Mathematical modeling of the short-term performance of railway track under train induced loading

Piyush Punetha[1][0000-0002-0812-4708], and Sanjay Nimbalkar[1][0000-0002-1538-3396]

[1] School of Civil and Environmental Engineering, FEIT, University of Technology Sydney, NSW-2007, Australia

Abstract. The accurate prediction of the track deformation under train induced repetitive loading is inevitable to assess the efficiency of a railway track. This paper presents an analytical technique to calculate the transient deformations in a railway track subjected to train induced loading. The method considers the track substructure as multi-layered media in which the behavior of an individual track layer is simulated using a mass-spring-dashpot model. Unlike existing approaches to model the track substructure as an equivalent single or double layer, the proposed analytical approach considers all the three layers of the ballasted track (i.e., ballast, capping or subballast and subgrade). The accuracy of the proposed technique is investigated by comparing the predicted values of track settlement with the published data available in the literature. The predicted results are found to be in good agreement with past studies. A parametric study on the substructure behavior revealed that the elastic modulus of track layers significantly influences the track response.

Keywords: Mathematical model, Recoverable Deformation, Railway Track.

1 Introduction

With an increase in demand for higher speed, the stress and deformations in the ballasted tracks have increased substantially [1]. To maintain an adequate level of passenger safety and comfort necessary for high-speed rail operations, the frequency of maintenance activities has increased manifolds. These maintenance operations are usually expensive due to poor understanding of the track behavior, inadequate planning, lack of time for the analysis of track inspection data or unavailability of an adequate database [2]. Thus, an accurate evaluation of the track substructure response is essential to plan the maintenance cycles and optimize track performance. The development of a reliable technique for prediction of track response would lead to significant cost savings in the operation of the railways at elevated train speed.

The field studies, laboratory investigations, numerical and analytical simulations can be used to understand the behavior of a railway track. The field and reduced scale laboratory investigations with proper instrumentation are reliable approaches to understand the track response. They also provide valuable data that can be used for validating the numerical or analytical models. However, these investigations are usually time consuming and expensive. The numerical and analytical techniques are promising alternatives to analyze the track response. Consequently, several researchers have utilized numerical simulations to evaluate the response of the railway tracks subjected to train induced repeated loads [3-12]. However, the numerical simulation of the railway tracks generally requires enormous computational time and resources.

The analytical techniques are relatively faster than the numerical simulations. Therefore, several researchers have developed analytical methodologies in an attempt to simulate the response of the railway track [13-16]. The track substructure in these methods is modeled using equivalent spring or dashpots, as a half-space (either homogenous or layered) or a combination of these two [17, 18]. Choudhury et al. [19] simulated the response of the railway track using a two degree of freedom mass-spring-dashpot model.

In this paper, a methodology is developed to calculate the transient (recoverable) response of the track substructure layers subjected to train induced repeated loads. The present approach employs a mass-spring-dashpot model to capture the track behavior. The method includes the ballast, capping (also known as subballast) and subgrade layers, and also considers the continuity of these layers in the longitudinal direction. The validity of the proposed technique is examined by comparing the model predictions with the published data available in the literature. Subsequently, a parametric investigation is carried out to study the effect of individual layer properties on the track behavior.

2 Development of mathematical model to predict the track response

Fig. 1 represents the schematic diagram of a ballasted railway track structure. It consists of two components: superstructure and substructure. The superstructure comprises of the rails, ties (sleepers), rail pads and fasteners. The substructure is the geotechnical component which comprises of ballast, capping (or subballast) and subgrade layers. To evaluate the transient response, the track substructure is modelled as a three degree of freedom (3DoF) system. Fig. 2 shows the 3DoF mass-spring-dashpot model of the track substructure. Each substructure layer is composed of lumped masses that are supported by visco-elastic elements such as springs and dashpots.

