

An efficient PU-based approach to model quasi-brittle fracture and interfaces

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Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy

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December 2020

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Certificate of Original Authorship

I, Seyedamir Latifaghili declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

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This research is supported by the Australian Government Research Training Program.

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To *Nazanin*

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Abstract

Partition of Unity (PU)-based approaches in Nonlinear Fracture Mechanics facilitated and improved the modelling of the fracture behaviour of quasi-brittle materials, such as mortar, concrete, masonry and rock. To this end, the discrete crack approach is assumed to localise the microcracks into the discontinuity surface represented by fictitious crack. The eXtended Finite Element Method (XFEM), as an advantageous modelling technique in the PU context, has been introduced and noticed in recent decades; however, its advantages have accompanied a number of difficulties such as ill-conditioned system, significant growth in the bandwidth of global matrix and inaccurate local solution around the crack path. Furthermore, the designation of reliable criterion, providing information about the strain localisation (i.e. crack initiation) and its orientation, can be considered another difficulty in using the discrete crack approaches.

Recently, various attempts have been made to overcome the difficulties in PU-based approaches. Regardless of all efforts, almost related to the improvement of convergence rate, numerical integration scheme and PU satisfaction, the difficulties of increasing additional degrees of freedom and inaccurate local solution at a discontinuity tip have gained little attention, and no efficient treatments have been presented.

In this study, a comparison between conventional cracking criteria and modified ones is drawn to obtain the appropriate criterion for different fracture modes. In addition, two innovative formulations are proposed: the XFEM with multi-layered Heaviside enrichment

and a polygonal enriched Partition of Unity Method. The main advantages are *i*) the cracking criterion can be employed in tensile and compressive states without any special consideration, *ii*) the bandwidth of global matrix and condition number decrease, and *iii*) the displacement jump and stress field are captured accurately.

The capability of the presented formulations is assessed by comparing them with standard XFEM. It is found that the formulation of XFEM with multi-layered Heaviside enrichment shows a remarkable agreement with standard XFEM locally and globally. The proposed formulation with polygonal enrichment overcomes the spurious behaviour of PU-based elements utilised in standard XFEM and opens the possibility of interface problems analysis. Furthermore, several benchmark tests concerning mode-I, mode-II and mixed-mode fracture are simulated, and the numerical results indicate remarkable similarities with the corresponding experimental data.

As a final result of this study, a robust and efficient numerical tool has been introduced to model the crack propagation and interfaces of quasi-brittle materials.

Acknowledgements

I would like to thank the people whose helps, supports and encouragement allowed me to accomplish the work presented in this thesis.

First, I would like to express Dr Nadarajah Gowripalan my deep sense of gratitude for comprehending my severe condition during this journey, supporting me wholeheartedly, believing in me and letting me follow my passion. I would also like to thank him for sharing his wide and invaluable experience with me generously. Without his substantial help, nothing could have been possible.

I appreciate the support of Dr Mina Mortazavi to allow me to meet my official commitment to the University of Technology Sydney.

I would like to thank Dr Emre Erkmén for helping me to take my initial and essential steps in this research field successfully. I am also grateful to him for implementing the Fortran code, helping me a lot since I started my study.

And Professor Daniel Dias-da-Costa, whose presence has profoundly influenced every second of this long journey. My sincere gratitude to him for giving me the chance to work with and learn from him. I have always admired his incredible generosity in sharing his knowledge; however, he kept mentioning, "There is no need to thank me!". He has always been there to help and guide me through various complications arisen, leading to the implementation of several innovative approaches. His passion, competence, care and professionalism have taught me how to avoid being just a knowledgeable person acting strictly with people, and

provide them with professional insight instead. I am proud of our friendship and collaboration in the last two years.

I am grateful for having had the chance of consulting with Professor M.E. Warkiani and Dr M. Asadnia. Their advice during this project changed the entire path of this study.

I appreciate the contribution of Milad Bybordiani to produce marvellous and brilliant ideas. Without our stimulating conversation during the process of writing, this work could not have been strong enough to be proud of.

