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An efficient PU-based approach to model quasi-brittle fracture and interfaces

by Seyedamir Latifaghili

Thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

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Certificate of Original Authorship

I, Seyedamir Latifaghili declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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To *Nazanin*

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Abstract

Partition of Unity (PU)-based approaches in Nonlinear Fracture Mechanics facilitated and improved the modelling of the fracture behaviour of quasi-brittle materials, such as mortar, concrete, masonry and rock. To this end, the discrete crack approach is assumed to localise the microcracks into the discontinuity surface represented by fictitious crack. The eXtended Finite Element Method (XFEM), as an advantageous modelling technique in the PU context, has been introduced and noticed in recent decades; however, its advantages have accompanied a number of difficulties such as ill-conditioned system, significant growth in the bandwidth of global matrix and inaccurate local solution around the crack path. Furthermore, the designation of reliable criterion, providing information about the strain localisation (i.e. crack initiation) and its orientation, can be considered another difficulty in using the discrete crack approaches.

Recently, various attempts have been made to overcome the difficulties in PU-based approaches. Regardless of all efforts, almost related to the improvement of convergence rate, numerical integration scheme and PU satisfaction, the difficulties of increasing additional degrees of freedom and inaccurate local solution at a discontinuity tip have gained little attention, and no efficient treatments have been presented.

In this study, a comparison between conventional cracking criteria and modified ones is drawn to obtain the appropriate criterion for different fracture modes. In addition, two innovative formulations are proposed: the XFEM with multi-layered Heaviside enrichment and a polygonal enriched Partition of Unity Method. The main advantages are *i*) the cracking criterion can be employed in tensile and compressive states without any special consideration, *ii*) the bandwidth of global matrix and condition number decrease, and *iii*) the displacement jump and stress field are captured accurately.

The capability of the presented formulations is assessed by comparing them with standard XFEM. It is found that the formulation of XFEM with multi-layered Heaviside enrichment shows a remarkable agreement with standard XFEM locally and globally. The proposed formulation with polygonal enrichment overcomes the spurious behaviour of PU-based elements utilised in standard XFEM and opens the possibility of interface problems analysis. Furthermore, several benchmark tests concerning mode-I, mode-II and mixed-mode fracture are simulated, and the numerical results indicate remarkable similarities with the corresponding experimental data.

As a final result of this study, a robust and efficient numerical tool has been introduced to model the crack propagation and interfaces of quasi-brittle materials.

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Amir Latif Aghili

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Contents

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List of Figures

List of Tables

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List of Algorithms

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List of Symbols

Latin symbols

$\mathbf{u}$	total displacement vector
$[\![\mathbf{u}]\!]$	jump vector
$\mathbb{R}$	Real Number
$\mathscr{H}$	Heaviside function
$\mathscr{P}^{(m)}$	polynomial of degree m
$\mathbf{A}$	standard assembly operator
b	width
c	cohesion
d	scalar damage
E	Young's modulus
f	function
h	characteristic element size
h	parameter defining the jump transmission to Ω^+ and Ω^-
i, j	finite element node
l	measure of significant distance around the tip
n	number of the finite element nodes
r	distance between the integration point and the tip

$\hat{\mathbf{u}}$ regular displacement field vector

τ tangential stress vector

$\mathbf{a}$ total displacement vector at the nodes

$\mathbf{B}$ strain-nodal displacement matrix

$\mathbf{D}$ constitutive matrix

$\mathbf{f}$ vector of global nodal forces

$\mathbf{L}$ differential operator matrix

$\mathbf{L}$ differential operator matrix

$\mathbf{N}$ shape function matrix

$\mathbf{n}$ unit vector normal to a boundary

$\mathbf{P}$ external load

$\mathbf{R}$ transformation matrix

$\mathbf{s}$, $\mathbf{n}$ unit vectors tangent and orthogonal to the discontinuity composing the local frame

