendum.

Majority voting is popular because of its many virtues. It is transparent and easy to execute, it is based on a simple principle of equal participation and equal power ("one man, one vote"), and it induces voters to vote sincerely.

However, majority voting is generally not efficient, as it only allows voters to indicate the alternative they prefer but not how intense their preference for that alterative is. In the examples above, it is typically the case that some voters care more about which outcome prevails than others. Changes in zoning laws might preclude some families from sending their kids to a good public school and cause the value of their homes to drop. Other families may simply be unaffected. Likewise, relaxing opening times in Zurich may be very valuable to store owners and a select group of shoppers while others may see little (or negative) value. The inherent weakness of majority voting lies in its failure to reflect the intensity of preferences: an almost indifferent majority prevails over an intense minority.

Another important shortcoming of majority voting is that there is no compensation for those that lose. Lack of monetary compensation may seem natural for national (e.g. Presidential) elections but becomes problematic when voting is used to settle local issues. For example, when a change in zoning restrictions causes house values to drop, it seems unreasonably harsh not to compensate its owners.

A final shortcoming is that some groups may repeatedly lose under a majority-based rule. Lack of representation may cause a minority group to lose interest in participating in the democratic process with adverse effects for overall welfare. This possibility is commonly referred to as the "tyranny of the majority."

2. The environment

Various proposals that aim to improve on majority voting have been put forward and are presented here. They all share some common assumptions about the environment in which they are implemented.

2.1 The voters

It is typically assumed that there is a finite number of voters, denoted by n. Some of the results in the literature depend on this number becoming very large. While this may be problematic when examining decisions in small groups, it does seem appropriate for national votes or even local referenda, where voters typically number in the thousands. Of course, it is not straightforward to determine what number of voters is "large enough." Interestingly, many of the theoretical results are corroborated by lab experiments in which group sizes rarely exceed $n = 15$ voters.

2.2 Issues and alternatives

The simplest setup in which one can study voting systems is that of a single issue, d, with two possible alternatives: $d \in \{x, y\}$. Tractability favors such an environment as a testbed for alternative voting systems. Besides being more amenable to mathematical modeling and analysis, this "single issue – binary alternative" case is perhaps the most common environment in which majority voting is encountered, making it particularly interesting for finding improved voting mechanisms.

2.3 Preferences and information

Each voter i is assumed to assign values $v_i(x)$ and $v_i(y)$ to each of the alternatives, which represent her preference intensity. The larger the value, the larger the voter's utility if the given alternative wins. These values can also be negative, representing disutility from a specific alternative. In fact, for the case of only two alternatives considered here, it is often assumed that each voter assigns a positive value to one alternative and its negative to the other: $v_i(x) = -v_i(y)$. Given this assumption one can simply refer to the value as v_i, where the sign determines the direction of preference: positive values indicate a preference for x, negative for y. Voters' values are assumed to be i.i.d. draws from some distribution, $F(v)$, with density, $f(v)$, and voters know their own private values.

An important factor when analyzing any voter system is the degree of knowledge voters have about others' values. Common knowledge of values seems unrealistic. In fact, were this the case, no voting system would be necessary: a benevolent planner could simply pick the best alternative that maximizes $\sum_i v_i$ and arrange for transfers between voters that compensate the losers.

A more plausible assumption is that voters only know the distribution from which values are drawn, but not the actual draws. This is the environment in which most proposed alternatives to majority voting are evaluated theoretically and empirically – through lab experiments.

A third possibility is the existence of aggregate uncertainty, where the distribution from which values are drawn is not known. That is, $f(v;\theta)$ may depend on some underlying unknown state of nature θ about which voters are imperfectly informed.

3. Inefficiency of majority voting

If we define $w(v) = \sum_i v_i$ then social welfare is given by:

$$W(v,d) = \begin{cases} w(v), \; if \; d = x \\ -w(v), \; if \; d = y \end{cases}$$

It is straightforward to see that the optimal decision d^{opt} depends on the sign of the sum of values or, equivalently, the mean $\bar{v} = \frac{1}{n}\sum_i v_i$. In particular:

$$d^{opt} = \begin{cases} x & \text{if } \bar{v} > 0 \\ \{x,y\} & \text{if } \bar{v} = 0 \\ y & \text{if } \bar{v} < 0 \end{cases}$$

Under majority voting it is not the mean that determines the outcome. In fact, preference intensities do not play a role, only the number of voters supporting each alternative. Hence, it is the position of the median voter v_{med} that becomes crucial. Since it is a dominant strategy to vote for one's preferred alternative:

$$d^{maj} = \begin{cases} x & \text{if } v_{med} > 0 \\ \{x,y\} & \text{if } v_{med} = 0 \\ y & \text{if } v_{med} < 0 \end{cases}$$

The above expressions make clear that the positions of $\bar{v}$ and v_{med} with respect to zero have important consequences. The position of the mean determines which alternative is socially optimal. The position of the median determines which alternative wins in a majority vote. Whenever the two are on the same side of zero, majority voting gives socially optimal results. When this is not true, majority voting fails completely. Assuming a symmetric value distribution, the probability that the latter happens is equal to[2]

$$\text{Prob}\big(\text{sign}(\bar{v}) \neq \text{sign}(v_{med})\big) = \frac{1}{\pi}\ \text{ArcTan}[\sqrt{\frac{E(v^2)}{E(|v|)^2} - 1}\,]$$

Here $E(v^2) = \int v^2 f(v)dv$ and $E(|v|) = \int |v| f(v)dv$ with $E(v^2) \geq E(|v|)^2$ by Jensen's inequality. For instance, $\text{Prob}\big(\text{sign}(\bar{v}) \neq \text{sign}(v_{med})\big)$ is approximately 21% for normally distributed values and approximately 17% for uniformly distributed values.