To Habib Rasouli, Faraz Sadeghi, Mina Ghanbari, Mohsen Ranjbar, Atila Sarikaya, Ralf Gonzales, Sloan Trad, Zhiyu Luo and all colleagues that directly or indirectly provided a warm social environment, our friendly chats will never be forgotten.

Ali Paknahad has always been there for me from the very beginning while starting at UTS. His deep concern has helped me to overcome the most difficult moments in my life. I would like to stress his valuable advice and strong friendship that we have built throughout these years.

I owe a special thanks to Mehdi Aghayarzadeh for all early mornings that I met him in the silent room, giving me indefatigable energy that I am not alone at the school. For all serious talks during lunch, my enduring gratitude.

To my siblings, Payam for giving me the most special gift in the whole world; Pegah for introducing me to beautiful mind; and Poone for being there to help and support me however tough it has been, my eternal and heartfelt gratitude.

My parents, Maryam and Esa, have encouraged me all along the way to not give up at the awkward moments. My mother, with her patience and smile, made me feel true love. My

father, whose endurance and perseverance have taught me the courage to pursue my dreams until they come true. To them, my deepest gratitude.

Words are not enough to thank Nazanin, star of my life, who has always stood by me all the way, encouraged me to go beyond my limits. For comforting me in all disappointing moments, my profoundest gratitude. I thank her for bearing my little availability, her wholehearted and boundless support. Nothing could have ever been possible without her love. We have had a chance to grow up together and find the meaning of life, Love and only Love.

Amir Latif Aghili

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Contents

Contents	xiii
List of Figures	xvii
List of Tables	xxv
List of Algorithms	xxvii
List of Symbols	xxix
Chapter 1 Introduction	1
1.1 Objectives	7
1.2 Outline	8
Chapter 2 Literature review	11
2.1 Strong discontinuity approach.....	12
2.2 Discrete constitutive models	19
2.3 eXtended Finite Element Method (XFEM)	32
2.4 Partition of unity-based discontinuous elements	44
2.5 Conclusions.....	54
Chapter 3 A comparative study on crack propagation criteria	55
3.1 Cracking criteria based on failure surfaces.....	56
3.2 Criterion based on the averaged effective stress tensor	62

3.3	Crack initiation	63
3.4	Crack propagation	64
3.5	Numerical examples	66
3.6	Conclusions	81
Chapter 4 An XFEM multi-layered Heaviside enrichment for fracture propagation		83
4.1	Theoretical formulation and finite element discretisation	84
4.2	Layer activation and enrichment procedure	87
4.3	Local Schur complement	89
4.4	Alternative Static condensation	92
4.5	Numerical examples	94
4.6	Conclusions	114
Chapter 5 Surmount spurious behaviour of PU-based discontinuous elements		117
5.1	Computational issue	117
5.2	Polygonal shape functions	120
5.3	XFEM enrichment by polygonal interpolant	122
5.4	Numerical examples	127
5.5	Conclusions	141
Chapter 6 Conclusions		143
6.1	Main Conclusions	145
6.2	Suggestions for future developments	150
Appendix A Polygonal Finite Element		153

A1	Wachspress interpolants	153
A2	Interpolant patch tests	155
References		157

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List of Figures

Figure 2.1	Domain Ω crossed by a strong and a weak discontinuity Γ_d	12
Figure 2.2	Representation of displacement and strain fields: (a) weak discontinuity (b) strong discontinuity	13
Figure 2.3	Domain Ω crossed by a discontinuity surface Γ_d	14
Figure 2.4	Discrete material models- (a) cohesive crack for a specimen under tension (b) material model for continuum and cohesive crack with different softening laws.	20
Figure 2.5	Bulk integration scheme for mid-point rule with three point (a) two sub-integrals on Ω^+ and Ω^- , and (b) one sub-integral on Ω^+ and one integral on Ω (the dots and crosses represent the additional and the regular Gaussian integration point respectively).	41
Figure 2.6	Representation of misalignment of the crack path in the parent element and the physical element.	41
Figure 2.7	Conventional interface element.	45
Figure 2.8	Traction-free notched beam test: (a) Schematic geometry (b) Traction profile oscillations (k_n measured in $\frac{\text{MPa}}{\text{mm}}$).	48
Figure 2.9	Different possibilities of horizontal discontinuity placement in quadrilateral element	51
Figure 2.10	Single element test.	52

