UNIVERSITY OF TECHNOLOGY SYDNEY

Development of a Novel Controllable Seismic Isolation System Based on Negative Stiffness Concept

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Huan Li declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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LIST OF PUBLICATIONS RELATED TO THIS THESIS

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6. Yu, Y., Li, J., Li, Y., Li, S., **Li, H.** and Wang, W., 2019. Comparative investigation of phenomenological modeling for hysteresis responses of magnetorheological elastomer devices. International journal of molecular sciences, 20(13), p.3216.
7. Yu, Y., Subhani, M., Hoshyar, A.N., Li, J. and **Li, H.**, 2020. Automated Health Condition Diagnosis of in situ Wood Utility Poles Using an Intelligent Non-Destructive Evaluation (NDE) Framework. International Journal of Structural

Stability and Dynamics, 20(10), p.2042002.

8. Yu, Y., Royel, S., Li, Y., Li, J., Yousefi, A.M., Gu, X., Li, S. and **Li, H.**, 2020. Dynamic modelling and control of shear-mode rotational MR damper for mitigating hazard vibration of building structures. Smart Materials and Structures, 29(11), p.114006.

Conference Papers

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AFR	Amplitude frequency response
ASNSD	Adaptive seismic negative stiffness device
ASNSD-FC	Adaptive seismic negative stiffness device with feedback control
ASNSD-OC	Adaptive seismic negative stiffness device with open-loop control
ASNSS	Adaptive seismic negative stiffness system
DCD	Dynamic crowding distance
GA	Genetic algorithm
LQR	Linear quadratic regulator
NSD	Negative stiffness damper
NSGA-II	Non-dominated sorting genetic algorithm type II
NSVID	Negative stiffness vibration isolation device
QZS	Quasi-zero stiffness
SDOF	Single degree of freedom
TSK	Takagi-Sugeno-Kang
SM	Sliding mode
UB	Uncontrolled building
UAP	UB with ASNSD featuring passive damper
UAF-LQR	UB with ASNSD-FC employing the LQR controller
UAF-H	UB with ASNSD-FC employing the $H\infty$ controller
UAF-SM	UB with ASNSD-FC employing the SM controller

ABSTRACT

Adaptive seismic negative stiffness device (ASNSD), as one of the effective seismic protection devices, has been proposed, investigated, and applied to preserve structures in both mechanical and civil fields. It can provide an effective seismic isolation effect through negative stiffness when it is mostly needed, which contributes to a significant reduction in acceleration. Besides, constantly changing stiffness endows it with the capacity to work equivalently as a smart base isolator and avoid the resonance problem. To mitigate the large displacement response across the ASNSD level accompanying the reduction of lateral stiffness, an ASNSD generally is designed to work with passive linear viscous dampers. Although supplementary damping is beneficial for improving energy dissipation capacity and suppressing the resonance response, it underpins the drawback of degrading the vibration isolation effect during the high-frequency region and amplifying acceleration. Besides, despite the variable stiffness property of the ASNSD, it still works in a passive control way, which cannot adequately adapt to a wider earthquake range. Moreover, the property of the ASNSD is geometrically achieved through mechanisms, which cannot achieve the desired seismic isolation and suppression performance simultaneously without comprehensively optimising the structural parameter combination. In addition, there is a lack of published investigations concerning the influence of enormous vertical seismic load associating with excessive lateral displacement on the structural stability of the ASNSD members' interlayer slips. Consequently, this thesis aims to address the above challenges that traditional base isolation devices have undergone with the achievements of novel modified ASNSD with different adaptabilities to contribute new knowledge to the seismic protection field.

Firstly, the principle of negative stiffness on vibration isolation is analysed to better comprehend the fundamentals of the ASNSD. Then, the beneficial effects of nonlinear damping and nonlinear stiffness on vibration isolation are investigated, which contributes to achieving a modified ASNSD with open-loop control (ASNSD-OC). It has the ability to generate both negative stiffness and nonlinear damping in the horizontal direction through a linkage mechanism configuring linear springs and linear viscous dampers. The dynamic characteristics of a single degree of freedom (SDOF) system with such a device are analysed and formulated using the Harmonic Balance Method, with a special focus on its amplitude-frequency response (AFR) and transmissibility. The system with damping nonlinearity as a function of displacement and velocity is proven to have attractive advantages over that with linear damping. It has the capacity to reduce transmissibility during the resonance region without increasing it in the high-frequency region. The effect of ASNSD-OC with nonlinear damping on suppressing displacement and acceleration responses is numerically verified under different sinusoidal excitations and earthquakes. It can achieve additional displacement and acceleration reductions under scaled earthquakes, especially intensive earthquakes, compared with the scenario only featuring linear damping.

Secondly, controllable damping is then introduced to the ASNSD to improve its adaptability and robustness to various earthquake events. The developed innovative ASNSD with feedback control (ASNSD-FC) integrates both negative stiffness and controllable damping characteristics to realise real-time controllable property. The force-displacement relationship of the controllable ASNSD-FC is derived as the forward model to present its nonlinear and hysteresis properties. Three representative control algorithms, i.e. Linear Quadratic Regular (LQR) control, $H\infty$ control, and Sliding Mode (SM) control are utilised to develop controllers to attain the optimal control force for the controllable ASNSD-FC. Based on the Takagi-Sugeno-Kang (TSK) fuzzy inference system optimised by Non-Dominated Sorted Genetic Algorithm-II (NSGA-II), a novel non-parametric inverse model is proposed

accordingly to obtain input current according to the required control force and real-time system responses. To demonstrate the feasibility and efficiency of the controllable ASNSD-FC for structural seismic protection, a numerical case study is conducted on a three-storey building model with the controllable ASNSD-FC installed on its first floor. Four scaled benchmark earthquakes are employed as excitations and ten evaluation criteria are adopted to quantitatively assess the performance for the case study. Numerical results indicate the proposed controllable ASNSD-FC can significantly improve vibration control performance on all evaluation criteria simultaneously in comparison with uncontrolled and conventional passive controlled systems.

Thirdly, to enhance the vibration isolation capacity of the ASNSD, it is innovatively implemented to multiple storeys of a building to develop an adaptive seismic negative stiffness system (ASNSS). It can achieve vibration isolation along the building height. Following that, to improve the performance of the ASNSS to reach its full potential on both seismic isolation and suppression, a comprehensive multi-objective optimisation with nonlinearity is conducted to obtain the optimal structural parameter combination of the ASNSD. Based on the parametric analysis, 6 optimisation variables and 1 constraint are determined accordingly. Four objective functions are also defined by considering the two adverse requirements simultaneously, i.e. enhancing vibration isolation and improving vibration suppression. Then, the complex nonlinear optimisation problem is adequately addressed by the modified NSGA-II with dynamic crowding distance (DCD) algorithm. To verify and evaluate the feasibility and effectiveness of the proposed optimisation approach and system, a numerical case study is conducted based on a five-storey benchmark building model subjected to six different earthquakes. The results demonstrated that the optimised ASNSS can effectively address the trade-off between vibration isolation and vibration suppression to achieve desirable seismic protection performance.

Last but not least, the quasi-zero stiffness (QZS) vibration isolator is adopted as a potential candidate for mitigating the impact of the vertical seismic load associating

with the excessive lateral displacement on structural stability across the ASNSD level. To illustrate its capacity to isolate the vertical load applied on the ASNSD, this thesis provides an in-depth analysis of the static and dynamic characteristics of a typical QZS vibration isolator. During this process, the importance of conducting design optimisation to compromise the contradiction between obtaining small maximum AFR and achieving low transmissibility and resonance frequency is determined. Genetic algorithm (GA) is employed to minimise the objective function so that the optimal stiffness ratios for different conditions are obtained. The general design guidelines and recommendations are provided and discussed. Moreover, the features and characteristics of four types of QZS vibration isolators with different negative stiffness elements are comparatively analysed, in terms of their effective stiffness region and transmissibility.

CHAPTER 1.

INTRODUCTION

1.1 MOTIVATION AND GAP OF KNOWLEDGE IN THIS RESEARCH

Earthquakes constitute one of the world's most destructive natural disasters and throughout history, they have caused massive damage, heavy casualties, and enormous material loss in terms of infrastructure, buildings, etc. Except for these direct damages, other accompanied disasters such as fire, flood, toxic gas leakage, and tsunamis often occur after earthquakes. One of the worst earthquakes that happened in Kobe with magnitude-7.2, Japan on January 17, 1995, killed more than 6,000 people, caused nearly 400,000 buildings to collapse, and finally resulted in a direct economic loss of $50 billion. The direct economic loss of the devastating earthquake with a magnitude-8.0 which occurred in Sichuan, China on May 12, 2008, reached $122 billion. It killed nearly 90,000 people and most of them were buried by collapsed buildings. In 2011, another serious earthquake with magnitude-9.0 hit the coast of Japan, which led to about 16,000 people's deaths and more than 2,600 missing. Following the earthquake, a severe tsunami was triggered. Over 8,000 people lost their lives during the Nepal earthquake in 2015 and the avalanche on Mount Everest occurred after the earthquake.

It is worth noticing from these devastating episodes that many thousands of deaths were caused by widespread collapsing buildings instead of the earthquake itself (Lewis 2005). Due to the unpreventable and uncontrollable characteristics of earthquakes, improving the seismic performance of building structures is of vital importance for reducing the loss of human lives and property. Over the past few decades, researchers and engineers have pursued great efforts in these regards. Various methods have been proposed and applied gradually, for instance, strengthening, adding energy dissipation devices, weakening and damping, attaching base isolators, etc (Pall et al. 1980; Reinhorn et al. 2005; Viti et al. 2006; Pasala et al. 2012; Basu et al. 2014; Li et al. 2016). Of these methods, installing the nonlinear negative stiffness vibration isolation device (NSVID) that belongs to smart passive control device, is one of the great potential candidates for the vibration protection approaches and technologies in both mechanical and civil fields.

Among these traditional seismic protection methods, adding retaining walls, frames, and braces or utilising high strength materials to directly increase the stiffness to reinforce a structure can decrease its displacement response, while the acceleration response is also magnified (Li et al. 2016). It is not economical to increase the stiffness of a structure infinitely. Similarly, the approach proposed by Reinhorn et al. (2005) and Viti et al. (2006), namely weakening and damping, can attenuate the acceleration and inter-storey drift of a structure at the same time. However, since it reduces the stiffness of the entire structure to a low value by disconnecting the frames or walls, the structure will yield early, which can result in permanent deformation. The principle of a base isolator is to insert a soft layer with a low lateral stiffness between the base and superstructure. It can mitigate the seismic energy transmitted from the ground motion to the superstructure, hence reducing its responses. Nevertheless, due to wide frequency range of earthquakes, it cannot completely avoid the occurrence of resonance especially under low and ultralow frequency range, which is smaller than the fundamental frequency of a standard isolation system. Most importantly, the implementation of the above passively controlled methods adds potential risk to the entire system since their parameters are predesigned for certain excitations and their properties cannot adapt in a timely way to different earthquake excitations, especially unknown attacks.

NSVID can effectively address the above drawbacks with the nonlinear force-displacement characteristic, which means its stiffness can constantly change between positive, negative, and zero according to its displacement. Different from positive stiffness, the negative stiffness force of a NSVID can assist its displacement instead of resisting it, which contributes to a significant reduction in acceleration if used properly. If the displacement exceeds a threshold, the NSVID will move to the stiffening region with considerable positive stiffness, which enables the structure to move back to its stable position. The variable stiffness endows it with the capacity to work equivalently as a smart base isolator to isolate the superstructure from hazardous ground motion, reduce seismic energy transfer, and subsequently protect the structure from seismic destruction. Most importantly, employing the NSVID ensures that the fundamental frequency of the entire structural system always varies from the predominant frequency of earthquake, hence the resonance issue can be avoided.

The negative stiffness of the NSVID is realised through mechanisms with sophisticated geometry designs. The core elements utilised for generating negative stiffness can be generally categorised into energy storage elements (spring and pre-buckled beam etc.), magnetic elements, geometrically nonlinear structures, as well as composite structures and metamaterials (Li et al. 2020). A great number of NSVIDs for vertical vibration isolation, usually named as quasi-zero-stiffness (QZS) vibration isolators or high-static & low-dynamic vibration isolators, have been proposed and investigated (Carrella et al. 2007b; Huang et al. 2014; Gao & Chen 2015; Zhou et al. 2015; Shi et al. 2016; Shi & Zhu 2017). The NSVIDs have also attracted a great deal of interest in the seismic protection of civil structures (Iemura et al. 2008; Toyooka et al. 2015; Sun et al. 2017b; Cain et al. 2020). In this thesis, to differentiate them, NSVIDs that can provide negative stiffness in the horizontal direction for seismic protection of civil structures are defined as adaptive seismic negative stiffness devices (ASNSDs).

To the best of the author's knowledge, the existing research results of ASNSDs are substantially less than that of the QZS vibration isolators. As one of few ASNSDs designed for seismic protection, the device proposed by Nagarajaiah and his co-workers (Sarlis et al. 2011) is the most attractive and recognised ASNSD. It can trigger negative stiffness when its displacement is larger than a predefined value to mitigate the acceleration of the structure. The large displacement induced by negative stiffness is eased by supplementary linear viscous dampers mounted in parallel. Sarlis et al. (2012) provided the force-displacement and stiffness-displacement characteristics of the ASNSD in detail. Following that, Pasala et al. (2014)

employed the ASNSD to a single degree of freedom (SDOF) building to verify its seismic protection effect. The effectiveness of the ASNSD on the seismic protection of the bridge was numerically and experimentally verified by Attary et al. (2013). The results showed that the peak base shear of the bridge was significantly reduced and the peak displacement was suppressed by the parallel-connected viscous damper.

However, despite the significant advantages of the representative ASNSD in structural seismic protection, some challenges remain to be overcome to improve its performance and fully exploit its potential. Firstly, the same intractable problem as the conventional base isolator remains, i.e. relatively large displacement across the ASNSD level, because the negative stiffness element further prompts the displacement to eliminate the excitation force. Therefore, the ASNSD is designed to work with supplemental passive linear viscous dampers to suppress the large displacement response and structural shear. However, adding damping into the base isolation system is at the expense of encouraging acceleration response and inter-storey drift (Kelly 1999). Besides, from the perspective of transmissibility, a large damping ratio is favourable under the resonance region, while it becomes harmful during the high-frequency region.

To address the foregoing contradiction, nonlinear damping can be a potential solution. Researchers have verified that the nonlinear damping force that is proportional to the cube or square of velocity can show more advantages than the linear damping force for a linear system (Jing & Lang 2009; Lang et al. 2009). Ravindra & Mallik (1994) investigated the effect of system damping nonlinearity that is a function of velocity, which can not only mitigate peak transmissibility and unstable region but also decrease the transmissibility during high-frequency region under force excitation. Tang & Brennan (2013) compared the vibration isolation effects of two QZS vibration isolators with nonlinear damping realised by horizontal placed linear viscous dampers and vertical inherent nonlinear damper respectively. Both were proved to be more desirable than the linear system under force excitation, while the former performed more effectively than the latter under displacement excitation.

It is reasonable to believe that introducing geometrically damping nonlinearity into ASNSD to realise both lateral negative stiffness and nonlinear damping can overcome the above problem faced by the conventional ASNSD with linear damping and accomplish more superior seismic protection effects. However, so far, research concentrating on the mechanism and practical design of such a device has barely been investigated. To realise ASNSD with nonlinear damping and demonstrate its feasibility and efficiency on vibration control, several obstacles need to be overcome in this research. The design of the configuration to achieve nonlinear damping with a function of velocity and displacement should be addressed first. Then, its mechanical property, i.e. nonlinear force-displacement relationship needs to be derived and analysed. Most importantly, investigating the influence of nonlinear damping associating with nonlinear stiffness on the dynamic characteristics of the ASNSD including amplitude-frequency response (AFR) and transmissibility is another challenge.

