Is the Allais Paradox Due to Appeal of Certainty or Aversion to Zero?*

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Abstract

We provide a novel but intuitive explanation for expected utility violations found in the Allais paradox: individuals are commonly averse to receiving nothing. We call this phenomenon the zero effect. Our laboratory experiments show support for the zero effect. By contrast, the evidence for the certainty effect is weak to nonexistent.

Keywords—Allais paradox, Certainty effect, Common consequence, Common ratio, Decision theory, Zero effect

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1 Introduction

The Allais (1953) paradox is a systematic pattern of choice under risk that violates expected utility theory. The pattern involves choosing a certain, strictly positive payment over a risky lottery that has positive probability of a zero outcome, then making the opposite choice if both alternatives are modified in an identical way such that both can yield a zero outcome. It is widely accepted that this behavior reflects an added appeal of a riskless decision, a phenomenon that Kahneman and Tversky (1979) call the *certainty effect* (defined precisely in Wakker, 2010, Schneider and Schonger, 2019, as an additional appeal of lotteries with zero variance). Our purpose in this paper is to challenge this interpretation. We argue that avoidance of zero, rather than attraction to certainty, is a more compelling explanation.

We call our alternative explanation the *zero effect*, and note that it reflects a previously unrecognized confound. In all four lotteries in standard Allais-type tasks, the only lottery without a possible zero outcome is the certain lottery. Therefore, the certainty effect and the zero effect lead to the same pattern of choice. By getting participants to make additional choices, we are able to separate the certainty effect from the zero effect, and to test the ability of each to explain the Allais paradox.

Before describing our experiments and analysis, we provide some illustrations. In the common consequence task Allais (1953) presents, participants choose between the following alternatives:

$$A = \begin{cases} \text{1 million francs,} & \text{with an 11\% chance} \\ 0, & \text{with an 89\% chance} \end{cases}$$

vs.

$$B = \begin{cases} \text{5 million francs,} & \text{with a 10\% chance} \\ 0, & \text{with a 90\% chance} \end{cases}$$

In a second problem, the same participants choose between

$$A' = \text{1 million francs} \qquad \text{or} \qquad B' = \begin{cases} \text{5 million francs,} & \text{with a 10\% chance} \\ \text{1 million francs,} & \text{with an 89\% chance} \\ 0, & \text{with a 1\% chance} \end{cases}$$

Decision makers who choose B in the first task and A' in the second violate expected utility theory. Both A and B have an 89% chance of paying 0. We refer to this as a *common consequence* of 0. Both A' and B' have an 89% chance of paying 1 million; i.e., the common consequence is 1 million, again received with 89% probability. On the remaining 11%, both decisions are the same, so expected utility theory predicts that the decision maker must choose either (A, A') or (B, B'). This conflicts with a common choice profile of B and A'.[1] This is seen more directly if we present the lotteries in a table format, used in many prior studies (for instance Savage, 1972, Starmer and Sugden, 1989, Starmer, 1992, Wakker et al., 1994). See Table 1.

[1] For example, a similar experiment in Kahneman and Tversky (1979) finds that 12 out of 72 participants chose A over B, but 59 out of 72 chose A' over B'. These numbers imply that at most 35% chose either both A and A' or both B and B', with the remainder violating expected utility theory.

Table 1 – Allais-type common consequence task, table presentation

Lottery	89%	10%	1%
A	0	1 million	1 million
B	0	5 million	0
A'	1 million	1 million	1 million
B'	1 million	5 million	0

Our main experiments exploit the structure of Table 1. By including additional non-zero common consequences, we are able to run a horse race between the zero effect and the certainty effect. To illustrate, consider an additional task, in which a participant chooses between a lottery A'', paying 8 million with an 89% chance and 1 million with an 11% chance, versus a lottery B'', paying 8 million with an 89% chance, 5 million with a 10% chance, and 0 with a 1% chance. See Table 2.

Table 2 – Allais-type common consequence task with an additional lottery pair

Lottery	89%	10%	1%
A	0	1 million	1 million
B	0	5 million	0
A'	1 million	1 million	1 million
B'	1 million	5 million	0
A''	8 million	1 million	1 million
B''	8 million	5 million	0

As is readily seen from the table, the decision maker cannot avoid a possible zero outcome in the choice between A and B, but can in the other choices. Therefore, a decision maker who chooses B over A while choosing A' over B' and A'' over B'' behaves consistently with the zero effect. This is different from the pattern of behavior that the certainty effect explains. Only the choice between A' and B' involves a riskless lottery. A decision maker who chooses A' over B' while choosing B over A and B'' over A'' behaves consistently with the certainty effect.

Adding the third lottery pair distinguishes the certainty effect from the zero effect, but is still not enough to rule out both of these as explanations for the Allais common consequence effect (henceforth referred to as the CCE). Any decision maker who makes the standard CCE choices by definition selects B over A and A' over B'. A choice of B'' over A'' is evidence of the certainty effect, and a choice of A'' over B'' is evidence of the zero effect. In order to reject both, it is necessary to have a fourth lottery pair, say A''' and B''', with an additional common consequence. The certainty effect could explain the CCE only if the decision maker chooses (B, A', B'', B'''), and the zero effect could explain the CCE only if the decision maker chooses (B, A', A'', A'''). Adding more lottery pairs, each with new common consequences, makes the tests of the certainty effect and the zero effect more powerful.

