UNIVERSITY
OF TECHNOLOGY
SYDNEY

A Suite of Analytical Solutions for Free Strain Consolidation of Soft Soil Reinforced by Stone Columns

by Sam Huu Doan

Thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

under the supervision of A/Prof Behzad Fatahi and

A/Prof Hadi Khabbaz

University of Technology Sydney
Faculty of Engineering and Information Technology

September 2020

CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Sam Huu Doan declare that this thesis, is submitted in fulfilment of the requirements for the award of the degree Doctor of Philosophy, in the School of Civil and Environmental Engineering, Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program and the Ministry of Education and Training (MoET) – Vietnam.

Production Note:
Signature: Signature removed prior to publication.

Date: 30 September 2020

ABSTRACT

Most existing analytical studies on the consolidation of stone column stabilised soft soils adopt equal strain hypothesis for the column and soil settlements, which cannot capture the response of the composite ground (i.e. unequal column and soil settlements) under a typical flexible embankment – platform system accurately. Thus, the differential settlement and column – soil interaction along the interface during the consolidation process were ignored in the available analytical studies. In contrast, several researchers proposed analytical models to investigate the final deformation of the composite stone column – soft ground considering free strain settlement of stone column and soft soil (i.e. differential settlement). Indeed, these studies were for time-independent deformation analysis of the composite ground or considered the effect of consolidation in an uncoupled fashion. To address the above mentioned shortcomings of analytical studies in the literature, this thesis presents a suite of analytical models to the consolidation of soft soils reinforced by stone columns, adopting unit cell concept and one-dimensional free strain condition (i.e. vertical deformation) for stone column and encircling soft soil. The thesis first examines the consolidation behaviour of the composite ground subjected to constant loadings under plane strain and axisymmetric conditions. Then, the mathematical model for the axisymmetric consolidation is developed to account for the effect of time-dependent loadings on consolidation response of the composite ground. Lastly, an analytical model for the coupled consolidation – deformation analysis of stone column reinforced soft soils is formulated. In this thesis, the mathematical formulations are integrated with the associated horizontal and vertical flows of pore water in stone column and soft soil regions with orthotropic permeability for each region. Various total vertical

stress distributions in the composite ground induced by external loadings are adopted including uniform and spatial variation patterns.

In an attempt to develop novel analytical solutions for the free strain consolidation with the incorporation of deformation analysis of the composite ground, the method of separation of variables in conjunction with eigenfunction expansion technique, Green's formula and Green's function method are employed for the analytical derivations. The obtained analytical solutions can capture the excess pore water pressure variations with time at any point in the composite ground. Thus, the column and soil settlements and accompanying differential settlements can be achieved along with other performance objectives such as average degrees of consolidation and normalised average surface settlements of stone column and soft soil. Furthermore, for the combined consolidation – deformation analysis, the transferring of total vertical stress from soft soil to stone column via their interface, as a result of differential settlement and the shear stress distribution in soft soil during the consolidation process, are also captured. Several worked examples and parametric analyses using the achieved analytical solutions are conducted thoroughly. The verifications of the obtained analytical solutions in this thesis against finite element simulations and field measurements show reasonable agreements, which validate the capability of the proposed analytical models and the attained analytical solutions. It can be noted that the proposed analytical solutions may be applicable to the consolidation and deformation analysis of soft soils supported by other pervious columns such as compacted sand columns and soil-cement mixing columns, taking consideration of corresponding physical and mechanical properties.

To my wife, *Sinh Thi Minh Nguyen*, and my daughter, *Linh Vu Nhat Doan*,

who shared love, trust and strength with me throughout this glorious journey.

ACKNOWLEDGEMENT

The PhD program at the University of Technology Sydney (UTS), which I have pursued up to date has afforded me academic experience and ability intensely. Throughout my PhD course, I have built up a variety of research skills and academic passion that promote mindset in addressing research questions conscientiously and creatively. Nevertheless, this wonderful achievement would be impossible without enormous supports from my supervisors, my family and fellow members.

First and foremost, I would like to express the deepest gratitude to my principal supervisor, A/Prof Behzad Fatahi, and my co-supervisor, A/Prof Hadi Khabbaz, for their professional supervision, enthusiasm and encouragement. They have been always willing to support and give me exhaustive advice to surmount challenging academic matters effectively. Particularly, my research project might not have been attainable without the valuable comments and recommendations, the patience and unmeasurable favour from A/Prof Behzad Fatahi.

Second of all, I would like to thank all geotechnical research fellows who shared with me academic experience, skill and support. I would also like to thank Dr Liem Huu Ho for his useful recommendations on my research topic at the beginning of my PhD study. I would like to sincerely acknowledge the scholarships and funding awarded by UTS and Ministry of Education and Training (MOET) – Vietnam, which allow me to accomplish my PhD course. Additionally, I would like to extend my appreciation to Van Le and Responsible Academic Officers at UTS Faculty of Engineering and Information Technology for their timely assistance.