Figure 3.1	Failure surfaces.	60
Figure 3.2	Determination of the propagation direction.	63
Figure 3.3	Three point bending beam.	67
Figure 3.4	Three point bending beam: load–displacement curves for the loaded node, obtained from studied criteria including averaged effective stress, Alfaiate’s, Carol’s and Rankine.	68
Figure 3.5	Stress field σ_{xx} in MPa for averaged effective stress criterion when the vertical displacement of the loaded node is (a) $u_y=0.08$ mm; (b) $u_y= 0.1$ mm.	68
Figure 3.6	Sensitivity to the length parameter for three point bending beam test. ...	69
Figure 3.7	L-shape panel: geometry, boundary conditions and finite element mesh.	70
Figure 3.8	L-shape panel: Comparison of crack path traced by different approaches.	70
Figure 3.9	L-shaped panel: load–vertical displacement curves at the loaded node for Averaged effective stress criterion, Alfaiate’s, Carol’s and Rankine cracking surfaces.....	71
Figure 3.10	Stress along x-axis for L-shaped panel: for (a) Averaged effective stress (b) Alfaiate (c) Carol (d) Rankine in $u_y=0.35$ mm.....	71
Figure 3.11	The sensitivity of averaged effective stress criterion to the length parameter characterised by the characteristic element size h for L-shape panel.	72
Figure 3.12	Mixed-mode three point bending beam test: geometry and mesh.....	73
Figure 3.13	the comparison between the crack paths predicted by different cracking surface with experimental results for $\bar{\lambda} = 0.5$	74

Figure 3.14	Load versus vertical displacement at the loaded node for mixed-mode three point bending beam with $\bar{\lambda} = 0.5$	74
Figure 3.15	Mixed-mode three point bending test $\bar{\lambda} = 0.5$: Stress field σ_{xx} in MPa and crack path during softening when $u_y=0.06$ mm: (a) Alfaiate cracking surface (b) Rankine (displacements magnified 100 times).	76
Figure 3.16	The comparison between the crack paths predicted by different cracking surface with experimental results for $\bar{\lambda} = 0.25$).	77
Figure 3.17	Load versus vertical displacement at the loaded node for mixed-mode three point bending beam with $\bar{\lambda} = 0.25$).	77
Figure 3.18	Mixed-mode three point bending test $\bar{\lambda} = 0.25$: Stress field σ_{xx} in MPa and crack path during softening when $u_y=0.06$ mm: (a) Alfaiate cracking surface (b) Rankine (displacements magnified 100 times).	78
Figure 3.19	Uniaxial compression test: geometry and mesh.	78
Figure 3.20	Uniaxial compression test: crack path at the end of softening stage for Alfaiate's and Carol's cracking surfaces.	80
Figure 3.21	Uniaxial compression test: stress field along horizontal axis for Alfaiate's surface in (a) $u_y=0.25$ mm (b) $u_y=0.35$ mm (displacements amplified 20 times).	81
Figure 4.1	Enrichment layers with crack propagation.	85
Figure 4.2	Multi-layer enrichment definitions; ● represents the enrichment nodes at different layers.	86
Figure 4.3	Multi-layer enrichment details: a) active length ($l_a = 2h$); and b) characteristic element size (h).	89

Figure 4.4	Instability of stiffness matrix in case of a crack separating a single node on one side. Almost softened and softened normal tangent stiffness represent respectively, $k_n = -1 \times 10^{-5}$ and $k_n = 0$. Arrows represent the prescribed Dirichlet boundary conditions.	98
Figure 4.5	Inclusion problem definition: a) analytical model; b) finite element mesh with variable inclusion radius range.	99
Figure 4.6	Inclusion problem enrichment layers.	99
Figure 4.7	Condition number as a function of inclusion radius.	100
Figure 4.8	Geometry and mesh (dimensions in ‘mm’).	101
Figure 4.9	Representation of enrichment layers for $\lambda = 1$ at an advanced stage of crack propagation.	102
Figure 4.10	Load <i>versus</i> CMSD curves for different active lengths and experimental results.	103
Figure 4.11	Single-edge notched beam: deformed mesh at CMOD=0.1mm (with displacements amplified 50 times): (a) $\lambda = 0$; (b) $\lambda = 1$	103
Figure 4.12	Single edge notched beam – maximum principal stress (N/mm ²) contour at $u_v=0.3$ mm (with displacements amplified 100 times): (a) $\lambda = 1$; (b) $\lambda = 2$; (c) XFEM.	104
Figure 4.13	Geometry, loading and boundary conditions, and mesh (dimensions in ‘mm’).	105
Figure 4.14	Load <i>versus</i> CMOD curves for the different active lengths and experimental results.	106