Secondly, although the stiffness of the passively controlled ASNSD can vary with its displacement, its entire force-displacement relationship is fixed once designed. Due to the complex frequency contents and magnitudes of earthquakes, it cannot fully adapt to a wide variety of earthquake episodes and achieve a desirable vibration control effect. This issue will degrade the performance, reduce the effectiveness, and constrain the application of the ASNSD. Considering that nonlinear damping plays an important role, to significantly improve the adaptability and robustness of the conventional ASNSD, the controllable nonlinear damper that has been investigated and applied for other occasions can inspire. The advantages of controllable damping on traditional vibration control designs and technologies have been investigated. For example, Johnson et al. (1998), Spencer et al. (2000), and Yoshioka et al. (2002) introduced the smart dampers (magnetorheological (MR) fluid damper) to a passive base isolator to improve the device's adaptability to different earthquakes. By controlling the clamping force with the electromagnetic field, Agrawal & Yang (2000) proposed a controllable friction damper for the seismic protection of buildings and results showed that it can achieve an obvious reduction in the peak drift. Kim et al. (2006) did an experimental investigation on a hybrid base isolation system consisting of friction pendulum bearings and MR damper, which showed that the system was highly robust to various earthquake excitations. Nagarajaiah et al.

(2009) induced MR damper controlled by the Lyapunov control strategy to a base-isolated bridge and achieved a significant base displacement suppression outcome.

Although the real-time feedback controllable property based on system responses will enable the ASNSD to be highly adjustable and effective to variable seismic events, this to date has not been investigated. In developing the novel ASNSD with high controllability and adaptability, the following critical problems have to be dealt with. First of all, an accurate and efficient forward model is required to describe the complicated nonlinear relationships between the force generated by the controllable ASNSD, input current, and system responses. Based on that premise, different controllers are needed to attain the desirable control force to minimise system responses based on the real-time feedbacks accordingly. Since the required control force cannot be taken as the direct input to command the controllable ASNSD, an accurate and compact inverse model should be designed to calculate the required current according to the optimal control force and system responses. However, due to the highly nonlinear and hysteretic characteristics of the controllable ASNSD, it is not easy to attain the parametric inverse model directly. For this reason, a non-parametric model based on the advanced fuzzy logical system is of great interest.

Furthermore, to maintain the stability and vibration suppression, it is impractical to design the ASNSD with zero stiffness through the entire excursion. Thus, one-level ASNSD cannot completely isolate the building structure from the excitations, especially strong earthquakes, which leads to the remaining seismic energy transmitted to the above floors. By comparison, the floors above the ASNSD will perform as a rigid body that amplifies the responses. Thus, establishing an adaptive seismic negative stiffness system (ASNSS) by mounting the ASNSD on multiple storeys of a building can be a desirable solution. It will enhance the vibration isolation over the height of the building structure step by step. Pasala et al. (2012) proved that the peak base shear, peak inter-storey drift, and peak acceleration of an ASNSS were significantly reduced by comparison with the scenario with only one ASNSD. Nagarajaiah & Sen (2020) investigated the desirable location of placing the ASNSD along building height. Nevertheless, it is not a straightforward task to apply the ASNSD to establish the ASNSS.

There is a trade-off between effective vibration isolation and favourable vibration suppression since they are two adverse requirements. Besides, as stated previously, the constantly varying stiffness of the ASNSD is achieved in a passive way. Its properties are determined by the stiffness and pre-compression force of its springs, as well as the distances between different joint points, etc. Hence, many factors decide the seismic protection effects of the ASNSS is favourable or not.

Therefore, the structural parameters of the ASNSD should be perfectly predesigned and optimised based on its applied occasion, e.g. the characteristics of the building structure and excitations, etc. Qu et al. (2017) conducted an investigation to obtain the optimal number of the ASNSD on multi-storey buildings but ignored the device's nonlinear characteristic. They innovatively changed the optimisation question into a static feedback controller design problem. Mathew et al. conducted a single parameter optimisation on three parameters of the ASNSD, including stiffness ratio, pre-compressed force, and damping coefficient (Mathew et al. 2015; Mathew & Jangid 2018). These three parameters were optimised one by one separately with the peak structural displacement and acceleration responses as evaluation criteria. However, the properties and performance of the ASNSD and ASNSS are decided by not only the structural parameters individually but also their coupling effects. Consequently, conducting a comprehensive optimisation that takes into account the relevant structural parameters, coupling effects between them, vibration isolation and vibration suppression effects, as well as the characteristics of earthquakes simultaneously is another important research topic for the ASNSD.

Finally, traditional structural seismic protection takes the horizontal earthquake as the excitation by default and has achieved favourable results. Nevertheless, a large number of earthquake records have indicated that the vertical shaking component of a seismic excitation can be in the same order and exceed the commonly assumed 2/3 times of the magnitude of the horizontal excitation. It has caused damage to a lot of building structures (Papazoglou & Elnashai 1996; Elgamal & He 2004). For instance, the vertical-to-horizontal peak acceleration ratio of the Kalamata earthquake in Greece on 13 September 1986 was as high as 1.26 (Elnashai et al. 1989). As one of the most devastating earthquakes, the peak vertical acceleration of the Northridge earthquake was 1.18g which was much larger than its horizontal acceleration. Its vertical-to-horizontal peak acceleration ratio reached 1.79 (Norton et al. 1994). The peak vertical acceleration of the Kobe earthquake, in Japan, was 0.322g that amounted to 121% of the horizontal acceleration (Tokimatsu & Asaka 1998). The vertical acceleration of an earthquake can reach or exceed 1g in a high earthquake intensity zone which cannot be ignored. A strong vertical seismic acceleration can greatly increase the axial load applied on the ASNSD, which can be twice the gravity.

As aforementioned, the decrease of lateral stiffness across the ASNSD level leads to a relatively large displacement, which is the safeguard of effective vibration isolation. When the enormous vertical load associates with the excessive lateral displacement, the integrated rotational moment could seriously threaten the structural stability across the ASNSD level. In this regard, their integration impact deserves to be explored and a potential solution should be provided accordingly. Due to the relatively large displacement is unavoidable, adopting a vibration isolator to mitigate the vertical load applied on the ASNSD is capable of addressing this challenge. However, it requires the capacity to achieve effective vibration isolation while maintaining sufficient load-carrying capacity to the superstructure, which is contradictory for a linear vibration isolator. Therefore, the QZS vibration isolator designed with high-static & low-dynamic stiffness property is a potential candidate. The effect of adopting the QZS vibration isolator to diminish the vertical seismic load exerted on the isolation level should be investigated. Besides, although plenty of QZS vibration isolators have been proposed and investigated individually, research on general design guidelines has barely been published. Also, the advantages and disadvantages of different types of QZS vibration isolators should be comparatively analysed.

1.2 OBJECTIVE OF THE PRESENT RESEARCH

The main objective of this research is to propose reliable and oriented solutions to fill the aforementioned research gaps faced by the conventional ASNSD as well as other traditional base isolators to significantly improve seismic protection performance. To cope with the problem that high damping can help suppress the resonance response while being undesirable during the high-frequency range, the impact of linear and nonlinear damping on the ASNSD will be investigated. To further improve the adaptability and robustness of the ASNSD to adapt well to various earthquakes, introducing controllable damping to the ASNSD should be studied to develop a controllable ASNSD with feedback control (ASNSD-FC). Accordingly, the accurate forward model, inverse model, and controllers need to be designed to make the real-time controllable property possible. To enhance seismic protection performance, an ASNSS will be developed to achieve effective vibration isolation along the building height. Following that, to solve the trade-off between vibration isolation and vibration suppression, a comprehensive multi-objective optimisation will be conducted to obtain the optimal structural parameter combination of the ASNSD. Finally, the influence of vertical load associating with excessive lateral displacement on the structural stability of the ASNSD will be investigated. Then a potential strategy will be proposed and explored.

The specific objectives of this research are as follows:

- To derive non-dimensional expressions of the nonlinear force-displacement relationship of the ASNSD for describing and analysing the influence of structural parameters, as well as their coupling effects on the properties of the ASNSD.
- To investigate the effect of linear and nonlinear damping on the static and dynamic characteristics of the ASNSD in the form of proposing an ASNSD with open-loop control (ASNSD-OC) to simultaneously realise lateral negative stiffness and nonlinear damping.
- To introduce the controllable property to the ASNSD so that a novel controllable ASNSD-FC to improve its controllability, adaptability, and robustness can be devised.
- To design different controllers for the ASNSD-FC to calculate the optimal control force to realise favourable real-time vibration control effects.

- To design an accurate and compact non-parametric inverse model to predict the input current for the controllable ASNSD-OC based on the required control force and system responses.
- To attach the ASNSD on every floor of a building structure to establish an ASNSS to achieve multi-storey vibration isolation.
- To conduct multi-objective optimisation for attaining the optimal structural parameter combination of the ASNSD to improve the performance of the ASNSS on seismic isolation and suppression.
- To analyse the impact of the vertical earthquake load combined with excessive displacement across the isolation level on the structural stability of the ASNSD members' interlayer slips.
- To investigate the properties of the QZS vibration isolator and its capacity to diminish the enormous vertical load, for the purpose of resolving system instability.
- To provide general design guidelines for designing an optimal QZS vibration isolator and comparatively analyse the pros and cons of different types of QZS vibration isolators.

1.3 ORGANISATION OF THIS THESIS

This thesis is structured into seven chapters. The backgrounds, motivation, research gaps, and objectives of this research are outlined in this chapter, i.e. Chapter 1. The following chapters are explained below.

Chapter 2 presents a comprehensive literature review on the existing research results related to this research topic. It starts with the fundamentals of the NSVIDs, e.g. concept, realisation, and characteristics. Following that, it pays attention to the classification of the NSVIDs and summarises the features and advantages of each device type. Besides, the applications of the NSVIDs in mechanical and civil engineering are also reported. Moreover, the state-of-the-art features of three vibration control strategies, including passive control, active control, and semi-active control are presented. Finally, based on the literature review, important conclusions are made and research gaps are identified which will be investigated and addressed in the subsequent four chapters.

Chapter 3 starts with the principle of negative stiffness on vibration isolation and the fundamentals of the ASNSD. Following that, it devises a method, i.e. adopting nonlinear damping, to address the contradictory effects of high damping on the resonance and high-frequency regions with an achievement of a modified ASNSD-OC. The configuration and nonlinear force characteristics of the ASNSD-OC, including nonlinear elastic force and nonlinear damping force are analysed first. Furthermore, the effectiveness of nonlinear damping integrated with nonlinear stiffness on the dynamic characteristics of systems, including AFR and transmissibility, are mathematically analysed. Moreover, the advantages of the ASNSD-OC with nonlinear damping over linear damping are numerically verified under both sinusoidal excitations and earthquakes.

Chapter 4 introduces the controllable property to the ASNSD to develop a controllable ASNSD-FC with high adaptability and robustness. Its properties can be adjusted in real-time based on the system responses. A brief introduction of the magnetorheological (MR) damper including the properties of MR fluid, its models and applications is given at the beginning of this chapter. The realisation of the controllable ASNSD-FC is demonstrated by installing it on a three-storey building model. Then, a forward model is derived to generate the nonlinear hysteresis control force for future states based on the current and system responses. A novel non-parametric inverse model is developed based on a fuzzy logic system for predicting the input current according to the system responses and required control force. Most importantly, three classic control algorithms are adopted to develop the controllers so that the optimal control force can be calculated in real-time for the controllable ASNSD-FC. To verify and evaluate the capacity and efficiency of the controllable ASNSD-FC with three different controllers, a numerical investigation is conducted and results are compared with the uncontrolled and passive control systems.

Chapter 5 conducts a comprehensive multi-objective optimisation on the structural parameters of the ASNSD to improve the performance of the ASNSS on seismic protection. The system description of the ASNSS that can realise vibration isolation over the height of the building is presented first. Based on the parametric analysis of the ASNSD, the optimisation variables and constraint are determined accordingly. Then, the objective functions are defined by considering both vibration isolation and vibration suppression. After that, the modified non-dominated sorting genetic algorithm type II (NSGA-II) with dynamic crowding distance (DCD) is employed to address the multi-parameter and multi-objective nonlinear optimisation problem. A case study is given based on a five-storey benchmark building model to demonstrate the advantages of the proposed optimisation method and ASNSS.

Chapter 6 investigates the influence of vertical seismic load integrating with excessive lateral displacement on the structural stability across the ASNSD level. QZS vibration isolator is then utilised as the potential solution to mitigate the enormous vertical load applied on the ASNSD to avoid system instability. The explicit formulae and corresponding parameter analysis of the force-displacement relationship, AFR, and force transmissibility of the QZS vibration isolator are obtained and discussed. Besides, to improve the vibration control effect and attain general design guidelines for the QZS vibration isolator, an optimisation of its stiffness ratio is conducted. Moreover, the characteristics and features of four different types of QZS vibration isolators are analysed and compared based on their effective stiffness region and transmissibility.

Chapter 7 summarises the conclusions and remarkable findings obtained in each chapter. Furthermore, some other studies that are not covered in this thesis but valuable are highly recommended to be completed in the future.

CHAPTER 2.

LITERATURE REVIEW

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Li, H., Li, Y. and Li, J., 2020. Negative stiffness devices for vibration isolation applications: A review. Advances in Structural Engineering, 23(8), pp.1739-1755.

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CHAPTER 3.

INVESTIGATE THE IMPACT OF DAMPING ON THE PERFORMANCE OF ASNSD

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Li, H., Li, J., Yu, Y. and Li, Y. 2020, Modified Adaptive Negative Stiffness Device with Variable Negative Stiffness and Geometrically Nonlinear Damping for Seismic Protection of Structures, International Journal of Structural Stability and Dynamics, p. 2150107.

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CHAPTER 4.

STUDY THE CONTROLLABLE DAMPING TO IMPROVE THE PERFORMANCE OF ASNSD

[Production Note: This chapter is not included in this digital copy due to copyright restrictions.]

Li, H., Aakari, M., Li, J., Li, Y. and Yu, Y. 2021, A novel structural seismic protection system with negative stiffness and controllable damping, Structural Control and Health Monitoring, 2021;e2810.

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CHAPTER 5.

IMPROVE THE PERFORMANCE OF ASNSS FOR SEISMIC PROTECTION UTILISING MULTI-OBJECTIVE OPTIMISATION

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Li, H., Yu, Y., Li, J., Li, Y. and Aakari, M. 2021, Multi-objective optimisation for improving the seismic protection performance of a multi-storey adaptive negative stiffness system based on modified NSGA-II with DCD, Journal of Building Engineering, 43, Article 103145.

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CHAPTER 6.

CONSIDER VERTICAL LOAD ON THE STRUCTURAL STABILITY OF ASNSD

[Production Note: This chapter is not included in this digital copy due to copyright restrictions.]

Li, H., Yu, Y., Li, J. and Li, Y. 2020, 2021, Analysis and optimisation of a typical quasi-zero stiffness vibration isolator. Smart Structures and Systems,27(3), pp.525-536.

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CHAPTER 7.

CONCLUSIONS AND FUTURE RESEARCH

This research makes contributions to address the challenges faced by conventional base isolation systems in the form of developing novel ASNSD based on nonlinear damping and semi-active control technic to realise superior seismic protection for civil structures. To have an in-depth understanding of the ASNAD, this thesis started with a comprehensive literature review and the research gaps were identified. The principle of negative stiffness on vibration isolation and the fundamentals of the prototype ASNSD were theoretically analysed. Subsequently, the novel modified ASNSDs with different adaptabilities that feature nonlinear damping and controllable properties were developed. Following that, a series of theoretical investigations and numerical verifications were conducted to demonstrate and evaluate the feasibility and effectiveness of the proposed modified ASNSDs. Furthermore, to improve the seismic protection performance of the ASNSS, a comprehensive multi-objective optimisation was done to obtain the optimal structural parameters of the ASNSD. Moreover, this research innovatively adopted the QZS vibration isolator as a potential resolution to mitigate the impact of vertical load associating with excessive lateral displacement on the structural stability across the ASNSD level. The major conclusions of this thesis are summarised and presented below. Some suggestions for future research are also proposed and given in the following sections.