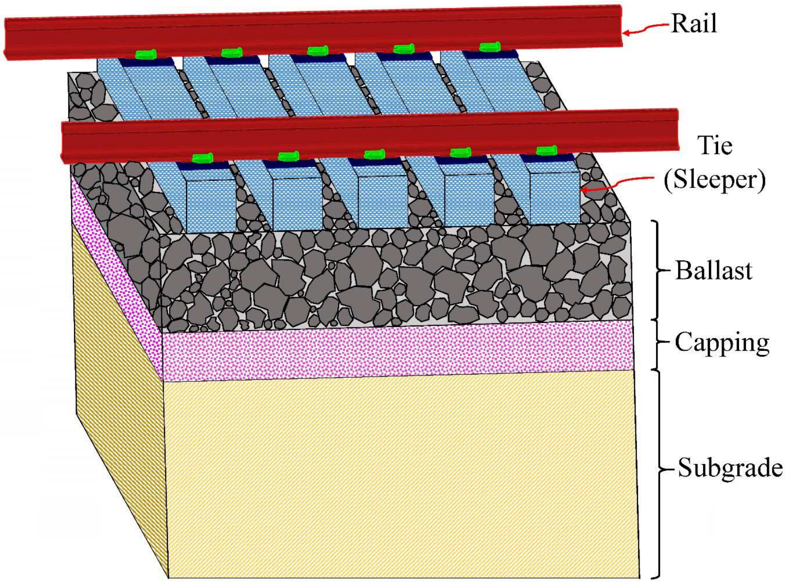

Fig. 1. Structure of a ballasted railway track

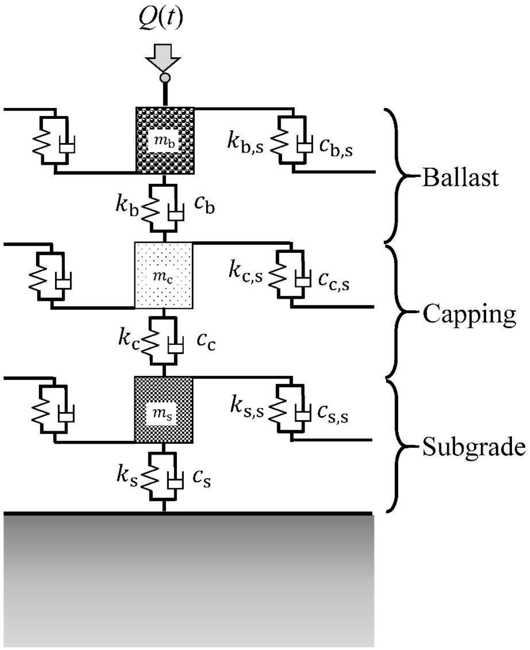

Fig. 2. Mass-spring-dashpot model of the track substructure

The equation of motion for the track below the n^{th} tie point is derived by imposing the dynamic equilibrium condition:

$$\boldsymbol{M}\ddot{\boldsymbol{Y}}_{\mathbf{n}} + \boldsymbol{C}\dot{\boldsymbol{Y}}_{\mathbf{n}} + \boldsymbol{K}\boldsymbol{Y}_{\mathbf{n}} = \boldsymbol{F}_{\mathbf{n}} + \boldsymbol{C}^{s}\left(\dot{\boldsymbol{Y}}_{\mathbf{n+1}} + \dot{\boldsymbol{Y}}_{\mathbf{n-1}}\right) + \boldsymbol{K}^{s}\left(\boldsymbol{Y}_{\mathbf{n+1}} + \boldsymbol{Y}_{\mathbf{n-1}}\right) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{C}^{s}$, $\boldsymbol{K}$ and $\boldsymbol{K}^{s}$ are the mass, damping and stiffness matrices; $\ddot{\boldsymbol{Y}}_{\mathbf{n}}$, $\dot{\boldsymbol{Y}}_{\mathbf{n}}$, $\boldsymbol{Y}_{\mathbf{n}}$ and $\boldsymbol{F}_{\mathbf{n}}$ are the acceleration, velocity, displacement and force vectors at the n^{th} tie, respectively. The Newmark's implicit scheme is used to solve Eq. (1) and evaluate the transient response of the substructure layers in terms of displacement, acceleration and velocity time-histories. The mass, stiffness and damping matrices in Eq. (1) are defined as:

$$\boldsymbol{M} = \begin{bmatrix} m_{\text{s}} & 0 & 0 \\ 0 & m_{\text{c}} & 0 \\ 0 & 0 & m_{\text{b}} \end{bmatrix} \tag{2}$$

$$\boldsymbol{C} = \begin{bmatrix} c_{\text{s}} + c_{\text{c}} + 2c_{\text{s,s}} & -c_{\text{c}} & 0 \\ -c_{\text{c}} & c_{\text{c}} + c_{\text{b}} + 2c_{\text{c,s}} & -c_{\text{b}} \\ 0 & -c_{\text{b}} & c_{\text{b}} + 2c_{\text{b,s}} \end{bmatrix}; \boldsymbol{C}^{s} = \begin{bmatrix} c_{\text{s,s}} & 0 & 0 \\ 0 & c_{\text{c,s}} & 0 \\ 0 & 0 & c_{\text{b,s}} \end{bmatrix} \tag{3}$$

$$\boldsymbol{K} = \begin{bmatrix} k_{\text{s}} + k_{\text{c}} + 2k_{\text{s,s}} & -k_{\text{c}} & 0 \\ -k_{\text{c}} & k_{\text{c}} + k_{\text{b}} + 2k_{\text{c,s}} & -k_{\text{b}} \\ 0 & -k_{\text{b}} & k_{\text{b}} + 2k_{\text{b,s}} \end{bmatrix}; \boldsymbol{K}^{s} = \begin{bmatrix} k_{\text{s,s}} & 0 & 0 \\ 0 & k_{\text{c,s}} & 0 \\ 0 & 0 & k_{\text{b,s}} \end{bmatrix} \tag{4}$$

where m_{s}, m_{c} and m_{b} are the vibrating mass of subgrade, capping and ballast, respectively; c_{s}, c_{c} and c_{b} are the damping coefficients of subgrade, capping and ballast, respectively; k_{s}, k_{c} and k_{b} are the stiffness of subgrade, capping and ballast, respectively; $k_{\text{s,s}}$, $k_{\text{c,s}}$ $k_{\text{b,s}}$ are the shear stiffness of subgrade, capping and ballast, respectively; $c_{\text{s,s}}$, $c_{\text{c,s}}$ and $c_{\text{b,s}}$ are the shear damping coefficients of subgrade, capping and ballast, respectively. The mass and stiffness of the track layers are evaluated by using a pyramidal load distribution model incorporating the overlapping effect along both longitudinal and transverse directions [20, 21]. The damping coefficients for each layer are evaluated using the principle of vibrations as [22]:

$$c_{\text{i}} = \sqrt{\frac{E_{\text{i}}\rho_{\text{i}}}{(1+\nu_{\text{i}})(1-\nu_{\text{i}})}} \tag{5}$$

where $i = b$, c and s for ballast, capping and subgrade, respectively; ρ_{i} and ν_{i} are the density and Poisson's ratio of the i^{th} substructure layer, respectively. The acceleration, velocity, displacement and force vectors are defined as:

$$\ddot{\boldsymbol{Y}}_{\mathbf{n}} = \begin{Bmatrix} \ddot{y}_{\text{s,n}}(t) \\ \ddot{y}_{\text{c,n}}(t) \\ \ddot{y}_{\text{b,n}}(t) \end{Bmatrix}; \dot{\boldsymbol{Y}}_{\mathbf{n}} = \begin{Bmatrix} \dot{y}_{\text{s,n}}(t) \\ \dot{y}_{\text{c,n}}(t) \\ \dot{y}_{\text{b,n}}(t) \end{Bmatrix}; \boldsymbol{Y}_{\mathbf{n}} = \begin{Bmatrix} y_{\text{s,n}}(t) \\ y_{\text{c,n}}(t) \\ y_{\text{b,n}}(t) \end{Bmatrix}; \boldsymbol{F}_{\mathbf{n}} = \begin{Bmatrix} f_{\text{s,n}}(t) \\ f_{\text{c,n}}(t) \\ f_{\text{b,n}}(t) \end{Bmatrix} \tag{6}$$

where the subscripts *s*, *c* and *b* represent the subgrade, capping and ballast, respectively; $\ddot{y}$, $\dot{y}$ and y are the acceleration, velocity and displacement of the track layers, respectively; f is the external load acting on the substructure layer. The external load acting on the capping and subgrade layers at a tie point are taken as zero, whereas, the external load acting on the ballast layer is equal to the rail-seat load. The rail-seat load $[Q(\mathrm{t})]$ is evaluated following the approach given in Doyle [23].