Figure 4.15	First principal stress (N/mm^2) contour at $\text{CMOD} = 0.2$ mm (with displacements amplified 200 times) for: (a) $\lambda = 1$; (b) $\lambda = 2$; and (c) XFEM.	106
Figure 4.16	Traction profile above the notch at: (a) peak load for $\text{CMOD} = 0.05$ mm; and (b) softening for $\text{CMOD} = 0.16$ mm.	107
Figure 4.17	Geometry, loading and boundary conditions, and mesh (dimensions in ‘mm’).	108
Figure 4.18	Load <i>versus</i> vertical displacement for different active lengths superposed with experimental results.	109
Figure 4.19	First principal stress (N/mm^2) maps at $u_v = 0.1$ mm (with displacements amplified 50 times): (a) $\lambda = 1$; (b) $\lambda = 2$; and (c) standard XFEM.	110
Figure 4.20	Traction vectors at each integration point at $\text{CMOD} = 0.013$ (displacements amplified 500 times) for: (a) $\lambda = 0$; (b) $\lambda = 1$; (c) $\lambda = 2$; and (d) standard XFEM.	111
Figure 4.21	Prenotched gravity dam model: (a) geometry (dimensions in ‘cm’); and (b) mesh.	112
Figure 4.22	Load <i>versus</i> CMOD curves superposed with experimental results.	113
Figure 4.23	Deformed mesh at $\text{CMOD} = 0.2$ mm for (a) $\lambda = 0$; (b) $\lambda = 1$, and $\text{CMOD} = 0.3$ for (c) $\lambda = 1$ (displacements amplified 500 times).	113
Figure 4.24	Notched gravity dam – maximum principal stress contour at $\text{CMOD} = 0.2$ mm : (a) $\lambda = 0$; (b) $\lambda = 1$; (c) $\lambda = 2$; (d) XFEM (displacements amplified 500 times).	114
Figure 4.25	Traction profile in front of the notch.	115

Figure 5.1	Representation of unrealistic moment couple in the transmission of the traction to the equivalent nodal forces.	118
Figure 5.2	Representation of polygonal parts formed in a quadrilateral element with corner cut (regular and additional nodes are represented by black circle and square respectively).	120
Figure 5.3	Voronoi diagram of point P.	122
Figure 5.4	Representation of the enrichment difference: (a) conventional XFEM and (b) XFEM enriched by polygonal interpolant (black and white nodes show regular and additional nodes respectively).	123
Figure 5.5	Isoparametric mapping for canonical element.	127
Figure 5.6	Single element test: geometry and boundary conditions.	129
Figure 5.7	Representation of adopted meshes for single element test: (a) fine mesh (b) coarse mesh.	129
Figure 5.8	Horizontal displacement jump for single element test: (a) uniform crack opening (b) nonuniform crack opening.	130
Figure 5.9	Traction profile for uniform crack opening: (a) along X axis (b) along Y axis.	131
Figure 5.10	Traction profile of uniform crack opening: (a) polygonal enrichment (b) standard XFEM enrichment for different penalty parameters.	132
Figure 5.11	Traction profile for non-uniform crack opening: (a) along X axis (b) along Y axis.	133
Figure 5.12	Traction profile of non-uniform crack opening: (a) polygonal enrichment (b) standard XFEM enrichment for different penalty parameters.	133