7.1 RESEARCH CONCLUSION

- In Chapter 3, the modified ASNSD-OC was proposed to investigate the effect of damping on vibration isolation. The non-dimensional elastic force-displacement relationship and damping force-displacement/velocity relationship of the proposed ASNSD-OC were derived. It could achieve negative stiffness and nonlinear damping in the horizontal direction by configuring springs and linear viscous dampers. The nonlinear damping term was verified to be a function of structural parameters, displacement, and velocity, which meant the contribution of the nonlinear damping force became significant with displacement increasing. That is a favourable property for a vibration isolation device since it can suppress large deformation to avoid overturning of the building structure.

- The AFR function and displacement transmissibility function of the ASNSD-OC were derived utilising the Harmonic Balance Method. Compared with linear damping, it was confirmed that the damping nonlinearity had attractive advantages in largely reducing transmissibility around the resonance region without increasing that in the high-frequency region. Besides, the nonlinear damping was demonstrated to be capable of diminishing the probability of unstable phenomenon by eliminating the unstable region and improving the threshold excitation amplitude. The performance of the ASNSD-OC on suppressing displacement and acceleration was numerically verified under different sinusoidal and earthquake excitations with different intensities. Compared with the system only featuring linear damping, the proposed ASNSD-OC with nonlinear damping could achieve additional displacement and acceleration reductions under all excitations, especially intensive sinusoidal excitations and earthquakes with large deformation or shear velocity involved.
- In Chapter 4, the highly adaptive controllable ASNSD-FC was developed which was realised with the integration of ASNSD and controllable damper (MR damper). It could achieve negative stiffness and controllable damping effects simultaneously. The skeleton curve of its control force was decided by the ASNSD and the MR damper determined its energy dissipation capacity. The classical and commonly used Bouc-Wen model was employed as the forward model of the controllable ASNSD-FC to calculate the required control force based on the real-time system responses and input current. Considering the complicated hysteresis characteristics of the controllable ASNSD-FC, a novel non-parametric inverse model was designed based on the TSK fuzzy system optimised by

NSGA-II with input selection to predict the required current for the forward model. It required 6 inputs, i.e. $x(t-1)$, $x(t)$, $v(t)$, $F(t-2)$, $F(t-1)$, and $F(t)$, which could accurately grasp the hysteresis behaviour of the controllable ASNSD-FC.

- In Chapter 4, three controllers, including the LQR controller, H∞ controller, and SM controller, were employed to attain the appropriate control force for the controllable ASNSD-FC to attenuate all the system responses to desirable ranges. A numerical case study was carried out on a three-storey building model to demonstrate the capacity of the ASNSD-FC on structural seismic protection. Four scaled earthquakes were employed as the excitations and ten evaluation criteria were defined to quantitatively evaluate the performance of the proposed ASNSD-FC. It was verified that the controllable ASNSD-FC could significantly improve the vibration control effect by largely diminishing all the evaluation criteria simultaneously in comparison with the uncontrolled building and the building with the conventional ASNSD. Among the three controllers, the controllable ASNSD-FC with SM controller was proved to be more superior in mitigating the structure shear and first-floor acceleration. Besides, the controllable ASNSD-FC with LQR controller required the least energy, while the ASNSD with SM controller consumed more current.
- In Chapter 5, an ASNSS was proposed to enhance the vibration isolation by implementing the ASNSD on multiple storeys of a building structure, which could isolate seismic energy along the height of the building step by step. Following that, the nonlinear multi-objective optimisation was conducted on the structural parameters of the ASNSD to improve the performance of the ASNSS on seismic protection. In the optimisation problem, 6 non-dimensional structural parameters were determined as the optimisation variables and one was utilised as the constraint. Four comprehensive objective functions were defined with two adverse requirements considered, i.e. vibration isolation and vibration suppression. The NSGA-II with DCD algorithm was implemented to address the nonlinear optimisation problem. Based on the attained Pareto front results, the optimal structural parameter combination for the five-storey ASNSS was obtained as [α_s c_{e1} c_{e2} χ ψ q] = [0.287 6.286

2.017 1.029 3.886 0.134]. The numerical case study demonstrated that the vibration suppression effect of the optimised ASNSS was more superior than the system with dampers attached to each floor. In the meantime, it presented better vibration isolation performance than the ASNSS without optimisation. Therefore, the optimised ASNSS could effectively balance the contradiction to realise desirable vibration isolation and vibration suppression performance simultaneously.

- In Chapter 6, the QZS vibration isolator was innovatively applied to mitigate the impact of enormous vertical load combining with the excessive lateral displacement on the structural stability across the ASNSD level. The nonlinear force-displacement and stiffness-displacement relationships of the compact QZS vibration isolator were analysed with a new derived formulation. The supporting force fluctuation rate and effective stiffness region were defined to assess its static property. The recommended ranges of stiffness ratio α_{QZS} was 0.35~2.00 and the corresponding ranges of incline ratio β_{QZS} was 0.45~1.33. The results implied that the QZS vibration isolator could significantly decrease the vertical load applied on the ASNSD, hence being able to effectively improve the structural stability. To achieve the above results, its dynamic characteristics optimisation was done to direct the design of the QZS vibration isolator. With GA, optimal stiffness ratios for different scenarios with various working conditions were attained. A small stiffness ratio was beneficial for the scenario with displacement limitation. In contrast, strong oblique springs were favourable for the situation with a large damping ratio and large excitation amplitude. Moreover, based on their effective stiffness and force transmissibility, the advantages and drawbacks of different types of QZS vibration isolators were analysed, compared, and evaluated at the end of Chapter 6.

7.2 RECOMMENDATIONS FOR FUTURE RESEARCH

- In this thesis, the feasibility and efficiency of the proposed ASNSD-FC with controllable property on seismic protection have been theoretically and numerically investigated and verified. To further demonstrate its seismic isolation and suppression effects for building structure, the experimental investigation of the proposed ASNSD-FC implemented to a three-storey building model is under preparation. However, due to the deep and continuing impact of COVID-19, it is regrettable that the experimental investigation has been delayed. It will involve the design and fabrication of the ASNSD, the installation and calibration of the MR dampers and sensors, as well as the settings of the data acquisition system. Due to the complexity of the mechanical structure of the controllable ASNSD-FC, its main component has been fabricated via 3D printing as shown in Figure 7.1, the configuration parameters of which were designed based on the results in Chapter 6. As Figure 7.2 depicts, the force-displacement relationships of all the springs were tested in Instron E10000.

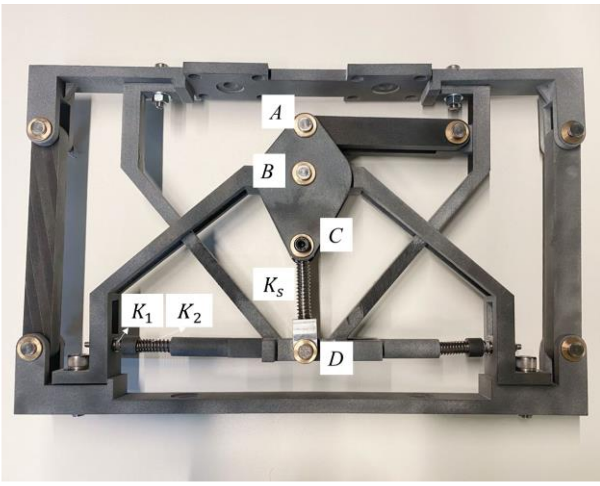


Figure 7.1 The photo of the ASNSD

Figure 7.2 The spring testing

- To achieve the desirable seismic protection effect, three controllers for the controllable ASNSD-FC have been developed for realising controllable property. The working process of these three controllers is to obtain the required optimal control force first. Then, the force and the system responses are delivered to the inverse model to predict the current to drive the device. In order to simplify the process, a novel control algorithm that can attain the current directly is preferred to be proposed.
- In most published research on the seismic protection of structure, the earthquake load is

always independently applied to the structural principle axis. However, the seismic waves generated by the vibration source spread in all directions. Generally, it is impossible to restrict the direction of an earthquake following the principle axis of structures and there will be a deviation angle between them. Considering the force provided by the ASNSD is limited to a particular plane, a multi-direction ASNSD should be designed and its seismic protection performance will be investigated in future research.

REFERENCE

Abdel-Rohman, M. & Leipholz, H.H. 1983, 'Active control of tall buildings', *Journal of Structural Engineering*, vol. 109, no. 3, pp. 628-45.

Agrawal, A. & Yang, J. 2000, 'A semi-active electromagnetic friction damper for response control of structures', *Advanced technology in structural engineering*, pp. 1-8.

Agrawal, A.K., Cha, Y.-J. & Xu, Z., 'SEMI-ACTIVE CONTROL OF STRUCTURAL SYSTEMS', *Structural Engineering and Geomechanics*.

Ahamed, R., Ferdaus, M.M. & Li, Y. 2016, 'Advancement in energy harvesting magneto-rheological fluid damper: A review', *Korea-Australia Rheology Journal*, vol. 28, no. 4, pp. 355-79.

Alabuzhev, P. 1989, *Vibration protection and measuring systems with quasi-zero stiffness*, CRC Press.

Antoniadis, I., Chronopoulos, D., Spitas, V. & Koulocheris, D. 2015, 'Hyper-damping properties of a stiff and stable linear oscillator with a negative stiffness element', *Journal of Sound and Vibration*, vol. 346, pp. 37-52.

Antoniadis, I.A., Kanarachos, S.A., Gryllias, K. & Sapountzakis, I.E. 2018, 'KDamping: a stiffness based vibration absorption concept', *Journal of Vibration and Control*, vol. 24, no. 3, pp. 588-606.

Askari, M., Li, J., Samali, B. & Gu, X. 2016, 'Experimental forward and inverse modelling of magnetorheological dampers using an optimal Takagi–Sugeno–Kang fuzzy scheme', *Journal of Intelligent Material Systems and Structures*, vol. 27, no. 7, pp. 904-14.

Attary, N., Symans, M., Nagarajaiah, S., Reinhorn, A.M., Constantinou, M.C., Sarlis, A.A., Pasala, D.T.R. & Taylor, D. 2014, 'Performance Evaluation of Negative Stiffness Devices for Seismic Response Control of Bridge Structures via Experimental Shake Table Tests', *Journal of Earthquake Engineering*, vol. 19, no. 2, pp. 249-76.

Attary, N., Symans, M., Nagarajaiah, S., Reinhorn, A.M., Constantinou, M.C., Sarlis, A.A., Pasala, D.T.R. & Taylor, D. 2015a, 'Numerical simulations of a highway bridge structure employing passive negative stiffness device for seismic protection', *Earthquake Engineering & Structural Dynamics*, vol. 44, no. 6, pp. 973-95.

Attary, N., Symans, M., Nagarajaiah, S., Reinhorn, A.M., Constantinou, M.C., Sarlis, A.A., Pasala, D.T.R. & Taylor, D.P. 2015b, 'Experimental Shake Table Testing of an Adaptive Passive Negative Stiffness Device within a Highway Bridge Model', *Earthquake Spectra*, vol. 31, no. 4, pp. 2163-94.

Attary, N., Symans, M., Nagarajaiah, S., Reinhorn, A.M., Constantinou, M.C., Taylor, D., Sarlis, A.A. & Pasala, D.T.R. 2013, 'Performance Assessment of a Highway Bridge Structure employing Adaptive Negative Stiffness for Seismic Protection', paper presented to the *Structures Congress 2013*.

Barbu, V. & Sritharan, S. 1998, 'H-infinity control of fluid dynamics', *Proc R Soc Lond Ser A*, vol. 545, pp. 3009-33.

Basu, B., Bursi, O.S., Casciati, F., Casciati, S., Del Grosso, A.E., Domaneschi, M., Faravelli, L., Holnicki-Szulc, J., Irschik, H. & Krommer, M. 2014, 'A European Association for the Control of Structures joint perspective. Recent studies in civil structural control across Europe', *Structural Control and Health Monitoring*, vol. 21, no. 12, pp. 1414-36.

Bazant, Z.P., Cedolin, L. & Tabbara, M.R. 1991, 'New method of analysis for slender columns', *ACI Structural Journal*, vol. 88, no. 4, pp. 391-401.

Braz-César, M. & Barros, R. 2010, 'Semi-active vibration control of buildings using MR dampers: numerical and experimental verification', paper presented to the *14th European Conference on Earthquake Engineering*.

Brennan, M., Kovacic, I., Carrella, A. & Waters, T. 2008, 'On the jump-up and jump-down frequencies of the Duffing oscillator', *Journal of Sound and Vibration*, vol. 318, no. 4-5, pp. 1250-61.

Cain, T., Harvey Jr, P. & Walsh, K. 2020, 'Modeling, Characterizing, and Testing a Simple, Smooth Negative-Stiffness Device to Achieve Apparent Weakening', *Journal of Engineering Mechanics*, vol. 146, no. 10, p. 04020114.

Cao, D., Guo, X. & Hu, W. 2019, 'A novel low-frequency broadband piezoelectric energy harvester combined with a negative stiffness vibration isolator', *Journal of Intelligent Material Systems and Structures*, vol. 30, no. 7, pp. 1105-14.

Carlson, J. 1996, 'Magneto-rheological fluid dampers for semi-active seismic control', pp. 35-40.

Carrella, A., Brennan, M., Kovacic, I. & Waters, T. 2009, 'On the force transmissibility of a vibration isolator with quasi-zero-stiffness', *Journal of Sound and Vibration*, vol. 322, no. 4-5, pp. 707-17.

Carrella, A., Brennan, M. & Waters, T. 2007a, 'Optimization of a quasi-zero-stiffness isolator', *Journal of Mechanical Science and Technology*, vol. 21, no. 6, p. 946.

Carrella, A., Brennan, M.J. & Waters, T.P. 2007b, 'Static analysis of a passive vibration isolator with quasi-zero-stiffness characteristic', *Journal of Sound and Vibration*, vol. 301, no. 3-5, pp. 678-89.

Carrella, A., Brennan, M.J., Waters, T.P. & Shin, K. 2008, 'On the design of a high-static–low-dynamic stiffness isolator using linear mechanical springs and magnets', *Journal of Sound and Vibration*, vol. 315, no. 3, pp. 712-20.

Chen, L., Sun, L. & Nagarajaiah, S. 2015, 'Cable with discrete negative stiffness device and viscous damper: Passive realization and general characteristics', *Smart Struct. Syst*, vol. 15, no. 3, pp. 627-43.

Chen, S., Wang, B., Zhu, S., Tan, X., Hu, J., Lian, X., Wang, L. & Wu, L. 2020, 'A novel composite negative stiffness structure for recoverable trapping energy', *Composites Part A: Applied Science and Manufacturing*, vol. 129, p. 105697.

Cheng, C., Li, S., Wang, Y. & Jiang, X. 2015, '1769. On the analysis of a piecewise nonlinear-linear vibration isolator with high-static-low-dynamic-stiffness under base excitation', *Journal of Vibroengineering*, vol. 17, no. 7.

Cheng, C., Li, S., Wang, Y. & Jiang, X. 2016, 'On the analysis of a high-static-low-dynamic stiffness vibration isolator with time-delayed cubic displacement feedback', *Journal of Sound and Vibration*, vol. 378, no. 2016, pp. 76-91.

Chu, S.Y., Yeh, S.W., Lu, L.Y. & Peng, C.H. 2017, 'A leverage-type stiffness controllable mass damper for vibration mitigation of structures', *Structural Control and Health Monitoring*, vol. 24, no. 4, p. e1896.

Chung, L., Lin, R., Soong, T. & Reinhorn, A. 1989, 'Experimental study of active control for MDOF seismic structures', *Journal of Engineering Mechanics*, vol. 115, no. 8, pp. 1609-27.

Chung, L., Reinhorn, A. & Soong, T. 1988, 'Experiments on active control of seismic structures', *Journal of Engineering Mechanics*, vol. 114, no. 2, pp. 241-56.