The dynamic effects of the rail-wheel interaction are also incorporated in the analysis using an impact factor (*IF*) [24], which is a multiplier to the static axle (or wheel) load (Q_{a}) [1].

$$IF = 1 + \alpha_1 \left(\frac{V}{D_{\mathrm{w}}}\right)^{\alpha_2} \tag{7}$$

where V and D_{w} are the train speed, and wheel diameter, respectively; α_1 and α_2 are the empirical coefficients.

3 Verification of the proposed model

The validity of the proposed technique is examined by comparing the predicted results with the field investigation results reported by Takemiya and Bian [25] and Priest et al. [26]. Takemiya and Bian [25] presented the dynamic response of a high-speed ballasted rail track section located along the West Coast line in Sweden. The track at the test section comprised of 60 kg/m rails supported by rail pad, ties, ballast and subgrade layers. The track response was expressed in terms of the vertical displacement and acceleration generated during the passage of a Swedish X-2000 high-speed train.

Fig. 3 compares the Fourier amplitude spectrum of the transient vertical ground displacement calculated using the present approach with the data recorded in the field investigations [25]. The values of the parameters used for the simulation are provided in Table 1. The thickness of top (ballast), middle (capping) and bottom (subgrade) layers are taken as 1, 13.5 and 36 m, respectively. It is shown that the response predicted using the proposed method is nearly identical to that observed in the field investigations. It can also be observed that the response is distributed over a frequency range of 0.1–9 Hz. The peaks are observed in the frequency range between 0.1 and 3 Hz (due to the combined effect of train geometry and speed), and at 3.2, 3.66, 4.12, 4.73, 5 and 6.7 Hz (associated with bogie positions).

Priest et al. [26] carried out extensive field investigations in a heavy haul track in South Africa. The track section comprised of 60 kg/m rails supported by ties, ballast (0.3 m thick) and formation layers. They employed geophones to monitor the velocity and the corresponding displacement of individual track layers. Fig. 4 compares the variation of transient displacement with time during the passage of 20-tonne axle load wagons, calculated using the present approach with the data recorded in the field investigations. Table 1 provides the values of the parameters employed in simulations. The model predictions slightly underestimate the magnitude of vertical displacement, however, the trend is similar to the field data.

Table 1. Parameters for evaluating track response

Parameter	Symbol	Unit	Takemiya and Bian [25]	Priest et al. [26]	Parametric study
Wheel diameter	D_w	m	1.016	0.954	1.016
Axle load	Q_a	kN	118–180	196	250
Empirical coefficient	α_1	–	0.0065	0.0065	0.0058
	α_2	–	1	1	0.89
Ballast:					
Elastic modulus	E_b	MPa	19	100	276 (138–551$^{\#}$)
Poisson's ratio	υ_b	–	0.49	0.3	0.3
Density	ρ_b	kg/m^3	1500	1800	1760
Capping:					
Elastic modulus	E_c	MPa	20	220	138 (69–276$^{\#}$)
Poisson's ratio	υ_c	–	0.5	0.3	0.35
Density	ρ_c	kg/m^3	1430	2175	1920
Subgrade:					
Elastic modulus	E_s	MPa	44	27000	14 (14–276$^{\#}$)
Poisson's ratio	υ_s	–	0.5	0.25	0.35
Density	ρ_s	kg/m^3	1475	2300	1920

Note: $^{\#}$Values indicate the range used for the parametric study.