Besides determining how often majority voting fails, it is important to measure the welfare consequences of such failures. Assuming a symmetric distribution of values around zero, Goeree and Li (2008) calculate the expected welfare under majority voting to be:

$$W_{voting} = n\binom{n-1}{(n-1)/2}\left(\frac{1}{2}\right)^{n-1} E(|v|)$$

The idea behind the calculation is that each voter creates a surplus of $|v|$ when $m \geq (n-1)/2$ others prefer the same alternative, and $-|v|$ when $m < (n-1)/2$ others do so. Since the event where m others prefer the alternative is equally likely as the event that $n-m-1$ others prefer it, all terms cancel out except for $m = (n-1)/2$.

If instead the decision is made through some mechanism delivering an optimal outcome, then a voter with value v generates a surplus of v when the sum of others' values is greater than $-v$. For n large, this occurs with probability $\Phi\left(\frac{v}{\sqrt{n-1}}\right)$ where $\Phi(\cdot)$ is the standard normal distribution. In all other cases the voter generates a surplus of $-v$. Expected welfare is thus given by:

$$W_{optimal} = n\int_{-\infty}^{+\infty} v\left(2\Phi\left(\frac{v}{\sqrt{n-1}}\right)-1\right)dF(v) \approx \sqrt{\frac{2n\,E(v^2)}{\pi}}$$

Goeree and Li (2008) show that, as the size of the electorate grows large, inefficiency persists and the ratio $W_{voting}/W_{optimal}$ limits to:

$$\lim_{n\to\infty} \frac{W_{voting}}{W_{optimal}} = \frac{E\left(\sqrt{v^2}\right)}{\sqrt{E(v^2)}}$$

which is less than 1 by Jensen's inequality. Moreover, the total surplus loss, i.e. the difference $W_{optimal} - W_{voting}$, diverges to infinity in the limit.

An alternative voting system would ideally overcome this problem and give outcomes that are optimal for any value distribution.

4. Alternative voting mechanisms that reflect preference intensity

Various mechanisms have been proposed to mitigate or eliminate voting inefficiencies by allowing the voters to express the strengths of their preferences. One can broadly distinguish three approaches: the use of storable votes, the introduction of markets for votes, and, inspired by the theory of mechanism design, the replacement of votes by bids. Each of these is reviewed in this section. The theoretical underpinnings are explained and experimental results from the lab are reported.

4.1 Storable votes

Often, collective decisions are not made in isolation. The same electorate or smaller groups of people need to make choices on different issues, either at the same time or in different points in time. One way to improve voting systems takes advantage of this characteristic by making votes "storable." The idea is to allow voters to store votes through abstaining on some issues, presumably the ones they care less about, and use them on the issues about which they care more.

The notion was introduced by Casella (2005). In her model, voters face a series of binary choices and are endowed with a budget of votes. To illustrate, suppose there are three voters and two binary issues: $\{x_1, y_1\}$ are the alternatives for issue 1 and $\{x_2, y_2\}$ are the alternatives for issue 2. Voters' preferences are represented by different private valuations for the alternatives as shown in Table 1 below, where both X and Y are positive. (Where the payoff of losing is 0 rather than minus the value to be in line with previously published papers on storable voting.)

The simple majority outcomes are y_1 for issue 1 and y_2 for issue 2. Under the storable votes mechanism, assuming ties are broken by the flip of a coin, there exists an equilibrium where voter 1 casts 2 votes on issue 1 and voters 2 and 3 each mix with equal probability between casting 2 votes on issue 1 and casting 1 vote on each issue. In this equilibrium, the outcomes x_1 and x_2 occur

Table 1. Voter preferences over two binary issues

	Alternative	Voter 1	Voter 2	Voter 3
Issue 1	x_1	7X	0	0
	y_1	0	Y	Y
Issue 2	x_2	X	0	0
	y_2	0	Y	Y

with probability $\frac{1}{8}$ and y_1 and y_2 with complementary probability. The net benefit of storable votes compared to majority voting is thus X – Y/2, which may be positive or negative depending on the relative sizes of X and Y. When X > Y/2, storable votes result in an efficiency gain. But the equilibrium also applies when X is small, say 7X < 2Y, in which case the majority outcomes y_1 and y_2 are socially optimal and storable votes cause efficiency to fall.

While the welfare effects of storable votes are ambiguous in this example, Casella (2005) argues that they are generally positive if one of the following conditions holds: i) the number of voters is above a minimum threshold, ii) their preferences are not extremely polarized, and iii) there is a long time horizon.

4.1.1. Experimental tests of storable voting

In Casella's model, voters know their own valuations for the current issue and the distribution about other voters' current and future valuations and their own future valuations. Thus, voters need to solve a complicated dynamic game, comparing the marginal effect of an extra vote on the probability of being pivotal for the current issue, to its effect on the probability of being pivotal for the next issue.

Would voters be able to solve such a complicated problem in practice? Casella and her coauthors tested a variation of this mechanism in laboratory experiments (Casella, Gelman, and Palfrey, 2006). The experiment consisted of 6 treatments: the committees of size $n \in \{2,3,6\}$ were asked to cast votes across two or three issues. In each session, subjects were randomly grouped every period with new value draws and played a total of 30 periods of the storable votes mechanism. In each period, voters were endowed with a "regular" vote that they must use in each issue and some "bonus" votes that they may cast in addition to the regular votes to express their strong preference. The efficiency-improvement over simple majority voting predicted by the theory were observed almost perfectly across all treatments even though the actual choice of subjects were substantially different from the equilibrium strategies. This is because, subjects followed approximately monotonic strategies. For instance, when faced with two issues, the number of bonus votes used on the first issue increased with voters' preference intensities.