Correa, D.M., Klatt, T., Cortes, S., Haberman, M., Kovar, D. & Seepersad, C. 2015, 'Negative stiffness honeycombs for recoverable shock isolation', *Rapid Prototyping Journal*, vol. 21, no. 2, pp. 193-200.

Danh, L.T. & Ahn, K.K. 2014, 'Active pneumatic vibration isolation system using negative stiffness structures for a vehicle seat', *Journal of Sound and Vibration*, vol. 333, no. 5, pp. 1245-68.

Deierlein, G.G., Reinhorn, A.M. & Willford, M.R. 2010, 'Nonlinear structural analysis for seismic design', *NEHRP seismic design technical brief*, vol. 4, pp. 1-36.

Den Hartog, J. & Ormondroyd, J. 1928, 'Theory of the dynamic vibration absorber', *ASME J. Appl. Mech*, vol. 50, no. 7, pp. 11-22.

Deshmukh, S. & Chandiramani, N. 2014, 'LQR control of wind excited benchmark building using variable stiffness tuned mass damper', *Shock and Vibration*, vol. 2014.

Dhadve, P., Rao, A., Rupanvar, A., Deokate, K., Admile, P. & Nemade, P. 2015, 'Assessment of P-delta effect on high rise buildings', *Int. J. Recent Innov. Trends Comput. Commun*, vol. 3, p. 3235.

Dhawale, P.J., Narule, G. & Engineer, M.S. 2016, 'Analysis of P-Delta Effect on High Rise Buildings', *Int J Eng Res General Sci*, vol. 4, no. 4, pp. 90-103p.

Dijkstra, K., Videc, B.P. & Huizinga, J. 1988, 'Mechanical spring having negative spring stiffness useful in an electroacoustic transducer', Series Mechanical spring having negative spring stiffness useful in an electroacoustic transducer Google Patents.

Dominguez, A., Sedaghati, R. & Stiharu, I. 2006, 'Semi-active vibration control of adaptive structures using magnetorheological dampers', *AIAA journal*, vol. 44, no. 7, pp. 1563-71.

Drugan, W. 2007, 'Elastic composite materials having a negative stiffness phase can be stable', *Physical review letters*, vol. 98, no. 5, p. 055502.

Du, H., Sze, K.Y. & Lam, J. 2005, 'Semi-active H∞ control of vehicle suspension with magneto-rheological dampers', *Journal of sound and vibration*, vol. 283, no. 3-5, pp. 981-96.

Dyke, S., Spencer Jr, B., Quast, P., Kaspari Jr, D. & Sain, M. 1996a, 'Implementation of an active mass driver using acceleration feedback control', *Computer-Aided Civil and Infrastructure Engineering*, vol. 11, no. 5, pp. 305-23.

Dyke, S., Spencer Jr, B., Sain, M. & Carlson, J. 1996b, 'Seismic response reduction using magnetorheological dampers', *IFAC Proceedings Volumes*, vol. 29, no. 1, pp. 5530-5.

Easu, D. & Siddharthan, A. 2013, 'Theoretical and Experimental Analysis of a Vibration Isolation System Using Hybrid Magnet', *Procedia Engineering*, vol. 64, pp. 1139-46.

Elgamal, A. & He, L. 2004, 'Vertical earthquake ground motion records: an overview', *Journal of Earthquake Engineering*, vol. 8, no. 05, pp. 663-97.

Elnashai, A., Pilakoutas, K. & Ambraseys, N. 1989, 'The Kalamata earthquake, performance of reinforced concrete buildings', *Civil engineering dynamics*, Thomas Telford Publishing, pp. 193-207.

Feynman, R.P., Leighton, R.B. & Sands, M.L. 1964, *The Feynman lectures on physics. Exercises*, vol. 1, Addison-Wesley Pub. Co.

Fisco, N. & Adeli, H. 2011, 'Smart structures: part I—active and semi-active control', *Scientia Iranica*, vol. 18, no. 3, pp. 275-84.

Fulcher, B.A., Shahan, D.W., Haberman, M.R., Conner Seepersad, C. & Wilson, P.S. 2014, 'Analytical and Experimental Investigation of Buckled Beams as Negative Stiffness Elements for Passive Vibration and Shock Isolation Systems', *Journal of Vibration and Acoustics*, vol. 136, no. 3.

Gao, X. & Chen, Q. 2015, 'Static and dynamic analysis of a high static and low dynamic stiffness vibration isolator utilising the solid and liquid mixture', *Engineering Structures*, vol. 99, pp. 205-13.

Gu, X., Li, J. & Li, Y. 2020, 'Experimental realisation of the real-time controlled smart magnetorheological elastomer seismic isolation system with shake table', *Structural Control and Health Monitoring*, vol. 27, no. 1, p. e2476.

Gu, X., Yu, Y., Li, J., Li, Y. & Alamdari, M.M. 2016, 'Semi-active storey isolation system employing MRE isolator with parameter identification based on NSGA-II with DCD', *Earthq. Struct*, vol. 11, pp. 1101-21.

Gu, X., Yu, Y., Li, Y., Li, J., Askari, M. & Samali, B. 2019, 'Experimental study of semi-active magnetorheological elastomer base isolation system using optimal neuro fuzzy logic control', *Mechanical Systems and Signal Processing*, vol. 119, pp. 380-98.

Ha, Q., Nguyen, M., Li, J. & Kwok, N. 2013, 'Smart structures with current-driven MR dampers: Modeling and second-order sliding mode control', *IEEE/ASME Transactions on Mechatronics*, vol. 18, no. 6, pp. 1702-12.

Ha, S.-H., Choi, S.-B. & You, W.-H. 2008, 'Vibration control and steering performance evaluation of railway vehicle using magnetorheological damper', *Transactions of the Korean Society for Noise and Vibration Engineering*, vol. 18, no. 5, pp. 524-32.

Haghpanah, B., Shirazi, A., Salari-Sharif, L., Izard, A.G. & Valdevit, L. 2017, 'Elastic architected materials with extreme damping capacity', *Extreme Mechanics Letters*, vol. 17, pp. 56-61.

Hao, Z. & Cao, Q. 2015, 'The isolation characteristics of an archetypal dynamical model with stable-quasi-zero-stiffness', *Journal of Sound and Vibration*, vol. 340, pp. 61-79.

Heidari, A.H., Etedali, S. & Javaheri-Tafti, M.R. 2018, 'A hybrid LQR-PID control design for seismic control of buildings equipped with ATMD', *Frontiers of Structural and Civil Engineering*, vol. 12, no. 1, pp. 44-57.

Huang, X., Liu, X., Sun, J., Zhang, Z. & Hua, H. 2014, 'Vibration isolation characteristics of a nonlinear isolator using Euler buckled beam as negative stiffness corrector: A theoretical and experimental study', *Journal of Sound and Vibration*, vol. 333, no. 4, pp. 1132-48.

Ibidapo-Obe, O. 1985, 'Optimal actuators placements for the active control of flexible structures', *Journal of mathematical analysis and applications*, vol. 105, no. 1, pp. 12-25.

Ibrahim, R.A. 2008, 'Recent advances in nonlinear passive vibration isolators', *Journal of Sound and Vibration*, vol. 314, no. 3-5, pp. 371-452.

Iemura, H., Igarashi, A., Pradono, M.H. & Kalantari, A. 2006, 'Negative stiffness friction damping for seismically isolated structures', *Structural Control and Health Monitoring*, vol. 13, no. 2-3, pp. 775-91.

Iemura, H., Kouchiyama, O., Toyooka, A. & Shimoda, I. 2008, 'Development of the friction-based passive negative stiffness damper and its verification tests using shaking table', vol. 12, pp. 01-0219.

Iemura, H. & Pradono, M.H. 2002a, 'Passive and semi-active seismic response control of a cable-stayed bridge', *Structural Control and Health Monitoring*, vol. 9, no. 3, pp. 189-204.

Iemura, H. & Pradono, M.H. 2002b, 'Passive and semi-active seismic response control of a cable-stayed bridge', *Journal of Structural Control*, vol. 9, no. 3, pp. 189-204.

Iemura, H. & Pradono, M.H. 2003, 'Application of pseudo-negative stiffness control to the benchmark cable-stayed bridge', *Structural Control and Health Monitoring*, vol. 10, no. 3-4, pp. 187-203.

Iemura, H. & Pradono, M.H. 2009, 'Advances in the development of pseudo-negative-stiffness dampers for seismic response control', *Structural Control and Health Monitoring*, vol. 16, no. 7-8, pp. 784-99.

Izard, A.G., Alfonso, R.F., McKnight, G. & Valdevit, L. 2017, 'Optimal design of a cellular material encompassing negative stiffness elements for unique combinations of stiffness and elastic hysteresis', *Materials & Design*, vol. 135, pp. 37-50.

Janocha, H. 1999, *Adaptronics and smart structures*, Springer.

Jing, X.J. & Lang, Z.Q. 2009, 'Frequency domain analysis of a dimensionless cubic nonlinear damping system subject to harmonic input', *Nonlinear Dynamics*, vol. 58, no. 3, p. 469.

Johnson, E.A., Ramallo, J.C., Spencer, B. & Sain, M.K. 1998, 'Intelligent base isolation systems', vol. 1, pp. 367-76.

Jolly, M.R., Bender, J.W. & Carlson, J.D. 1998, 'Properties and applications of commercial magnetorheological fluids', vol. 3327, International Society for Optics and Photonics, pp. 262-76.

Kashdan, L., Conner Seepersad, C., Haberman, M. & Wilson, P.S. 2012, 'Design, fabrication, and evaluation of negative stiffness elements using SLS', *Rapid Prototyping Journal*, vol. 18, no. 3, pp. 194-200.

Kelly, J.M. 1999, 'The role of damping in seismic isolation', *Earthquake engineering & structural dynamics*, vol. 28, no. 1, pp. 3-20.

Kelly, J.M., Skinner, R. & Heine, A. 1972, 'Mechanisms of energy absorption in special devices for use in earthquake resistant structures', *Bulletin of NZ Society for Earthquake Engineering*, vol. 5, no. 3, pp. 63-88.

Kim, H.-S., Roschke, P.N., Lin, P.-Y. & Loh, C.-H. 2006, 'Neuro-fuzzy model of hybrid semi-active base isolation system with FPS bearings and an MR damper', *Engineering Structures*, vol. 28, no. 7, pp. 947-58.

Knowles, J. & Corne, D. 1999, 'The pareto archived evolution strategy: A new baseline algorithm for pareto multiobjective optimisation', vol. 1, IEEE, pp. 98-105.

Koh, C.G. & Balendra, T. 1989, 'Seismic response of base isolated buildings including p-Δ effects of isolation bearings', *Earthquake engineering & structural dynamics*, vol. 18, no. 4, pp. 461-73.

Kolovsky, M.Z. 2013, *Nonlinear dynamics of active and passive systems of vibration protection*, Springer Science & Business Media.

Kordonski, W. & Golini, D. 1999, 'Fundamentals of magnetorheological fluid utilization in high precision finishing', *Journal of Intelligent Material Systems and Structures*, vol. 10, no. 9, pp. 683-9.

Kumbhar, B.K., Patil, S.R. & Sawant, S.M. 2015, 'Synthesis and characterization of magneto-rheological (MR) fluids for MR brake application', *Engineering Science and Technology, an International Journal*, vol. 18, no. 3, pp. 432-8.

Kwok, N., Ha, Q., Nguyen, T., Li, J. & Samali, B. 2006, 'A novel hysteretic model for magnetorheological fluid dampers and parameter identification using particle swarm optimization', *Sensors and Actuators A: Physical*, vol. 132, no. 2, pp. 441-51.

Lai, Z., Sun, T. & Nagarajaiah, S. 2019, 'Adjustable template stiffness device and SDOF nonlinear frequency response', *Nonlinear Dynamics*, vol. 96, no. 2, pp. 1559-73.

Lakes, R. 2001, 'Extreme damping in compliant composites with a negative-stiffness phase', *Philosophical Magazine Letters*, vol. 81, no. 2, pp. 95-100.

Lakes, R. & Drugan, W. 2002, 'Dramatically stiffer elastic composite materials due to a negative stiffness phase?', *Journal of the Mechanics and Physics of Solids*, vol. 50, no. 5, pp. 979-1009.

Lakes, R.S., Lee, T., Bersie, A. & Wang, Y. 2001, 'Extreme damping in composite materials with negative-stiffness inclusions', *Nature*, vol. 410, no. 6828, p. 565.

Lan, C.-C., Yang, S.-A. & Wu, Y.-S. 2014, 'Design and experiment of a compact quasi-zero-stiffness isolator capable of a wide range of loads', *Journal of Sound and Vibration*, vol. 333, no. 20, pp. 4843-58.

Lang, Z., Jing, X., Billings, S., Tomlinson, G. & Peng, Z.K. 2009, 'Theoretical study of the effects of nonlinear viscous damping on vibration isolation of sdof systems', *Journal of Sound and Vibration*, vol. 323, no. 1-2, pp. 352-65.

Le, T.D. & Ahn, K.K. 2011, 'A vibration isolation system in low frequency excitation region using negative stiffness structure for vehicle seat', *Journal of Sound and Vibration*, vol. 330, no. 26, pp. 6311-35.

Le, T.D. & Ahn, K.K. 2013a, 'Experimental investigation of a vibration isolation system using negative stiffness structure', *International Journal of Mechanical Sciences*, vol. 70, pp. 99-112.

Le, T.D. & Ahn, K.K. 2013b, 'Fuzzy sliding mode controller of a pneumatic active isolating system using negative stiffness structure', *Journal of Mechanical Science and Technology*, vol. 26, no. 12, pp. 3873-84.

Lee, C.-M., Goverdovskiy, V., Sim, C.-S. & Lee, J.-H. 2016, 'Ride comfort of a high-speed train through the structural upgrade of a bogie suspension', *Journal of Sound and Vibration*, vol. 361, pp. 99-107.

Lee, C.M. & Goverdovskiy, V.N. 2012, 'A multi-stage high-speed railroad vibration isolation system with “negative” stiffness', *Journal of Sound and Vibration*, vol. 331, no. 4, pp. 914-21.

Lee, C.M., Goverdovskiy, V.N. & Temnikov, A.I. 2007, 'Design of springs with “negative” stiffness to improve vehicle driver vibration isolation', *Journal of Sound and Vibration*, vol. 302, no. 4-5, pp. 865-74.

Lewis, J. 2005, 'Earthquake destruction: Corruption on the fault line', *Global Corruption Report*, pp. 23-30.

Li, H.-N., Qu, C., Huo, L. & Nagarajaiah, S. 2016, 'Equivalent bilinear elastic single degree of freedom system of multi-degree of freedom structure with negative stiffness', *Journal of Sound and Vibration*, vol. 365, pp. 1-14.

Li, H., Li, J., Li, Y. & Yu, Y. 2021a, 'Dynamic Property Optimization of a Vibration Isolator with Quasi-Zero Stiffness', *Vibration Engineering for a Sustainable Future*, Springer, pp. 289-95.

Li, H., Li, J., Yu, Y. & Li, Y. 2021b, 'Modified Adaptive Negative Stiffness Device with Variable Negative Stiffness and Geometrically Nonlinear Damping for Seismic Protection of Structures', *International Journal of Structural Stability and Dynamics*, p. 2150107.

Li, H., Li, Y. & Li, J. 2020, 'Negative stiffness devices for vibration isolation applications: A review', *Advances in Structural Engineering*, p. 1369433219900311.

Li, H., Liu, J. & Ou, J. 2011, 'Seismic response control of a cable-stayed bridge using negative stiffness dampers', *Structural Control and Health Monitoring*, vol. 18, no. 3, pp. 265-88.

Li, H., Liu, M. & Ou, J. 2008, 'Negative stiffness characteristics of active and semi-active control systems for stay cables', *Structural Control and Health Monitoring*, vol. 15, no. 2, pp. 120-42.

