Optimal and Stable Fuzzy Controllers for Nonlinear Systems based on an Improved Genetic Algorithm

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Abstract: This paper addresses the optimization and stabilization problems of nonlinear systems subject to parameter uncertainties. The methodology is based on a fuzzy logic approach and an improved genetic algorithm (GA). In order to analyze the system stability, the TSK fuzzy plant model is employed to describe the dynamics of the uncertain nonlinear plant. A fuzzy controller is then obtained to close the feedback loop. The stability conditions are derived. The feedback gains of the fuzzy controller and the solution for meeting the stability conditions are determined using the improved GA. In order to obtain the optimal fuzzy controller, the membership functions of the fuzzy controller are obtained automatically by minimizing a defined fitness function using the improved GA under the consideration of the system stability. An application example on stabilizing a two-link robot arm will be given.

I. INTRODUCTION

Fuzzy control is one of the useful control techniques for uncertain and ill-defined nonlinear systems. Control actions of a fuzzy controller are described by some linguistic rules. This property makes the control algorithm easy to understand. The early design of fuzzy controllers is heuristic. It incorporates the experience or knowledge of the designer into the rules of the fuzzy controller, which is fine tuned based on trial and error. In order to have a systematic tuning procedure, a fuzzy controller implemented by a neural network was proposed in [8-9]. Through the use of tuning methods, fuzzy rules can be generated automatically. Genetic algorithm (GA) is a powerful searching algorithm [6]. It has been applied to fuzzy systems in order to generate the membership functions and/or the rule sets [28-31]. These methodologies make the design simple; however, they do not guarantee the system stability and robustness.

In order to investigate the system stability, the TSK fuzzy plant model approach was proposed [1-2, 16, 24, 39-40]. A nonlinear system is modelled as a weighted sum of some simple sub-systems. It gives a fixed structure to some of the nonlinear systems and facilitates the analysis of the systems. There are two ways to obtain the fuzzy plant model: 1) by performing identification methods through the use of the input-output data of the plant [1-2, 16, 24], 2) by deriving directly from the mathematical model of the nonlinear plant. Stability of a fuzzy system formed by a fuzzy plant model and a fuzzy controller has been investigated recently. Different stability conditions have been obtained by employing Lyapunov stability theory [4, 7, 11], passivity theory [20] and other methods [17, 23, 26]. Most of the fuzzy controllers proposed are functions of the grades of membership of the fuzzy plant model's membership functions. Hence, the membership functions of the fuzzy plant model must be known. It implies that the parameters of the nonlinear plant must be known, or be constant when the identification method is used to derive the fuzzy plant model. Practically, the parameters of many nonlinear plants will change during the operation, e.g. the load of a dc-dc converter, the number of passengers on board a train. In these cases, the robustness property of the fuzzy controller is an important concern. Robust analysis of fuzzy control systems can be found [10, 12-15, 18-19, 21-22, 25, 27, 32, 34-35]. In most of these works, only a stability and robustness testing condition is provided. The determination of the parameters (e.g. gains and membership functions) of the fuzzy controller and the system performance have seldom been discussed.

In order to have a systematic method to obtain a fuzzy controller that guarantees the system stability, robustness, optimality and good performance, a fuzzy controller derived from an improved GA [36] is proposed. In this paper, the contributions are sevenfold. 1) Stability conditions for fuzzy control system subject to parameter uncertainties are derived [18, 22, 32, 34-35]. 2) The parameters of the fuzzy controller are obtained using an improved GA based on the stability conditions. 3) The derived stability conditions are solved using the improved GA. 4) The membership functions of the fuzzy controller are obtained automatically using an improved GA to achieve the optimal system performance. The concern is not only on stability but also performance. 5) New genetic operators are introduced in the improved GA [36]. It will be shown that the improved GA performs better than the traditional GA [6] based on the benchmark De Jong's test functions [37]. 6) The improved GA is implemented in floating-point numbers. Without coding and decoding, the processing time is shorter than that of the traditional GA [6, 37]. 7) The improved GA needs only one user-input parameter (population size), instead of three, for its implementation. This makes the improved GA simple and easy to use, especially for the users who do not have too much knowledge on tuning.

This paper is organized as follows. The fuzzy plant model and fuzzy controller are presented in section II. The fuzzy control system subject to parameter uncertainties are analyzed in section III. The stability conditions will be derived. The improved GA are presented in section IV. De Jong's Test Functions [3-4, 6, 17] are used as the benchmark test functions to examine the applicability and efficiency of the improved GA in section V. The problems of solving the derived stability conditions, obtaining the feedback gains of the fuzzy controller, and optimizing the system performance using the improved GA are presented in section VI. An application example on stabilizing a two-link robot arm using the proposed fuzzy controller will be presented. A conclusion will be drawn in section VIII.

II. TSK Fuzzy Plant Model with Parameter Uncertainties and Fuzzy Controller

A fuzzy control system can be regarded as a nonlinear plant subject to parameter uncertainties connected with a fuzzy controller in closed loop. In order to obtain a fuzzy controller, a TSK fuzzy plant model is employed to describe the dynamics of the nonlinear plant subject to parameter uncertainties.

A. TSK Fuzzy Plant Model with Parameter Uncertainties

Let p be the number of fuzzy rules describing the uncertain nonlinear plant. The i-th rule is of the following format,

Rule i : IF $f_1(\mathbf{x}(t))$ is M_1^i and … and $f_\Psi(\mathbf{x}(t))$ is M_Ψ^i

$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\,\mathbf{u}(t) \tag{1}$$

where M_α^i is a fuzzy term of rule i corresponding to the function $f_\alpha(\mathbf{x}(t))$ containing the parameter uncertainties of the nonlinear plant, $\alpha = 1, 2, \ldots, \Psi$, $i = 1, 2, \ldots, p$, Ψ is a positive integer; $\mathbf{A}_i \in \Re^{n\times n}$ and $\mathbf{B}_i \in \Re^{n\times m}$ are known constant system and input matrices respectively; $\mathbf{x}(t) \in \Re^{n\times 1}$ is the system state vector and $\mathbf{u}(t) \in \Re^{m\times 1}$ is the input vector. The inferred system is given by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p} w_i(\mathbf{x}(t))\big(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\big), \tag{2}$$

where,

$$\sum_{i=1}^{p} w_i(\mathbf{x}(t)) = 1,\; w_i(\mathbf{x}(t)) \in [0,\quad 1] \text{ for all } i \tag{3}$$

$$w_i(\mathbf{x}(t)) = \frac{\mu_{\mathrm{M}_1^i}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^i}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{M}_\Psi^i}(f_\Psi(\mathbf{x}(t)))}{\sum_{k=1}^{p}\Big(\mu_{\mathrm{M}_1^k}(f_1(\mathbf{x}(t))) \times \mu_{\mathrm{M}_2^k}(f_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{M}_\Psi^k}(f_\Psi(\mathbf{x}(t)))\Big)} \tag{4}$$

is a nonlinear function of $\mathbf{x}(t)$ and $\mu_{\mathrm{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ is the membership function corresponding to M_α^i. The value of $\mu_{\mathrm{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ is unknown as $f_\alpha(\mathbf{x}(t))$ is related to parameter uncertainties of the nonlinear plant. A fuzzy controller will be obtained based on the TSK fuzzy plant model of (2).

B. Fuzzy Controller

A fuzzy controller with *c* fuzzy rules is to be designed for the plant. The *j*-th rule of the fuzzy controller is of the following format,

Rule *j*: IF $g_1(\mathbf{x}(t))$ is N_1^j and … and $g_\Omega(\mathbf{x}(t))$ is N_Ω^j

THEN $\mathbf{u}(t) = \mathbf{G}_j\mathbf{x}(t) + \mathbf{r}$ (5)

where N_β^j is a fuzzy term of rule *j* corresponding to the function $g_\beta(\mathbf{x}(t))$, $\beta = 1, 2, \ldots, \Omega$, $j = 1, 2, \ldots, c$, Ω is a positive integer; $\mathbf{G}_j \in \Re^{m\times n}$ is the feedback gain of rule *j* to be designed, $\mathbf{r} \in \Re^{n\times 1}$ is the reference input vector. The inferred output of the fuzzy controller is given by,

$$\mathbf{u}(t) = \sum_{j=1}^{c} m_j(\mathbf{x}(t))\big(\mathbf{G}_j\mathbf{x}(t) + \mathbf{r}\big) \tag{6}$$

where,

$$\sum_{j=1}^{c} m_j(\mathbf{x}(t)) = 1,\; m_j(\mathbf{x}(t)) \in [0,\quad 1] \text{ for all } j \tag{7}$$

$$m_j(\mathbf{x}(t)) = \frac{\mu_{\mathrm{N}_1^j}(g_1(\mathbf{x}(t))) \times \mu_{\mathrm{N}_2^j}(g_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{N}_\Omega^j}(g_\Omega(\mathbf{x}(t)))}{\sum_{k=1}^{c}\Big(\mu_{\mathrm{N}_1^k}(g_1(\mathbf{x}(t))) \times \mu_{\mathrm{N}_2^k}(g_2(\mathbf{x}(t))) \times \cdots \times \mu_{\mathrm{N}_\Omega^k}(g_\Omega(\mathbf{x}(t)))\Big)} \tag{8}$$

is a nonlinear function of $\mathbf{x}(t)$ and $\mu_{\mathrm{N}_\beta^j}(g_\beta(\mathbf{x}(t)))$ is the membership function corresponding to N_β^j to be designed.

