

One-shot resource distillation in quantum resource theories, and W-state encoding for optical QEC

by Madhav Krishnan Vijayan

Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy

under the supervision of Dr. Min-Hsiu Hsieh and Dr. Peter Rohde

University of Technology Sydney
Faculty of Engineering and Information Technology

November 2020

Certificate of Authorship/Originality

I, Madhav Krishnan Vijayan declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

Signature: **Production Note:**
Signature removed prior to publication.

Date: 23/11/2020

© Copyright 2020 Madhav Krishnan Vijayan

ABSTRACT

One-shot resource distillation in quantum resource theories, and W-state encoding for optical QEC

by

Madhav Krishnan Vijayan

Part I of this thesis studies the problem of optimally converting a single copy of an arbitrary quantum state into maximally resourceful states. Specifically, in the resource theory of coherence, the ideal rate for this conversion is found when assisted by a distant party with whom one shares a quantum state. The optimal distillation of a target pure state in a more general resource theory framework is then studied with minimal assumptions regarding the physical resource. Part II of this thesis studies the problem of error correction in linear quantum optics. An encoding using W-states is introduced which is easily implementable using current technology without feed-forward and it is shown that this encoding is robust against independent dephasing errors.

Dedication

To Vijayan and Syama, my parents who always loved and supported me even when I troubled them by never going to school.

Acknowledgements

There are many people I wish to thank without whose help this thesis would not have been written. First, I would like to thank my wife Kiran who's love and support got me through this journey. I thank my Teacher who had a deep impact on my life and well-being. I thank my brother who has always encouraged my interest in science.

I thank my supervisors Dr. Min-Hsiu Hsieh and Dr. Peter Rohde for being the most supportive mentors I could have hoped for.

Min-Hsiu, I am deeply grateful for the opportunity you gave me to come to Sydney and work with you. I could always depend on your patience, encouragement, pragmatism and advice to navigate this PhD.

Peter, working with you has been a pleasure and has helped me so much professionally and personally. I will always be grateful for the joy and positivity that you have brought to our work.

I wish to thank my co-authors and their substantial contribution in uplifting the quality of the work in this thesis. Thanks, Dr. Eric Chitambar and Dr. Austin Lund.

Sydney, Australia, 2020.

List of Publications

Journal Papers

- J-1. **Vijayan, M.K.**, Chitambar, E. and Hsieh, M.H., 2018. One-shot assisted concentration of coherence. *Journal of Physics A: Mathematical and Theoretical*, 51(41), p.414001.
- J-2. **Vijayan, M.K.**, Chitambar, E. and Hsieh, M.H., 2020. Simple bounds for one-shot pure-state distillation in general resource theories. *Physical Review A*, 102(5), p.052403.
- J-3. **Vijayan, M.K.**, Lund, A.P. and Rohde, P.P., 2020. A robust W-state encoding for linear quantum optics. *Quantum*, 4, p.303.

Conference Papers

- C-1. Ramakrishnan, R.K., Ravichandran, A.B., Talabattula, S., **Vijayan, M.K.**, Lund, A.P. and Rohde, P.P., 2020, August. Photonic Quantum Error Correction of Qudits Using W-state Encoding. In *Conference on Lasers and Electro-Optics/Pacific Rim* (p. P5_23). Optical Society of America.

Contents

| | |
|--------------------------------------------------|----------|
| Certificate | ii |
| Abstract | iii |
| Dedication | iv |
| Acknowledgments | v |
| List of Publications | vi |
| List of Figures | xi |
| Abbreviation | xii |
| Notation | xiii |
| 1 Introduction | 1 |
| 1.1 Research Aims | 1 |
| 1.2 Scope and significance | 2 |
| 1.3 Thesis Organisation | 2 |
| I Resource Theories | 4 |
| 2 Background and Literature Survey | 5 |
| 2.1 Introduction | 5 |
| 2.2 Resource theory of Coherence | 5 |
| 2.2.1 Classes of incoherent operations | 6 |
| 2.2.2 Coherence quantifiers | 8 |
| 2.2.3 Operational tasks | 11 |

| | | |
|-----------|------------------------------------------------------------------|-----------|
| 2.3 | General resource theories | 13 |
| 3 | One-shot Assisted Coherence distillation | 16 |
| 3.1 | Introduction | 16 |
| 3.2 | Definitions | 18 |
| 3.3 | Pure state concentration | 22 |
| 3.4 | Coherence concentration for an ensemble of pure states | 25 |
| 3.5 | Asymptotic coherence of assistance | 28 |
| 3.6 | Conclusions | 36 |
| 4 | General One-Shot Distillation | 37 |
| 4.1 | Introduction | 37 |
| 4.2 | Definitions | 38 |
| 4.3 | General Distillation Bounds | 42 |
| 4.4 | Examples | 47 |
| 4.5 | Conclusions | 50 |
| 5 | Discussion | 52 |
| 5.1 | Main Results | 52 |
| 5.2 | Open problems | 52 |
| 5.3 | Future Directions | 53 |
| II | Optical Quantum Error Correction | 54 |
| 6 | Introduction | 55 |
| 6.1 | Overview | 55 |
| 6.2 | Preliminaries | 56 |