Li, H., Yu, Y., Li, J. & Li, Y. 2021c, 'Analysis and optimization of a typical quasi-zero stiffness vibration isolator', *Smart Structures and Systems*, vol. 27, no. 3, p. 525.

Li, Q., Zhu, Y., Xu, D., Hu, J., Min, W. & Pang, L. 2013, 'A negative stiffness vibration isolator using magnetic spring combined with rubber membrane', *Journal of Mechanical Science and Technology*, vol. 27, no. 3, pp. 813-24.

Li, Y., Li, J., Li, W. & Du, H. 2014, 'A state-of-the-art review on magnetorheological elastomer devices', *Smart materials and structures*, vol. 23, no. 12, p. 123001.

Liu, X., Huang, X. & Hua, H. 2013, 'On the characteristics of a quasi-zero stiffness isolator using Euler buckled beam as negative stiffness corrector', *Journal of Sound and Vibration*, vol. 332, no. 14, pp. 3359-76.

Luo, B., Zheng, J., Xie, J. & Wu, J. 2008, 'Dynamic crowding distance? A new diversity maintenance strategy for MOEAs', vol. 1, IEEE, pp. 580-5.

Lynch, J., Wang, Y., Swartz, R., Lu, K.-C. & Loh, C. 2008, 'Implementation of a closed-loop structural control system using wireless sensor networks', *Structural Control and Health Monitoring: The Official Journal of the International Association for Structural Control and Monitoring and of the European Association for the Control of Structures*, vol. 15, no. 4, pp. 518-39.

Mahmoodi, P. 1969, 'Structural dampers', *Journal of the Structural division*, vol. 95, no. 8, pp. 1661-72.

Malatkar, P. & Nayfeh, A. 2002, 'Calculation of the jump frequencies in the response of sdof non-linear systems', *Journal of Sound Vibration*, vol. 254, pp. 1005-11.

Mathew, G.M. & Jangid, R. 2018, 'Seismic response control of a building by negative stiffness devices', *Asian Journal of Civil Engineering*, vol. 19, no. 7, pp. 849-66.

Mathew, G.M., Qureshi, A. & Jangid, R. 2015, 'Optimal placement of negative stiffness damping system', vol. 57298, American Society of Mechanical Engineers, p. V001T03A18.

Mizuno, T., Murashita, M., Takasaki, M. & Ishino, Y. 2005, 'Pneumatic three-axis vibration isolation system using negative stiffness', vol. 44, IEEE; 1998, p. 8254.

Mizuno, T., Takasaki, M., Kishita, D. & Hirakawa, K. 2007, 'Vibration isolation system combining zero-power magnetic suspension with springs', *Control Engineering Practice*, vol. 15, no. 2, pp. 187-96.

Mizuno, T., Toumiya, T. & Takasaki, M. 2003, 'Vibration isolation system using negative stiffness', *JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing*, vol. 46, no. 3, pp. 807-12.

Modi, V. 1987, 'Vibration control using nutation dampers', pp. 369-76.

Molyneaux, W. 1957, 'Supports for Vibration Isolation, ARC/CP-322'.

Mori, H., Waters, T., Saotome, N., Nagamine, T. & Sato, Y. 2016, 'The effect of beam inclination on the performance of a passive vibration isolator using buckled beams', vol. 744, IOP Publishing, p. 012229.

Muhammad, A., Yao, X.-l. & Deng, Z.-c. 2006, 'Review of magnetorheological (MR) fluids and its applications in vibration control', *Journal of Marine Science and Application*, vol. 5, no. 3, pp. 17-29.

Naeim, F. 1989, *The seismic design handbook*, Springer Science & Business Media.

Nagarajaiah, S., Narasimhan, S., Agrawal, A. & Tan, P. 2009, 'Benchmark structural control problem for a seismically excited highway bridge—Part III: Phase II Sample controller for the fully base-isolated case', *Structural Control and Health Monitoring: The Official Journal of the International Association for Structural Control and Monitoring and of the European Association for the Control of Structures*, vol. 16, no. 5, pp. 549-63.

Nagarajaiah, S., Pasala, D.T.R., Reinhorn, A., Constantinou, M., Sirilis, A.A. & Taylor, D. 2013, 'Adaptive Negative Stiffness: A New Structural Modification Approach for Seismic Protection', *Advanced Materials Research*, vol. 639-640, pp. 54-66.

Nagarajaiah, S. & Sen, D. 2020, 'Apparent-weakening by adaptive passive stiffness shaping along the height of multistory building using negative stiffness devices and dampers for seismic protection', *Engineering Structures*, vol. 220, p. 110754.

Narasimhan, S., Nagarajaiah, S., Johnson, E.A. & Gavin, H.P. 2006, 'Smart base-isolated benchmark building. Part I: problem definition', *Structural Control and Health Monitoring*, vol. 13, no. 2-3, pp. 573-88.

Niu, F., Meng, L., Wu, W., Sun, J., Zhang, W., Meng, G. & Rao, Z. 2014, 'Design and analysis of a quasi-zero stiffness isolator using a slotted conical disk spring as negative stiffness structure', *Journal of Vibroengineering*, vol. 16, no. 4.

Norton, J., King, A., Bull, D., Chapman, H., McVerry, G., Larkin, T. & Spring, K. 1994, 'Northridge earthquake reconnaissance report', *Bulletin of the New Zealand Society for Earthquake Engineering*, vol. 27, no. 4, pp. 235-344.

Orečný, M., Segľa, Š., Huňady, R. & Ferková, Ž. 2014, 'Application of a magneto-rheological damper and a dynamic absorber for a suspension of a working machine seat', *Procedia Engineering*, vol. 96, pp. 338-44.

Oyelade, A.O. 2019, 'Vibration isolation using a bar and an Euler beam as negative stiffness for vehicle seat comfort', *Advances in Mechanical Engineering*, vol. 11, no. 7, p. 1687814019860983.

Pall, A.S., Marsh, C. & Fazio, P. 1980, 'Friction joints for seismic control of large panel structures', *Journal of Prestressed Concrete Institute*, vol. 25, no. 6, pp. 38-61.

Papazoglou, A. & Elnashai, A. 1996, 'Analytical and field evidence of the damaging effect of vertical earthquake ground motion', *Earthquake Engineering & Structural Dynamics*, vol. 25, no. 10, pp. 1109-37.

Parulekar, Y. & Reddy, G. 2009, 'Passive response control systems for seismic response reduction: A state-of-the-art review', *International Journal of Structural Stability and Dynamics*, vol. 9, no. 01, pp. 151-77.

Pasala, D., Sarlis, A., Nagarajaiah, S., Reinhorn, A., Constantinou, M. & Taylor, D. 2011, 'True adaptive negative stiffness: a new structural modification approach for seismic protection', paper presented to the *Proceedings of 2011 NSF Engineering Research and Innovation Conference*.

Pasala, D., Sarlis, A., Nagarajaiah, S., Reinhorn, A., Constantinou, M. & Taylor, D. 2012, 'Negative stiffness device for seismic response control of multi-story buildings', pp. 83-96.

Pasala, D.T.R., Sarlis, A.A., Nagarajaiah, S., Reinhorn, A.M., Constantinou, M.C. & Taylor, D. 2013, 'Adaptive Negative Stiffness: New Structural Modification Approach for Seismic Protection', *Journal of Structural Engineering*, vol. 139, no. 7, pp. 1112-23.

Pasala, D.T.R., Sarlis, A.A., Reinhorn, A.M., Nagarajaiah, S., Constantinou, M.C. & Taylor, D. 2014, 'Simulated Bilinear-Elastic Behavior in a SDOF Elastic Structure Using Negative Stiffness Device: Experimental and Analytical Study', *Journal of Structural Engineering*, vol. 140, no. 2.

Piersol, A.G. & Harris, C.M. 2017, *Harri's Shock and Vibration Handbook Fifth Edition*, McGraw-Hill.

Platus, D.L. 1992, 'Negative-stiffness-mechanism vibration isolation systems', vol. 1619, International Society for Optics and Photonics, pp. 44-55.

Poynor, J.C. 2001, 'Innovative designs for magneto-rheological dampers', Virginia Tech.

Qu, C., Li, H.-N., Huo, L. & Yi, T.-H. 2017, 'Optimum value of negative stiffness and additional damping in civil structures', *Journal of Structural Engineering*, vol. 143, no. 8, p. 04017068.

Rabinow, J. 1948, 'The magnetic fluid clutch', *Electrical Engineering*, vol. 67, no. 12, pp. 1167-.

Rahman, M., Ong, Z.C., Julai, S., Ferdaus, M.M. & Ahamed, R. 2017, 'A review of advances in magnetorheological dampers: their design optimization and applications', *Journal of Zhejiang University-Science A*, vol. 18, no. 12, pp. 991-1010.

Ramesh, S., Kannan, S. & Baskar, S. 2012, 'Application of modified NSGA-II algorithm to multi-objective reactive power planning', *Applied Soft Computing*, vol. 12, no. 2, pp. 741-53.

Ravaud, R., Lemarquand, G., Babic, S., Lemarquand, V. & Akyel, C. 2010, 'Cylindrical magnets and coils: Fields, forces, and inductances', *IEEE Transactions on Magnetics*, vol. 46, no. 9, pp. 3585-90.

Ravindra, B. & Mallik, A. 1994, 'Performance of non-linear vibration isolators under harmonic excitation', *Journal of Sound and Vibration*, vol. 170, no. 3, pp. 325-37.

Reinhorn, A.M., Viti, S. & Cimellaro, G. 2005, 'Retrofit of structures: Strength reduction with damping enhancement', pp. 16-21.

Ren, C., Yang, D. & Qin, H. 2018, 'Mechanical Performance of Multidirectional Buckling-Based Negative Stiffness Metamaterials: An Analytical and Numerical Study', *Materials*, vol. 11, no. 7, p. 1078.

Robertson, W.S., Kidner, M.R.F., Cazzolato, B.S. & Zander, A.C. 2009, 'Theoretical design parameters for a quasi-zero stiffness magnetic spring for vibration isolation', *Journal of Sound and Vibration*, vol. 326, no. 1-2, pp. 88-103.

Samali, B. 1999, 'System identification of a five storey benchmark model using modal analysis'.

Samali, B., Yang, J. & Liu, S. 1985, 'Active control of seismic-excited buildings', *Journal of Structural Engineering*, vol. 111, no. 10, pp. 2165-80.

Sarlis, A.A., Pasala, D.T.R., Constantinou, M., Reinhorn, A., Nagarajaiah, S. & Taylor, D. 2012, 'Negative stiffness device for seismic protection of structures', *Journal of Structural Engineering*, vol. 139, no. 7, pp. 1124-33.

Sarlis, A.A., Pasala, D.T.R., Constantinou, M.C., Reinhorn, A.M., Nagarajaiah, S. & Taylor, D. 2011, 'Negative stiffness device for seismic protection of structures–an analytical and experimental study', paper presented to the *COMPDYN 2011, Proc. of 3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Corfu, Greece*.

Sarlis, A.A., Pasala, D.T.R., Constantinou, M.C., Reinhorn, A.M., Nagarajaiah, S. & Taylor, D.P. 2013, 'Negative Stiffness Device for Seismic Protection of Structures', *Journal of Structural Engineering*, vol. 139, no. 7, pp. 1124-33.

Sarlis, A.A., Pasala, D.T.R., Constantinou, M.C., Reinhorn, A.M., Nagarajaiah, S. & Taylor, D.P. 2016, 'Negative Stiffness Device for Seismic Protection of Structures: Shake Table Testing of a Seismically Isolated Structure', *Journal of Structural Engineering*, vol. 142, no. 5.

Sato, E. & Fujita, T. 2008, 'STUDY ON APPLICATION OF CONTROLLABLE DAMPERS USING SMART MATERIALS TO FULL-SCALE BASE-ISOLATED BUILDINGS', paper presented to the *The 14th World Conference on Earthquake Engineering*, Beijing, China 2008.

Shan, Y., Wu, W. & Chen, X. 2015, 'Design of a miniaturized pneumatic vibration isolator with high-static-low-dynamic stiffness', *Journal of Vibration and Acoustics*, vol. 137, no. 4, p. 045001.

Shen, Y., Golnaraghi, M. & Heppler, G. 2007, 'Load-leveling suspension system with a magnetorheological damper', *Vehicle System Dynamics*, vol. 45, no. 4, pp. 297-312.

Shi, X., Shi, W. & Xing, L. 2019, 'Performance analysis of vehicle suspension systems with negative stiffness', *SMART STRUCTURES AND SYSTEMS*, vol. 24, no. 1, pp. 141-55.

Shi, X. & Zhu, S. 2015, 'Magnetic negative stiffness dampers', *Smart Materials and Structures*, vol. 24, no. 7, p. 072002.

Shi, X. & Zhu, S. 2017, 'Simulation and optimization of magnetic negative stiffness dampers', *Sensors and Actuators A: Physical*, vol. 259, pp. 14-33.

Shi, X., Zhu, S., Li, J.-Y. & Spencer Jr, B.F. 2016, 'Dynamic behavior of stay cables with passive negative stiffness dampers', *Smart Materials and Structures*, vol. 25, no. 7, p. 075044.

Shi, X., Zhu, S., Ni, Y. & Li, J. 2018, 'Vibration suppression in high-speed trains with negative stiffness dampers', *Smart Structures and Systems*, vol. 21, no. 5, pp. 653 - 68.

Shi, X., Zhu, S. & Spencer Jr, B.F. 2017, 'Experimental study on passive negative stiffness damper for cable vibration mitigation', *Journal of Engineering Mechanics*, vol. 143, no. 9, p. 04017070.

Shin, D.-H. & Kim, H.-J. 2019, 'Influence of the lateral restoring force of isolation system to the seismic performance of isolated buildings in low-to-moderate seismicity regions', *Soil Dynamics and Earthquake Engineering*, vol. 125, p. 105706.

Silva, P.F., Sangtarashha, A. & Burgueño, R. 2012, 'P-Delta effects in limit state design of slender RC bridge columns', paper presented to the *15th WCEE World Conference on Earthquake Engineering, Lisbon, Portugal*, Lisbon, Portugal.

Snowdon, J.C. 1979, 'Vibration isolation: use and characterization', *The Journal of the Acoustical Society of America*, vol. 66, no. 5, pp. 1245-74.

Soong, T.T. & Costantinou, M.C. 2014, *Passive and active structural vibration control in civil engineering*, vol. 345, Springer.

Soong, T.T. & Dargush, G.F. 1997, * *Passive Energy Dissipation Systems in Structural Engineering*, Wiley.

Spencer, J.B.F., Johnson, E.A. & Ramallo, J.C. 2000, '" Smart" Isolation for Seismic Control', *JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing*, vol. 43, no. 3, pp. 704-11.

Spencer Jr, B., Dyke, S., Sain, M. & Carlson, J. 1997, 'Phenomenological model for magnetorheological dampers', *Journal of engineering mechanics*, vol. 123, no. 3, pp. 230-8.

Sun, M., Song, G., Li, Y. & Huang, Z. 2019, 'Effect of negative stiffness mechanism in a vibration isolator with asymmetric and high-static-low-dynamic stiffness', *Mechanical Systems and Signal Processing*, vol. 124, pp. 388-407.

Sun, T., Lai, Z., Nagarajaiah, S. & Li, H.N. 2017a, 'Negative stiffness device for seismic protection of smart base isolated benchmark building', *Structural Control and Health Monitoring*, vol. 24, no. 11.

Sun, T., Lai, Z., Nagarajaiah, S. & Li, H.N. 2017b, 'Negative stiffness device for seismic protection of smart base isolated benchmark building', *Structural Control and Health Monitoring*, vol. 24, no. 11, p. e1968.

Symans, M.D. & Constantinou, M.C. 1999, 'Semi-active control systems for seismic protection of structures: a state-of-the-art review', *Engineering structures*, vol. 21, no. 6, pp. 469-87.

Tang, B. & Brennan, M. 2013, 'A comparison of two nonlinear damping mechanisms in a vibration isolator', *Journal of Sound and Vibration*, vol. 332, no. 3, pp. 510-20.