Figure 5.13	Linear elastic notched beam: geometry and boundary condition.....	134
Figure 5.14	Representation of adopted meshes for Linear elastic notched beam: (a) structured mesh (b) unstructured mesh.	134
Figure 5.15	Linear elastic notched beam – traction profile in front of the notch with polygonal enrichment and Newton-Cotes/Lobatto with two points for different penalty parameters: (a) structured mesh (b) unstructured mesh.	135
Figure 5.16	Linear elastic notched beam – traction profile in front of the notch with standard XFEM enrichment and Newton-Cotes/Lobatto with two points for different penalty parameters: (a) structured mesh (b) unstructured mesh.	135
Figure 5.17	Linear elastic notched beam – representation of traction vectors for unstructured mesh by means of Newton-Cotes/Lobatto : (a) Polygonal enrichment (b) standard XFEM enrichment.	136
Figure 5.18	Peel test: geometry and boundary conditions.....	136
Figure 5.19	Representation of adopted meshes for peel test: (a) structured mesh (b) unstructured mesh.....	137
Figure 5.20	Peel test: load <i>versus</i> vertical displacement curves for different formulations.....	138
Figure 5.21	Peel test: traction profile of unstructured mesh for different penalty parameters in vertical displacement (a) $u_y = 0.4$ mm and (b) $u_y = 4$ mm.	138
Figure 5.22	Peel test: stress map of (a) polygonal enrichment and (b) standard XFEM enrichment in $u_y = 4$ mm.	139
Figure 5.23	Four-point shear test: geometry and mesh.	140

Figure 5.24	Four-point shear test: Comparison of polygonal enrichment with standard XFEM and experimental results.	140
Figure 5.25	Four-point shear test: stress map of polygonal enrichment for: (a) CMSD = 0.025 mm; (b) CMSD = 0.07 mm; and (c) CMSD = 0.085 mm.	141
Figure A.1	Wachspress shape functions on a pentagon.	154
Figure A.2	Polygonal domain.	156

List of Tables

Table 3.1	Uniaxial compression test: comparison of peak axial stress-displacement.	80
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List of Algorithms

Algorithm 4.1 Enrichment procedure during crack propagation	88
Algorithm 4.2 Updating stiffness matrix in crack propagation problems.	91
Algorithm 5.1 Integration over Ω^+	128

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List of Symbols

Latin symbols

\mathbf{u}	total displacement vector
$[[\mathbf{u}]]$	jump vector
\mathbb{R}	Real Number
\mathcal{H}	Heaviside function
$\mathcal{P}^{(m)}$	polynomial of degree m
\mathbf{A}	standard assembly operator
b	width
c	cohesion
d	scalar damage
E	Young's modulus
f	function
h	characteristic element size
h	parameter defining the jump transmission to Ω^+ and Ω^-
i, j	finite element node
l	measure of significant distance around the tip
n	number of the finite element nodes
r	distance between the integration point and the tip

$\hat{\mathbf{u}}$	regular displacement field vector
$\boldsymbol{\tau}$	tangential stress vector
\mathbf{a}	total displacement vector at the nodes
\mathbf{B}	strain-nodal displacement matrix
\mathbf{D}	constitutive matrix
\mathbf{f}	vector of global nodal forces
\mathbf{L}	differential operator matrix
\mathbf{L}	differential operator matrix
\mathbf{N}	shape function matrix
\mathbf{n}	unit vector normal to a boundary
\mathbf{P}	external load
\mathbf{R}	transformation matrix
\mathbf{s}, \mathbf{n}	unit vectors tangent and orthogonal to the discontinuity composing the local frame
\mathbf{T}	discontinuity constitutive matrix
\mathbf{t}	traction vector
\mathbf{w}	nodal jump vector
\mathbf{x}	global coordinates of a material point
D_{sk}	shear stiffness for an advanced state of damage
f_c	compressive strength
f_t	tensile strength
G_F	fracture energy
G_{F_c}	compressive fracture energy
h_j	distance