Finally, I am extremely grateful to my wife, daughter, parents, relatives and friends for their love, trust, favour and encouragement dedicating to me. They all and my wife and daughter, in particular, have been the strong spiritual supports, which motivates me to work hard, overcome difficult time and achieve my goals successfully.

LIST OF PUBLICATIONS

❖ Journal Articles

Doan S, Fatahi B. Analytical solution for free strain consolidation of stone column-reinforced soft ground considering spatial variation of total stress and drain resistance. Computers and Geotechnics. 2020; 118:103291.

Doan S, Fatahi B. Green's function analytical solution for free strain consolidation of soft soil improved by stone columns subjected to time-dependent loading. Computers and Geotechnics. 2021; 136:103941.

❖ Conference Papers

Doan S, Fatahi B, Khabbaz H. Exact series solution for plane strain consolidation of stone column improved soft soil accounting for space-dependent total stresses. Proceedings of the 16th International Conference of the International Association for Computer Methods and Advances in Geomechanics (IACMAG). Turin, Italy: Springer International Publishing. 2021; 794-802.

Doan S, Fatahi B, Khabbaz H, Rasekh H. Analytical solution for plane strain consolidation of soft soil stabilised by stone columns. Proceedings of the 4th International Conference on Transportation Geotechnics (ICTG). Chicago, USA: Springer International Publishing. 2021. (in press)

TABLE OF CONTENTS

ABSTRACT i

ACKNOWLEDGEMENT iv

LIST OF PUBLICATIONS vi

TABLE OF CONTENTS..vii

LIST OF TABLES xiii

LIST OF FIGURES xiv

LIST OF NOTATIONS xxi

CHAPTER 1 INTRODUCTION 1

1.1 General...1

1.2 Stone columns to stabilise soft soil foundations..................................2

1.3 Statement of problem..6

1.4 Objectives and scope of research..9

1.5 Organisation of thesis 11

CHAPTER 2 LITERATURE REVIEW 14

2.1 General...14

2.2 Terzaghi's theory of consolidation 17

2.3 Biot's theory of consolidation 20

2.4 Consolidation of soft soils assisted by pervious elements..................25

2.5 Equal strain and free strain consolidations of soft soil stabilised by stone columns 32

2.6 Effects of drain resistance, clogging, smear and partially drained boundaries 34

2.7 Impacts of total stress variations against space and time on consolidation 37

2.8 Effects of layered soils and partially penetrated columns 39

2.9 Effects of nonlinearity, rheology and unsaturation of soil on consolidation 40

2.10 Deformation of the composite stone column stabilised soft ground supporting embankments 42

2.11 Analytical methods for consolidation of soft soils supported by pervious columns 45

2.12 Summary 49

CHAPTER 3 ANALYTICAL SOLUTION FOR PLANE STRAIN CONSOLIDATION OF SOFT SOIL STABILISED BY STONE COLUMNS 53

3.1 Introduction 53

3.2 Problem description 54

3.3 Analytical solution 57

3.4 Validation of the proposed analytical solution 63

3.5 Summary 72

CHAPTER 4 ANALYTICAL SOLUTION FOR FREE STRAIN CONSOLIDATION OF STONE COLUMN-REINFORCED SOFT GROUND

CONSIDERING SPATIAL VARIATION OF TOTAL STRESS AND DRAIN RESISTANCE 74

4.1 Introduction 74

4.2 Basic assumptions and mathematical model 75

4.3 Analytical solution 80

4.4 Worked example and verification 86

4.4.1 Worked example 87

4.4.2 Verification against finite element simulation 96

4.4.3 Verification against field measurements and existing analytical studies 102

4.5 Parametric study 110

4.6 Summary 121

CHAPTER 5 GREEN'S FUNCTION ANALYTICAL SOLUTION FOR FREE STRAIN CONSOLIDATION OF SOFT SOIL IMPROVED BY STONE COLUMNS SUBJECTED TO TIME-DEPENDENT LOADING 123

5.1 Introduction 123

5.2 Description of the problem 124

5.3 Analytical solution for excess pore water pressure dissipation and consolidation settlement 127

5.4 Worked examples 129

5.4.1 Step loading 134

5.4.2 Ramp loading 140

5.4.3 Sinusoidal loading 146

5.5 Verification against field measurements 153

5.6 Summary 160

CHAPTER 6 SIMPLIFIED ANALYTICAL SOLUTION FOR COUPLED ANALYSIS OF CONSOLIDATION AND DEFORMATION OF STONE COLUMN IMPROVED SOFT SOIL 162