Tavakoli, H., Gilani, H. & Abdollahzadeh, G. 2012, 'Comparative evaluation of seismic parameters for near-fault and far-fault earthquakes', pp. 24-8.

Thanikachalam, J., Nagaraj, P. & Hikku, G. 2016, 'Effect of graphene as anti-settling agent for magnetorheological fluid', *Journal of Achievements in Materials and Manufacturing Engineering*, vol. 77, no. 2, pp. 49--56.

Tobias, S. 1959, 'Design of small isolator units for the suppression of low-frequency vibration', *Journal of Mechanical Engineering Science*, vol. 1, no. 3, pp. 280-92.

Tokimatsu, K. & Asaka, Y. 1998, 'Effects of liquefaction-induced ground displacements on pile performance in the 1995 Hyogoken-Nambu earthquake', *Soils and Foundations*, vol. 38, pp. 163-77.

Toyooka, A., MOTOYAMA, H., KOUCHIYAMA, O. & IWASAKI, Y. 2015, 'Development of Autonomous Negative Stiffness Damper for Reducing Absolute Responses', *Quarterly Report of RTRI*, vol. 56, no. 4, pp. 284-90.

Trimboli, M.S., Wimmel, R. & Breitbach, E.J. 1994, 'Quasi-active approach to vibration isolation using magnetic springs', vol. 2193, International Society for Optics and Photonics, pp. 73-84.

Utami, D., Mazlan, S.A., Imaduddin, F., Nordin, N.A., Bahiuddin, I., Aziz, A., Aishah, S., Mohamad, N. & Choi, S.-B. 2018, 'Material characterization of a magnetorheological fluid subjected to long-term operation in damper', *Materials*, vol. 11, no. 11, p. 2195.

van Eijk, J. & Dijksman, J.F. 1979, 'Plate spring mechanism with constant negative stiffness', *Mechanism and Machine Theory*, vol. 14, no. 1, pp. 1-9.

Virk, K., Monti, A., Trehard, T., Marsh, M., Hazra, K., Boba, K., Remillat, C.D.L., Scarpa, F. & Farrow, I.R. 2013, 'SILICOMB PEEK Kirigami cellular structures: mechanical response and energy dissipation through zero and negative stiffness', *Smart Materials and Structures*, vol. 22, no. 8.

Viti, S., Cimellaro, G.P. & Reinhorn, A.M. 2006, 'Retrofit of a hospital through strength reduction and enhanced damping', *Smart Structures and Systems*, vol. 2, no. 4, pp. 339-55.

Wang, D. & Liao, W.H. 2011, 'Magnetorheological fluid dampers: a review of parametric modelling', *Smart materials and structures*, vol. 20, no. 2, p. 023001.

Wang, K., Zhou, J., Chang, Y., Ouyang, H., Xu, D. & Yang, Y. 2020, 'A nonlinear ultra-low-frequency vibration isolator with dual quasi-zero-stiffness mechanism', *Nonlinear Dynamics*, vol. 101, no. 2, pp. 755-73.

Wang, M., Sun, F.-f., Yang, J.-q. & Nagarajaiah, S. 2019, 'Seismic protection of SDOF systems with a negative stiffness amplifying damper', *Engineering Structures*, vol. 190, pp. 128-41.

Wang, X., Liu, H., Chen, Y. & Gao, P. 2018, 'Beneficial stiffness design of a high-static-low-dynamic-stiffness vibration isolator based on static and dynamic analysis', *International Journal of Mechanical Sciences*, vol. 142, pp. 235-44.

Wang, Y. & Lakes, R. 2004, 'Extreme stiffness systems due to negative stiffness elements', *American Journal of Physics*, vol. 72, no. 1, pp. 40-50.

Weber, F., Boston, C. & Maślanka, M. 2011, 'An adaptive tuned mass damper based on the emulation of positive and negative stiffness with an MR damper', *Smart Materials and Structures*, vol. 20, no. 1.

Winterflood, J., Blair, D.G. & Slagmolen, B. 2002, 'High performance vibration isolation using springs in Euler column buckling mode', *Physics Letters A*, vol. 300, no. 2-3, pp. 122-30.

Wolpert, D.M., Ghahramani, Z. & Jordan, M.I. 1995, 'An internal model for sensorimotor integration', *Science*, vol. 269, no. 5232, pp. 1880-2.

Wu, T.-H. & Lan, C.-C. 2016, 'A wide-range variable stiffness mechanism for semi-active vibration systems', *Journal of Sound and Vibration*, vol. 363, pp. 18-32.

Wu, W., Chen, X. & Shan, Y. 2014, 'Analysis and experiment of a vibration isolator using a novel magnetic spring with negative stiffness', *Journal of Sound and Vibration*, vol. 333, no. 13, pp. 2958-70.

Wu, Y.M. & Samali, B. 2002, 'Shake table testing of a base isolated model', *Engineering Structures*, vol. 24, no. 9, pp. 1203-15.

Xiang, S. & Songye, Z. 2019, 'A comparative study of vibration isolation performance using negative stiffness and inerter dampers', *Journal of the Franklin Institute*, vol. 356, no. 14, pp. 7922-46.

Xu, D., Yu, Q., Zhou, J. & Bishop, S.R. 2013, 'Theoretical and experimental analyses of a nonlinear magnetic vibration isolator with quasi-zero-stiffness characteristic', *Journal of Sound and Vibration*, vol. 332, no. 14, pp. 3377-89.

Xu, J. & Sun, X. 2015, 'A multi-directional vibration isolator based on Quasi-Zero-Stiffness structure and time-delayed active control', *International Journal of Mechanical Sciences*, vol. 100, pp. 126-35.

Yang, J., Li, Z., Wu, J. & Young, K. 1993, 'A discontinuous control method for civil engineering structures', *Dynamics and control of large structures*, pp. 167-80.

Yang, J., Wu, J., Agrawal, A. & Hsu, S. 1997, 'Sliding mode control with compensator for wind and seismic response control', *Earthquake engineering & structural dynamics*, vol. 26, no. 11, pp. 1137-56.

Ye, K., Ji, J. & Brown, T. 2020, 'A novel integrated quasi-zero stiffness vibration isolator for coupled translational and rotational vibrations', *Mechanical Systems and Signal Processing*, vol. 149, p. 107340.

Yoshioka, H., Ramallo, J. & Spencer Jr, B. 2002, '"Smart" base isolation strategies employing magnetorheological dampers', *Journal of engineering mechanics*, vol. 128, no. 5, pp. 540-51.

Yu, Y., Li, J., Li, Y., Li, S., Li, H. & Wang, W. 2019, 'Comparative investigation of phenomenological modeling for hysteresis responses of magnetorheological elastomer devices', *International journal of molecular sciences*, vol. 20, no. 13, p. 3216.

Yu, Y., Li, Y. & Li, J. 2014, 'Parameter identification of an improved Dahl model for magnetorheological elastomer base isolator based on enhanced genetic algorithm', paper presented to the *23RD AUSTRALASIAN CONFERENCE ON THE MECHANICS OF STRUCTURES AND MATERIALS*.

Yuan, S., Sun, Y., Wang, M., Ding, J., Zhao, J., Huang, Y., Peng, Y., Xie, S., Luo, J. & Pu, H. 2020, 'Tunable negative stiffness spring using Maxwell normal stress', *International Journal of Mechanical Sciences*, p. 106127.

Zhang, F., Xu, M., Shao, S. & Xie, S. 2020, 'A new high-static-low-dynamic stiffness vibration isolator based on magnetic negative stiffness mechanism employing variable reluctance stress', *Journal of Sound and Vibration*, p. 115322.

Zheng, Y., Zhang, X., Luo, Y., Yan, B. & Ma, C. 2016, 'Design and experiment of a high-static–low-dynamic stiffness isolator using a negative stiffness magnetic spring', *Journal of Sound and Vibration*, vol. 360, pp. 31-52.

Zheng, Y., Zhang, X., Luo, Y., Zhang, Y. & Xie, S. 2018, 'Analytical study of a quasi-zero stiffness coupling using a torsion magnetic spring with negative stiffness', *Mechanical Systems and Signal Processing*, vol. 100, pp. 135-51.

Zhou, J., Wang, X., Xu, D. & Bishop, S. 2015, 'Nonlinear dynamic characteristics of a quasi-zero stiffness vibration isolator with cam–roller–spring mechanisms', *Journal of Sound and Vibration*, vol. 346, pp. 53-69.

Zhou, J., Xiao, Q., Xu, D., Ouyang, H. & Li, Y. 2017, 'A novel quasi-zero-stiffness strut and its applications in six-degree-of-freedom vibration isolation platform', *Journal of Sound and Vibration*, vol. 394, pp. 59-74.

Zhou, N. & Liu, K. 2010, 'A tunable high-static–low-dynamic stiffness vibration isolator', *Journal of Sound and Vibration*, vol. 329, no. 9, pp. 1254-73.

Zhou, P. & Li, H. 2016, 'Modeling and control performance of a negative stiffness damper for suppressing stay cable vibrations', *Structural Control and Health Monitoring*, vol. 23, no. 4, pp. 764-82.

Zhou, S., Walker, P., Tian, Y. & Zhang, N. 2020, 'Mode switching analysis and control for a parallel hydraulic hybrid vehicle', *Vehicle System Dynamics*, pp. 1-21.

Zhou, S., Walker, P., Wu, J. & Zhang, N. 2021, 'Power on gear shift control strategy design for a parallel hydraulic hybrid vehicle', *Mechanical Systems and Signal Processing*, vol. 159, p. 107798.

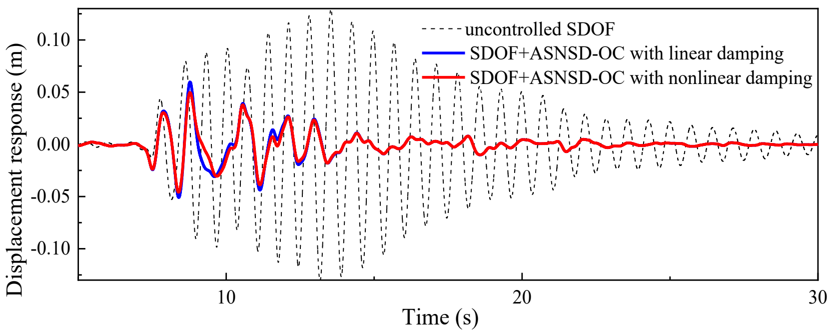
Zhu, X., Jing, X. & Cheng, L. 2012a, 'Magnetorheological fluid dampers: a review on structure design and analysis', *Journal of intelligent material systems and structures*, vol. 23, no. 8, pp. 839-73.

Zhu, Y., Li, Q., Xu, D., Hu, C. & Zhang, M. 2012b, 'Modeling and analysis of a negative stiffness magnetic suspension vibration isolator with experimental investigations', *Rev Sci Instrum*, vol. 83, no. 9, p. 095108.

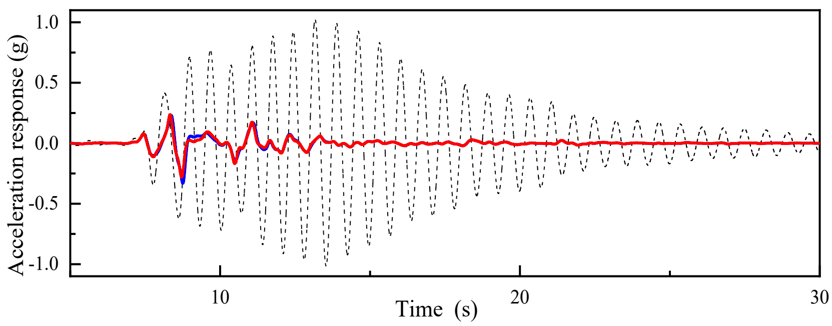
Zinober, A.S. 1994, *Variable structure and Lyapunov control*, vol. 193, Springer.

APPENDIX

A.1 THE TIME HISTORY RESULTS OF ASNSD-OC UNDER DIFFERENT EARTHQUAKES

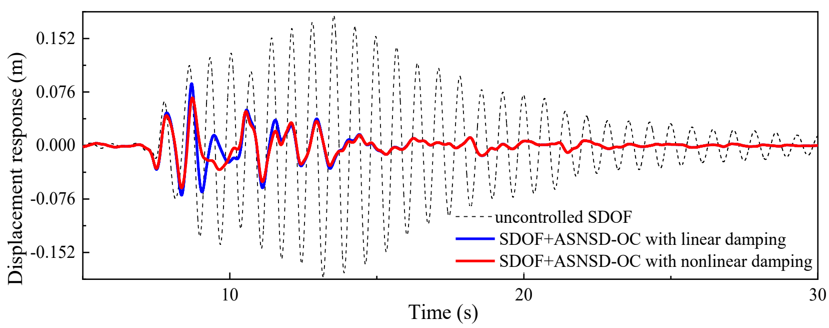


(a)



(b)

Figure A. 1 The time history responses of the SDOF system under the Kobe Earthquake (scale=0.35) (a) displacement response (b) acceleration response



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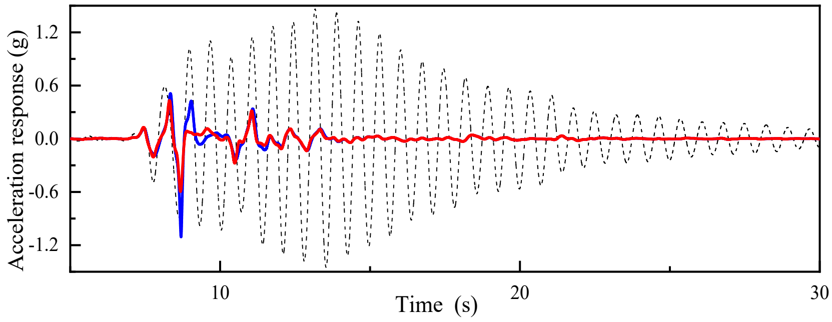
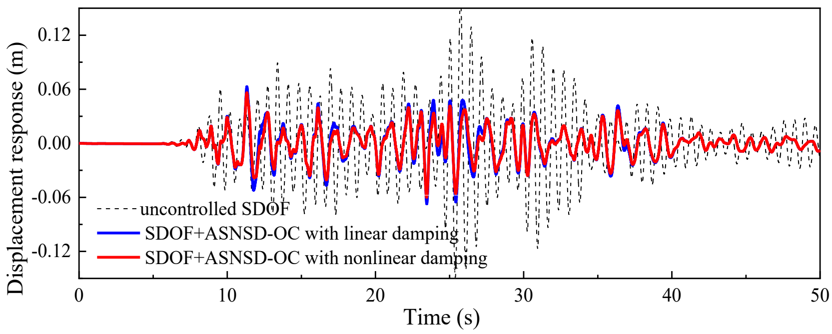


(b)

Figure A. 2 The time history responses of the SDOF system under the Kobe Earthquake (scale=0.5) (a) displacement response (b) acceleration response