h_s	parameter defined by: $-\ln(D_{s\kappa}/D_{s\kappa_0})$
k_n, k_s	normal and shear penalty parameters
l_a	active length scale parameter
l_{ch}	Hillerborg's characteristic length
s_j	length
u_v	vertical displacement
$\bar{\mathbf{b}}$	body forces vector
$\bar{\mathbf{N}}$	shape function matrix
$\bar{\mathbf{t}}$	natural forces vector
$\bar{\mathbf{u}}$	essential boundary conditions vector
$\hat{\mathbf{a}}$	regular displacement vector at the nodes
$\hat{\mathbf{f}}$	regular external vector force at the regular nodes
$\tilde{\mathbf{a}}$	enhanced displacement vector at the nodes
$\tilde{\mathbf{f}}$	enhanced external vector force at the regular nodes
\mathbf{d}_i	vector connecting discontinuity tip to integration points
\mathbf{f}_{cond}	condensed vector force at the nodes
\mathbf{H}_{Γ_d}	diagonal matrix containing the Heaviside function evaluated at each degree of freedom
\mathbf{K}_d	discontinuity stiffness matrix
$\mathbf{K}_{\hat{\mathbf{a}}\hat{\mathbf{a}}}$	bulk stiffness matrix
$\mathbf{K}_{\tilde{\mathbf{a}}\hat{\mathbf{a}}}, \mathbf{K}_{\tilde{\mathbf{a}}\tilde{\mathbf{a}}}, \mathbf{K}_{\hat{\mathbf{a}}\tilde{\mathbf{a}}}$	enhanced bulk stiffness matrices for the XFEM
\mathbf{K}_{aa}	stiffness matrix for the interface elements
\mathbf{K}_{cond}	condensed stiffness matrix

\mathbf{T}^d	damage stiffness matrix
\mathbf{T}^e	initial isotropic stiffness matrix
\mathbf{x}_{dis}	location of the discontinuity

Greek symbols

α	history variable
$\bar{\lambda}$	offset ratio
β	shear contribution parameter
$\delta(\cdot)$	admissible or virtual variation of (\cdot)
$\Delta(\cdot)$	change of (\cdot)
Γ	boundary
κ	scalar variable
λ	active length
$\nabla(\cdot)$	gradient of (\cdot)
ν	Poisson ratio
Ω	elastic domain
ω	weight coefficient
ϕ	internal friction angle
$\Psi(\mathbf{x})$	function
ρ	dead-weight
θ	angle
α_i	weight function
$\hat{\varepsilon}$	regular strain tensor

σ	stress tensor
ε	total strain tensor
δ_a	Dirac's delta function at point a
δ_{Γ_d}	Dirac's delta function along the surface Γ_d
η_L	circle diameter
Γ_d	discontinuity surface
Γ_t	boundary with natural forces
Γ_u	boundary with essential conditions
κ_0	scalar parameter denoting the beginning of the softening
$\delta(\cdot)$	admissible or virtual variation of (\cdot)
σ_1	first principal stress
δ_i	angle
γ_i	angle
φ_i	function

Indices

$(\cdot)^+, (\cdot)^-$	(\cdot) at the Ω^+ and Ω^-
$(\cdot)^\cdot$	rate form of (\cdot)
$(\cdot)^s$	symmetric part of $(\cdot)^s$
$(\cdot)^T$	transpose of (\cdot)
$(\cdot)^{-1}$	inverse of (\cdot)
$(\cdot)_0$	initial value of (\cdot)
$(\cdot)_L$	L^{th} enrichment layer of (\cdot)

$(\cdot)_n, (\cdot)_s$ normal and shear components of (\cdot)

$(\cdot)_n, (\cdot)_s, (\cdot)_e$ normal, tangential and equivalent components of (\cdot)

$(\cdot)_{el}$ elastic (\cdot)

$(\cdot)^{elL}$ (\cdot) belonging to the layer L

$(\cdot)_{cond}$ Schur complement of (\cdot)

Acronyms

BVP Boundary Value Problem

CMOD Crack Mouth Opening Displacement

CMSD Crack Mouth Sliding Displacement

DSDA Discrete Strong Discontinuity Approach

GFEM Generalised Finite Element Method

GSDA Generalised Strong Discontinuity Approach

LEFM Linear Elastic Finite Element

MLF Moving Least-Squares

NR Newton-Raphson

PU Partition of Unity

PUM Partition of Unity Method

SDA Strong Discontinuity Approach

XFEM eXtended Finite Element Method