Can storable votes help overcome the "tyranny of the majority"? If so, is it at the expense of the efficiency gain over majority voting? Casella, Palfrey and Riezman (2008) show that storable votes can increase the power of minorities without sacrificing aggregate efficiency. This main theoretical finding is confirmed by a series of experiments.

It is important to note that with storable votes the majority does not directly compensate the minority. Instead, the accumulation of a large number of votes by the minority allows it to "beat" the majority on the issues for which it has a strong preference.

4.1.2 Extensions

Jackson and Sonnenschein (2007) and Hortala-Vallve (2012) generalize the "storable votes" idea to mechanisms where agents can effectively reflect their relative intensities and improve over majority rule by linking decisions across multiple dimensions through a common budget constraint.

Engelmann and Grimm (2012) and Hortala-Vallve and Llorente-Saguer (2010) test the performance of these mechanisms in the lab and find that efficiency levels are improved as predicted by theory. In Hortala-Vallve and Llorente-Saguer (2010), the experiment consisted of three treatments varying the number of issues. Two members of a committee are endowed with 6 votes each that they can allocate across K issues, where $K \in \{2,3,6\}$. One member is always in favor and one is always opposed in all issues and the intensities of the preferences are realized draws from a uniform distribution. Members cast votes simultaneously on all issues after being told their private value draws for all issues but not the values for the other member. This is different from the storable votes model where voters only know the value for the current issue and voting is done sequentially, issue by issue. Efficiency under this mechanism is predicted to be increasing in K and above 80%. The realized experimental efficiency tracked this prediction well as a vast majority of the subjects used monotonic strategies.

4.2 Markets for votes

Markets that allow for the free trade of goods generally lead to efficient outcomes. Can their power be used to obtain more efficient outcomes when making collective choices? While the idea of trading votes in the same way as a simple commodity is generally not viewed favorably, several scholars have argued that it should not be dismissed (Buchanan and Tullock, 1962; Coleman, 1966; Haefele, 1971; Mueller, 1973). Voters that care more for an issue would buy more votes from voters that care less about it and are therefore willing to sell. Voters becoming traders means they can express their preference intensity, thus leading to more efficient outcomes.

This view has been challenged by pointing out that votes are not simple goods: they involve externalities. While a trade might be beneficial for the voters involved, it may be detrimental for overall efficiency (Riker and Brams, 1973). Early experimental tests of this hypothesis lend some support, but are far from conclusive (McKelvey and Ordeshook, 1980).

Table 2. Voter preferences that lead to non-existence of CE

	Alternative	Voter 1	Voter 2	Voter 3
Issue 1	x	30	0	0
	y	0	10	12

This earlier work mainly focused on legislatures where informal vote trading (known as "logrolling") is a common phenomenon that is facilitated by the frequent voting over a variety of issues.[3] More recently, scholars examined the idea of vote-markets that could be used in single-issue votes such as local or national referenda. One important issue in such markets is that a competitive equilibrium (CE) may not exist (Ferejohn, 1974; Philipson and Snyder, 1996). This is illustrated by the example in Table 2 above (where the payoff of losing is 0 instead of minus the value to be in line with previously published papers).

The simple majority outcome is y whereas the efficient outcome is x. Voter 1 demands at most one vote at any positive price. Voter 1 can buy voter 2's vote at price of 11. But the market will not clear because voter 3's vote now is worth nothing and therefore voter 3 is willing to sell it at any positive price. There will be excessive supply driving the price down to 0. But a price of 0 cannot be an equilibrium because neither voter 2 nor voter 3 will be willing to sell a vote so there is excessive demand of 1 vote from voter 1.

4.2.1 Experimental tests of vote markets

Casella, Llorente-Saguer, and Palfrey (2012) proposed the "ex-ante competitive equilibrium" notion for such vote markets which allow the possibility that markets will not clear exactly. The equilibrium condition does not require supply to equal demand with probability one, but instead requires market clearing in expectation. Ex post, the market is cleared through a rationing rule. The existence of such an equilibrium is proved by construction. In any equilibrium with trade there is a dictator – a voter owning more than half of the votes. The dictator will be the voter with the highest or the next-to-highest value draw. However, unless the dictator's value is overwhelmingly large, or there is some correlation between preferences and intensities, this equilibrium will not generally be efficient. In fact, it generates significant welfare losses relative to simple majority voting.

Casella et al. (2012) collect experimental data to test their theoretical predictions. As is the case with competitive equilibrium notions for traditional markets, the definition of "ex-ante competitive equilibrium" for vote markets does not specify a particular trading protocol. In the lab, the authors have participants trade in a continuous double auction, which is perhaps the most common format in market experiments and typically results in prices and allocations that closely adhere to theoretical predictions (Friedman, 1993). Significant overpricing (with respect to equilibrium prices) is observed in the experimental vote markets. This is reduced with experience and prices converge to levels that can be explained by allowing for voters being risk averse. The intuition is that risk averse voters assign a higher value to votes as they can get a sure outcome if they buy enough of them.

Concerning the emergence of dictators, the evidence from the experiment is mixed. In smaller markets ($n = 5$) the voters with the highest preference intensity buy more than half votes in about 62% of rounds. This tends to happen more often in later rounds. In larger markets ($n = 9$), dictators are observed only in about 10% of all rounds. Again this increases in later rounds, going up to 20% in the last round. Despite the deviations from equilibrium, welfare in the experimental vote markets is typically lower than what could be achieved by majority voting (albeit not as bad as predicted by the equilibrium analysis).