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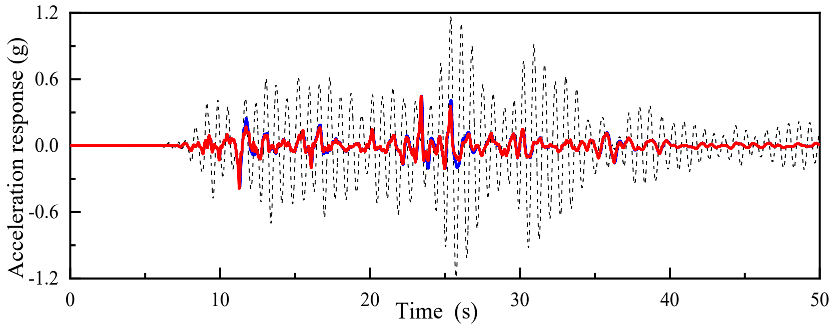
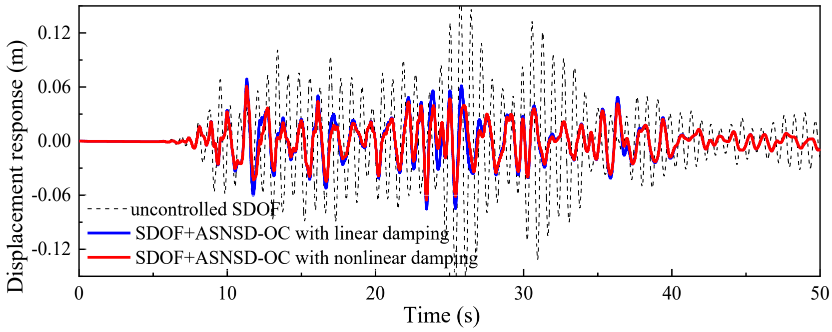
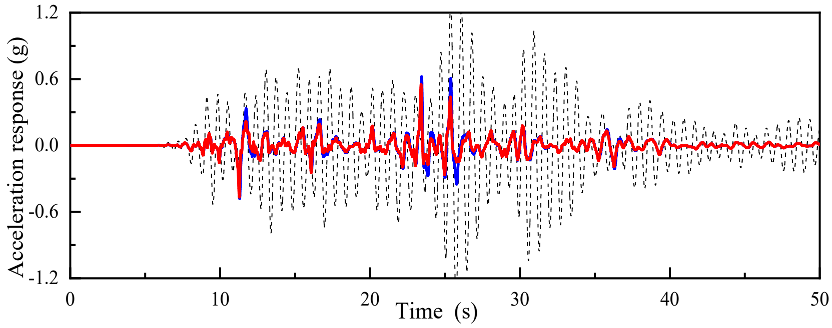


(b)

Figure A. 3 The time history responses of the SDOF system under the Chi-Chi Earthquake (scale=1.1) (a) displacement response (b) acceleration response



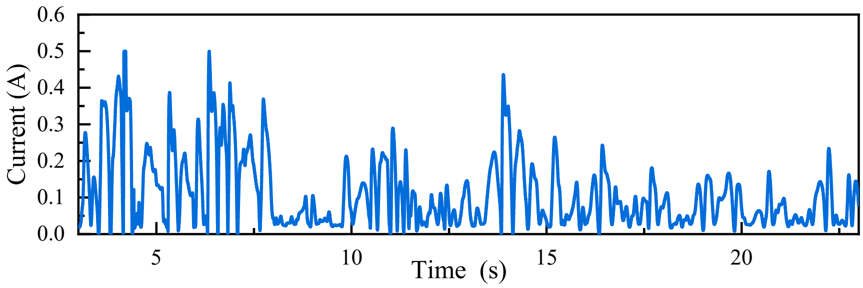
(a)



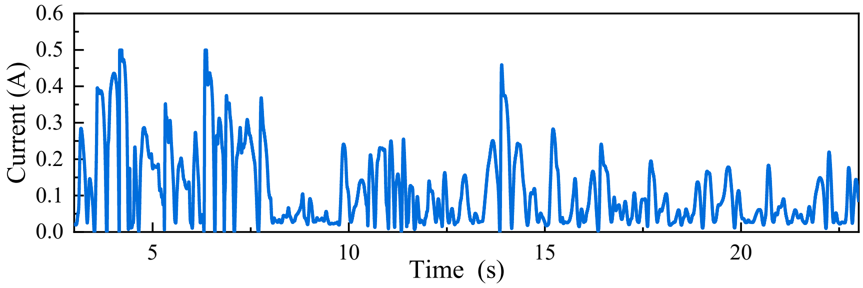
(b)

Figure A. 4 The time history responses of the SDOF system under the Chi-Chi Earthquake (scale=1.3) (a) displacement response (b) acceleration response

A.2 THE CURRENTS AND TIME HISTORY RESULTS OF ASNSD-FC UNDER DIFFERENT EARTHQUAKES



(a)



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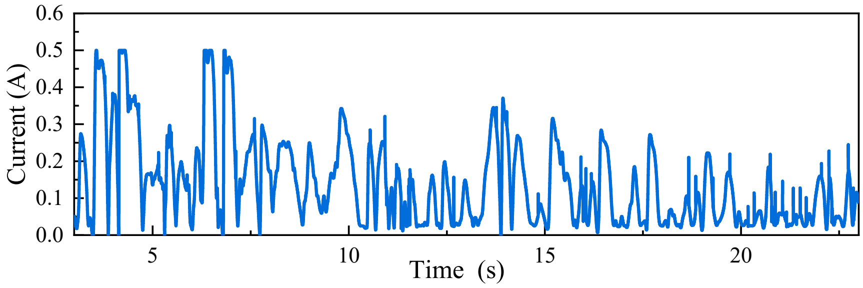
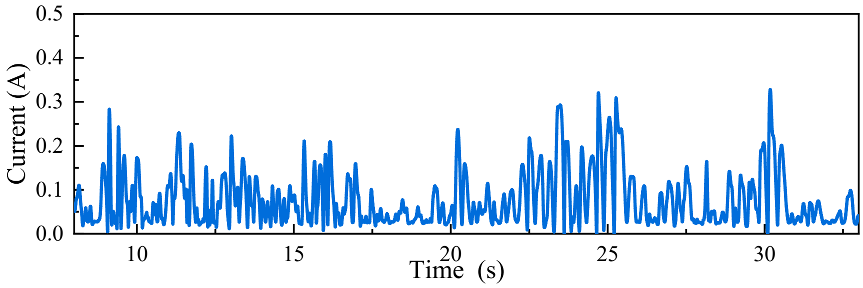
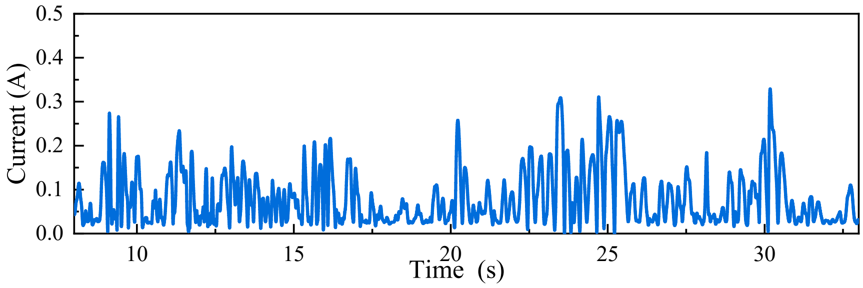


(c)

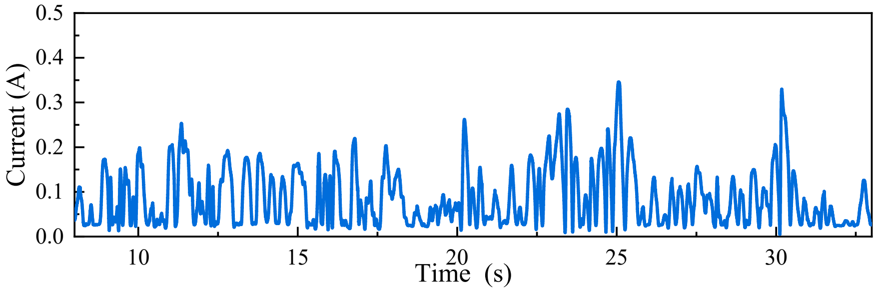
Figure A. 5 The currents of the systems under the El Centro Earthquake (scale=0.25) (a) UAF-LQR (b) UAF-H (c) UAF-SM



(a)

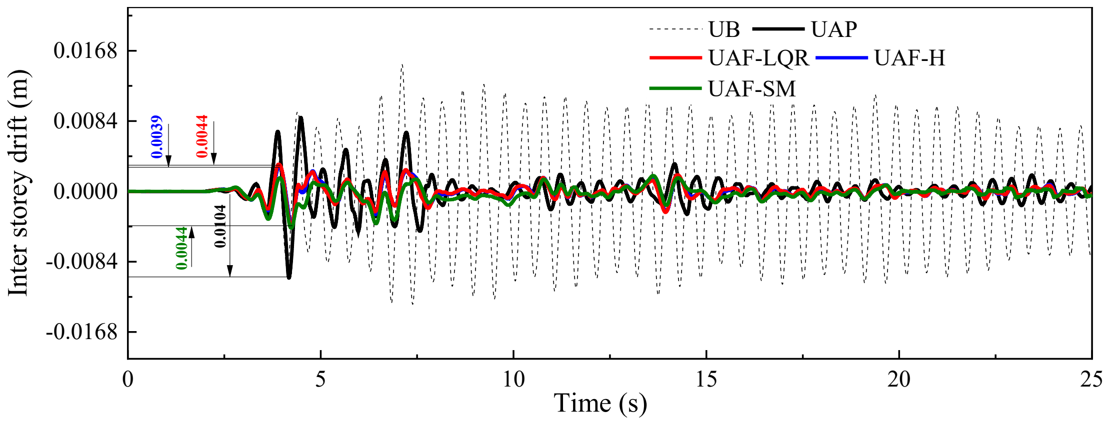


(b)

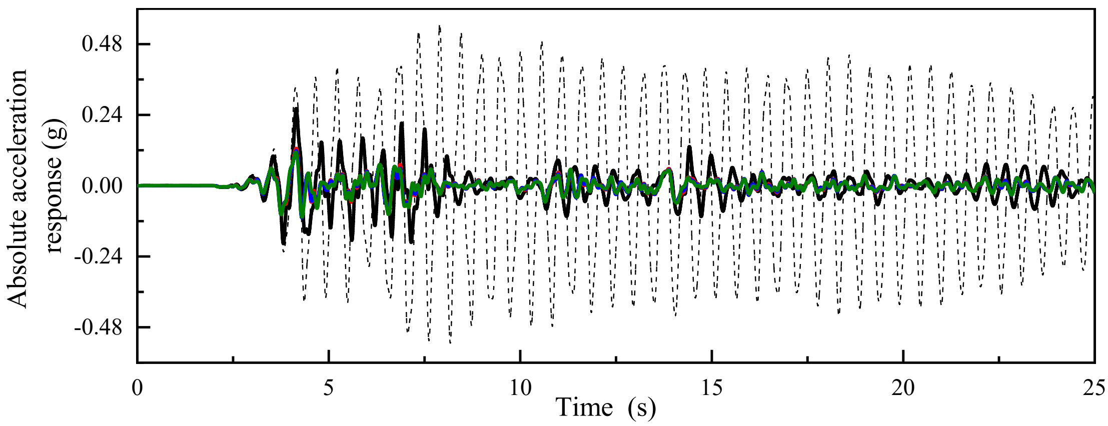


(c)

Figure A. 6 The currents of the systems under the Chi-Chi Earthquake (scale=0.10) (a) UAF-LQR (b) UAF-H (c) UAF-SM

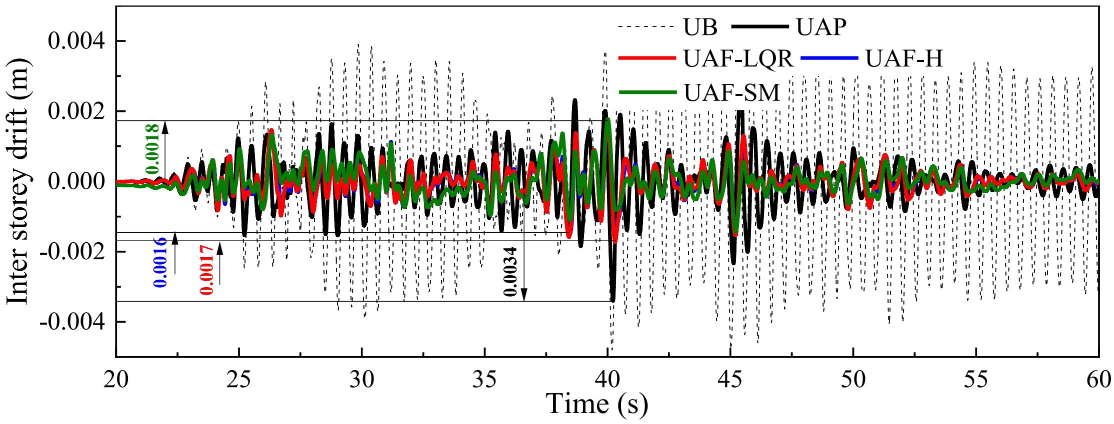


(a)

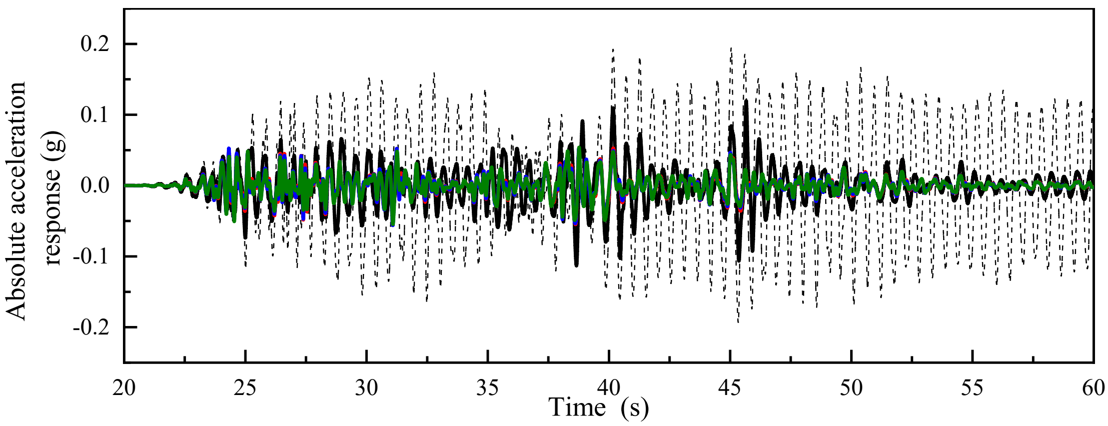


(b)

Figure A. 7 The time history responses of the systems under the El Centro Earthquake (scale=0.25) (a) first-floor inter-storey drift (b) top-floor absolute acceleration response



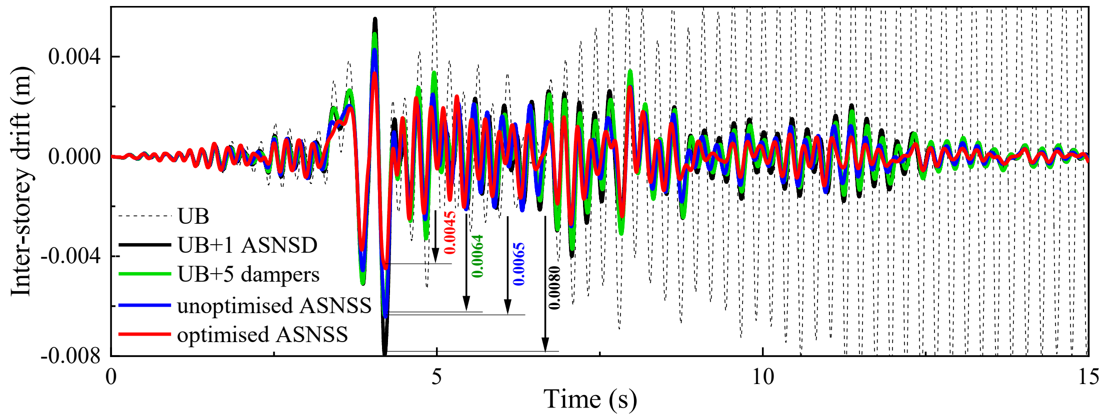
(a)