C. Fuzzy Control System

In order to carry out the analysis, the closed-loop fuzzy system should be obtained. From (2) and (6), the fuzzy control system is given by,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(t))m_j(\mathbf{x}(t))\big(\mathbf{H}_{ij}\mathbf{x}(t) + \mathbf{B}_i\mathbf{r}\big) \tag{9}$$

where,

$$\mathbf{H}_{ij} = \mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j \tag{10}$$

III. Stability Analysis

In the following, the stability of the fuzzy control system of (9) subject to parameter uncertainties will be analysed [18, 32, 34-35]. Consider the Taylor series,

$$\mathbf{x}(t+\Delta t)=\mathbf{x}(t)+\dot{\mathbf{x}}(t)\Delta t+\mathbf{o}(\Delta t) \tag{11}$$

where $\mathbf{o}(\Delta t)=-\mathbf{x}(t)-\dot{\mathbf{x}}(t)\Delta t+\mathbf{x}(t+\Delta t)$ is the error term and $\Delta t>0$,

$$\lim_{\Delta t\to 0^+}\frac{\|\mathbf{o}(\Delta t)\|}{\Delta t}=0 \tag{12}$$

From (9) and (11), writing $w_i(\mathbf{x}(t))$ as w_i and $m_j(\mathbf{x}(t))$ as m_j, and multiplying a transformation matrix $\mathbf{T}\in\Re^{n\times n}$ of rank n to both sides, we have

$$\begin{aligned}\mathbf{Tx}(t+\Delta t)&=\mathbf{Tx}(t)+\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{T}\big(\mathbf{H}_{ij}\mathbf{x}(t)+\mathbf{B}_i\mathbf{r}\big)\Delta t+\mathbf{To}(\Delta t)\\&=\left(\mathbf{I}+\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right)\mathbf{Tx}(t)+\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\Delta t+\mathbf{To}(\Delta t)\end{aligned}$$

The reason for introducing $\mathbf{T}$ will be given at the end of this section. Taking norm on both sides of the above equation,

$$\|\mathbf{Tx}(t+\Delta t)\|\le\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\left(\mathbf{I}+\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right)\right\|\|\mathbf{Tx}(t)\|+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\Delta t\right\|+\|\mathbf{To}(\Delta t)\| \tag{13}$$

where $\|\cdot\|$ denotes the l_2 norm for vectors and l_2 induced norm for matrices. From (13),

$$\|\mathbf{Tx}(t+\Delta t)\|\le\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\left\|\mathbf{I}+\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right\|\|\mathbf{Tx}(t)\|+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\Delta t\right\|+\|\mathbf{To}(\Delta t)\|$$

$$\begin{aligned}\Rightarrow\|\mathbf{Tx}(t+\Delta t)\|-\|\mathbf{Tx}(t)\|\le&\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\left(\left\|\mathbf{I}+\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right\|-1\right)\|\mathbf{Tx}(t)\|\\&+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\Delta t\right\|+\|\mathbf{To}(\Delta t)\|\end{aligned}$$

$$\begin{aligned}\Rightarrow\lim_{\Delta t\to 0^+}\frac{\|\mathbf{Tx}(t+\Delta t)\|-\|\mathbf{Tx}(t)\|}{\Delta t}\le&\lim_{\Delta t\to 0^+}\Big[\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\left(\left\|\mathbf{I}+\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right\|-1\right)\|\mathbf{Tx}(t)\|\\&+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\Delta t\right\|+\|\mathbf{To}(\Delta t)\|\Big]/\Delta t\end{aligned} \tag{14}$$

From (12) and (14),

$$\begin{aligned}\frac{\mathrm{d}\|\mathbf{Tx}(t)\|}{\mathrm{d}t}&\le\lim_{\Delta t\to 0^+}\frac{\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\left(\left\|\mathbf{I}+\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right\|-1\right)}{\Delta t}\|\mathbf{Tx}(t)\|+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\right\|\\&\le\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right]\|\mathbf{Tx}(t)\|+\left\|\sum_{i=1}^{p}\sum_{j=1}^{c}w_i m_j\mathbf{TB}_i\mathbf{r}\right\|\end{aligned} \tag{15}$$

where,

$$\mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right]=\lim_{\Delta t\to 0^+}\frac{\left\|\mathbf{I}+\mathbf{TH}_{ij}\mathbf{T}^{-1}\Delta t\right\|-1}{\Delta t}=\lambda_{\max}\left(\frac{\mathbf{TH}_{ij}\mathbf{T}^{-1}+(\mathbf{TH}_{ij}\mathbf{T}^{-1})^{*}}{2}\right) \tag{16}$$

is the corresponding matrix measure [5] of the induced matrix norm of $\left\|\mathbf{TH}_{ij}\mathbf{T}^{-1}\right\|$ (or the logarithmic derivative of $\left\|\mathbf{TH}_{ij}\mathbf{T}^{-1}\right\|$); $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue, * denotes the conjugate transpose. From (15),

$$\frac{\mathrm{d}\|\mathbf{Tx}(t)\|}{\mathrm{d}t} \le \sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right]\|\mathbf{Tx}(t)\| + \left\|\sum_{i=1}^{p} w_i \mathbf{TB}_i\mathbf{r}\right\|$$

$$\le \sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right]\|\mathbf{Tx}(t)\| + \left\|\sum_{i=1}^{p} w_i \mathbf{TB}_i\mathbf{r}\right\| \tag{17}$$

If $\mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right]$ satisfies the following inequality,

$$\mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right] \le -\varepsilon \text{ for all } i \text{ and } j. \tag{18}$$

where ε is a designed nonzero positive constant, it can be proved that (17) implies a stable system of (9). Before conducting this proof, consider the following inequality obtained from (17) and (18).

$$\frac{\mathrm{d}\|\mathbf{Tx}(t)\|}{\mathrm{d}t} \le -\sum_{i=1}^{p}\sum_{j=1}^{c} w_i m_j \varepsilon\|\mathbf{Tx}(t)\| + \sum_{i=1}^{p} w_j \|\mathbf{TB}_i\mathbf{r}\|$$

$$= -\varepsilon\|\mathbf{Tx}(t)\| + \sum_{i=1}^{p} w_i \|\mathbf{TB}_i\mathbf{r}\|$$

$$\Rightarrow \left(\frac{\mathrm{d}\|\mathbf{Tx}(t)\|}{\mathrm{d}t} + \varepsilon\|\mathbf{Tx}(t)\|\right)e^{\varepsilon(t-t_o)} \le \sum_{i=1}^{p} w_i \|\mathbf{TB}_i\mathbf{r}\| e^{\varepsilon(t-t_o)}$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\|\mathbf{Tx}(t)\|e^{\varepsilon(t-t_o)}\right) \le \sum_{i=1}^{p} w_i \|\mathbf{TB}_i\mathbf{r}\| e^{\varepsilon(t-t_o)} \tag{19}$$

where $t_o < t$ is an arbitrary initial time. Based on (19), there are two cases to investigate the system behavior: $\mathbf{r} = \mathbf{0}$ and $\mathbf{r} \ne \mathbf{0}$. For the former case, it can be shown that if the condition of (18) is satisfied, the closed-loop system of (9) is exponentially stable, and $\|\mathbf{x}(t)\| \to 0$ as $t \to \infty$.

Proof. For $\mathbf{r} = \mathbf{0}$, from (19),

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\mathbf{Tx}(t)\|e^{\varepsilon(t-t_o)} \le 0$$

$$\Rightarrow \|\mathbf{Tx}(t)\|e^{\varepsilon(t-t_o)} \le \|\mathbf{Tx}(t_o)\|$$

$$\Rightarrow \|\mathbf{Tx}(t)\| \le \|\mathbf{Tx}(t_o)\|e^{-\varepsilon(t-t_o)} \tag{20}$$

Since ε is a positive value, $\|\mathbf{Tx}(t)\| \to 0$ as $t \to \infty$. And,

$$\sigma_{\min}(\mathbf{T}^{\mathrm{T}}\mathbf{T})\|\mathbf{x}(t)\|^2 \le \|\mathbf{Tx}(t)\|^2 = \mathbf{x}(t)^{\mathrm{T}}\mathbf{T}^{\mathrm{T}}\mathbf{Tx}(t) \le \sigma_{\max}(\mathbf{T}^{\mathrm{T}}\mathbf{T})\|\mathbf{x}(t)\|^2 \tag{21}$$

where $\sigma_{\max}(\mathbf{T}^{\mathrm{T}}\mathbf{T})$ and $\sigma_{\min}(\mathbf{T}^{\mathrm{T}}\mathbf{T})$ denote the maximum and minimum singular values of $\mathbf{T}^{\mathrm{T}}\mathbf{T}$ respectively. As $\mathbf{T}^{\mathrm{T}}\mathbf{T}$ is symmetric positive definite ($\mathbf{T}$ has rank n), from (21), $\|\mathbf{Tx}(t)\| \to 0$ only when $\|\mathbf{x}(t)\| \to 0$. **QED**

For the latter case of $\mathbf{r} \ne \mathbf{0}$, the closed-loop system of (9) is input-to-state stable, i.e. the system states are bounded if the condition of (18) is satisfied and **r** is bounded.