Do vote markets help intense minorities to win more often? Casella, Palfrey and Turban

(2014) compared the vote-trading outcomes and efficiency when vote-trading is coordinated by group leaders and when vote-trading takes place in competitive market. The main theoretical results highlight the tradeoff between increasing minority voice and efficiency. With market trades, vote trading can be welfare reducing because the minority wins too often. With group leaders, vote trading improves over no-trade, but still falls short of full efficiency because the minority does not win frequently enough. These predictions are tested in a set of lab experiments and the results strongly support them.

In terms of compensation for the losers, vote markets offer a direct way for it to happen. Voters can get compensated by selling their vote. Nevertheless, such compensation is unlikely to be fair. If everyone starts off with one vote and sells it at the market price, then for compensation to be proportional to a loser's preference intensity it must be the case that the probability of selling must be increasing to said intensity. It is hard to imagine a market where this happens in equilibrium. For any given price, one would expect voters with lower intensities to be the ones more willing to sell their vote, as they have "less to lose". To conclude, markets for votes do not appear to have the same appeal as those for standard goods.[4]

4.3 Replacing votes with bids

Inspired by the success of mechanism design to come up with games that implement desired outcomes in a variety of economic environments, several scholars have explored the possibility to use this idea in the context of elections. In broad terms, the idea is to allow voters to indicate an amount they are willing to pay for their preferred alternative to be implemented or by how much they need to be compensated in case it does not. The mechanism then would determine the outcome based on these "bids" and also indicate appropriate transfers. Ideally, the outcome will be efficient, and the transfers will adequately compensate the loser.

4.3.1 Linear voting

The first to explore this idea is Smith (1977) who proposed a direct mechanism that he dubbed the "auction election" (which later became the "compensation election"). This mechanism was independently rediscovered by Pérez-Castrillo and Wettstein (2002) who were the first to provide a thorough equilibrium analysis. Voters submit a single bid for one of two alternatives. The alternative with the largest total bid is implemented. Those that win pay their bids and those that lose get paid their (own) bids. Because payments are linear in bids, we refer to this mechanism as "linear voting."

Smith (1977) does not specify what happens with the surplus money when the sum of winning bids exceeds the sum of losing bids. Pérez-Castrillo and Wettstein (2002), in contrast, balance the budget by rebating to all bidders an equal share of the money surplus. Another difference is that Pérez-Castrillo and Wettstein allow voters to indicate a preferred outcome besides making a monetary bid.

Pérez-Castrillo and Wettstein (2002) show that, with complete information about voters' values, the election ends up in a tie but the additional information about voters' preferred outcomes can be used to break the tie efficiently. In other words, linear voting is efficient in a complete information environment.

Veszteg (2010) extends the analysis to incomplete information environments. He shows that when voters' values are privately known, and symmetrically distributed, Bayes-Nash equilibrium bids are increasing in value and the resulting outcome is efficient with two voters. No analytic solutions exist for more than two voters, but in the limit when the number of voters grows large, the Bayes-Nash equilibrium is shown to be proportional to value, again resulting in efficient outcomes.

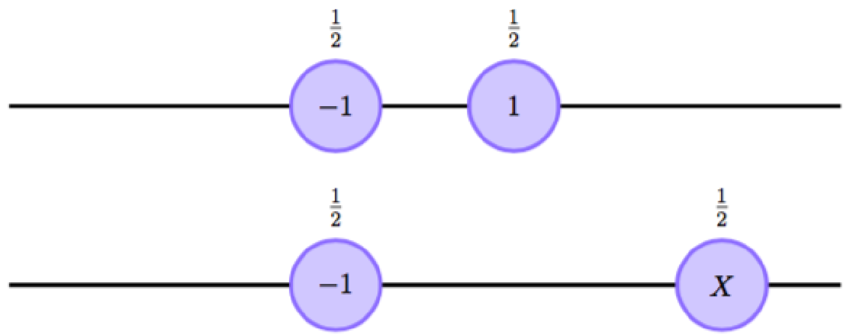

Figure 1. A symmetric (top) and asymmetric (bottom) electorate with only two types. In the symmetric case, there is no ex ante winner. For the asymmetric case, implementing right is socially optimal since $X > 1$.

The positive results for the incomplete-information case apply when values are symmetrically distributed around zero, but not necessarily when there are value asymmetries. To illustrate the difference, consider a simple setup with only two types $v \in \{-1, X\}$, that are either symmetric ($X = 1$) or asymmetric ($X > 1$), illustrated in Figure 1 above. For the case shown in the top line of Figure 1, there is no ex ante winner. For the case shown in the bottom line of Figure 1, it is socially optimal to implement right since $X > 1$.

Assuming a type symmetric equilibrium, there are two bids, $b(-1)$ and $b(X)$, to be determined. When the electorate size, *n*, gets large, the payoff of a voter with value $v \in \{-1, X\}$ who bids b can be approximated as (recall a voter receives $-v$ when losing plus her own bid)

$$\pi(v, b) = (v - b)(2\Phi\left(\frac{b + n\mu}{\sqrt{n}\sigma}\right) - 1)$$

which yields the first-order condition

$$b(v) = v - \frac{\Phi\left(\frac{b + n\mu}{\sqrt{n}\sigma}\right) - \frac{1}{2}}{\frac{1}{\sqrt{n}\sigma}\,\phi\left(\frac{b + n\mu}{\sqrt{n}\sigma}\right)}$$

with $\mu = \frac{1}{2}(b(-1) + b(X))$ and $\sigma^2 = \frac{1}{2}(b(-1)^2 + b(X)^2) - \mu^2$.