We find support for the zero effect and no support for the certainty effect. Across all of our treatments in which the zero effect can occur, the zero effect is always more common than the certainty effect. These findings are robust to allowing for stochastic choice.

Having established that the zero effect plays a stronger role than the certainty effect in explaining Allais-type behavior, we turn to robustness questions. A natural question is whether there is anything special about a zero outcome.

To investigate whether zero (as opposed to any relatively small outcome) plays a special role in the CCE, we conduct several additional tests. First, we change the lowest outcome in the risky lottery (corresponding to Lotteries B, B', and B'' in the examples above) from 0 to \$1, keeping everything else the same. With this change, either both lotteries in a given pair avoid a zero outcome or neither does. In a second test, we change the lowest common consequence from 0 to \$1, keeping everything else the same as in our main experiment.

Our results of both these tests suggest that participants respond specifically to zero, rather than just to a small outcome.

Our last question addresses the broader significance of the zero effect, by investigating whether it plays a role in the common ratio effect. We find indirect evidence for the zero effect in the common ratio task.

The structure of the rest of this paper is as follows: Section 2 provides formal definitions of the CCE, the certainty effect, and the zero effect. Section 3 presents our experiments and results. Section 4 shows the results of our robustness checks, including our extension to test the role of the zero effect in explaining the Allais common ratio effect. Section 5 concludes. Full results from all our sessions, along with possible extensions, are in the online appendices.

2 The Zero Effect, the Certainty Effect, and the Common Consequence Effect

We define the zero effect, the certainty effect, and the common consequence effect in revealed preference terms, allowing for expressions of indifference.[2] For convenience, we write the lottery that pays x_1 with probability $p_1, \ldots, x_n$ with probability p_n as $(x_1, p_1; \ldots; x_n, p_n)$. Our definitions are as follow:

Definition 1. *Consider lotteries* $s := (m, q; c, 1-q)$, $r := (h, p; c, 1-q; \ell, q-p)$ *where* $c \geq 0$, $h > m > \ell = 0$, *and* $1 > q > p > 0$. *For a decision maker (DM) who can choose between these lotteries or express indifference, we say*

- *The DM's choices display the* zero effect *if one of the following holds:*

[2] In our extension to the common ratio effect (CRE) below in Section 4, we give the formal definition of the CRE due to Battalio et al. (1990).

  - *The DM chooses r when $c = 0$ and s whenever $c \neq 0$*
  - *The DM expresses indifference when $c = 0$ and chooses s whenever $c \neq 0$*
  - *The DM chooses r when $c = 0$ and expresses indifference whenever $c \neq 0$*

- *The DM's choices display the* certainty effect *if one of the following holds:*
  - *The DM chooses s when $c = m$ and r whenever $c \neq m$*
  - *The DM expresses indifference when $c = m$ and chooses r whenever $c \neq m$*
  - *The DM chooses s when $c = m$ and expresses indifference whenever $c \neq m$*

- *The DM's choices display the* common consequence effect (CCE) *if one of the following holds*
  - *The DM chooses s when $c = m$ and r when $c = 0$*
  - *The DM expresses indifference when $c = m$ and chooses r when $c = 0$*
  - *The DM chooses s when $c = m$ and expresses indifference when $c = 0$*

Definition 1 shows that there are two values of c involved in the CCE, both 0 and m. The focus of the literature on the certainty effect is based on the presumption that the case of $c = m$ is the reason for the CCE. We observe, however, that the main cause of the CCE may instead be the case of $c = 0$.

3 Experiments

To separate the importance of the zero effect from the certainty effect, we run experiments with additional common consequences. Our hypotheses, stated in alternative form, are as follow:

H_A^1 The zero effect occurs more frequently than attributable to chance.

H_A^2 The certainty effect occurs more frequently than attributable to chance.

We test H_A^1 and H_A^2 using T-tests.

3.1 Overview of the experimental design

We presented participants with a list of lottery pairs, and asked them to choose one lottery from each pair. The participants made several decisions, in which the pairs differed in the common consequence c and had the same values for the parameters (p, q, h, m, ℓ) (see Definition 1). One lottery pair had $c = 0$, another $c = m$, and one or more pairs had additional values of c.

Although this design is used in prior studies (an example is Wakker et al., 1994), a more common way to study the CCE is to restrict the common consequence c to $\{\ell, m, h\}$ and to vary the probabilities p and q. This approach (known as a *triangle* design due to the graphical depiction of Marschak, 1950, Machina, 1982) is commonly used because of its visual appeal and because it provides a sharp test of expected utility theory, due to the linearity of expected utility in probabilities. To test the certainty effect against the zero effect, however, a triangle design is less useful. Having values of the common consequence c that differ from ℓ and m provides a direct test of which effect has greater explanatory power. To allow for the possibility that neither effect explains the CCE, it is necessary to include at least four values of c (see the remarks in the introduction).[3]

To illustrate, consider a task with six common consequences, $c_1 = 0 < c_2 < c_3 = m < c_4 < c_5 < c_6$, where m is as in Definition 1. Let R indicate a choice of lottery r from

[3]For approaches staying within a triangle design that provide indirect tests by considering multiple triangles, see Appendix B.