Proof. For $\mathbf{r} \ne \mathbf{0}$, from (19),

$$\|\mathbf{Tx}(t)\|e^{\varepsilon(t-t_o)} \le \|\mathbf{Tx}(t_o)\| + \int_{t_o}^{t}\sum_{i=1}^{p} w_i \|\mathbf{TB}_i\mathbf{r}\| e^{\varepsilon(\tau-t_o)}\mathrm{d}\tau$$

$$\Rightarrow \|\mathbf{Tx}(t)\|e^{\varepsilon(t-t_o)} \le \|\mathbf{Tx}(t_o)\| + \|\mathbf{TB}_i\mathbf{r}\|_{\max}\int_{t_o}^{t} e^{\varepsilon(\tau-t_o)}\mathrm{d}\tau$$

where $\max_i \|\mathbf{TB}_i\mathbf{r}\|_{\max} \ge \|\mathbf{TB}_i\mathbf{r}\|$. Then

$$\|\mathbf{Tx}(t)\|e^{\varepsilon(t-t_o)} \le \|\mathbf{Tx}(t_o)\| + \frac{\|\mathbf{TB}_i\mathbf{r}\|_{\max}}{\varepsilon}(e^{\varepsilon(t-t_o)}-1)$$

$$\Rightarrow \|\mathbf{Tx}(t)\| \le \|\mathbf{Tx}(t_o)\|e^{-\varepsilon(t-t_o)} + \frac{\|\mathbf{TB}_i\mathbf{r}\|_{\max}}{\varepsilon}(1-e^{-\varepsilon(t-t_o)}) \qquad (22)$$

Since the right hand side of (22) is finite if $\mathbf{r}$ is bounded, the system states are also bounded.

QED

The stability conditions of the closed-loop fuzzy system can be summarized by the following lemma:

Lemma 1. *The fuzzy control system, subject to parameter uncertainties, as given by* (9) *is exponentially stable for* $\mathbf{r} = \mathbf{0}$ *or input-to-state stable for* $\mathbf{r} \ne \mathbf{0}$ *if* $\mathbf{TH}_{ij}\mathbf{T}^{-1}$ *is designed such that,*

$\mu\left[\mathbf{TH}_{ij}\mathbf{T}^{-1}\right] \le -\varepsilon$ *for all i and j*

where ε *is a nonzero positive constant scalar.*

It should be noted that with the use of a suitable transformation matrix $\mathbf{T}$, any Hurwitz matrix having a positive or zero matrix measure can be transformed into another matrix having a negative matrix measure (see (18)). The stability conditions derived can then be applied. The problem left is how to find such a matrix $\mathbf{T}$ for a given system. This will be discussed later. From the above derivation and Lemma 1, we also see the system stability is not affected by the membership functions of the fuzzy controller. So, the membership functions of the fuzzy controller can be determined using GA to obtain the optimal system performance.

IV. IMPROVED GENETIC ALGORITHM

Genetic algorithms (GAs) are powerful searching algorithms that can be used to solve optimization problems. The traditional GA process [6] is shown in Fig. 1. First, a population of chromosomes is created. Second, the chromosomes are evaluated by a defined fitness function. Third, some of the chromosomes are selected for performing genetic operations. Forth, genetic operations of crossover and mutation are performed. The produced offspring replace their parents in the initial population. This GA process repeats until a user-defined criterion is reached. However, a superior offspring is not guaranteed to produce in each reproduction process. In this paper, the traditional GA is modified and new genetic operators are introduced to improve its performance. Our improved GA is implemented by floating-point numbers, and the processing time is shorter than the GA implemented by binary numbers as the coding and decoding processes are not needed [6]. Two parameters, the probabilities of crossover and mutation, in the traditional GA are no longer needed in the improved GA. Only the population size is needed to be defined. The improved GA process is shown in Fig. 2. Its details will be given as follows.

A. Initial Population

The initial population is a potential solution set *P*. The first set of population is usually generated randomly.

$$P = \{\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_{pop\_size}\} \quad (23)$$

$$\mathbf{p}_i = \begin{bmatrix} p_{i_1} & p_{i_2} & \cdots & p_{i_j} & \cdots & p_{i_{no\_vars}} \end{bmatrix},\ i = 1, 2, \ldots, pop\_size;\ j = 1, 2, \ldots, no\_vars \quad (24)$$

$$para_{\min}^{j} \le p_{i_j} \le para_{\max}^{j}\ \ i = 1, 2, \ldots, pop\_size;\ j = 1, 2, \ldots, no\_vars \quad (25)$$

where *pop_size* denotes the population size; *no_vars* denotes the number of variables to be tuned; p_{i_j}, $i = 1, 2, \ldots, pop\_size$; $j = 1, 2, \ldots, no\_vars$, are the parameters to be tuned; $para_{\min}^{j}$ and $para_{\max}^{j}$ are the minimum and maximum values of the parameter p_{i_j}. It can be seen from (23) to (25) that the potential solution set *P* contains some candidate solutions $\mathbf{p}_i$ (chromosomes). The chromosome $\mathbf{p}_i$ contains some variables p_{i_j} (genes).

B. Evaluation

Each chromosome in the population will be evaluated by a defined fitness function. The better chromosomes will return higher values in this process. The fitness function to evaluate a chromosome in the population can be written as,

$$fitness = f(\mathbf{p}_i) \quad (26)$$

The form of the fitness function depends on the application.

C. Selection

Two chromosomes in the population will be selected to undergo genetic operations for reproduction. It is believed that the high potential parents will produce better offspring (survival of the best ones). The chromosome having a higher fitness value should therefore have a higher chance to be selected. The selection can be done by assigning a probability q_i to the chromosome $\mathbf{p}_i$:

$$q_i = \frac{f(\mathbf{p}_i)}{\sum_{j=1}^{pop\_size} f(\mathbf{p}_j)},\ i = 1, 2, \ldots, pop\_size \quad (27)$$

The cumulative probability $\hat{q}_i$ for the chromosome $\mathbf{p}_i$ is defined as,

$$\hat{q}_i = \sum_{j=1}^{i} q_j,\ i = 1, 2, \ldots, pop\_size \quad (28)$$

The selection process starts by randomly generating a nonzero floating-point number, $d \in [0 \quad 1]$, for each chromosome. Then, the chromosome $\mathbf{p}_i$ is chosen if $\hat{q}_{i-1} < d \le \hat{q}_i$, $i = 1, 2, \ldots, pop\_size$, and $\hat{q}_0 = 0$. It can be observed from this selection process that a chromosome having a larger $f(\mathbf{p}_i)$ will have a higher chance to be selected. Consequently, the best chromosomes will get more copies, the average will stay and the worst will die off. In the selection process, only two chromosomes will be selected to undergo the genetic operations.

D. Genetic Operations

The genetic operations are to generate some new chromosomes (offspring) from their parents after the selection process. They include the averaging and the mutation operations. The average operation is mainly for exchanging information from the two parents obtained in the selection process. The operation is realized by taking the average of the parents. For instance, if the two selected chromosomes are $\mathbf{p}_1$ and $\mathbf{p}_2$, the offspring generated by the averaging process is given by,

$$\mathbf{os} = \begin{bmatrix} os_1 & os_2 & \cdots & os_{no\_vars} \end{bmatrix} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \tag{29}$$

This offspring (29) will then undergo the mutation operation. The mutation operation is to change the genes of the chromosomes. Consequently, the features of the chromosomes inherited from their parents can be changed. Three new offspring will be generated by the mutation operation,

$$\mathbf{nos}_j = \begin{bmatrix} os_1^j & os_2^j & \cdots & os_{no\_vars}^j \end{bmatrix} + \begin{bmatrix} b_1 \Delta nos_1 & b_2 \Delta nos_2 & \cdots & b_{no\_vars} \Delta nos_{no\_vars} \end{bmatrix},\ j = 1, 2, 3 \tag{30}$$

where b_i, i = 1, 2, …, *no_vars*, can only take the value of 0 or 1, Δnos_i, i = 1, 2, …, *no_vars*, are randomly generated floating-point numbers such that $para_{\min}^i \le os_i^j + \Delta nos_i \le para_{\max}^i$. The first new offspring (j = 1) is obtained according to (30) with that only one b_i (i being randomly generated within the range) is allowed to be 1 and all the others are zeros. The second new offspring is obtained according to (30) with that some b_i chosen randomly are set to be 1 and others are zeros. The third new offspring is obtained according to (30) with all b_i = 1. These three new offspring will then be evaluated using the fitness function of (26). The one with the largest fitness value f_l will replace the chromosome with the smallest fitness value f_s in the population if $f_l > f_s$.

After the operation of selection, averaging, and mutation, a new population is generated. This new population will repeat the same process. Such an iterative process can be terminated when the result reaches a defined condition, e.g. the change of the fitness values between the current and the previous iteration is less than 0.001. For the traditional GA process depicted in Fig. 1, the offspring generated may not be better than their parents. This implies that the searched target is not necessarily approached monotonically after each iteration. Under the proposed improved GA process, however, if $f_l < f_s$, the previous population is used again in the next genetic cycle.