For the symmetric case with $X = 1$, the optimal bids are $b(-1) = -\frac{1}{2}$ and $b(1) = \frac{1}{2}$ so that $n\mu = 0$ while $\sqrt{n}\sigma$ diverges as n grows large. The selected outcome will be the one that receives more bids, which is efficient (as would majority voting be in this symmetric two-type case). For the asymmetric case with $X > 1$, the optimal bids are, to first order, $b(-1) = -\frac{1}{4}(X + 1)$ and $b(X) = \frac{1}{4}(X + 1)$, where we ignore terms of order $\frac{1}{n}$, which make it such that $n\mu = \frac{X-1}{2}$ while $\sqrt{n}\sigma$ diverges when n grows large. Again, the outcome is determined by the number of voters on each side (as in majority voting), which is not efficient as right has a higher expected value.

4.3.2 Experimental tests of linear voting

Oprea, Smith, and Winn (2007) tested the linear voting mechanism in a laboratory experiment that varied the size of the electorate (18 versus 6) and the distribution of the voters' private values (resulting in even, close, or landslide elections). They report that there is a positive relationship between bids and values in all treatments and the observed bid function was roughly linear. While this finding is encouraging with respect to the efficiency of outcomes, their design did not allow for a comparison between the compensation election and majority voting. A second encouraging finding is that they did not find evidence of "off-preference bid-behavior", i.e. voters biding more than their value or against their preferred outcome. Such behavior is undesirable as voters may end up making monetary losses.

Perez-Castrillo and Veszteg (2007) conducted experiments to test the linear bidding mechanism in a symmetric incomplete-information environment with group sizes of 2, 8, and 10. They find that the linear bidding mechanism is efficient in 75% of all the cases. With only two voters, a large faction of the subjects played according to the Bayes-Nash equilibrium identified in Veszteg (2010) while others used the safe maximin strategy (bidding half one's value) that ensures equal payoffs for losing and winning. For larger group sizes, the experiments do not provide conclusive evidence for Bayes-Nash equilibrium behavior.

4.3.3 Quadratic voting

Goeree and Zhang (2017) and Lalley and Weyl (2018) independently proposed a mechanism where voters can express the intensity of their preference by buying votes at a quadratic cost. Weyl termed this mechanism "quadratic voting."

The intuition for using quadratic costs is that in a large electorate, the impact of a bid on the probability that the preferred outcome is selected is small, and, hence, approximately linear in the bid. As a result, the marginal benefit of raising one's bid is proportional to one's value. With a quadratic payment rule the marginal cost is linear in the bid. Equating the marginal cost and benefit yields optimal bids that are proportional to value. And when bids are proportional to value, the alternative with the largest total bid is also the one with the larges total value: bidding is efficient in the limit.

Goeree and Zhang (2017) formalize this intuition by deriving quadratic voting as the limit (for large n) of the well-known AGV "expected externality" mechanism.[5] They show that quadratic voting inherits several useful properties from the AGV mechanism: it is budget-balanced, individually rational, and fully efficient in the limit. And the rebate, introduced to make the mechanism budget-balanced, offers compensation to losing bidders.

Goeree and Zhang (2017) derive these positive results for the symmetric case, which begs the question what happens in the presence of value asymmetries. Lalley and Weyl (2018) study quadratic voting in general asymmetric settings, i.e. with non-zero mean, for which the equilibrium is much harder to characterize. To illustrate, consider again the asymmetric setup in the bottom line of Figure 1. When the electorate size, n, gets large, the expected payoff of a voter

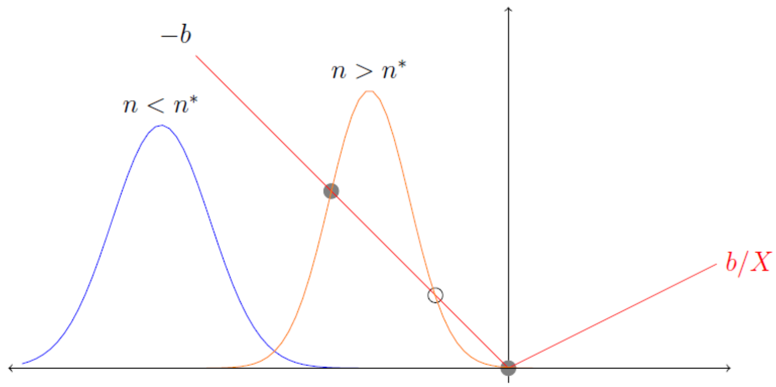

Figure 2. The first-order conditions for $b(-1)$ and $b(X)$ have unique solutions around $b = 0$ for $n < n^*$ but multiple solutions exist for $b(-1)$ when $n > n^*$.

with value $v \in \{-1, X\}$ who makes a bid b can be approximated as

$$\pi(v,b) = v\left(2\Phi\left(\frac{b+n\mu}{\sqrt{n}\sigma}\right)-1\right)-\frac{1}{2}b^2$$

which yields the first-order condition

$$b(v) = \frac{v}{\sqrt{n}\sigma}\,\phi\left(\frac{b+n\mu}{\sqrt{n}\sigma}\right)$$

with, as before, $\mu = \frac{1}{2}(b(-1)+b(X))$ and $\sigma^2 = \frac{1}{2}(b(-1)^2+b(X)^2)-\mu^2$.

For the symmetric case with $X = 1$, the optimal bids are $b(-1) = -(2\pi n)^{-\frac{1}{4}}$ and $b(1) = (2\pi n)^{-\frac{1}{4}}$. The selected outcome is the one that receives more bids, which is efficient (as would majority voting be in this symmetric two-type case). For the asymmetric case with $X > 1$, the optimal bids differ across two regimes that are characterized by a critical electorate size n^*.