Definition 1. Similarly, let S indicate a choice of lottery s, and let I indicate an expression of indifference.[4] We write a participant's choice pattern in increasing order of the common consequence. For instance, a choice pattern of $RSRSRS$ represents a participant choosing R for common consequences c_1, c_3, and c_5, and choosing S for c_2, c_4, and c_6. Our main variable of interest is the frequency of choice patterns (for similar approaches, see Starmer and Sugden, 1989, Starmer, 1992, Wakker et al., 1994, Harless and Camerer, 1994, Camerer and Ho, 1994).

There are $2^6 = 64$ possible choice patterns between r and s across the common consequences, if we exclude indifferences. Only two patterns are consistent with expected utility: $RRRRRR$ and $SSSSSS$. A participant whose choice pattern is $RRSRRR$ expresses a preference for s when $c = m$ and for r whenever $c \neq m$. We refer to $RRSRRR$ as the *certainty effect pattern*. One whose choice pattern is $RSSSSS$ expresses a preference for r when $c = 0$ and s whenever $c \neq 0$. We refer to $RSSSSS$ as the *zero effect pattern*. We test for the frequency of the zero effect and the certainty effect patterns.

3.2 Main Experiment

In our main experiment, participants made six choices, as described in subsection 3.1. We used the original Allais probabilities, set the medium outcome $m = \$8$, and set the high outcome $h = \$10$.[5] The common consequence c took values in $\{0, 5, 8, 10, 16, 20\}$. The four patterns of interest were $RRRRRR$ and $SSSSSS$ (expected utility), $RRSRRR$ (certainty effect), and $RSSSSS$ (zero effect).[6]

[4] Although allowing for indifference adds a complication, we permit it to rule out indifference as a cause of Allais-type behavior. See Harrison (1994).

[5] We deliberately kept the differences in the expected values of the lotteries small, as it is known that participants facing large differences in expected real payments in common consequence tasks overwhelmingly choose the risky lottery (see Conlisk, 1989, Fan, 2002).

[6] If we include indifferences, there are five additional patterns of interest: $IIIIII$ (expected utility); $RRIRRR$ and $IISIII$ (certainty effect), and $ISSSSS$ and $RIIIII$ (zero effect). We run our tests both

We ran our main experiment in three sessions, in order to include two controls. First, we control for presentation. Previous studies have shown that context matters in eliciting risk preferences in general (Zhou and Hey, 2018, Loomes and Pogrebna, 2014, Crosetto and Filippin, 2016, Lévy-Garboua et al., 2012), with presentation in particular affecting choices in Allais-type tasks (Moskowitz, 1974, Keller, 1985, Camerer, 1989, Gottlieb et al., 2007). To address any concerns about presentation, we varied whether participants saw the lottery pairs in a narrative or in a table format. All participants in a given session saw only one of the two formats for all lottery pairs. Second, we control for incentive structure, as prior work has presented mixed results on the effects of real versus hypothetical incentives (Camerer, 1989, Conlisk, 1989, Battalio et al., 1990, Harless and Camerer, 1994, Beattie and Loomes, 1997, Fan, 2002). To address this concern, we ran our experiments with real incentives, then replicated the payoff structure with hypothetical ones.[7] We counter-balanced the order of presentation in all sessions.

We presented all six choices on a single sheet of paper.[8] For each pair of lotteries, a participant indicated whether he or she preferred lottery $A = (c, 0.89; \$8, 0.11)$ or lottery $B = (c, 0.89; \$10, 0.1; \$0, 0.01)$. We also allowed participants to indicate indifference. After the participants made all six choices, an experimenter collected the sheets of paper. If the treatment included real incentives, we used a random lottery incentive system to determine payments (Azrieli et al., 2020, Starmer and Sugden, 1991). We resolved all uncertainty by rolling dice.

For the treatments involving real incentives, we ran our laboratory sessions at two univer-

including observations with indifferences and excluding observations with indifferences.

[7]We manipulate the presentation and the incentive structure to improve the generalizability of our results rather than to test their direct effects. See also Moskowitz (1974), Keller (1985), Gottlieb et al. (2007), Conlisk (1989), Beattie and Loomes (1997), Fan (2002), Blavatskyy et al. (2020).

[8]Littenberg et al. (2003) establish the reliability of pencil-and-paper instruments for tasks involving decisions under risk.

sities in a mid-Atlantic state in the United States. We recruited the participants through university administered pools. After reading and signing consent forms, participants read instructions, which an experimenter then read aloud. Subsequently, the participants made their choices. Reading the instructions, making the decisions, and resolving uncertainty took approximately 15 minutes. In addition, seating and privately paying the participants took just under another 15 minutes. The participants received a $5 show-up fee in addition to money earned during the experiment. Overall, earnings ranged from $5 to $25, with average earnings of approximately $14. Participants in the treatments with hypothetical incentives were undergraduate students at the same universities.

Results

In our main experiment, we recruited 96 participants across all three sessions. We included controls for presentation format and incentive type as discussed above. We had 32 participants in a treatment with real incentives and a table presentation, 29 participants in a treatment with real incentives and a narrative presentation, and 35 participants in a treatment with hypothetical incentives and a table presentation. Table 3 summarizes the results of our main experiment and reports the numbers for our patterns of interest.

