

UNIVERSITY OF TECHNOLOGY SYDNEY
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**Autonomous Navigation and Planning Technology
for Quad-rotors Unmanned Aerial Vehicle (UAV)
System**

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

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Certificate of Authorship/Originality

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degree except as fully acknowledged within the text.

I also certify that this thesis has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis.

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ABSTRACT

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In an unknown environment, a robot needs to keep estimating its pose and, simultaneously, building a map of its surrounding environment using only on-board sensors. This problem is called as simultaneous localization and mapping (SLAM), which is one of the key robotics problems that have been studied in the past decades. Meanwhile, many related research problems, including active SLAM, semantic SLAM and so on, are studied to further extend the applications of SLAM. This thesis aims to investigate the graph structure of SLAM and applies it in the related problems, including anchor selection and active SLAM, with the applications for Quad-rotors UAV system. The thesis is composed of three parts:

First, we explore the relation between the graphical structure of 2D and 3D pose-graph SLAM and Fisher information matrix (FIM), Cramér-Rao lower bounds (CRLB), and its optimal design metrics (T-/D-optimality). Based on the assumption of isotropic Langevin noise for rotation and block-isotropic Gaussian noise for translation, the FIM and CRLB are derived and shown to be closely related to the graph structure, in particular, the weighted Laplacian matrix. We also prove that the total node degrees and the weighted number of spanning trees, as two graph connectivity metrics, are closely related to the trace and determinant of FIM, respectively. We also present upper and lower bounds for the D-optimality metric, which can be efficiently computed and are almost independent of the estimation results. The proposed conclusions are verified with several well-known datasets.

Second, we consider 2D/3D pose-graph SLAM problem when accurate ground

truth for some poses, termed anchors, can be obtained. We present a high-efficient algorithm for the problem of choosing a set of anchored poses from a set of possible or potential poses, that minimizes estimated error in pose-graph SLAM. Using the tree-connectivity, the anchor selection problem is re-formulated as a sub-matrix selection problem for reduced weighted Laplacian matrix and belongs to maximization problem of a sub-modular function with a cardinality-fixed constraint. Two improved greedy methods, using Cholesky decomposition, approximate minimum degree permutation (AMDP), order re-use, and rank-1 update technologies, are presented to solve this problem with a performance guarantee between the chosen subset and the optimal solution. Simulations with public-domain datasets and real-world experiments are presented to demonstrate the efficiency of the proposed techniques.

Third, as an application of the graph structure results, based on map joining, two active SLAM methods with two different frameworks are presented: one for 2D feature-based SLAM and the other one for 3D pose-graph SLAM. For the 2D feature-based SLAM, we present a detached method based on model predictive control (MPC) framework. For the uncertainty minimization problem, a non-convex constrained least-squares problem is presented to approximate the original problem using graph topology. Using convex relaxation, it is further transformed into a convex problem, and then solved by a convex optimization method and a rounding procedure based on the singular value decomposition (SVD). For the area coverage problem, it is solved by the sequential quadratic programming (SQP) method. For the 3D pose-graph active SLAM problem, weighted node degree (T-optimality metric) and weighted tree-connectivity (D-optimality metric) are introduced to choose a candidate trajectory and several key poses. With the help of the key poses, a sampling-based path planning method and a continuous-time trajectory optimization method are combined hierarchically. In simulations and experiments, we validate these two approaches by comparing against existing methods, and we demonstrate the off/on-line planning part using a quad-rotor unmanned aerial vehicle (UAV).

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