For $n < n^*$, the solutions to the first-order conditions for $b(-1)$ and $b(X)$ are unique. It can be shown that, in equilibrium, the ratio $\rho = -\frac{b(-1)}{b(X)}$ satisfies

$$\rho = \frac{1}{X}\ \text{Exp}[2\ \frac{1-\rho}{1+\rho}\ \frac{n+1}{n}]$$

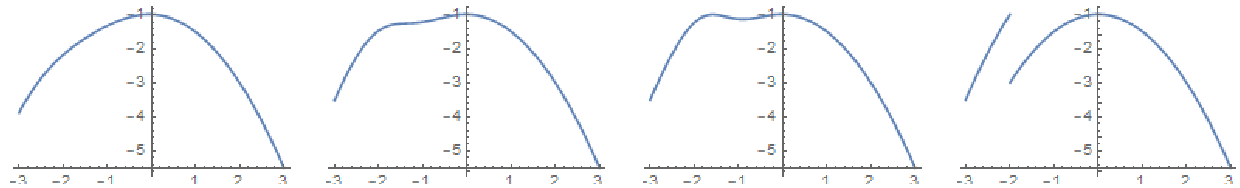

Figure 3. Payoff of the left voter with value $v = -1$ as a function of her bid b for various values of n. When $n < n^*$ the optimal bid is unique and small (i.e. close to 0), see the leftmost panel. When $n > n^*$, there are two local maxima and one local minimum that correspond to the disks and circle respectively in Figure 2. When the local maxima yield equal payoffs, the left voter is indifferent between the small and large bids, see the third and fourth panels. The chance of a large bid falls with n but it occurs just frequently enough, i.e. it creates enough variance in the win probability, to make others' bids be of order $\frac{1}{n}$. The final panel shows the limit case for $n \to \infty$.

which implies $0 < \rho < 1$ so quadratic voting is efficient. However, as can be seen from Figure 2, the equilibrium bids are determined by the exponential tail of the normal density and fall fast with n, pulling the mean μ to 0 as n grows large.

For $n > n^*$, there are multiple solutions to the first-order conditions, see Figure 2. Now the approximate equilibrium bids are $b(X) = 4X/(n(X-1))$ while $b(-1) = -4/(n(X-1))$ with probability $1-\epsilon$ and $b(-1) = -2$ with probability ϵ. Here ϵ vanishes when n grows large but introduces just enough variance for finite n to ensure others' equilibrium bids remain of order $\frac{1}{n}$.

Figure 3 shows the expected payoff for a left voter with value $v = -1$ as a function of her bid. For $n < n^*$, the payoff is single peaked, and the equilibrium bid is unique. For $n > n^*$, the payoff has two local maxima and one local minimum, which correspond to the disks and circle in Figure 2 respectively. For n large enough, the payoff of the large bid becomes equal to that of a small bid (third and fourth panels of Figure 3), in which case the left voter mixes between a small and large bid, with probabilities ϵ and $1-\epsilon$ respectively. In other words, the left voter occasionally "buys the election." However, ϵ falls with n, so quadratic voting is efficient in the limit when the electorate size diverges, also when voters' values are asymmetrically distributed.

Note that quadratic voting mechanism results in little to no compensation in equilibrium when electorates grow large. The mechanism is budget balanced as it redistributes any money surplus to the bidders, but this surplus is only 2 in a large electorate giving each voter a vanishingly small rebate of $\frac{2}{n}$.

4.3.4 Experimental tests of quadratic voting

The quadratic voting mechanism is not just of theoretical interest but is also simple enough to work in practice. Goeree and Zhang (2017) tested the quadratic voting mechanism in the laboratory and compared its performance to majority voting. The experiments employ a setup with "moderate" (low-value) and "extremist" (high-value) voters who are equally likely to prefer one of two alternatives. In the first twenty periods of the experiment, subjects make choices under both the bidding and voting mechanism. Then they collectively choose whether the bidding or voting mechanism applies in the final twenty periods of the experiment. With $n = 11$, 90% of the groups (18 out of 20) opt for the bidding mechanism. Asked to explain their choice, the subjects said they liked getting the cash rebate, and they had a greater sense of control the outcome. With standard voting they said they felt it was very unlikely their vote would be pivotal.

An additional benefit of this experimental design is that subjects have ample opportunity to learn in the first part, which makes equilibrium behavior in the second part more likely. Goeree and Zhang (2017) find that voting is near-perfect in this second part: only 0.7% of all votes are "mistakes" that go against the preferred alternative. As a result, observed efficiency losses under voting are as theory predicts: 22% on average with a group size of three and 28% with a group size of eleven. Observed efficiency losses are much smaller under the bidding mechanism: 7% with $n = 3$ and 13% with $n = 11$.

5. Practical and other issues

A planner wishing to implement any of the mentioned mechanisms in practice would need to overcome a series of issues that theorists can abstract away from and experimentalists can avoid through design. These are related to issues of feasibility in specific environments, as well as objections of a moral nature to some of the schemes proposed.

In the case of storable votes, while the approach looks promising, its main limitation in terms of applicability in real collective choice scenarios is the necessity to link different issues together. Casella (2005) considers this system as a viable alternative for decision making in the European Union, where member countries need to make decisions through vote repeatedly and with a relatively high frequency. Alas, this is not always the case. In other setups decisions are often one-off events and it is hard to predict when and if there will be a similar situation in the near future. One such example is the Brexit referendum. Even in cases where local referenda are more frequent, other issues need to be addressed. Demographic changes mean that the electorate is not fixed across time, which adds more layers of complexity to the voters' decision of whether or not to store their vote. A different issue is the possibility for strategic agenda setting. Whoever controls the order in which the issues are put to vote across time may do so in a way that favors her preferred outcomes. While these limitations hamper the use of storable votes in many scenarios, it remains a viable alternative to majority voting in many settings.