Only three patterns occurred more than three times: two expected utility patterns, $RRRRRR$ and $SSSSSS$, and a zero effect pattern $RSSSSS$. Across all sessions, 52% ($p < 0.01$) made expected utility consistent choices and 14%($p < 0.01$) chose the zero effect pattern.[9] In all sessions, not a single participant chose the certainty effect pattern. By contrast, regardless of our controls, we find consistent rates of zero effect choices (roughly 14%). Excluding indifferences did not change any of these results. Specifically, the expected utility choices

[9] Our null hypotheses are that each predicted pattern occurs no more than due to chance, i.e., with probability 1/64. We also tested the non-EU patterns to see if they occur no more than due to chance conditional on non-EU choice, i.e., with probability 1/62. The results were the same.

Table 3 – Main Experiment Summary: $n = 96$, 82 without indifferences. $^{*}p < 0.1,^{**}p < 0.05,^{***}p < 0.01$.

		Overall	Table Real	Narrative Real	Table Hypothetical
Consistent with EU					
	All obs.	50*** (52%)	18*** (56%)	11*** (38%)	21*** (60%)
	Excl. indiff.	47*** (57%)	18*** (64%)	11*** (46%)	21*** (70%)
Consistent with ZE					
	All obs.	13*** (14%)	4***(13%)	4***(14%)	5*** (14%)
	Excl. indiff.	11*** (13%)	4***(16%)	3***(13%)	4***(13%)
Consistent with CE		0	0	0	0
n (excl. indiff.)		96 (82)	32 (28)	29 (24)	35 (30)

were 57% (47/82, $p < 0.01$), the zero effect was 13% (11/82, $p < 0.01$), and there was no certainty effect.

Our rates of expected utility conforming choice, and the effects of our controls, match findings in prior work. In the table presentation with real incentives, the expected utility consistent choices are 56% ($p < 0.01$, in line with Starmer and Sugden, 1989, Starmer, 1992, Wakker et al., 1994, Fan, 2002, Harman and Gonzalez, 2015, who use a similar presentation). In the narrative presentation with real incentives, it is 38% ($p < 0.01$, consistent with the 35% inferred from Kahneman and Tversky, 1979, as noted in the introduction; they also use a narrative presentation). Lastly, in the table presentation with hypothetical rewards, the expected utility consistent choices make up 60% of the choices ($p < 0.01$, in line with Fan, 2002, who uses both hypothetical and real incentives).

In terms of risk tolerance, we find that the expected utility choices under hypothetical incentives favored the risky lottery, consistent with prior literature (Slovic, 1969, Camerer and Hogarth, 1999, Harrison, 2006) (see online Appendix A for details). Together, these results show that choices in our task are consistent with previous studies. The only common choice pattern consistent with the CCE we observe is the zero effect pattern.[10]

We see evidence of the CCE in our data. Excluding indifferences, we observed 18 participants whose choices were consistent with the CCE, compared with only 4 whose choices included the reverse patterns, $S\_R\_\_\_$. If we include indifferences, these numbers change to 26 observations of the CCE versus 9 observations of the reverse patterns. Thus there was a clear, systematic bias in favor of the CCE, which we cannot attribute purely to noise ($p < 0.01$ with and without indifferences).

As we note above, however, no patterns other than the two expected utility patterns and the zero effect pattern occurred more than three times in our data set. Moreover, if we exclude the zero effect pattern, the frequency of the CCE becomes marginally significant or insignificant ($p = 0.09$ on a one-tail test excluding indifferences and $p = 0.11$ including indifferences). Put simply, there was no evidence for the certainty effect or for any other systematic cause of the CCE. The only systematic departure from expected utility was the zero effect pattern.

Stochastic choice analysis

Excluding the two expected utility and the zero effect patterns, the frequencies with which the remaining choice patterns occur are indistinguishable from random choice. These results are based on an exact choice theory, under which a participant's decisions reflect the participant's true preferences. To adjust for possible noise in a participant's observed decisions, we extend our analysis to consider stochastic choice.

[10]To elaborate, we note that there are additional patterns consistent with the CCE, aside from the certainty effect and zero effect patterns. Recall from Definition 1 that the CCE depends only on decisions under two common consequences, $c \in \{0, m\}$. Therefore, any pattern with riskier choice when c=0 than when c=m would be consistent with the CCE (e.g. $R\_S\_\_\_$).

There are several approaches to allow for a random choice (for an overview, see Blavatskyy and Pogrebna, 2010, Bhatia and Loomes, 2017). We use a tremble model based on Harless and Camerer (1994), as it fits well with our design.[11]

The Harless and Camerer (1994) approach is based on maximum likelihood, and has the advantage of not requiring estimation of individual-specific parameters. As an example, consider the decisions of an EU-maximizer. There are two choice patterns consistent with EU: all risky and all safe ($RRRRRR$ and $SSSSSS$). Let p_1 be the probability that the decision maker prefers $RRRRRR$ and p_2 be the probability that the decision maker prefers $SSSSSS$. On any given choice, the decision maker might tremble and circle the wrong alternative. Assume the participant trembles with probability ε, independently across all six choices. Let ρ be the number of the choices on which the decision maker chooses the risky lottery, and let $\sigma = 6 - \rho$ be the number of choices on which the decision maker chooses the safe lottery. The ex ante probability of observing the decision maker's choice pattern is

$$p_1\varepsilon^{\sigma}(1-\varepsilon)^{\rho} + p_2\varepsilon^{\rho}(1-\varepsilon)^{\sigma} \tag{1}$$

We test two additional tremble models to allow for different types of decision makers, each extending expected utility by allowing for an additional preference profile. In one of these additional models, we allow for a decision maker whose true preferences are the certainty effect pattern, and in the other, we allow for one whose true preferences are the zero effect pattern. The expected utility model therefore is nested in both of these extensions.