V. BENCHMARK TEST FUNCTIONS

De Jong's Test Functions [3-4, 6, 17] are used as the benchmark test functions to examine the applicability and efficiency of the improved GA. Five test functions, $f_i(\mathbf{x})$, i = 1, 2, 3, 4, 5, will be used, where $\mathbf{x} = \begin{bmatrix} x_1 & x_1 & \cdots & x_n \end{bmatrix}$. n is an integer denoting the dimension of the vector $\mathbf{x}$.

The five test functions are defined as follows,

$$f_1(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 \,, \; -5.12 \le x_i \le 5.12 \tag{31}$$

where n = 3 and the minimum point is at $f_1(0, 0, 0) = 0$

$$f_2(\mathbf{x}) = \sum_{i=1}^{n-1} \left(100 \times \left(x_{i+1} - x_i{}^2\right)^2 + \left(x_i - 1\right)^2\right), \; -2.048 \le x_i \le 2.048 \tag{32}$$

where n = 2 and the minimum point is at $f_2(0, 0) = 0$.

$$f_3(\mathbf{x}) = 6 \times n + \sum_{i=1}^{n} \mathit{floor}(x_i)\,, \; -5.12 \le x_i \le 5.12 \tag{33}$$

where n = 5 and the minimum point is at $f_3([-5.12, -5], \ldots, [-5.12, -5]) = 0$. The floor function, $\mathit{floor}(\cdot)$, is to round down the argument to an integer.

$$f_4(\mathbf{x}) = \sum_{i=1}^{n} i \times x_i^4 + \mathit{Gauss}(0,1)\,, \; -1.28 \le x_i \le 1.28 \tag{34}$$

where n = 30 and the minimum point is at $f_4(0, \ldots, 0) = 0$. *Gauss*(0, 1) is a function to generate uniformly a floating-point number between 0 and 1 inclusively.

$$f_5(\mathbf{x}) = \frac{1}{k} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} \left(x_i - a_{ij}\right)^6}\,, \quad -65.356 \le x_i \le 65.356 \tag{35}$$

where

$$\mathbf{a} = \{a_{ij}\} = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & -16 & 0 & 16 & 32 & -32 & -16 & 0 & 16 & 32 & -32 & -16 & 0 & 16 & 32 & -32 & -16 & 0 & 16 & 32 \\ 32 & 32 & 32 & 32 & 32 & -16 & -16 & -16 & -16 & -16 & 0 & 0 & 0 & 0 & 0 & 16 & 16 & 16 & 16 & 16 & 32 & 32 & 32 & 32 & 32 \end{bmatrix},$$

k = 500 and the minimum point is at $f_5(-32, -32) \approx 1$. It can be seen that the minimum values of all functions in the defined domain are zero except for $f_5(\mathbf{x})$. The fitness function for f_1 to f_4 is defined as,

$$\mathit{fitness} = \frac{1}{1 + f_i(\mathbf{x})}\,, \; i = 1, 2, 3, 4. \tag{36}$$

and the fitness function for f_5 is defined as,

$$\mathit{fitness} = \frac{1}{f_5(\mathbf{x})} \tag{37}$$

The improved GA goes through these five test functions. The results are compared with those obtained by the traditional GA [5]. For each test function, the simulation takes 500 iterations and the population size is 20. Each parameter of the traditional GA is encoded into a 40-bit chromosome and the probabilities of crossover and mutation are 0.25 and 0.03 respectively. The initial values of $\mathbf{x}$ in the population for a test function are set to be the same for both GAs. For tests 1 to 5, the initial values are $[1 \;\; 1 \;\; 1]$, $[0.5 \;\; 0.5]$, $[1 \;\; \cdots \;\; 1]$, $[0.5 \;\; \cdots \;\; 0.5]$ and $[10 \;\; \cdots \;\; 10]$

respectively. The results of the average fitness values over 30 times of simulations of the improved and traditional GAs are shown in Fig. 3 and tabulated in Table I. It can be seen from Fig. 3 that the performance of the improved GA is better than that of the traditional GA. As seen from Table I, the processing time of the improved GA is much shorter than that of the traditional GA.

VI. Solving the Stability Conditions, Obtaining the Feedback Gains and Optimizing the System Performance

In this section, the problems of solving the stability conditions derived in the previous section, obtaining the feedback gains of the fuzzy controller and optimizing the system performance will be tackled using the improved GA.

A. Solving the Stability Conditions and Obtaining the Feedback Gains

From Lemma 1, the uncertain fuzzy control system is stable if there exists a transformation matrix **T** satisfying the following conditions,

$$\mu\left[\mathbf{T}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\mathbf{T}^{-1}\right] \leq -\varepsilon,\ i = 1, 2, \ldots, p,\ j = 1, 2, \ldots, c \tag{38}$$

The objectives are to find the $\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1n} \\ T_{21} & T_{22} & \cdots & T_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_{n1} & T_{n2} & \cdots & T_{nn} \end{bmatrix}$ and $\mathbf{G}_j = \begin{bmatrix} G_{11}^j & G_{11}^j & \cdots & G_{1n}^j \\ G_{21}^j & G_{22}^j & \cdots & G_{2n}^j \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}^j & G_{m2}^j & \cdots & G_{mn}^j \end{bmatrix}$ such that the above conditions are satisfied. The fitness function is defined as follows,

$$fitness = \sum_{i=1}^{p}\sum_{j=1}^{c} n_{ij}\,\mu\left[\mathbf{T}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\mathbf{T}^{-1}\right] \tag{39}$$

where $n_{ij} \geq 0$, i = 1, 2, …, p, j =1, 2, …, c, are constant scalar. The problems of finding **T** and $\mathbf{G}_j$ are now formulated to a minimization problem. The aim is to minimize the fitness function of (39) with **T** and $\mathbf{G}_j$ using the improved GA. As **T** and $\mathbf{G}_j$ are the variables of the fitness function of (39), they will be used to form the genes of the chromosomes. The finding of the solution to this minimization problem, however, does not imply that the conditions of (38) are satisfied. Hence, different n_{ij}, i = 1, 2, …, p, j =1, 2, …, c, may need to be tried to weight the conditions of (38) in order to change the significance of different terms on the right hand side of (39). For instance, one of the terms in (39) is very positive, which returns a very large fitness value. Under this case, the conditions of (38) are not satisfied. A large value of n_{ij} corresponding to that term can be used to attenuate the effect of that term to the fitness function. This may help the GA process to find a solution which satisfies the conditions of (38) during the minimizing process.

B. Optimizing the System Performance

After **T** and $\mathbf{G}_j$ have been determined, what follows is to determine the membership functions of the fuzzy controller using the improved GA such that the performance of the uncertain fuzzy control system is optimal subject to a defined performance index. The dynamics of the uncertain fuzzy control system is restated with modification as follows,

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(t)) m_j(\mathbf{x}(t), \mathbf{z})\left(\mathbf{H}_{ij}\mathbf{x}(t) + \mathbf{B}_i\mathbf{r}\right) \tag{40}$$

where $\mathbf{z}$ is the parameter vector governing the membership functions of the fuzzy controller, e.g. the values of the means and the standard deviations of various gaussian membership functions. The fitness function (performance index) is defined as follows,

$$fitness = \int \mathbf{x}(t)^{\mathrm{T}}\mathbf{W}_x\mathbf{x}(t) + \mathbf{u}(t)^{\mathrm{T}}\mathbf{R}_u\mathbf{u}(t)\mathrm{d}t \tag{41}$$

where $\mathbf{W}_x \in \Re^{n\times n}$ and $\mathbf{R}_u \in \Re^{m\times m}$ are constant semi-positive or positive definite matrices. This fitness function is the performance index used in the conventional optimal control [3]. The optimization problem formulated here will be handled by the improved GA. $\mathbf{z}_j$, j = 1, 2, …, c, will be used to form the genes of the chromosomes for the GA process. As the fuzzy control system is an uncertain system, the nominal system parameters will be used for determining the membership functions of the fuzzy plant model. Hence, all $w_i(\mathbf{x}(t))$ in (40) are known.

The procedure to obtain the fuzzy controller using the improved GA can be summarized into the following steps.

1) Obtain the mathematical model of the nonlinear plant subject to parameter uncertainties. Convert the mathematical model into the fuzzy plant model of (2).
2) Determine the number of rules for the fuzzy controller. Solve $\mathbf{T}$ and $\mathbf{G}_j$ with the fitness function defined in (39) and $n_{ij} = 1$, i = 1, 2, …, p, j =1, 2, …, c using the improved GA. If $\mathbf{T}$ and $\mathbf{G}_j$ cannot be found, adjust n_{ij} accordingly.
3) Determine the membership functions of the fuzzy controller. Obtain the parameters of the membership functions using the improved GA to optimize the system performance with respect to the performance index of (41).

VII. Application Example

An application example will be given in this section. An MIMO two-link robot arm [38] shown in Fig. 4 is taken as the nonlinear plant. Refer to Fig. 4, m_1 is the centre of mass of link 1, m_2 is the centre of mass of link 2, m_3 is the mass of the load; l_1 is the length of link 1, l_2 is the length of link 2; l_{c_1} is the length from the joint of link 1 to its centre of mass, l_{c_2} is the length from the joint of link 2 to its centre of mass; I_1 is the lengthwise centroidal inertia of link 1, I_2 is the lengthwise centroidal inertia of link 2; θ_1 and θ_2 are the angles of the joints as shown in Fig. 4. A fuzzy controller will be obtained to stabilize the two-link robot arm using the design procedure in the previous section.