Markets for votes do not face significant hurdles in way of practical application. Today's technology offers tools to setup online markets for a range of goods and services. Furthermore, there have been significant advancements in the development of smart contracts, with the use of technologies such as blockchain, that can help avoid issues of multiple voting. On the other hand, vote markets often raise objections on moral grounds. The issues raised with respect to this approach are of a more philosophical nature. The kneejerk reaction to a proposal to allow for the buying and selling of votes is that this gives too much power to the wealthiest members of society.[6] In some contexts, such as corporate governance, one would not expect these objections to be relevant. Overall though, it seems unlikely for vote markets to become a standard feature of collective choice making.

As with vote markets, bidding for votes, i.e. linear or quadratic voting, also seems to be well within the capabilities of modern technology. At first sight, one might also object to such bidding schemes for the same reason as for vote markets. Namely, that they give too much power to the rich members of society. However, there is reason to question this argument, as the mechanisms do impose indirect restrictions to avoid such a possibility. "Buying" the election can be overly costly, especially in the case of quadratic voting where the cost of extra votes is increasing. In fact, laboratory data in Goeree and Zhang (2017) shows that moderate voters benefit most under quadratic voting compared to majority voting. In linear voting, voters can insure themselves against a loss through their bid and make sure they receive adequate compensation. At the same time, all proceeds are redistributed, which further enhances equity.

Quadratic voting faces another potential criticism, namely that people might collude and buy votes from fellow supporters of the same alternative to increase their side's total vote tally. Consider an extreme voter who buys 100 votes in equilibrium and pays (something proportional to) 10,000 as a result. If this extreme voter would instead buy only 10 votes and convince 99 others to do the same (paid for by the extreme voter) then the total number of votes would be a 1,000 while the cost would be the same. Weyl (2017) provides three reasons why the impact of collusion on the efficiency of the bidding mechanism is small in large electorates: (i) the collusive group must be large, (ii) individual members of large collusive groups have strong incentives to deviate from the collusive agreement, and (iii) large collusive groups provoke strong reactions from the rest of the electorate. It is important to note that this criticism does not apply to the linear voting mechanism.

6. Extensions and outlook

The first goal of any alternative to majority voting would be to improve efficiency by delivering outcomes in a way that accounts for voters' preference intensities. From the discussion above it becomes clear that both the storable votes and especially the 'bids instead of votes' approaches show some promise with respect to this point. Storable votes allow this by letting voters to store their votes on issues that are not important to them and cast them on the ones they care most about. While not achieving full efficiency, this approach has the potential to improve efficiency with respect to majority voting outcomes in some cases. Bidding mechanisms allow voters to directly express their preference intensity on any issue they vote upon. As long as voters bid proportionally to their preference intensity, such a mechanism could deliver full efficiency. In fact, this is the equilibrium prediction for quadratic voting, as well as for linear voting under preferences that are symmetrically distributed around zero. On the other hand, vote markets offer no guarantee of delivering more efficient outcomes. In equilibrium, only the voters with the most intense preferences determine the outcome. All others' preferences do not matter.

The second goal of an alternative voting mechanism is to avoid the tyranny of the majority. All the approaches discussed manage to avoid this problem, even if this sometimes happens at the expense of efficiency. Storable votes and vote markets still rely on the machinery of majority voting for delivering the final outcome. But the mechanisms allow for a redistribution of votes across issues or voters, the ex-ante majority of voters and the majority of vote holders at the time of the vote do not coincide. Both linear and quadratic voting dispense of majority voting entirely, freeing minorities from the majority's hold.

In terms of compensation for losing voters, it is necessary to make a distinction between whether the mechanism allows for, or even requires, monetary transfers between voters and the degree to which this happens in the theoretical equilibrium. The storable votes mechanism makes no provision for it. In vote markets, voters can receive some compensation by selling their vote. Notice however that in equilibrium this is expected to be decreasing in the size of the electorate and therefore become negligible for large electorates. Another interesting point related to this is that compensation in vote markets is not limited to voters on the losing side, as anyone is allowed to sell their vote and almost everyone does so in equilibrium. Quadratic voting also includes some compensation for all voters, irrespective of their preference, as all proceeds are redistributed equally across voters. Unfortunately, with large electorates, as bids decrease with its size, the equilibrium proceeds of the mechanism are very small and there is no effective redistribution and compensation. Linear voting also includes redistribution of earnings, but more importantly, provides for direct compensation of losers. Furthermore, in equilibrium this compensation remains substantial, see Table 3 below for an overview.

If efficiency is considered the most important criterion, quadratic voting scores best among the mechanisms considered here. For large electorates, it is efficient when values are symmetrically or asymmetrically distributed. However, it is important to qualify that, thus far, we have considered environments without any aggregate uncertainty. The distribution of values is commonly known and, hence, with a large electorate, the optimal outcome is known. If given the authority, a benevolent planner could simply implement the optimal outcome. In other words, the election merely ratifies what is commonly known to be the best outcome.

Table 3. Comparing various mechanisms with majority voting along three main criteria.

Mechanism	1. Improving efficiency *Symmetric distribution*	*Asymmetric distribution*	2. Avoiding "tyranny of the majority"	3. Compensating losers *Built into mechanism*	*In equilibrium*
Storable votes	+/-	+/-	+	-	-
Vote markets	-	-	+	+	0
Linear voting	+	0	+	+	+
Quadratic voting	+	+	+	+	0

A more realistic setup is where voters face some uncertainty about the socially optimal outcome. A simple example is shown in Figure 4. There are four possible states of nature: two are symmetric, i.e. both left and right voters' intensities are either 1 or *X*, and two are asymmetric, i.e. one side's intensity is 1 the other *X*. We assume that $1 < X < \frac{p}{1-p}$.