| | | |
|----------|-------------------------------------------------------------|-----------|
| 6.3 | Background and Motivation | 57 |
| 7 | W-state encoding for linear optics | 60 |
| 7.1 | Introduction | 60 |
| 7.1.1 | GHZ states | 60 |
| 7.1.2 | Cluster states | 61 |
| 7.1.3 | W-states | 61 |
| 7.2 | Protocol | 63 |
| 7.3 | Error heralding | 68 |
| 7.3.1 | Heralding probability | 69 |
| 7.3.2 | Heralded fidelity | 72 |
| 7.3.3 | Probability and fidelity plots | 76 |
| 7.4 | Single-qubit unitary operations | 76 |
| 7.5 | Discussion | 78 |
| 7.5.1 | Pros and cons | 78 |
| 7.5.2 | Robustness against different noise models | 79 |
| 7.6 | Conclusion | 79 |
| 8 | Discussion | 81 |
| 8.1 | Main Results | 81 |
| 8.2 | Comparison with other schemes | 81 |
| 8.3 | Open problems | 82 |
| 8.4 | Future Directions | 83 |
| A | Resource theory of coherence | 84 |
| A.1 | \mathcal{M} satisfies generalized Stein's lemma | 84 |

| | |
|--------------------------------------------------------------------------------|-----------|
| A.2 Upper bound for trace distance | 85 |
| B General resource theory | 87 |
| B.1 Converse for of one-shot concentration of entanglement and coherence . . . | 87 |
| B.1.1 Entanglement | 87 |
| B.1.2 Coherence | 89 |

List of Figures

| | | |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 3.1 | The general task considered in this chapter involves assisted coherence concentration. In phase (i), Alice and Bob share some entangled state $ \psi\rangle^{AB}$. In phase (ii), Alice makes a measurement on her system and communicates the measurement result to Bob. Bob then performs local incoherent operations to maximize the coherence $ \Phi_M\rangle$ of his system. . . . | 17 |
| 7.1 | Photonic W-state error-correction and -detection protocol. Encoding of a single dual-rail photonic qubit proceeds via a Quantum Fourier Transform ($Q\hat{F}T$), which maps the 2-mode encoding across a larger number of redundant modes. The independent dephasing noise channel is denoted by \mathcal{E} . Decoding proceeds via the inverse Quantum Fourier Transform ($Q\hat{F}T^\dagger$). Post-selection upon detecting the single photon within the desired 2 output modes defining the single qubit, projects the logical state into one with reduced noise action. | 63 |
| 7.2 | Analytic heralded error-correction results for the <i>absence heralding</i> technique. Results for <i>presence heralding</i> are given by simple transformations of these results (shift by η for P_H , and scale by $(1 - \eta)$ for F_H). (left) Heralding probability and (right) post-selected fidelity, parameterized in terms of loss-rate, and dephasing in terms of (top) phase-variance δ , (bottom) T_2 -time for $t_p = 1$ | 77 |

Abbreviation

CPTP : Completely Positive Trace Preserving

LOCC: Local Operations and Classical Communication

LOCC-1: Local Operations and Classical Communication with a single round of one-way communication.

MIO : Maximally Incoherent Operations

MCS : Maximally Coherent State

QRT : Quantum Resource Theory

Nomenclature and Notation

| | |
|------------------------------------------------------------------------|------------------------------|
| Set of real numbers | : \mathbb{R} |
| Hilbert space of dimension d | : \mathcal{H}_d |
| The set of density operators acting on the Hilbert space \mathcal{H} | : $\mathcal{D}(\mathcal{H})$ |
| The set of incoherent states | : \mathcal{I} |
| Largest eigenvalue | : $\lambda_{max}(\cdot)$ |
| Integer floor | : $\lfloor \cdot \rfloor$ |
| Projection operator onto the space of the state ρ | : Π_ρ |
| ψ majorizes ϕ | : $\psi \succ \phi$ |

Definitions

| | |
|--------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|
| von-Neumann entropy | : $S(\rho) := -\text{Tr} \rho \log \rho$ |
| Relative entropy | : $S(\rho \sigma) := \text{Tr} \rho \log \rho - \text{Tr} \rho \log \sigma$ |
| Relative entropy of coherence. | : $C_r(\rho) := \min_{\delta \in \mathcal{I}} S(\rho \delta)$ |
| Fidelity | : $F(\rho, \sigma) := \text{Tr} \sqrt{\sqrt{\sigma} \rho \sqrt{\sigma}}$ |
| Completely dephasing map | : $\Delta(\rho) = \sum_i i\rangle \langle i \rho i\rangle \langle i $ |
| Min-entropy | : $S_{min}(\rho) = -\log_2(\lambda_{max}(\rho))$ |
| Relative Rényi entropy of order 0 | : $S_0(\rho \sigma) = -\log_2(\text{Tr} \Pi_\rho \sigma)$ |
| ϵ -close ball | : $b(\rho, \epsilon) = \{\sigma : \sigma \geq 0, \text{Tr}[\sigma] = 1, F(\rho, \sigma) \geq 1 - \epsilon\}$ |
| Sub-normalised ϵ -close pure state ball | : $b'_*(\rho, \epsilon) = \{\bar{\psi} : \text{Tr}(\bar{\psi}) \leq 1, F(\bar{\psi}, \rho) \geq 1 - \epsilon\}$ |
| ϵ -close pure state ball | : $b_*(\rho, \epsilon) = \{\bar{\psi} : \text{Tr}(\bar{\psi}) = 1, \bar{\psi} \in b'_*(\rho, \epsilon)\}$ |