One-shot resource distillation in quantum resource theories, and W-state encoding for optical QEC

by Madhav Krishnan Vijayan

Thesis submitted in fulfilment of the requirements for
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Faculty of Engineering and Information Technology

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Certificate of Authorship/Originality

I, Madhav Krishnan Vijayan declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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ABSTRACT

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Part I of this thesis studies the problem of optimally converting a single copy of an arbitrary quantum state into maximally resourceful states. Specifically, in the resource theory of coherence, the ideal rate for this conversion is found when assisted by a distant party with whom one shares a quantum state. The optimal distillation of a target pure state in a more general resource theory framework is then studied with minimal assumptions regarding the physical resource. Part II of this thesis studies the problem of error correction in linear quantum optics. An encoding using W-states is introduced which is easily implementable using current technology without feed-forward and it shown that this encoding is robust against independent dephasing errors.

Dedication

To Vijayan and Syama, my parents who always loved and supported me even when I troubled them by never going to school.

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List of Publications

Journal Papers

J-1. **Vijayan, M.K.**, Chitambar, E. and Hsieh, M.H., 2018. One-shot assisted concentration of coherence. Journal of Physics A: Mathematical and Theoretical, 51(41), p.414001.

J-2. **Vijayan, M.K.**, Chitambar, E. and Hsieh, M.H., 2020. Simple bounds for one-shot pure-state distillation in general resource theories. Physical Review A, 102(5), p.052403.

J-3. **Vijayan, M.K.**, Lund, A.P. and Rohde, P.P., 2020. A robust W-state encoding for linear quantum optics. Quantum, 4, p.303.

Conference Papers

C-1. Ramakrishnan, R.K., Ravichandran, A.B., Talabattula, S., **Vijayan, M.K.**, Lund, A.P. and Rohde, P.P., 2020, August. Photonic Quantum Error Correction of Qudits Using W-state Encoding. In Conference on Lasers and Electro-Optics/Pacific Rim (p. P5_23). Optical Society of America.

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Abbreviation

CPTP : Completely Positive Trace Preserving

LOCC: Local Operations and Classical Communication

LOCC-1: Local Operations and Classical Communication with a single round of one-way communication.

MIO : Maximally Incoherent Operations

MCS : Maximally Coherent State

QRT : Quantum Resource Theory

Nomenclature and Notation

Set of real numbers	:	$\mathbb{R}$
Hilbert space of dimension d	:	$\mathcal{H}_d$
The set of density operators acting on the Hilbert space $\mathcal{H}$	:	$\mathcal{D}(\mathcal{H})$
The set of incoherent states	:	$\mathcal{I}$
Largest eigenvalue	:	$\lambda_{max}(\cdot)$
Integer floor	:	$\lfloor\cdot\rfloor$
Projection operator onto the space of the state ρ	:	Π_ρ
ψ majorizes ϕ	:	$\psi \succ \phi$

Definitions

von-Neumann entropy	:	$S(\rho) := -\operatorname{Tr}\rho\log\rho$
Relative entropy	:	$S(\rho\|\sigma) := \operatorname{Tr}\rho\log\rho - \operatorname{Tr}\rho\log\sigma$
Relative entropy of coherence.	:	$C_r(\rho) := \min_{\delta\in\mathcal{I}} S(\rho\|\|\delta)$
Fidelity	:	$F(\rho,\sigma) := \operatorname{Tr}\sqrt{\sqrt{\sigma}\rho\sqrt{\sigma}}$
Completely dephasing map	:	$\Delta(\rho) = \sum_i \|i\rangle\langle i\|\rho\|i\rangle\langle i\|$
Min-entropy	:	$S_{min}(\rho) = -\log_2(\lambda_{max}(\rho))$
Relative Rényi entropy of order 0	:	$S_0(\rho\|\sigma) = -\log_2(\operatorname{Tr}\Pi_\rho\sigma)$
ϵ-close ball	:	$b(\rho,\epsilon) = \{\sigma : \sigma \geq 0, \operatorname{Tr}[\sigma] = 1, F(\rho,\sigma) \geq 1-\epsilon\}$
Sub-normalised ϵ-close pure state ball	:	$b'_*(\rho,\epsilon) = \{\overline{\psi} : \operatorname{Tr}(\overline{\psi}) \leq 1, F(\overline{\psi},\rho) \geq 1-\epsilon\}$
ϵ-close pure state ball	:	$b_*(\rho,\epsilon) = \{\overline{\psi} : \operatorname{Tr}(\overline{\psi}) = 1, \overline{\psi} \in b'_*(\rho,\epsilon)\}$