[11] We also consider numerous alternative tests, allowing for heterogeneity in trembles across common consequences, in the spirit of the true-and-error model of Birnbaum and Schmidt (2008), and tests that can be considered a simple version of the true-and-error model as in Loomes et al. (1991). In addition, we modified our main tests to include other forms of heterogeneity in trembles, such as errors that depend on presentation order or presentation style. We also allowed for individual heterogeneity in errors based on preferences, e.g., allowing for an EU type who prefers a safe lottery to have different rates of trembles than an EU type who prefers a risky lottery. None of these variations changed our main conclusions (details available from the authors on request).

Table 4 – Likelihood ratio comparison: $n = 82$ excluding indifferences

Model	Log-likelihood	Free parameters	p-value
EU	-223.041	2	NA
ZE	-201.065	3	< 0.001
CE	-223.041	3	1

For the certainty effect and the zero effect, the null model in the likelihood ratio test is expected utility theory. The p-value comes from a χ^2-test.

The models are similar, though they each require an extra free parameter. We let p_3 be the probability that a decision maker prefers the zero effect pattern, $RSSSSS$. Likewise, we let p_4 be the probability that a decision maker prefers the certainty effect pattern, $RRSRRR$.

Table 4 shows the results of the estimation, with likelihood ratio tests based on models that nest. Both the zero effect and certainty effect include EU as a special case, but do not nest each other. The table shows that the zero effect pattern significantly improves the fit over EU. The certainty effect pattern generates no improvement over EU. To summarize, the results of the stochastic choice model are consistent with the analysis above based on deterministic choice.[12]

Table 5 shows the parameter estimates for all three tremble models.[13] The results show that stochastic choice cannot explain the lack of observed certainty effect, and that the zero effect pattern is robust to stochastic choice.

[12]Tables 4 and 5 show the results with indifferences excluded. We also analyzed the data including indifferences. The results do not change.

[13]We use the bootstrap method to estimate the standard error of the estimated parameters. For the bootstrap estimation, we re-sampled the experimental data 10,000 times.

Table 5 – Parameter estimates for three tremble models: $n = 82$ excluding indifferences, $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$.

Model	P(RRRRRR)	P(SSSSSS)	P(RSSSSS)	P(RRSRRR)	P(error)
EU	0.556***	0.444***	NA	NA	0.112***
	(0.058)	(0.058)			(0.019)
ZE	0.519***	0.220***	0.261***	NA	0.075***
	(0.057)	(0.057)	(0.053)		(0.016)
CE	0.556***	0.444***	NA	0.000	0.112***
	(0.058)	(0.058)		(0.002)	(0.019)

The patterns *RRRRRR* and *SSSSSS* are the predictions of expected utility. The pattern *RSSSSS* is the zero effect pattern. The pattern *RRSRRR* is the certainty effect pattern.

Original Allais setting

The results we report above go against the orthodox view of the CCE. Given the lack of evidence for the certainty effect and the strong support for the zero effect, it is natural to ask how generalizable our findings are.

Our first extension addresses the payoff structure. The problem Allais (1953) originally posed involves life-changing hypothetical rewards, in contrast to the small rewards in our main experiments. This difference is potentially important; for instance, Fan (2002, p. 417) reports that the CCE is considerably less frequent with small rewards, real or hypothetical, than with large hypothetical rewards. Blavatskyy et al. (2020) summarizes the evidence on reward structure, and concludes that small real rewards can make the CCE disappear. Although the CCE occurred in our main experiments, it is worth investigating whether the lack of evidence for the certainty effect and the explanatory power of the zero effect are still found in the original Allais task.

We ran an additional treatment, using the hypothetical incentive structure shown in Table 2 in the introduction. This treatment shows the original Allais choices along with one additional choice, in order to separate the zero effect from the certainty effect. We recruited

Table 6 – Summary of results on original Allais task: $n = 153$. $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$

	Original Allais
Consistent with EU	75*** (49%)
Consistent with ZE	30*** (20%)
Consistent with CE	10 (7%)

153 participants using the crowdsourcing Internet marketplace Mechanical Turk (MTurk). Participants were paid a flat rate of $1.25 for a task that took under 5 minutes. Our hypotheses for this treatment were as follow:

H_A^3 In the original Allais setting, the zero effect occurs more frequently than attributable to chance.

H_A^4 In the original Allais setting, the certainty effect occurs more frequently than attributable to chance.

Results Table 6 summarizes the results, which are again based on T-tests. As the table shows, the zero effect pattern is highly significant ($p < 0.01$) in the original Allais setting. We note also that the frequency of the CCE was significant (40 Allais versus 21 of the reverse pattern, $p = 0.01$), and when the zero effect choices are dropped, the frequency becomes insignificant. Compared with our main experiment, we see roughly the same frequency of expected utility conforming choice, a slightly higher frequency of the zero effect pattern, and no significant evidence of the certainty effect. Thus we find strong support for the alternative H_A^3 and no evidence in favor of alternative H_A^4, in line with our earlier results and expectations.

4 Robustness Analysis

Having established that we find no observable role for the certainty effect in the CCE, we turn our attention to studying the zero effect in greater detail.