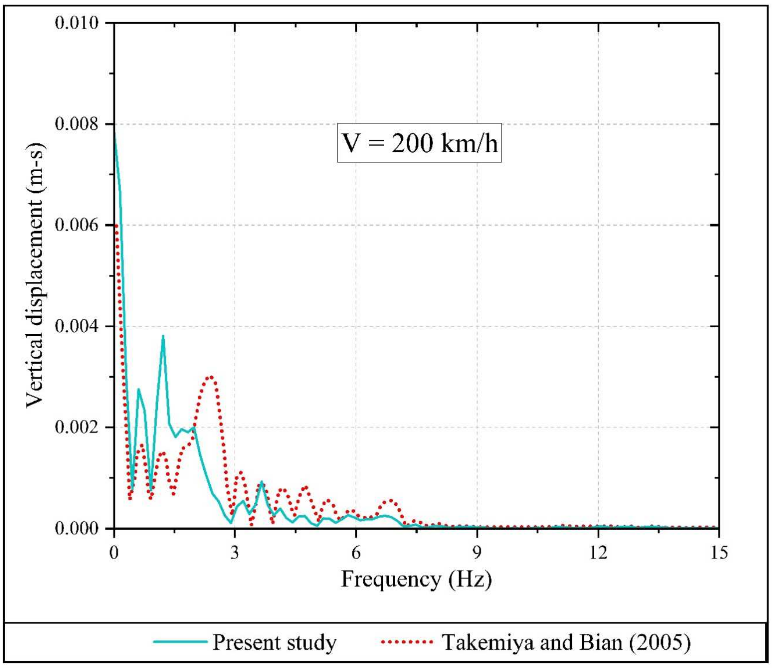

Fig. 3. Comparison of Fourier amplitudes of vertical ground displacement evaluated using the present approach with field data reported by Takemiya and Bian [25].

Thus, the present approach can accurately evaluate the transient or short-term response of the track substructure layers. This method can be employed to optimize track performance, improve the efficiency and consequently, reduce the operating cost of the railways. This approach may be of great interest to the practicing railway engineers owing to the simplicity and less computational power requirements.

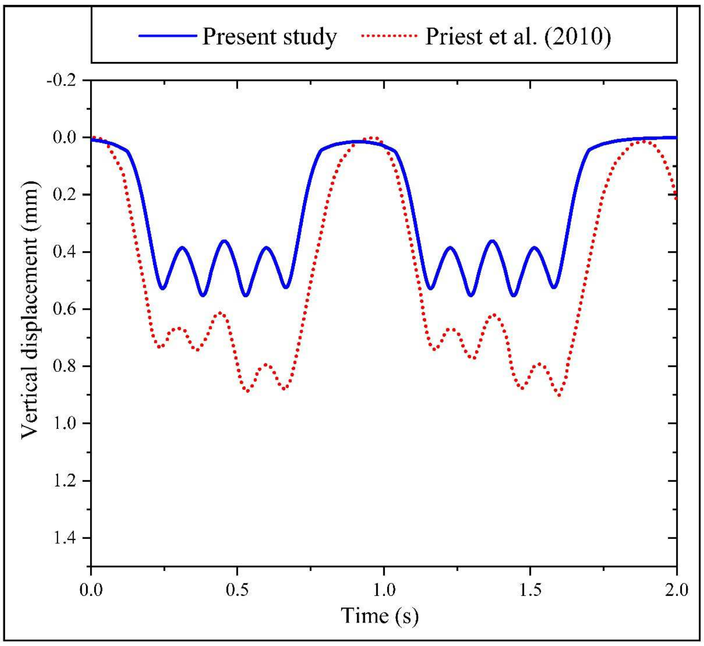

Fig. 4. Comparison of transient displacement evaluated using the present approach with field data reported by Priest et al. [26]

4 Parametric study

The proposed model is used to investigate the influence of elastic modulus on the transient response of track substructure layers. The elastic modulus of ballast (E_b), capping (E_c) and subgrade (E_s) is varied in the range of 138–551 MPa, 69–276 MPa and 14–276 MPa, respectively. A similar range of values has also been used by [27-29] in their parametric investigations. The track response is evaluated in terms of average recoverable vertical strain, which is the ratio of vertical deformation to the initial thickness of the substructure layer. The predictions are carried out for the Thalys high-speed train travelling at a speed of 150 km/h.