$\mathbf{T}$ discontinuity constitutive matrix

$\mathbf{t}$ traction vector

$\mathbf{w}$ nodal jump vector

$\mathbf{x}$ global coordinates of a material point

D_{sk} shear stiffness for an advanced state of damage

f_c compressive strength

f_t tensile strength

G_F fracture energy

G_{F_c} compressive fracture energy

h_j distance

h_s parameter defined by: $-\ln(D_{s\kappa}/D_{s\kappa_0})$

k_n, k_s normal and shear penalty parameters

l_a active length scale parameter

l_{ch} Hillerborg's characteristic length

s_j length

u_v vertical displacement

$\bar{\mathbf{b}}$ body forces vector

$\bar{\mathbf{N}}$ shape function matrix

$\bar{\mathbf{t}}$ natural forces vector

$\bar{\mathbf{u}}$ essential boundary conditions vector

$\hat{\mathbf{a}}$ regular displacement vector at the nodes

$\hat{\mathbf{f}}$ regular external vector force at the regular nodes

$\tilde{\mathbf{a}}$ enhanced displacement vector at the nodes

$\tilde{\mathbf{f}}$ enhanced external vector force at the regular nodes

$\mathbf{d}_i$ vector connecting discontinuity tip to integration points

$\mathbf{f}_{cond}$ condensed vector force at the nodes

$\mathbf{H}_{\Gamma_d}$ diagonal matrix containing the Heaviside function evaluated at each degree of freedom

$\mathbf{K}_d$ discontinuity stiffness matrix

$\mathbf{K}_{\hat{a}\hat{a}}$ bulk stiffness matrix

$\mathbf{K}_{\tilde{a}\hat{a}}, \mathbf{K}_{\hat{a}\tilde{a}}, \mathbf{K}_{\tilde{a}\tilde{a}}$ enhanced bulk stiffness matrices for the XFEM

$\mathbf{K}_{aa}$ stiffness matrix for the interface elements

$\mathbf{K}_{cond}$ condensed stiffness matrix

$\mathbf{T}^d$	damage stiffness matrix
$\mathbf{T}^e$	initial isotropic stiffness matrix
$\mathbf{x}_{dis}$	location of the discontinuity

Greek symbols

α	history variable
$\bar{\lambda}$	offset ratio
β	shear contribution parameter
$\delta(.)$	admissible or virtual variation of $(.)$
$\Delta(\cdot)$	change of $(\cdot)$
Γ	boundary
κ	scalar variable
λ	active length
$\nabla(.)$	gradient of $(.)$
ν	Poisson ratio
Ω	elastic domain
ω	weight coefficient
ϕ	internal friction angle
$\Psi(\mathbf{x})$	function
ρ	dead-weight
θ	angle
α_i	weight function
$\hat{\varepsilon}$	regular strain tensor

σ stress tensor

ε total strain tensor

δ_a Dirac's delta function at point a

δ_{Γ_d} Dirac's delta function along the surface Γ_d

η_L circle diameter

Γ_d discontinuity surface

Γ_t boundary with natural forces

Γ_u boundary with essential conditions

κ_0 scalar parameter denoting the beginning of the softening

$\delta(.)$ admissible or virtual variation of $(.)$

σ_{I} first principal stress

δ_i angle

γ_i angle

φ_i function

Indices

$(\cdot)^+,(\cdot)^-$ $(\cdot)$ at the Ω^+ and Ω^-

$(\cdot)^{\cdot}$ rate form of $(\cdot)$

$(\cdot)^s$ symmetric part of $(\cdot)^s$

$(\cdot)^T$ transpose of $(\cdot)$

$(\cdot)^{-1}$ inverse of $(\cdot)$

$(\cdot)_0$ initial value of $(\cdot)$

$(\cdot)_L$ L^{th} enrichment layer of $(\cdot)$

$(\cdot)_n$, $(\cdot)_s$ normal and shear components of $(\cdot)$

$(\cdot)_n$, $(\cdot)_s$, $(\cdot)_e q$ normal, tangential and equivalent components of $(\cdot)$

$(\cdot)_{el}$ elastic $(\cdot)$

$(.)^{elsL}$ (.) belonging to the layer L

$(.)_{cond}$ Schur complement of (.)

Acronyms

BVP Boundary Value Problem

CMOD Crack Mouth Opening Displacement

CMSD Crack Mouth Sliding Displacement

DSDA Discrete Strong Discontinuity Approach

GFEM Generalised Finite Element Method

GSDA Generalised Strong Discontinuity Approach

LEFM Linear Elastic Finite Element

MLF Moving Least-Squares

NR Newton-Raphson

PU Partition of Unity

PUM Partition of Unity Method

SDA Strong Discontinuity Approach

XFEM eXtended Finite Element Method