1) The system dynamics of the two-link robot arm is governed by the following dynamical equations,

$$\dot{\mathbf{x}}(t) = \left(\mathbf{A} + \Delta\mathbf{A}(\mathbf{x}(t))\right)\mathbf{x}(t) + \mathbf{B}(\mathbf{x}(t))\mathbf{u}(t) + \mathbf{E} \tag{42}$$

where $\mathbf{x}(t)=[x_1(t)\quad x_2(t)\quad x_3(t)\quad x_4(t)]^{\mathrm{T}}=[\theta_1(t)\quad \dot{\theta}_1(t)\quad \theta_2(t)\quad \dot{\theta}_2(t)]^{\mathrm{T}}$, $x_1(t)\in[x_{1_{\min}}\quad x_{1_{\max}}]=[-\pi\quad \pi]$, $x_2(t)\in[x_{2_{\min}}\quad x_{2_{\max}}]=[-3\quad 3]$, $x_3(t)\in[x_{3_{\min}}\quad x_{3_{\max}}]=[-\pi\quad \pi]$, $x_4(t)\in[x_{4_{\min}}\quad x_{4_{\max}}]=[-3\quad 3]$; $\mathbf{A}=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$,

$\Delta\mathbf{A}(\mathbf{x}(t))=\begin{bmatrix}0&0&0&0\\0&f_2(\mathbf{x}(t))hx_2(t)&0&f_1(\mathbf{x}(t))h(2x_2(t)+x_4(t))\\0&0&0&0\\0&-f_3(\mathbf{x}(t))hx_2(t)&0&-f_2(\mathbf{x}(t))h(2x_2(t)+x_4(t))\end{bmatrix}$,

$\mathbf{B}(\mathbf{x}(t))=\begin{bmatrix}0&0\\f_1(\mathbf{x}(t))&-f_2(\mathbf{x}(t))\\0&0\\-f_2(\mathbf{x}(t))&f_3(\mathbf{x}(t))\end{bmatrix}$; $\mathbf{E}=\begin{bmatrix}0\\-f_1(\mathbf{x}(t))g_1+f_2(\mathbf{x}(t))g_2\\0\\f_2(\mathbf{x}(t))g_1-f_3(\mathbf{x}(t))g_2\end{bmatrix}$;

$f_1(\mathbf{x}(t))=\dfrac{M_{22}}{M_{11}M_{22}-M_{12}M_{21}}$, $f_2(\mathbf{x}(t))=\dfrac{M_{12}}{M_{11}M_{22}-M_{12}M_{21}}$,

$f_3(\mathbf{x}(t))=\dfrac{M_{11}}{M_{11}M_{22}-M_{12}M_{21}}$;

$M_{11}=I_1+I_2+m_1l_{c_1}{}^2+m_2\left(l_1{}^2+l_{c_2}{}^2+2l_1l_{c_2}\cos(x_3(t))\right)+m_3\left(l_1{}^2+l_2{}^2+2l_1l_2\cos(x_3(t))\right)$,

$M_{12}=M_{21}=I_1+m_2\left(l_{c_2}{}^2+l_1l_{c_2}\cos(x_3(t))\right)+m_3\left(l_2{}^2+l_1l_2\cos(x_3(t))\right)$,

$M_{22}=I_2+m_2l_{c_2}{}^2+m_3l_2{}^2$, $h=m_2l_1l_{c_2}\sin(x_3(t))$,

$g_1=m_1l_{c_1}g\cos(x_1(t))+m_2g\left(l_{c_2}\cos(x_1(t)+x_3(t))+l_1\cos(x_1(t))\right)$,

$g_2=m_2l_{c_2}g\cos(x_1(t)+x_3(t))$; $m_1=10kg$, $m_2=10kg$, $m_3\in[0\text{kg}\quad 3\text{kg}]$, $I_1=5kgm^2$, $I_2=3.5\text{kgm}^2$, $l_1=1\text{m}$, $l_2=0.5\text{m}$, $l_{c_1}=0.8\text{m}$, $l_{c_2}=0.2\text{m}$ $g=9.8\text{ms}^{-2}$; $\mathbf{u}(t)=[u_1(t)\quad u_2(t)]^{\mathrm{T}}$ is the control inputs. It should be noted that $M_{11}M_{22}-M_{12}M_{21}>0$ [38] and the parameter uncertainties of m_3 are included in $f_1(\mathbf{x}(t))$, $f_2(\mathbf{x}(t))$ and $f_3(\mathbf{x}(t))$. Letting,

$$\mathbf{u}(t)=\mathbf{u}_1(t)+\mathbf{R}(\mathbf{x}(t)) \tag{43}$$

where,

$$\mathbf{R}=\begin{bmatrix}-hx_4(t)(2x_2(t)+x_4(t))+g_1\\hx_2(t)^2+g_2\end{bmatrix} \tag{44}$$

such that $\Delta\mathbf{A}\mathbf{x}(t)+\mathbf{B}(\mathbf{x}(t))\mathbf{R}(\mathbf{x}(t))+\mathbf{E}=\mathbf{0}$. From (43) and (42), we have,

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}(\mathbf{x}(t))\mathbf{u}_1(t) \tag{45}$$

The nonlinear plant of (45) can be represented by an 8-rule TSK fuzzy plant model. The 8 rules are shown as follows:

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i AND $f_2(\mathbf{x}(t))$ is M_2^i AND $f_3(\mathbf{x}(t))$ is M_3^i

THEN $\dot{\mathbf{x}}(t)=\mathbf{A}_i\mathbf{x}(t)+\mathbf{B}_i\mathbf{u}_1(t)$, i =1, 2, …, 8. (46)

where,

$$\mathbf{A}_1=\mathbf{A}_2=\mathbf{A}_3=\mathbf{A}_4=\mathbf{A}_5=\mathbf{A}_6=\mathbf{A}_7=\mathbf{A}_8=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix};\ \mathbf{B}_1=\begin{bmatrix}0&0\\f_{1_{\min}}&-f_{2_{\min}}\\0&0\\-f_{2_{\min}}&f_{3_{\min}}\end{bmatrix},$$

$$\mathbf{B}_2=\begin{bmatrix}0&0\\f_{1_{\min}}&-f_{2_{\min}}\\0&0\\-f_{2_{\min}}&f_{3_{\max}}\end{bmatrix},\quad \mathbf{B}_3=\begin{bmatrix}0&0\\f_{1_{\min}}&-f_{2_{\max}}\\0&0\\-f_{2_{\max}}&f_{3_{\min}}\end{bmatrix},\quad \mathbf{B}_4=\begin{bmatrix}0&0\\f_{1_{\min}}&-f_{2_{\max}}\\0&0\\-f_{2_{\max}}&f_{3_{\max}}\end{bmatrix},$$

$$\mathbf{B}_5=\begin{bmatrix}0&0\\f_{1_{\max}}&-f_{2_{\min}}\\0&0\\-f_{2_{\min}}&f_{3_{\min}}\end{bmatrix},\quad \mathbf{B}_6=\begin{bmatrix}0&0\\f_{1_{\max}}&-f_{2_{\min}}\\0&0\\-f_{2_{\min}}&f_{3_{\max}}\end{bmatrix},\quad \mathbf{B}_7=\begin{bmatrix}0&0\\f_{1_{\max}}&-f_{2_{\max}}\\0&0\\-f_{2_{\max}}&f_{3_{\min}}\end{bmatrix},$$

$$\mathbf{B}_8=\begin{bmatrix}0&0\\f_{1_{\max}}&-f_{2_{\max}}\\0&0\\-f_{2_{\max}}&f_{3_{\max}}\end{bmatrix};\quad f_{1_{\min}}=0.0794,\quad f_{1_{\max}}=0.1050,\quad f_{2_{\min}}=-0.0306,$$

$f_{2_{\max}}=0.2237$, $f_{3_{\min}}=0.7752$, $f_{3_{\max}}=2.0490$. The membership functions of M_j^i, i = 1, 2, …, 8, j = 1, 2, 3, which are shown in Fig. 5, are defined as follows:

$$\mu_{\mathrm{M}_1^1}(f_1(\mathbf{x}(t))=\mu_{\mathrm{M}_1^2}(f_1(\mathbf{x}(t))=\mu_{\mathrm{M}_1^3}(f_1(\mathbf{x}(t))=\mu_{\mathrm{M}_1^4}(f_1(\mathbf{x}(t))=\frac{-f_1(\mathbf{x}(t))+f_{1_{\max}}}{f_{1_{\max}}-f_{1_{\min}}}\tag{47}$$

$$\mu_{\mathrm{M}_1^5}(f_1(\mathbf{x}(t))=\mu_{\mathrm{M}_1^6}(f_1(\mathbf{x}(t))=\mu_{\mathrm{M}_1^7}(f_1(\mathbf{x}(t))=\mu_{\mathrm{M}_1^8}(f_1(\mathbf{x}(t))=\frac{f_1(\mathbf{x}(t))-f_{1_{\min}}}{f_{1_{\max}}-f_{1_{\min}}}\tag{48}$$

$$\mu_{\mathrm{M}_2^1}(f_2(\mathbf{x}(t))=\mu_{\mathrm{M}_2^2}(f_2(\mathbf{x}(t))=\mu_{\mathrm{M}_2^5}(f_2(\mathbf{x}(t))=\mu_{\mathrm{M}_2^6}(f_2(\mathbf{x}(t))=\frac{-f_2(\mathbf{x}(t))+f_{2_{\max}}}{f_{2_{\max}}-f_{2_{\min}}}\tag{49}$$