Our first robustness check evaluates whether there is a special reaction to zero, rather than to any low outcome or to a lottery's smallest outcome. We then extend the analysis to the role of zero in a common ratio task.

4.1 Zero or small outcomes

To verify that the results we attribute to the zero effect are indeed due to a zero outcome, we ran several additional treatments. We begin by discussing a modification to our earlier treatment with large, hypothetical rewards, designed to separate a reaction to small outcomes from a reaction that is specific to zero.

Our change consists of making the smallest value of the common consequence $c = \$1$ instead of zero, keeping everything else the same. See Table 7. In this treatment, the only lotteries that involve a zero outcome are B, B', and B''. The zero effect, therefore could not explain a choice of B over A, A' over B', and A'' over B'' in this task, in contrast to the task described in Table 2.

On the other hand, a decision maker who is sensitive to small outcomes might view a \$1 prize as essentially the same as a zero prize, particularly given that the other prizes are in millions. A decision maker who avoids small outcomes could justify the choices of B, A', and A''. If this single change of 0 to \$1 suffices to make the pattern (B, A', A'') less frequent, then there is reason to view zero as special.

Our hypothesis for this treatment, stated in alternative form, is as follow:

Table 7 – Modified Allais experiment

Lottery	89%	10%	1%
A	1	1,000,000	1,000,000
B	1	5,000,000	0
A'	1,000,000	1,000,000	1,000,000
B'	1,000,000	5,000,000	0
A''	8,000,000	1,000,000	1,000,000
B''	8,000,000	5,000,000	0

H_A^5 The zero effect pattern RSS occurs less frequently in the modified Allais experiment (described in Table 7) than in the original Allais experiment (described in Table 2).

We recruited 129 participants for this session, once more using MTurk. To test H_A^5, we compare the results to those shown in Table 6.

Results Table 8 summarizes the results. A T-test shows that the zero effect pattern is lower in the modified task ($p = 0.07$). Moreover, the zero effect pattern is insignificant in the modified task, despite being highly significant in the original Allais task. We also found no evidence of the certainty effect or the CCE. (There were 23 Allais pattern choices, compared with 15 of the reverse pattern, $p = 0.13$.) These findings suggest that zero in particular plays an important role.

Table 8 – Summary of results on original and modified Allais experiments: $n = 153$ (original Allais task) and $n = 129$ (modified Allais task). $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$

	Original Allais	Modified Allais
Consistent with EU	75*** (49%)	75*** (58%)
Consistent with ZE	30*** (20%)	17 (13%)
Consistent with CE	10 (7%)	6 (5%)

Additional tests of turning the zero effect off We conducted two additional tests of reactions to small rewards, using the design from our main experiment as a starting point. We modified our first main session (table presentation, real rewards) by changing the lowest value of the common consequence c to \$1 instead of zero, as in our previous test.

For our other test, we changed ℓ, the lowest outcome specific to the risky lottery, from 0 to \$1, and kept everything else the same as in our third main session (table presentation, hypothetical rewards). With this change, the risky lottery becomes $r = (c, 0.89; 10, 0.1; 1, 0.01)$ and the safe remains $s = (c, 0.89; 8, 0.10; 8, 0.01)$. The two lotteries both have a zero outcome if $c = 0$; otherwise, neither has a zero outcome. The zero effect could not explain the choice pattern $RSSSSS$ with this change, though a desire to avoid possible small outcomes could.

Our hypothesis is as follows:

H_A^6 The zero effect pattern $RSSSSS$ occurs less frequently in the modified experiment than in the main experiment.

We recruited two groups of 50 additional participants each, using the same pools as in our main sessions. We summarize the results below in Table 9, focusing for the sake of brevity on participants who did not express indifference. If we include indifferences, the conclusions are unchanged. For the consistency, we compare the new sessions to the ones in our main experiment that included a table presentation; including the narrative presentation does not meaningfully change the frequency of the zero effect.

As the table shows, it did not matter how we switched the zero effect off. The zero effect went from being highly significant in the original treatment to insignificant ($p < 0.01$). There was no meaningful increase in the certainty effect and no meaningful change in expected utility conforming choice.

Table 9 – Summary of results, main sessions versus modified sessions, excluding indifferences. $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$.

		Table Pooled	Table Real	Table Hypothetical
Consistent with EU				
	Main sessions	67%***	64%***	70%***
	Modified sessions	68%***	69%***	66%***
Consistent with ZE				
	Main sessions	14%***	16%***	13%***
	Modified sessions	3%	3%	2%
Consistent with CE				
	Main sessions	0	0	0
	Modified sessions	0	3%	5%
n main (modified)		58 (80)	28 (39)	30 (41)

To summarize our results: the zero effect is an important factor in Allais-type behavior, the only one we are able to observe consistently. Changing an outcome from 0 to $1 is enough to disrupt the zero effect, indicating that the specific value of zero plays a key role.