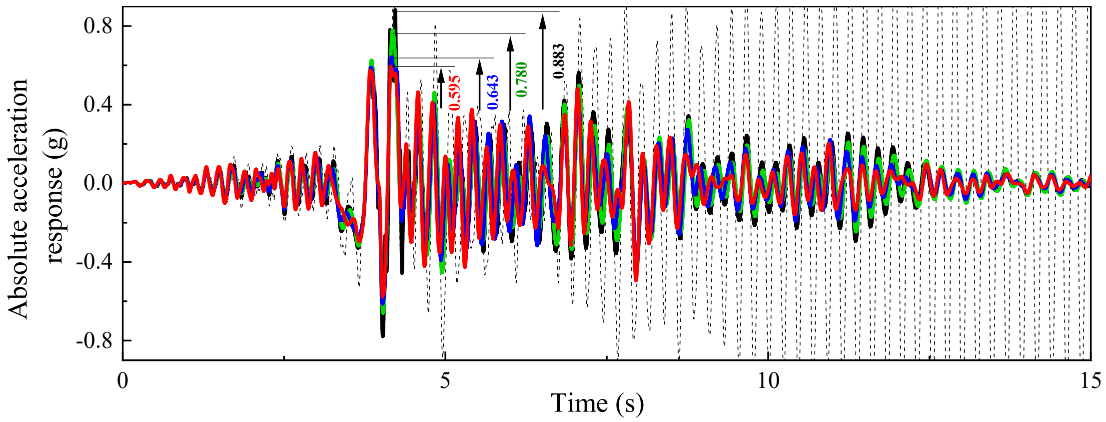
(b)

Figure A. 8 The time history responses of the systems under the Chi-Chi Earthquake (scale=0.10)
(a) first-floor inter-storey drift (b) top-floor absolute acceleration response

A.3 THE TIME HISTORY RESULTS OF ASNSS WITH AND WITHOUT OPTIMISATION UNDER DIFFERENT EARTHQUAKES

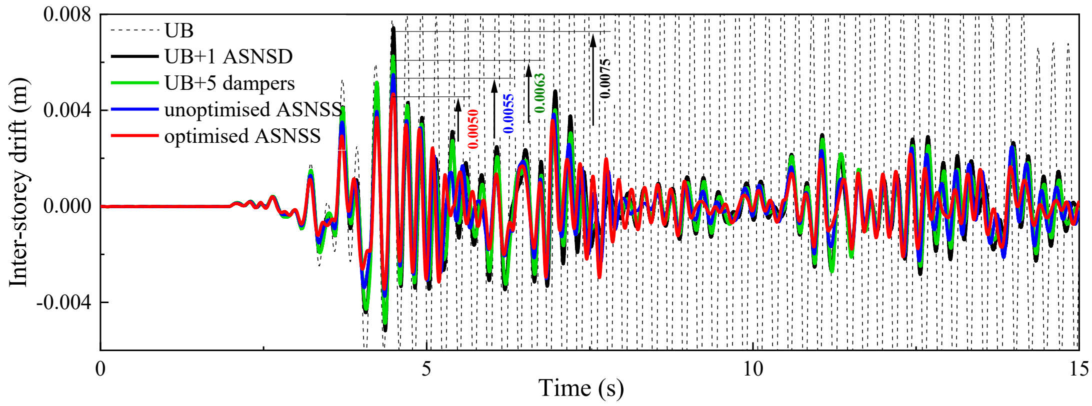


(a)

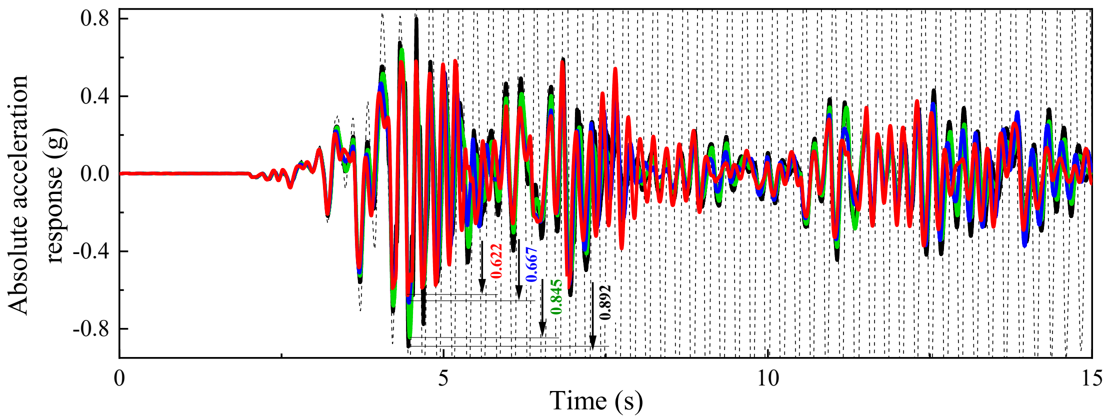


(b)

Figure A. 9 The time history responses of systems under the Northridge Earthquake (a) first-floor inter-storey drift (b) top-floor absolute acceleration response

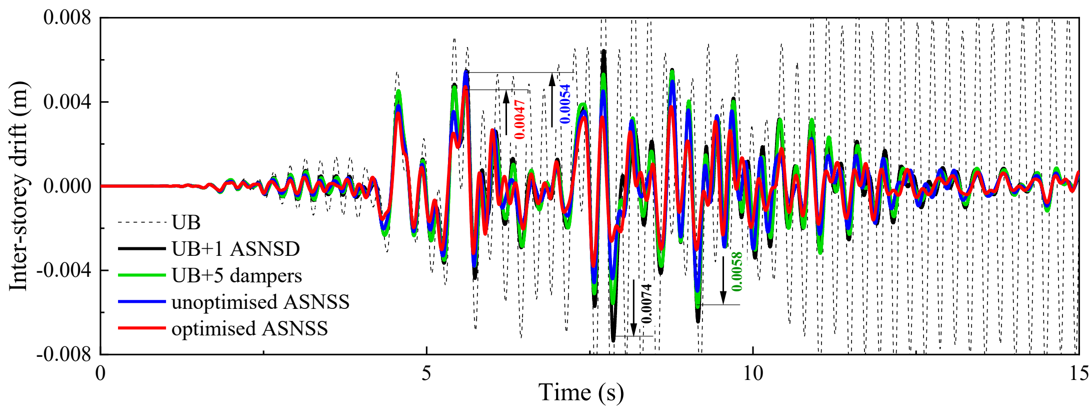


(a)

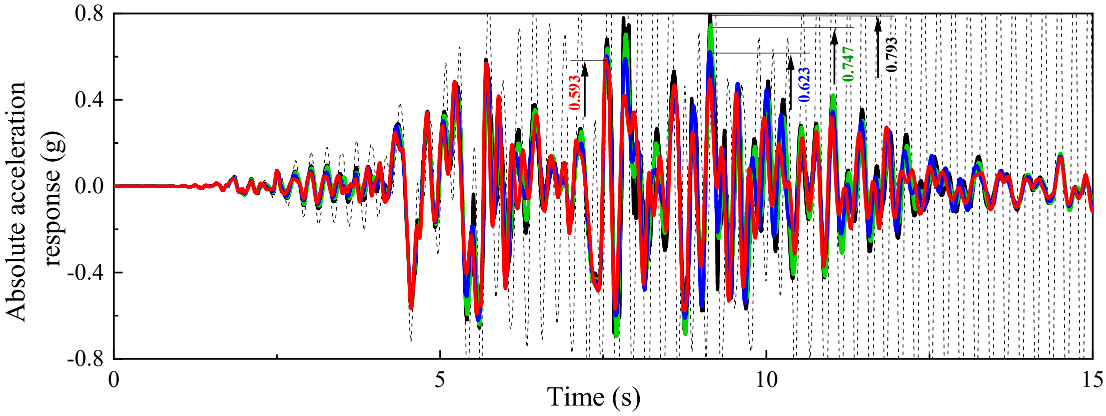


(b)

Figure A. 10 The time history responses of systems under the El Centro Earthquake (a) first-floor inter-storey drift (b) top-floor absolute acceleration response



(a)



(b)

Figure A. 11 The time history responses of systems under the Kobe Earthquake (a) first-floor inter-storey drift (b) top-floor absolute acceleration response

A.4 THE EXPRESSIONS AND EQUATIONS OF THE REPRESENTATIVE QZS VIBRATION ISOLATORS

Table A.1 The expressions and equations of the representative devices

No.	Expressions	Eq. No.
	$\hat{F}_{\mathrm{QZS-1}} = \sqrt{\gamma_{11}^2 - (\gamma_{12}-1)^2} - \hat{y} + 2\alpha_{\mathrm{QZS-1}}(\frac{1-\gamma_{12}}{\sqrt{\gamma_{11}^2 - \hat{y}^2}} + 1)\hat{y}$	Eq.A-1 (a)
1	$\hat{K}_{\mathrm{QZS-1}} = 1 + 2\alpha_{\mathrm{QZS-1}}(\frac{(\gamma_{12}-1)}{\sqrt{\gamma_{11}^2 - \hat{y}^2}} + \frac{\hat{y}^2(\gamma_{12}-1)}{(\gamma_{11}^2 - \hat{y}^2)^{1.5}} - 1)$	Eq.A-1 (b)
	${\hat{F}_{\mathrm{QZS-1}}}^{*} = \alpha_{\mathrm{QZS-1}} \frac{\gamma_{12}-1}{\gamma_{11}^3} \hat{y}^3$	Eq.A-1 (c)
	$\alpha_{\mathrm{QZS-1}} = \frac{\gamma_{11}}{2(1+\gamma_{11}-\gamma_{12})}$	Eq.A-1 (d)
	$\hat{F}_{\mathrm{QZS-2}} = 2\alpha_{\mathrm{QZS-2}}\left(1 - \frac{\pi\tilde{\delta}_2}{\sqrt{(\pi\tilde{\delta}_2)^2 + 4\left(1-\sqrt{{\gamma_2}^2+\hat{y}^2}\right)}}\right)\left(\frac{\sqrt{{\gamma_2}^2+\hat{y}^2}}{2} - \frac{12+(\pi\tilde{\delta}_2)^2}{8}\right)\frac{\hat{y}}{\sqrt{{\gamma_2}^2+\hat{y}^2}} + (\hat{y} + \sqrt{1-{\gamma_2}^2})$	Eq.A-2 (a)
2	$\hat{K}_{\mathrm{QZS-2}} = \frac{2\alpha_{\mathrm{QZS-2}}\pi\tilde{\delta}_2}{((\pi\tilde{\delta}_2)^2 + 4\left(1-\sqrt{{\gamma_2}^2+\hat{y}^2}\right))^{1.5}} \frac{\hat{y}^2(3+\frac{(\pi\tilde{\delta}_2)^2}{4} - \sqrt{{\gamma_2}^2+\hat{y}^2})}{{\gamma_2}^2+\hat{y}^2} + \alpha_{\mathrm{QZS-2}}\left(1 - \frac{\pi\tilde{\delta}_2}{\sqrt{(\pi\tilde{\delta}_2)^2 + 4\left(1-\sqrt{{\gamma_2}^2+\hat{y}^2}\right)}}\right)\left(\frac{({\gamma_2}^2+\hat{y}^2)^{1.5} - {\gamma_2}^2(3+\frac{(\pi\tilde{\delta}_2)^2}{4})}{({\gamma_2}^2+\hat{y}^2)^{1.5}}\right) + 1$	Eq.A-2 (b)
	${\hat{F}_{\mathrm{QZS-2}}}^{*} = (\left(\frac{1}{2{\gamma_2}^2} + \frac{2}{\gamma_2((\pi\tilde{\delta}_2)^2 - 4\gamma_2 + 12)}\right) + \frac{\pi\tilde{\delta}_2}{\gamma_2(((\pi\tilde{\delta}_2)^2 - 4\gamma_2 + 4)^{1.5} - \pi\tilde{\delta}_2((\pi\tilde{\delta}_2)^2 - 4\gamma_2 + 4))})\hat{y}^3$	Eq.A-2 (c)

	$\alpha_{\mathrm{QZS-2}}=\frac{-1}{(1-\frac{\pi\tilde{\delta}_2}{\sqrt{(\pi\tilde{\delta}_2)^2+4-4\gamma_2}})(1-\frac{1}{\gamma_2})(3+\frac{(\pi\tilde{\delta}_2)^2}{4})},\ \tilde{\delta}_2=\frac{q_0}{L}$	Eq.A-2 (d)
3	$\hat{F}_{\mathrm{QZS-3}}=\hat{y}+\sqrt{1-\gamma_3{}^2}+\tilde{\delta}_3-2\frac{\alpha_{\mathrm{QZS}_{31}}\left(\gamma_3+\beta_3-\sqrt{1-\hat{y}^2}\right)+\alpha_{\mathrm{QZS}_{32}}}{(\gamma_3+\beta_3+\tilde{p}_3-\sqrt{1-\hat{y}^2})}\frac{\hat{y}}{\sqrt{1-\hat{y}^2}}$	Eq.A-3 (a)
	$\hat{K}_{\mathrm{QZS-3}}=1-(\frac{2\left(\alpha_{\mathrm{QZS}_{32}}+\alpha_{\mathrm{QZS}_{31}}\left(\gamma_3+\beta_3-\sqrt{1-\hat{y}^2}\right)\right)}{(1-\hat{y}^2)^{1.5}\left(\gamma_3+\beta_3+\tilde{p}_3-\sqrt{1-\hat{y}^2}\right)}+\frac{2\hat{y}^2(\tilde{p}_3\alpha_{\mathrm{QZS}_{31}}-\alpha_{\mathrm{QZS}_{32}})}{(1-\hat{y}^2)(\gamma_3+\beta_3+\tilde{p}_3-\sqrt{1-\hat{y}^2})^2})(\frac{\gamma_3+\beta_3+\tilde{p}_3-1}{2\alpha_{\mathrm{QZS}_{32}}+2\alpha_{\mathrm{QZS}_{31}}(\gamma_3+\beta_3-1)})$	Eq.A-3 (b)
	$\hat{F}_{\mathrm{QZS-3}}{}^*=(\frac{\alpha_{\mathrm{QZS}_{32}}+\alpha_{\mathrm{QZS}_{31}}L(\gamma_3+\beta_3-1)}{L^3(\gamma_3+\beta_3+\tilde{p}_3-1)^2}-\frac{\alpha_{\mathrm{QZS}_{32}}+\alpha_{\mathrm{QZS}_{31}}L(\gamma_3+\beta_3)}{L^3(\gamma_3+\beta_3+\tilde{p}_3-1)})\hat{y}^3$	Eq.A-3 (c)
	$\alpha_{\mathrm{QZS}_{31}}=\frac{P_1}{kL},\ \alpha_{\mathrm{QZS}_{32}}=\frac{P_2}{kL^2},\ \tilde{p}_3=\frac{P_3}{L},\ \beta_3=\frac{D_{\mathrm{QZS}}}{L},\ \gamma_3=\frac{a}{L}$	Eq.A-3 (d)
4	$\hat{F}_{\mathrm{QZS-4}}=\begin{cases}(1-2\alpha_{\mathrm{QZS-4}}(1+\frac{\tilde{\delta}_4-1}{\sqrt{1-\hat{x}^2}}))\hat{y} & ,(\lvert\hat{y}\rvert\le\hat{y}_4)\\ \hat{y} & ,(\lvert\hat{y}\rvert>\hat{y}_4)\end{cases}$	Eq.A-4 (a)
	$\hat{K}_{\mathrm{QZS-4}}=\begin{cases}1-2\alpha_{\mathrm{QZS-4}}(1+\frac{\tilde{\delta}_4-1}{(1-\hat{y}^2)^{1.5}}) & ,(\lvert\hat{y}\rvert\le\hat{y}_4)\\ 1 & ,(\lvert\hat{y}\rvert>\hat{y}_4)\end{cases}$	Eq.A-4 (b)
	$\hat{F}_{\mathrm{QZS-4}}{}^*=\begin{cases}\alpha_{\mathrm{QZS-4}}(1-\tilde{\delta}_4)\hat{y}^3 & ,(\lvert\hat{y}\rvert\le\hat{y}_4)\\ \hat{y} & ,(\lvert\hat{y}\rvert>\hat{y}_4)\end{cases}$	Eq.A-4 (c)
	$\alpha_{\mathrm{QZS-4}}=\frac{1}{2\tilde{\delta}_4}$	Eq.A-4 (d)

Note: The definitionof parameters can be found in the references given in Table 6.1.