$$\mu_{\mathrm{M}_2^3}(f_2(\mathbf{x}(t))=\mu_{\mathrm{M}_2^4}(f_2(\mathbf{x}(t))=\mu_{\mathrm{M}_2^7}(f_2(\mathbf{x}(t))=\mu_{\mathrm{M}_2^8}(f_2(\mathbf{x}(t))=\frac{f_2(\mathbf{x}(t))-f_{2_{\min}}}{f_{2_{\max}}-f_{2_{\min}}}\tag{50}$$

$$\mu_{\mathrm{M}_3^1}(f_3(\mathbf{x}(t))=\mu_{\mathrm{M}_3^3}(f_3(\mathbf{x}(t))=\mu_{\mathrm{M}_3^5}(f_3(\mathbf{x}(t))=\mu_{\mathrm{M}_3^7}(f_3(\mathbf{x}(t))=\frac{-f_3(\mathbf{x}(t))+f_{3_{\max}}}{f_{3_{\max}}-f_{3_{\min}}}\tag{51}$$

$$\mu_{\mathrm{M}_3^2}(f_3(\mathbf{x}(t))=\mu_{\mathrm{M}_3^4}(f_3(\mathbf{x}(t))=\mu_{\mathrm{M}_3^6}(f_3(\mathbf{x}(t))=\mu_{\mathrm{M}_3^8}(f_3(\mathbf{x}(t))=\frac{f_3(\mathbf{x}(t))-f_{3_{\min}}}{f_{3_{\max}}-f_{3_{\min}}}\tag{52}$$

In order to regulate $x_1(t)$ and $x_3(t)$ of the two-link robot arm, a fuzzy controller with integral control will be employed. So, the TSK fuzzy plant model of (46) has to be augmented to the one with the following rules:

Rule i: IF $f_1(\mathbf{x}(t))$ is M_1^i AND $f_2(\mathbf{x}(t))$ is M_2^i AND $f_3(\mathbf{x}(t))$ is M_3^i

THEN $\dot{\bar{\mathbf{x}}}(t)=\overline{\mathbf{A}}_i\bar{\mathbf{x}}(t)+\overline{\mathbf{B}}_i\mathbf{u}_1(t)$, i =1, 2, …, 8. (53)

where $\bar{\mathbf{x}}(t)=\begin{bmatrix}x_1(t) & x_2(t) & x_3(t) & x_4(t) & x_5(t) & x_6(t)\end{bmatrix}^{\mathrm{T}}$,

$x_5(t) = x_5(t_o) + \int_{t_o}^{\infty}\left(\theta_{1_{ref}} - x_1(t)\right)dt$ and $x_6(t) = x_6(t_o) + \int_{t_o}^{\infty}\left(\theta_{2_{ref}} - x_3(t)\right)dt$, $x_5(t_o)$ and $x_6(t_o)$ are the initial values of $x_5(t)$ and $x_6(t)$ at an arbitrary time t_o, respectively. $\theta_{1_{ref}}$ and $\theta_{2_{ref}}$ are the reference values of $x_1(t)$ and $x_3(t)$ respectively.

$$\overline{\mathbf{A}}_1 = \overline{\mathbf{A}}_2 = \overline{\mathbf{A}}_3 = \overline{\mathbf{A}}_4 = \overline{\mathbf{A}}_5 = \overline{\mathbf{A}}_6 = \overline{\mathbf{A}}_7 = \overline{\mathbf{A}}_8 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix};$$

$$\overline{\mathbf{B}}_1 = \begin{bmatrix} 0 & 0 \\ f_{1_{\min}} & -f_{2_{\min}} \\ 0 & 0 \\ -f_{2_{\min}} & f_{3_{\min}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_2 = \begin{bmatrix} 0 & 0 \\ f_{1_{\min}} & -f_{2_{\min}} \\ 0 & 0 \\ -f_{2_{\min}} & f_{3_{\max}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_3 = \begin{bmatrix} 0 & 0 \\ f_{1_{\min}} & -f_{2_{\max}} \\ 0 & 0 \\ -f_{2_{\max}} & f_{3_{\min}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{\mathbf{B}}_4 = \begin{bmatrix} 0 & 0 \\ f_{1_{\min}} & -f_{2_{\max}} \\ 0 & 0 \\ -f_{2_{\max}} & f_{3_{\max}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_5 = \begin{bmatrix} 0 & 0 \\ f_{1_{\max}} & -f_{2_{\min}} \\ 0 & 0 \\ -f_{2_{\min}} & f_{3_{\min}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_6 = \begin{bmatrix} 0 & 0 \\ f_{1_{\max}} & -f_{2_{\min}} \\ 0 & 0 \\ -f_{2_{\min}} & f_{3_{\max}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{\mathbf{B}}_7 = \begin{bmatrix} 0 & 0 \\ f_{1_{\max}} & -f_{2_{\max}} \\ 0 & 0 \\ -f_{2_{\max}} & f_{3_{\min}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{\mathbf{B}}_8 = \begin{bmatrix} 0 & 0 \\ f_{1_{\max}} & -f_{2_{\max}} \\ 0 & 0 \\ -f_{2_{\max}} & f_{3_{\max}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The fuzzy controller will be found based on the TSK fuzzy plant model of (53).

2) A four-rule fuzzy controller will be found based on the TSK fuzzy plant model of (53). The rules of the fuzzy controllers are as follows:

Rule 1: IF $x_1(t)$ is N_1^j AND $x_3(t)$ is N_2^j THEN $\mathbf{u}_1(t) = \mathbf{G}_j\overline{\mathbf{x}}(t)$, j = 1, 2, 3, 4 (54)

The output of the fuzzy controller is given by,

$$\mathbf{u}_1(t) = \sum_{j=1}^{4} m_j(\mathbf{x}(t))\mathbf{G}_j\overline{\mathbf{x}}(t) \tag{55}$$

where $m_j(\mathbf{x}(t))$ defined in (8) with $\mu_{\mathrm{N}_1^1}(x_1(t)) = \mu_{\mathrm{N}_1^2}(x_1(t))$ = $e^{\frac{-(|x_1(t)|-\overline{m}_1)^2}{2\overline{\sigma}_1^2}}$; $\mu_{\mathrm{N}_1^3}(x_1(t)) = \mu_{\mathrm{N}_1^4}(x_1(t))$ = $1-e^{\frac{-(|x_1(t)|-\overline{m}_1)^2}{2\overline{\sigma}_1^2}}$; $\mu_{\mathrm{N}_2^1}(x_2(t)) = \mu_{\mathrm{N}_2^3}(x_2(t))$ = $e^{\frac{-(|x_3(t)|-\overline{m}_2)^2}{2\overline{\sigma}_2^2}}$;

$\mu_{\mathrm{N}_2^2}(x_2(t)) = \mu_{\mathrm{N}_2^4}(x_2(t)) \; = \; 1 - e^{\frac{-(|x_3(t)| - \overline{m}_2)^2}{2\overline{\sigma}_2^{\,2}}}$. $\overline{m}_1$, $\overline{m}_2$, $\overline{\sigma}_1$ and $\overline{\sigma}_2$ are the parameters governing the gaussian membership functions. These 4 parameters will be tuned by the improved GA to optimize the system performance. From (43) and (55), the final fuzzy controller is defined as,

$$\mathbf{u}(t) = \sum_{j=1}^{4} m_j(\mathbf{x}(t))\mathbf{G}_j\overline{\mathbf{x}}(t) + \mathbf{R}(\mathbf{x}(t)) \tag{56}$$

To obtain the **T** and $\mathbf{G}_j$, we chose the minimum and maximum values of each element **T** to be –1.5 and 1.5 respectively. The minimum and maximum values of each element of $\mathbf{G}_j$, j = 1, 2, 3, 4, are chosen to be –1500 and 1500 respectively. The fitness function is defined as follows,

$$fitness = \sum_{i=1}^{8}\sum_{j=1}^{4} n_{ij}\mu\left[\mathbf{T}\left(\mathbf{A}_i + \mathbf{B}_i\mathbf{G}_j\right)\mathbf{T}^{-1}\right] \tag{57}$$

where $n_{ij} = 1$, i = 1, 2, 3, 4, 5, 6, 7, 8; j =1, 2, 3, 4.