Fig. 5 shows the variation of average recoverable strain in the ballast, capping and subgrade layers with elastic moduli E_b, E_c and E_s. The values of parameters used in the prediction are given in Table 1. The horizontal dashed line in the figure represents the strain when the parameters are assigned the nominal values. The downward (blue) and upward (red) arrows indicate a reduction and increment in strain, respectively. The results indicate that the average recoverable strain in the ballast and capping layers decrease by 77.9% and 7.9%, respectively, with a rise in E_b from 138–551 MPa. The subgrade layer showed a marginal effect. This reduction in resilient response is attributed to the increase in stiffness of ballast layer (consequently, an increment in load distribution area) with a rise in the elastic modulus.

The recoverable strain in the ballast layer increases by 17% with a rise in E_c from 69–276 MPa. Whereas, the strain in the capping layer decreases by 73% with an increase in E_c from 69–276 MPa. The capping modulus has an insignificant influence on the subgrade strain. The average recoverable strain in the subgrade layer decreases by 73% with a rise in E_s from 14–276 MPa. However, the recoverable strain in the ballast and capping layers increases by 69% and 70%, respectively with a rise in E_s from 14–276 MPa. This indicates that the presence of a stiff subgrade may increase the deformation in the granular (ballast and capping) layers.

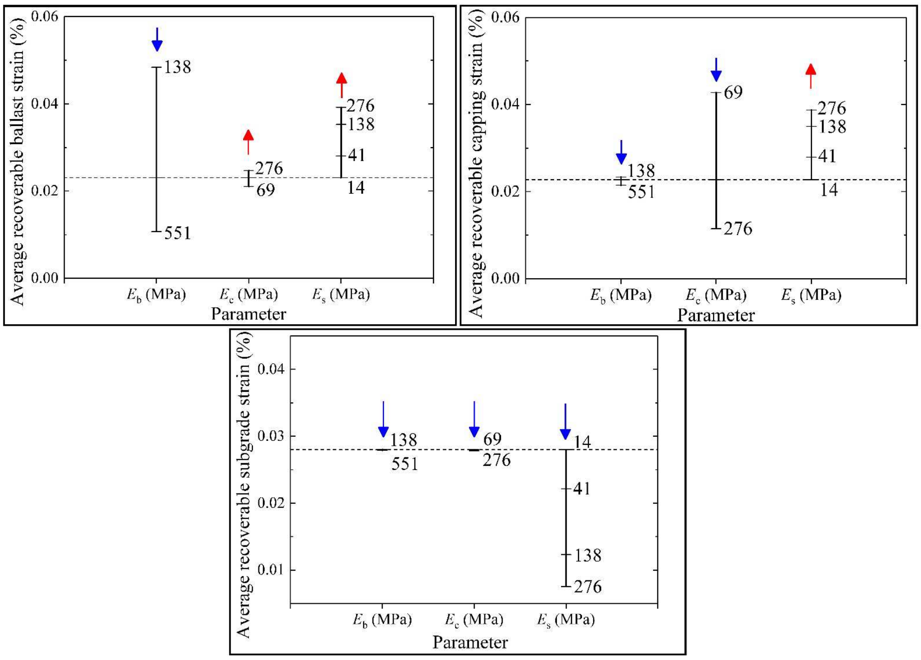

Fig. 5. Influence of elastic modulus on the average transient vertical strain in the track substructure layers.

Therefore, it is clear that the elastic modulus of the substructure layers plays a significant role in the transient response of the railway track. The present approach can be used to evaluate the track response for different train, track and substructure properties. This analysis may provide the value of parameters that are required to achieve optimum track performance.

5 Conclusions

An analytical method is developed to predict the transient response of substructure layers in ballasted railway tracks. The approach evaluates the recoverable response using a three degree of freedom mass-spring-dashpot model. The validity of the technique is examined by comparing the predicted results with the field investigation results reported in the literature. A close agreement between the observed and predicted results demonstrates the accuracy of the proposed approach in evaluating the track substructure response. The parametric study on the track behavior shows that the elastic modulus of track layers significantly influence their response. The presence of stiff subgrade may increase the deformation in the granular substructure layers (ballast and capping). The present approach can be adopted by railway engineers to improve track performance.

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