

**A hierarchical model to detect
differential gene expression
distributions, and their investigation
as a reflection of dysregulation in
cancer**

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Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy

under the supervision of Professor Paul Kennedy and
Associate Professor Daniel Catchpoole

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Certificate of Original Authorship

I, Aedan Roberts, declare that this thesis is submitted in fulfilment of the requirements for the award of Doctor of Philosophy in the School of Computer Science, Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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List of Publications

Listed below are the publications and other outputs associated with the research presented in this thesis.

Roberts, A. G. K., Catchpoole, D. R. & Kennedy, P. J., 2021, 'Identification of differentially distributed gene expression and distinct sets of cancer-related genes identified by changes in expression mean and variability'. (Submitted) Available as a preprint: <https://www.biorxiv.org/content/10.1101/2021.02.15.431343v2>.

Roberts, A. G. K., 2021, 'DiffDist', <https://github.com/aedanr/DiffDist>. R package.

Roberts, A. G. K., Catchpoole, D.R. & Kennedy, P.J., 2018, 'Variance-based Feature Selection for Classification of Cancer Subtypes Using Gene Expression Data', *2018 International Joint Conference on Neural Networks (IJCNN)*, Rio de Janeiro.

List of Abbreviations and Symbols

Abbreviation	Description
ALL	Acute lymphoblastic leukaemia
AP	Anti-profiles
AUC	Area under the ROC curve
BFDR	Bayesian false discovery rate
BRCA	Breast invasive adenocarcinoma
CGC	Cancer Gene Census
COAD	Colon adenocarcinoma
CV	Coefficient of variation
DE	Differential expression
DMD	Differences in means and deviations
$D\phi$	Differential dispersion
FDR	False discovery rate
FPR	False positive rate
GAMLSS	Generalised additive models for location, scale and shape
GEO	Gene Expression Omnibus
GO	Gene Ontology
GTE _x	Genotype–Tissue Expression
HM	Hierarchical model
HMM	Hierarchical mixture model
HPD	Highest posterior density
KEGG	Kyoto Encyclopedia of Genes and Genomes
KIRC	Kidney renal clear cell carcinoma
LDA	Linear discriminant analysis
LFC	Log fold change
LIHC	Liver hepatocellular carcinoma
LSVM	Linear SVM
LUAD	Lung adenocarcinoma
LUSC	Lung squamous cell carcinoma

Abbreviation	Description
MAD	Median absolute deviation
MCMC	Markov chain Monte Carlo
mRNA	messenger RNA
MSE	Mean squared error
NB	Negative binomial
NCBI	National Center for Biotechnology Information
NOS	Not otherwise specified
NSCLC	Non-small cell lung carcinoma
PRAD	Prostate adenocarcinoma
PSVM	SVM with polynomial kernel
RBF	Radial basis function
RF	Random forest
RIN	RNA integrity number
RLE	Relative log expression
RNA-seq	RNA sequencing
ROC	Receiver operating characteristic
RSVM	SVM with RBF kernel
SAGE	Serial analysis of gene expression
SAM	Significance Analysis of Microarrays
SVM	Support vector machine
TCGA	The Cancer Genome Atlas
THCA	Thyroid carcinoma
TMM	Trimmed mean of M-values
TPR	True positive rate
TRF	Random forest with distance-to-median transformed features

Symbol	Description
g	Number of genes represented in a gene expression dataset
m_μ	Location hyperparameter for log-normal prior on mean
m_ϕ	Location hyperparameter for log-normal prior on dispersion
n	Number of samples in a dataset
n_A	Number of samples in group A
n_B	Number of samples in group B
\hat{R}	Gelman–Rubin diagnostic
s	Sample standard deviation
s^2	Sample variance
v_μ	Scale hyperparameter for log-normal prior on mean
v_ϕ	Scale hyperparameter for log-normal prior on dispersion
y	Set of observed RNA-seq counts for all genes and samples in a dataset
\bar{y}	Sample mean of y
y_{ij}	Observed count for gene j in sample i
z_j	Mixture component indicator for gene j
$\Gamma(\cdot)$	Gamma function
γ	Set of all hyperparameters
γ_μ	Set of hyperparameters for prior on mean
γ_ϕ	Set of hyperparameters for prior on dispersion
θ	Set of means and dispersions for all genes: (μ_j, ϕ_j) , $j = 1, \dots, g$
θ_j	Set of mean and dispersion for gene j : (μ_j, ϕ_j)
λ	Proportion of differentially distributed genes in HMM or Poisson rate parameter
μ	Mean
ϕ	Negative binomial dispersion
σ	Standard deviation
σ^2	Variance

Abstract

Data from genome-wide gene expression studies provides a wealth of information on diseases such as cancer, which can lead to insights into disease mechanisms and advances in diagnosis and treatment. Analysis of expression data is most commonly aimed at identifying genes whose mean expression levels are increased or decreased in disease compared to normal tissue, or between disease subtypes – differential expression analysis. However, there is strong evidence that changes in the variability of gene expression, without a difference in mean, can also be relevant. Genes related to cancer have been shown to have changes in the variability of their expression between normal and tumour tissue, and these differentially variable genes have also been found to be informative for diagnostic and prognostic cancer classification. The research presented in this thesis addresses several aspects of research on differential gene expression variability, and the broader concept of differential distribution, defined as any difference in the distribution of expression values between groups.

This work makes three contributions to knowledge, relating to cancer classification, identification of differentially variable or distributed genes, and the biology of differential variability and distribution in cancer. Contribution 1 extends previous work by demonstrating that genes identified by differential variability or distribution can be used to classify closely related cancer subtypes, rather than purely diagnostic or prognostic classification. Contribution 2 is a Bayesian hierarchical model for RNA-seq data that provides tests for differential expression, variability and distribution. The performance of each test is compared with existing methods on simulated data and on real RNA-seq datasets modified to artificially introduce changes in expression between groups. The differential expression test is competitive with state-of-the-art methods, and the differential variability test improves on existing methods, particularly for small sample sizes. The differential distribution test is the first such test available for RNA-seq data. Contribution 3

builds on previous work by providing the first clear demonstration that differential variability and differential distribution analyses can identify cancer-related genes, and that differential expression and differential variability analyses identify distinct sets of cancer-related genes, each with different biological functions.

Overall, this research confirms and extends previous findings showing that changes in expression variability and distribution in cancer are both of biological significance and informative for classification. As well as further demonstrating the need to look beyond differential expression to a comprehensive assessment of changes in gene expression distributions, this work provides a method that enables the identification of these differentially distributed genes.