

UNIVERSITY OF TECHNOLOGY SYDNEY  
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**Research on 2D general feature based SLAM  
algorithm for mobile robot**

by

**Jiaheng Zhao**

A THESIS SUBMITTED  
IN PARTIAL FULFILLMENT OF THE  
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## ABSTRACT

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Simultaneous Localization and Mapping (SLAM) is a fundamental research problem for autonomous robot navigation and map construction. This thesis studied the problem of improving the performance of localization and mapping for mobile robots, including pre-fitting features with ellipse representation, representing features with implicit functions, parameterization in Fourier series, and submap joining.

The conventional planar scan matching approach cannot cope well with the open environment as lacking of sufficient edges and corners. A SLAM algorithm with pre-fitted conic features via 2D lidar is presented, which is named as Pre-fit SLAM and can be adapted to an open environment nicely. The novelty of this work includes threefold: (1) defining a conic feature based parameterization approach; (2) developing a SLAM method to utilize feature's conic geometric information and odometry information since open environments are short of regular linear geometric features. Synthetic and practical experiments demonstrated that the proposed SLAM algorithm can get accurate and convincing results for the open environment and the map in our representation can express accurately the environment situation.

In order to avoid information loss during pre-fitting progress and to enlarge the scope of feature representation, a post-count framework for 2D lidar SLAM with implicit functions on general features is studied. Since 2D laser data reflect the distances from the robot to the boundary of objects in the environment, it is natural to use the boundary of the general objects/features within the 2D environment to describe features. Implicit functions can be used to represent almost arbitrary shapes from simple (e.g. circle, ellipse, line) to complex (e.g. a cross-section of a bunny

model), thus it is worth studying implicit-expressed feature in 2D laser SLAM. The main contributions are the specific problem formulation and algorithm framework for 2D laser SLAM with general features represented by implicit functions (named as Implicit-SLAM). Furthermore, ellipses and lines are used as examples to compare the proposed SLAM method with the traditional pre-fit method. Simulation and experimental results show that Implicit-SLAM has a better performance compared with Pre-fit SLAM and other methods, demonstrating the potential of this new SLAM formulation and method.

A 2D laser SLAM approach with Fourier series based feature parameterization (called Fourier-SLAM) and submap joining is studied to improve the efficiency of convergence and the accuracy of method using implicit functions. The Fourier series are introduced to parameterize irregular closed shape features and the SLAM problem with Fourier series feature parameterization is formulated. A submap joining process is also derived in order to reduce the high dependence on precise initial guess and the computing time. Fourier-SLAM has been evaluated on both synthetic and actual data and is able to obtain accurate trajectory and feature boundaries. We also prove that submap joining method can improve the calculation efficiency without losing too much accuracy.

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## List of Publications

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4. **Zhao, Jiaheng**, Shoudong Huang, and Liang Zhao. “Constrained Gaussian mixture models based scan matching method.” In *Australasian Conference on Robotics and Automation, ACRA*. 2018.
5. Kong, Felix H., **Jiaheng Zhao**, Liang Zhao, and Shoudong Huang. “Analysis of Minima for Geodesic and Chordal Cost for a Minimal 2-D Pose-Graph SLAM Problem.” *IEEE Robotics and Automation Letters* 5, no. 2 (2019): 323-330.
6. Jia, Yan, Xiao Luo, Baoling Han, Guanhao Liang, **Jiaheng Zhao**, and Yuting Zhao. “Stability criterion for dynamic gaits of quadruped robot.” *Applied Sciences* 8, no. 12 (2018): 2381.

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# Abbreviation

SLAM - Simultaneous Localization and Mapping

RMSE - Root Mean Square Error

2D: Two-dimensional

3D: Three-dimensional

ICP - Iterated Closest Point

NDT - Normal Distribution Transform

GMM - Gaussian Mixture Models

ToF - Time-of-Flight



## Nomenclature and Notation

The semicolon is to represent vertical vector concatenation. Lowercase letters indicate scalars, bold lowercase letters indicate vectors, and uppercase letters indicate matrices. Some special symbols are listed below.

The observed points have zero-mean Gaussian noise  $\mathbf{n}_z \in \mathbb{R}^2 \sim N(\mathbf{0}, \Sigma_z)$ .

$\{j\}\mathbf{p} \in \mathbb{R}^2$  denotes an observed point in the frame  $j$ .

$\{j\}\mathbf{P}_k = [\{j\}\mathbf{p}_1^k, \dots, \{j\}\mathbf{p}_M^k]$  is a 2D point set observed of feature  $k$  at the frame  $j$ .

$\{G\}\mathbf{P}_k$  is usually abbreviated as  $\mathbf{P}_k$  since it is relative to the global frame.

$\phi$  is an angle within the range  $[-\pi, \pi)$ .

$R(\phi) \in SO(2) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$  is the corresponding rotation matrix which is abbreviated as  $R$ .

$\mathbf{t} = [t_x, t_y]^\top$  is the translation vector.

$R_{ij}, \mathbf{t}_{ij}$  means the rotation and translation from frame  $i$  to frame  $j$ .

If  $i$  is the global frame  $\{G\}$ , it is usually omitted in order to simplify the formula.

$\Xi_j = [\mathbf{t}_j; \phi_j]$  is a robot pose.

$T(\Xi_j, \{j\}\chi)$  represents the process of transforming a point/point cloud/feature from frame  $\{j\}$  to global frame  $\{G\}$ .

$T^{-1}(\Xi_j, \{G\}\chi)$  represents the process of transforming a point/point cloud/feature from global frame  $\{G\}$  to frame  $\{j\}$ .

Feature  $\Phi_k$  is in closed shape, whose boundary point set is denoted by  $\mathbf{P}_k$ .

$\|\mathbf{e}\|_{\Sigma}^2$  is the weighted L2 vector norm with a covariance of  $\Sigma$ .