In a symmetric Bayes-Nash equilibrium, $b(-1) = -b(1)$ and $b(-X) = -b(X)$, so that $\mu_{11} = \mu_{XX} = 0$, $\sigma_{11} = b(1)$ and $\sigma_{XX} = b(X)$. The first-order conditions are

$$b(X) = \frac{2X(1-p)}{\sqrt{2\pi n}\, b(X)} \phi(\frac{1}{\sqrt{n}}) + \frac{2Xp}{\sqrt{n}\sigma_{1X}} \phi(\frac{b(X) + n\mu_{1X}}{\sqrt{n}\sigma_{1X}})$$

and

$$b(1) = \frac{2p}{\sqrt{2\pi n}\, b(1)} \phi(\frac{1}{\sqrt{n}}) + \frac{2(1-p)}{\sqrt{n}\sigma_{1X}} \phi(\frac{b(1) - n\mu_{1X}}{\sqrt{n}\sigma_{1X}})$$

For large n, there is an equilibrium with $b(1) = \sqrt{2p} \left(\frac{1}{2\pi n}\right)^{\frac{1}{4}}$ and $b(X) = \sqrt{2X(1-p)} \left(\frac{1}{2\pi n}\right)^{\frac{1}{4}}$. (In this equilibrium, $|\frac{n\mu_{1X}}{\sqrt{n}\sigma_{1X}}|$ diverges, so only the first terms on the right side of the first-order conditions matter, from which the result follows.) This equilibrium is not always efficient.

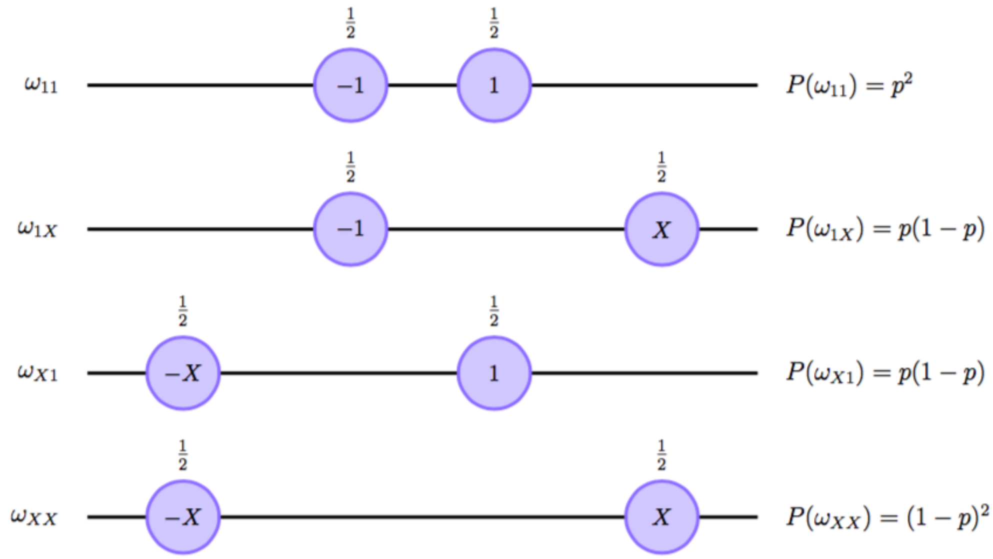

Figure 4. A setup with aggregate uncertainty. There are four possible states, two symmetric and two asymmetric, with prior probabilities shown on the right. We assume that $X > 1$, so that the X-type should optimally win in the asymmetric states, and $X < \frac{p}{1-p}$.

Specifically, in the symmetric states, the equilibrium is efficient and welfare gains are of the order $\sqrt{n}$. However, in the asymmetric states, $b(X) < b(1)$, because the voter with intensity 1 puts a higher probability on the symmetric state ω_{11} than the voter with intensity X puts on the symmetric state ω_{XX}. As a result, the "weaker" voter bids more than the "stronger" one and the equilibrium outcome is the wrong one causing losses of the order n.[7] In other words, quadratic voting is not necessarily efficient in the realistic case when voters face aggregate uncertainty.[8]

Research on improving voting systems has just scratched the surface of possible alternatives. No clear "winner" has emerged so far, and additional theoretical analysis and empirical testing are needed to design a mechanism that robustly produces efficient and equitable outcomes. Improving voting systems is an exciting application of market design, with potentially important consequences for collective decision making.

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[2] The joint distribution of the mean and median, which was used to compute this probability, was first derived by Laplace 200 years ago.

[3] In a recent article, Casella and Palfrey (2019) show that vote for vote trading can lead to a stable equilibrium and be efficiency improving.

[4] It is still possible that in different settings vote markets have more desirable properties. Xefteris and Ziros (2017) show that in the context of power sharing, where voters care about the vote share and not only about the winner of the election, vote trading leads to efficiency gains.

[5] The AGV mechanism was proposed by Arrow (1979) and independently by d'Aspremont and Gerard-Varet (1979a,b) and is also known as the "expected externality" mechanism. The mechanism is Bayesian incentive compatible, individually rational, budget balanced and results in efficient provision of public goods. The basic idea is that agents are taxed the expected negative externality their choice imposes on others and receive rebates that depend on others' tax payments so that the budget can be balanced without affecting incentives.

[6] There is an active literature in philosophy debating the morality of vote markets. For example, Freiman (2014), Brennan and Jarowski (2015), and Taylor (2018) all offer philosophical arguments in favor of such markets. Counterarguments can be found in Satz (2010), Archer and Wilson (2014) and Archer, Engelen and Ivancovic (2018).

[7] Myatt (2012) has dubbed this phenomenon the "Bayesian underdog effect."

[8] See Weyl (2017) for an analysis of a related environment where the inefficiencies under quadratic voting are less pronounced.