4.2 The Common Ratio Effect and Additional Tests

As a last robustness test, we investigate whether the zero effect is involved in the common ratio effect, which we review here in an example based on Kahneman and Tversky (1979). Consider the choices between the following pairs of lotteries:

$$A = \begin{cases} \$30 & \text{with a 100\% chance} \\ 0, & \text{with a 0\% chance} \end{cases} \qquad \text{or} \qquad B = \begin{cases} \$40 & \text{with an 80\% chance} \\ 0, & \text{with a 20\% chance} \end{cases}$$

and

$$A' = \begin{cases} \$30 & \text{with a 25\% chance} \\ 0, & \text{with a 75\% chance} \end{cases} \qquad \text{or} \qquad B' = \begin{cases} \$40 & \text{with a 20\% chance} \\ 0, & \text{with an 80\% chance} \end{cases}$$

Lottery A' is a mixture of Lottery A with probability 1/4 and 0 with probability 3/4. Lottery B' is a mixture of lottery B with probability 1/4 and 0 with probability 3/4. The common ratio effect is an expressed preference for A over B and for B' over A'. A formal definition is provided in Battalio et al. (1990): let $s_1 := (m, p; \ell, 1-p)$ and $r_1 := (h, q; \ell, 1-q)$. Let $s_2 := (m, tp; \ell, 1-tp)$ and $r_2 := (h, tq; \ell, 1-tq)$ where $p > q$, $h > m > \ell$, and $0 < t < 1$. The common ratio effect is defined as the expressed preference for s_1 over r_1 and for r_2 over s_2.

The usual presumption is that an appeal of certainty is the reason for the common ratio effect. Notice, however, that Lottery A is the only lottery that has no zero outcome. It is therefore possible that aversion to zero, rather than attraction to certainty, is involved in the common ratio effect.

We limit ourselves here to testing the role of zero aversion, without a test of the certainty effect. Our test involves giving the decision makers four choices between lottery pairs. The first two are the choices above between A and B and between A' and B'. The second two present a second common ratio task, in which we change the lowest outcome ℓ from 0 to \$1. The second pair of choices is therefore as follows:

$$A'' = \begin{cases} \$30 & \text{with a 100\% chance} \\ \$1, & \text{with a 0\% chance} \end{cases} \qquad \text{or} \qquad B'' = \begin{cases} \$40 & \text{with an 80\% chance} \\ \$1, & \text{with a 20\% chance} \end{cases}$$

and

$$A''' = \begin{cases} \$30 & \text{with a 25\% chance} \\ \$1, & \text{with a 75\% chance} \end{cases} \quad \text{or} \quad B''' = \begin{cases} \$40 & \text{with a 20\% chance} \\ \$1, & \text{with an 80\% chance} \end{cases}$$

This common ratio test makes use of a triangle design, in that it gives the decision maker choices involving a fixed set of three outcomes. In one triangle (which we call Triangle CRE), the outcomes are $\{0, 30, 40\}$, and in the other triangle (Triangle CRE Modified), the outcomes are $\{1, 30, 40\}$. If we observe the common ratio effect in Triangle CRE and do not observe it in Triangle CRE Modified, then we have indirect evidence that the zero effect is an important reason for observing the common ratio effect. A limitation of this design is that it does not enable us to obtain evidence on the certainty effect.

Our hypotheses are as follows:

H_A^7 The common ratio pattern occurs more frequently than the reverse pattern in Triangle CRE.

H_A^8 The common ratio pattern occurs more frequently than the reverse pattern in Triangle CRE Modified.

If we can reject the null in favor of H_A^7 and cannot reject the null in favor of H_A^8, then we can say there is indirect support for the zero effect playing a role in the common ratio effect.

We made use of a within-participants design, which allows us also to test whether the common ratio pattern occurs more frequently than violations of first-order stochastic dominance (FOSD). To see this, observe that Lottery A and Lottery A'' both give \$30 with certainty. Lottery $B'' = (40, .8; 1, .2)$ is an improvement over Lottery $B = (40, .8; 0, .2)$ in the sense of

FOSD. Any participant who chooses B over A and A'' over B'' violates first-order stochastic dominance. This observation gives us the following additional test of the significance of the CRE:

H_A^9 The common ratio pattern occurs more frequently than FOSD violations in Triangle CRE.

H_A^{10} The common ratio pattern occurs more frequently than FOSD violations in Triangle CRE Modified.

In order to conclude that we found the CRE, we would need to reject the null in favor of H_A^9. We include H_A^{10} for completeness.

We recruited 198 participants from a university research pool in Australia. Participants made all their choices in a laboratory, making all decisions on the Qualtrics platform. Earnings including the A\$10 show-up fee averaged A\$21.14. The experiment, including an additional task described below, took a total of 45 minutes (including everything from showing up to leaving).

Among the 198 participants, we found that 13% made the common ratio pattern choices of A and B' in Triangle CRE, compared with 7% who made the reverse choices ($p = 0.04$). This finding is comparable to prior research using real incentives in common ratio tasks; for instance, MacDonald and Wall (1989) use real incentives and report 12% of their participants made the common ratio effect choice; results in Cubitt et al. (1998), while not directly reporting the number of common ratio choices, are of a similar magnitude. We also found that the rate of common ratio choices in our setting was significantly higher than the rate of FOSD violations ($p = 0.01$).

In Triangle CRE Modified, the common ratio effect disappears. Although the rate of common ratio choices remains higher than that of FOSD violations ($p = 0.04$), an identical number of participants chose the reverse pattern (11% in each case, $p = 0.56$). Thus the change of ℓ from 0 to \$1 is enough to eliminate the common ratio effect.