T and $\mathbf{G}_j$, j = 1, 2, 3, 4, will be obtained automatically using the improved GA under the consideration of the system stability conditions of (38). The population size is 100 and the initial values of **T** and $\mathbf{G}_j$, j = 1, 2, 3, 4, are randomly generated for the GA process. After applying the improved GA process, we obtain

$$\mathbf{T} = \begin{bmatrix} -0.3535 & 0.4190 & 0.8451 & 0.4185 & -0.4937 & -0.9036 \\ -0.7364 & -0.8006 & -0.1412 & 0.2738 & 0.3417 & -0.2373 \\ -1.4941 & -0.0884 & -1.0955 & -0.1150 & 1.1466 & 0.8027 \\ -1.4976 & -0.5144 & 1.0884 & -0.0660 & 1.4986 & -0.2179 \\ -0.5464 & -0.2925 & -0.4113 & -0.4133 & -0.5956 & -0.2079 \\ 0.3071 & -0.0250 & 0.1705 & 0.0163 & -1.3510 & 0.8340 \end{bmatrix}, \quad \text{and}$$

$$\mathbf{G}_1 = \begin{bmatrix} -1086.6560 & -831.5020 & 0 & 0 & 824.9274 & 0 \\ 0 & 0 & -1057.7074 & -899.7480 & 0 & 837.0547 \end{bmatrix},$$

$$\mathbf{G}_2 = \begin{bmatrix} -995.7281 & -735.9732 & 0 & 0 & 746.4286 & 0 \\ 0 & 0 & -948.5759 & -770.1179 & 0 & 736.6712 \end{bmatrix},$$

$$\mathbf{G}_3 = \left[302.1633 \begin{matrix} -301.3946 & -199.9928 & 0 & 0 & 250.0390 & 0 \\ 0 & 0 & -250.0412 & -174.9477 & 0 & 183.9495 \end{matrix} \right],$$

$$\mathbf{G}_4 = \begin{bmatrix} -274.2215 & -179.9977 & 0 & 0 & 225.0594 & 0 \\ 0 & 0 & -225.0425 & -157.4589 & 0 & 164.4405 \end{bmatrix}.$$

The stability analysis result is tabulated in Table II. It can be seen that the values of $\mu[\mathbf{TH}_{ij}\mathbf{T}^{-1}]$, i = 1, 2, 3, 4, 5, 6, 7, 8; j =1, 2, 3, 4, are all negative. According to Lemma 1, the fuzzy control system is guaranteed to be stable.

3) The optimal performance of the fuzzy control system will be obtained by tuning the membership functions of the fuzzy controller. The tunable parameters of the membership functions are $\overline{m}_1 \in [-\pi \quad \pi]$, $\overline{\sigma}_1 \in [0 \quad 5]$, $\overline{m}_2 \in [-\pi \quad \pi]$ and $\overline{\sigma}_2 \in [0 \quad 5]$. The population size is 10 and the initial values are $\overline{m}_1 = 0$, $\overline{\sigma}_1 = 5$, $\overline{m}_2 = 0$ and $\overline{\sigma}_2 = 5$. The fitness function is chosen as follows,

$$fitness = \int_0^{20} \overline{\mathbf{x}}(t)^{\mathrm{T}} \mathbf{W}_x \overline{\mathbf{x}}(t) + \mathbf{u}(t)^{\mathrm{T}} \mathbf{R}_u \mathbf{u}(t) \mathrm{d}t \quad (58)$$

where,

$$\mathbf{W}_x = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{R}_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (59)$$

After the improved GA process, $\overline{m}_1 = 1.9633$, $\overline{\sigma}_1 = 0.2770$, $\overline{m}_2 = 1.2695$ and $\overline{\sigma}_2 = 1.5105$.

Fig. 6 shows the membership functions of the fuzzy controller before and after the GA tuning respectively. Fig. 7 shows the responses of the system states, with m_3 = 0kg, $\theta_{1_{ref}} = \frac{\pi}{2}$ and $\theta_{2_{ref}} = \frac{\pi}{2}$ for $0 \le t < 10$ and m_3 = 3kg, $\theta_{1_{ref}} = -\frac{\pi}{2}$ and $\theta_{2_{ref}} = -\frac{\pi}{2}$ for $t \ge 10$, before (solid line) and after (dotted line) the GA process has tuned the membership functions under the initial condition of $\overline{\mathbf{x}}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$. Fig. 8 shows the control signals of the fuzzy controller before and after the tuning process. It can be seen from the simulation results that the responses after optimization are better than those before optimization using the improved GA.

VIII. CONCLUSION

Fuzzy control of nonlinear systems subject to parameter uncertainties has been presented. Stability conditions have been derived for this class of uncertain fuzzy control system. An improved genetic algorithm has been proposed to help finding the solution to the stability conditions and determining the feedback gains of the fuzzy controller. Moreover, the membership functions of the fuzzy controller can be determined automatically to optimize the system performance. An application example on stabilizing a two-link robot arm has been presented to illustrate the merits of the proposed fuzzy controller.

Acknowledgement

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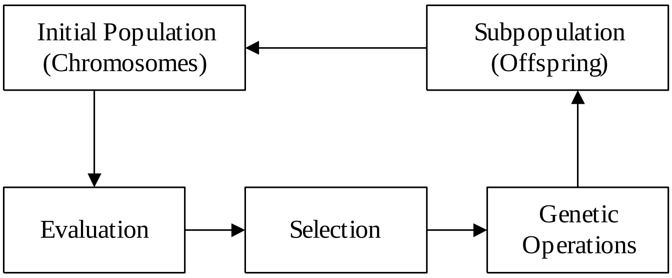


Fig. 1. Traditional GA.

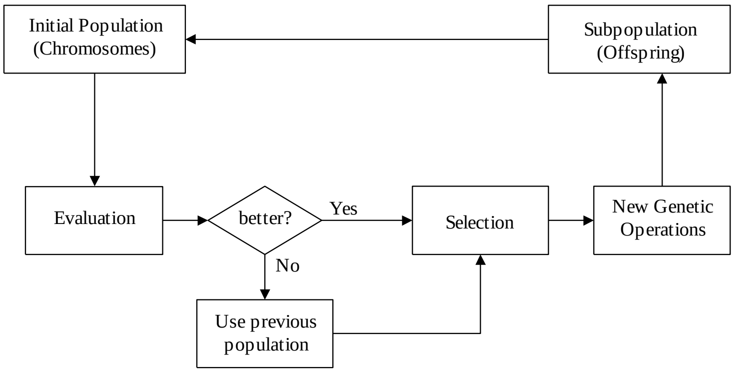
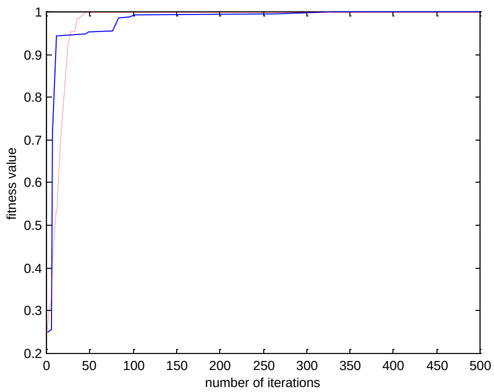
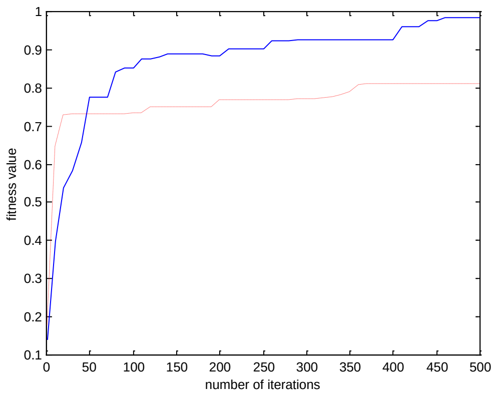


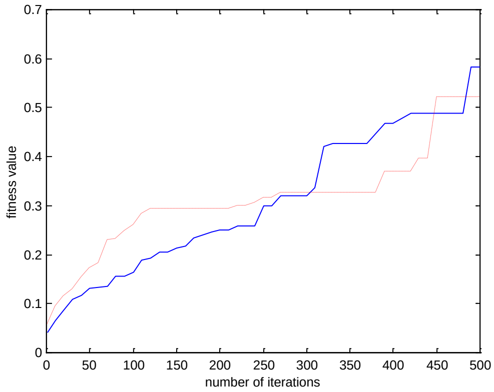
Fig. 2. Improved GA.



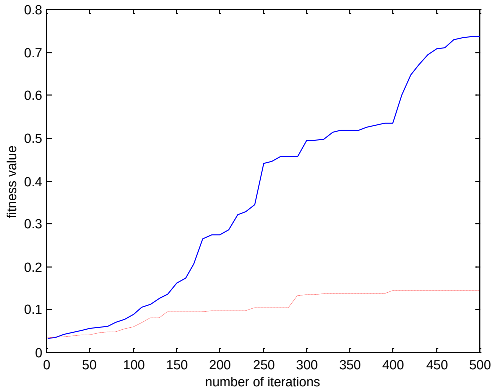
(a). The averaged fitness value of the test function $f_1(\mathbf{x})$ obtained by the improved (solid line) and traditional (dotted line) GAs.



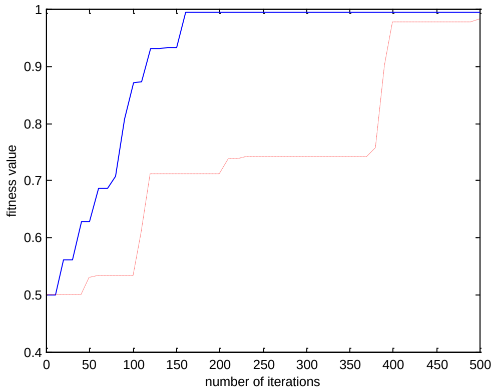
(b). The averaged fitness value of the test function $f_2(\mathbf{x})$ obtained by the improved (solid line) and traditional (dotted line) GAs.



(c). The averaged fitness value of the test function $f_3(\mathbf{x})$ obtained by the improved (solid line) and traditional (dotted line) GAs.