To compare the strength of the results on the common ratio effect to those on the CCE, we ran a similar indirect test of the zero effect in a common consequence task. We used a triangle CCE design with the same 198 participants in the same session. For this task, we used two triangles. In Triangle CCE, the participants made choices with outcomes in $\{0, 8, 10\}$, with the same probabilities as in our main sessions. In Triangle CCE Modified, participants made choices with outcomes in $\{1, 8, 10\}$. Six common consequence choices, three from each triangle, were shown on the same screen in random order, and on a separate screen from the common ratio task described above.

The results of this test were mixed. We present more details below in Table 10, and limit ourselves to some brief remarks here. Our hypotheses are analogous to H_A^7–H_A^{10} above, and our measurement of FOSD violations follows the same logic as above. In Triangle CCE, the CCE did not occur significantly more frequently than the reverse pattern ($p = 0.21$). In Triangle CCE Modified, there was greater frequency of the CCE than the reverse pattern ($p = 0.04$). However, the triangle design enables us to estimate the frequency of FOSD violations, and neither the CCE nor the reverse pattern was more common than FOSD violations in either triangle ($p = 0.35$ for Triangle CCE and $p = 0.14$ for Triangle CCE Modified). In terms of magnitude, 19% of our participants violated FOSD, versus 21% choosing the CCE in Triangle CCE and 24% in Triangle CCE Modified. For the reverse pattern, the frequencies were 17% and 16%, respectively.

The weak evidence for the CCE is not surprising, as the CCE is known to be fragile (see Blavatskyy et al., 2020). Even so, we note that the zero effect pattern is more common than the certainty effect pattern, and this difference is marginally significant in Triangle CCE ($p = 0.07$) but insignificant in Triangle CCE Modified ($p = 0.22$).

Overall, our robustness tests show support for the zero effect, with the possible exception of a task in which the CCE did not occur. We find no support for the certainty effect in any of our tests. Finally, we find evidence that the zero effect also plays a role in the common ratio effect.

5 Conclusion

Studies of the Allais paradox take as given that systematic expected utility violations are due to an appeal of certainty. The certainty effect is the basic building block of extensions of expected utility that aim to explain the Allais paradox. But as we have argued, there is an alternative explanation for Allais-type behavior: aversion to receiving nothing. We have named this alternative explanation the zero effect.

Overall, our findings support the zero effect. Table 10 provides an overview of all our sessions and findings.

As the table shows, the CCE occurs in our main and our original Allais treatments. The zero effect explains a significant portion of the CCE in these cases; the certainty effect is statistically meaningless. In the associated robustness treatments, in which (by design) the zero effect cannot occur, the CCE becomes insignificant.

The CCE did not occur in our triangle treatment. For the modified triangle treatment, the CCE is statistically meaningful; however, neither the zero effect nor the certainty effect can explain the CCE in this setting. Moreover, because the triangle experiments involve a within-subject design, we can observe the frequency of FOSD violations. The appearance

Table 10 – Summary of all treatments and results

Treatment	c_{min}	ℓ	pairs	n	$\frac{\text{Allais}}{\text{Allais+Reverse Allais}}$ (vs. Binomial)	$\frac{\text{ZE}}{\text{Allais}}$ (vs. Binomial)	$\frac{\text{CE}}{\text{Allais}}$ (vs. Binomial)	Allais vs FOSD Violations
Main	0	0	6	82	**0.002**	**0.000**	0.313	Cannot Test
Orig. Allais	0	0	3	153	**0.010**	**0.001**	1.000	Cannot Test
Mod. Allais	1	0	3	129	0.128	N/A	N/A	Cannot Test
Main, Mod. c_{min}	1	0	6	39	0.688	N/A	N/A	Cannot Test
Main, Mod. ℓ	0	1	6	41	0.344	N/A	N/A	Cannot Test
Triang. CRE	0	0	2	198	**0.040**	Cannot Test	Cannot Test	**0.008**
Triang. CRE Mod.	1	1	2	198	0.560	Cannot Test	Cannot Test	**0.036**
Triang. CCE	0	0	3	198	0.208	N/A	N/A	0.353
Triang. CCE Mod.	1	1	3	198	**0.044**	0.280	0.809	0.135

The columns reporting p-values computed under the binomial distribution have a null of random choice. For instance, in the test of the Allais (CCE or CRE, depending on the treatment) against reverse Allais patterns, the null is that each occurs with probability 1/2. In the tests of the ZE and CE as a fraction of the Allais patterns, the null is that each choice pattern consistent with Allais behavior is equally likely. Tests against the rate of FOSD violations use a t-test on differences in proportions.

of the CCE in this session is statistically indistinguishable from that of FOSD violations. To summarize, to the extent that the CCE occurs without the zero effect, it is weak, and not explained by the certainty effect.

One benefit of the triangle design is that it provides us with a test of the common ratio effect (CRE). We find that the CRE occurs in the standard treatment, in which the zero effect is possible, and differs significantly from the rate of FOSD violations. In the treatment that replaces 0 with $1, the CRE vanishes. Although this test is indirect, the evidence suggests that the zero effect plays a role in the CRE as well as in the CCE.

In all of our treatments in which aversion to zero can affect choice, the zero effect is the only pattern other than the expected utility ones to occur commonly. In particular, we find no evidence of the certainty effect. These results hold in the original Allais task and in tasks with much smaller prizes, whether real or hypothetical, whether in a table or a narrative presentation. To summarize, the zero effect appears involved in the Allais paradox. By contrast, all of our experiments reject any role for the certainty effect.

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