(d). The averaged fitness value of the test function $f_4(\mathbf{x})$ obtained by the improved (solid line) and traditional (dotted line) GAs.



(e). The averaged fitness value of the test function $f_5(\mathbf{x})$ obtained by the improved (solid line) and traditional (dotted line) GAs.

Fig. 3. Simulation results of the improved and traditional GAs.

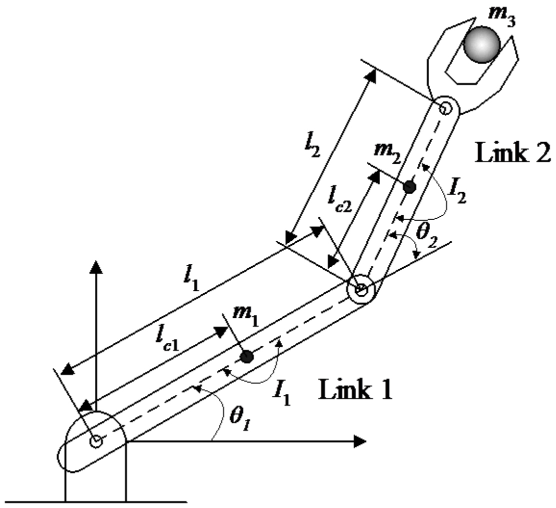


Fig. 4. Two-link robot arm.

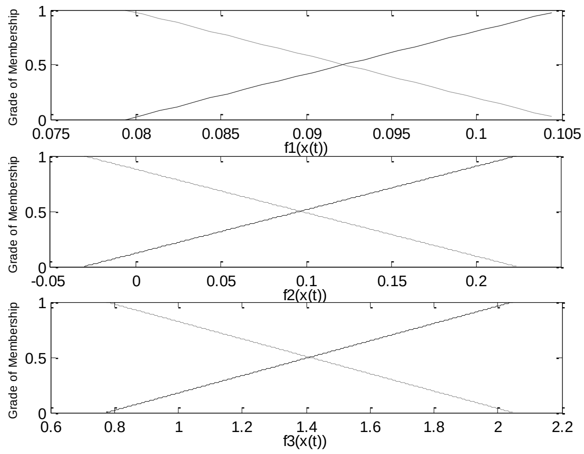
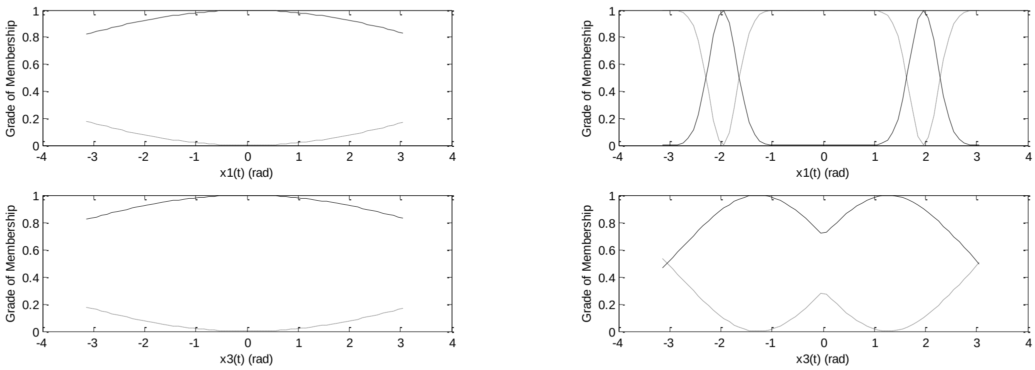
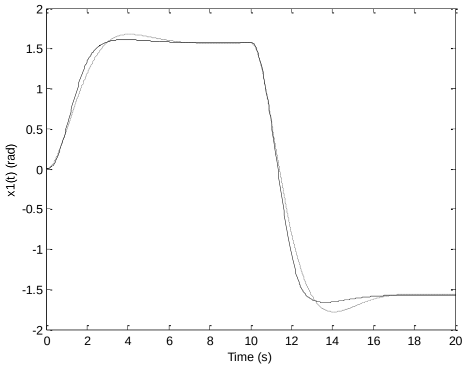


Fig. 5. Membership functions of the fuzzy plant model of the two-link robot arm.

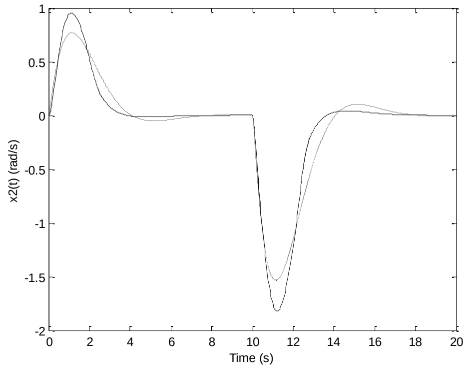


(a). Before tuning   (b). After tuning.

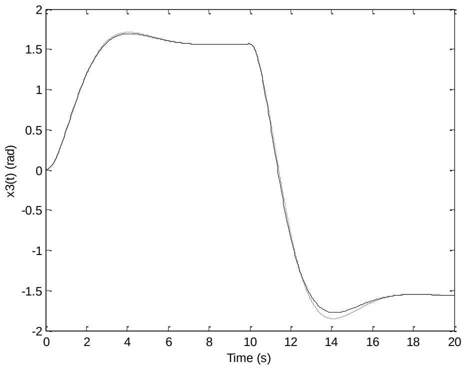
Fig. 6. Membership functions of the fuzzy controller before(dotted lines) and after (solid lines) tuning.



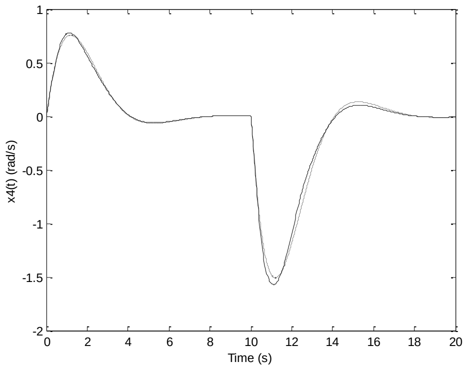
(a)



(b)

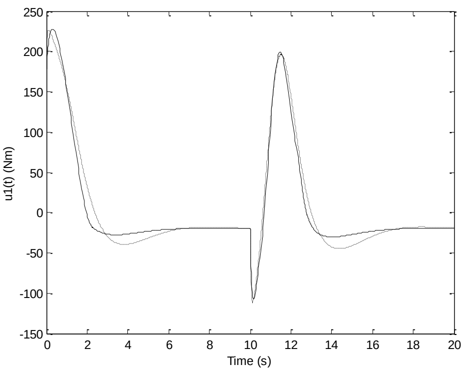


(c)

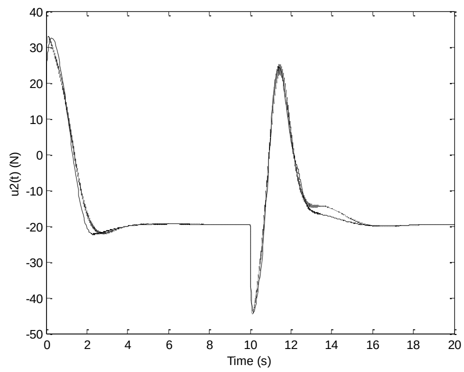


(d)

Fig. 7. Responses of $\mathbf{x}(t)$ of the two-link robot arm with the fuzzy controller before (dotted lines) and after (solid lines) tuning.



(a)



(b)

Fig. 8. Control signals of the fuzzy controller before (dotted lines) and after (solid lines) tuning.

	Improved GA		Traditional GA	
Test Functions	Fitness Values	Searching Time (s)	Fitness Values	Searching Time (s)
$f_1(\mathbf{x})$	0.999955	1.21	0.999382	8.35
$f_2(\mathbf{x})$	0.984039	1.22	0.810813	8.36
$f_3(\mathbf{x})$	0.583333	1.09	0.520833	10.93
$f_4(\mathbf{x})$	0.737526	2.69	0.14211	74.21
$f_5(\mathbf{x})$	0.995509	6.15	0.982912	21.13

Table I. Simulation results of the improved and the traditional GAs based on the De Jong's test functions.

i, j	$\mu[\mathbf{TH}_{ij}\mathbf{T}^{-1}]$	i, j	$\mu[\mathbf{TH}_{ij}\mathbf{T}^{-1}]$
1, 1	–0.8126	5, 1	–0.8124
1, 2	–0.8135	5, 2	–0.8133
1, 3	–0.8074	5, 3	–0.7647
1, 4	–0.8142	5, 4	–0.7903
2, 1	–0.8011	6, 1	–0.8085
2, 2	–0.8132	6, 2	–0.8130
2, 3	–0.2805	6, 3	–0.3454
2, 4	–0.4125	6, 4	–0.4641
3, 1	–0.8071	7, 1	–0.8110
3, 2	–0.8055	7, 2	–0.8128
3, 3	–0.2109	7, 3	–0.7806
3, 4	–0.0471	7, 4	–0.7762
4, 1	–0.8014	8, 1	–0.8020
4, 2	–0.8142	8, 2	–0.8142
4, 3	–0.7323	8, 3	–0.7472
4, 4	–0.7684	8, 4	–0.7703

Table II. Stability analysis results.