6.1 Introduction 162

6.2 Governing equations, boundary and initial conditions of the consolidation 164

6.3 Deformation of the stone column reinforced ground at the end of consolidation 167

6.4 Coupled analysis for the consolidation and deformation of soft ground reinforced by stone columns 174

6.5 Worked example 180

6.5.1 Total vertical stresses in stone column and soft soil 181

6.5.2 Excess pore water pressure dissipations in stone column and soft soil .. 185

6.5.3 Consolidation settlements in stone column and soft soil 189

6.5.4 Distribution of shear stresses in soft soil 192

6.6 Verification against finite element modelling 197

6.7 Verification against full-scale test 206

6.8 Summary 210

CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS 212

7.1 Summary 212

7.2 Key concluding remarks 216

7.2.1 General 216

7.2.2 Analytical investigation of the two-dimensional plane strain and axisymmetric consolidations under constant loadings 218

7.2.3 Analytical examination of the axisymmetric consolidation under time-dependent loadings 221

7.2.4 Analytical evaluation of the associated consolidation – deformation response under constant loading condition 223

7.3 Recommendations for further studies 225

REFERENCES 228

APPENDICES 250

Appendix A. Derivation of the solutions for excess pore water pressure $(u_c^{(i)}, u_s^{(i)})$ and $(u_c^{(ii)}, u_s^{(ii)})$ corresponding to the eigenfunctions $(R_{cmn}^{(i)}, R_{smn}^{(i)})$ and $(R_{cmn}^{(ii)}, R_{smn}^{(ii)})$ 250

Appendix B. Derivation of the eigenvalues pairs $(\nu_{cmn}^{(i)}, \nu_{smn}^{(i)})$ and $(\nu_{cmn}^{(ii)}, \nu_{smn}^{(ii)})$ and their corresponding eigenfunctions $(R_{cmn}^{(i)}, R_{smn}^{(i)})$ and $(R_{cmn}^{(ii)}, R_{smn}^{(ii)})$ 254

Appendix C. Derivation of Green's function for the non-homogeneous consolidation problem 261

Appendix D. Derivation of the excess pore water pressure $u_i^{(*)}$ in Equation (5.6) corresponding to the investigation loadings in the example 265

Appendix E. Derivation of the average excess pore water pressure in soft soil $\bar{\bar{\Theta}}_2$ corresponding to the homogeneous consolidation formulation 268

Appendix F. Derivation of the excess pore water pressure solutions for the non-homogeneous consolidation formulation 270

LIST OF TABLES

Table 5.1. Selected loading parameters for the worked example 133

Table 5.2. Geometric and material parameters for the verification 155

Table 6.1. Parameters for the verification against case study 209

LIST OF FIGURES

Figure 1.1. Stone column installation methods: (a) wet top feed and (b) dry bottom feed (modified after Taube and Herridge [3])....4

Figure 1.2. Arrangements of stone columns on soft soils and the corresponding influence zone of each stone column (after Balaam and Booker [4])....5

Figure 2.1. Saturated soil stratum endures (a) one-way drainage and (b) two-way drainage of excess pore water pressure induced by uniform surcharge....17

Figure 2.2. Arrangement pattern of drain wells on plan and unit cell model for the consolidation of soil (after Barron [64]) 28

Figure 2.3. Conversion of (a) axisymmetric unit cell into (b) plane strain unit cell (after Indraratna and Redana [103])....29

Figure 2.4. Consolidation settlement of the composite stone column – soft ground under equal strain and free strain conditions....34

Figure 2.5. Deformation of a typical embankment fill – soft soil foundation system stabilised by stone columns (after Deb [184] and Basack et al. [185])....45

Figure 3.1. The model of the problem 55

Figure 3.2. Excess pore water pressure isochrones against width at depth $z = 0.5H$ for the stone column and soft soil regions 65

Figure 3.3. Dissipation of excess pore water pressure against time at different points in the stone column and soft soil regions 66

Figure 3.4. Plane strain finite element model of the example....67

Figure 3.5. Comparison in average degree of consolidation obtained from proposed analytical solution and finite element result....68

Figure 3.6. Effect of permeability ratio k_{1h}/k_{2h} on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 70

Figure 3.7. Effect of modulus ratio E_1/E_2 on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 71

Figure 4.1. The unit cell model of the problem... 76

Figure 4.2. The variation of total vertical stresses against depth for the stone column and soft soil regions 88

Figure 4.3. Excess pore water pressure isochrones against radius at depth $z = 0.5H$ for the stone column and soft soil regions 91

Figure 4.4a. Excess pore water pressure isochrones against depth at radius $r = 0.5a$ within the stone column region 92

Figure 4.4b. Excess pore water pressure isochrones against depth at radius $r = 0.5(a+b)$ within the soft soil region.. 92

Figure 4.5. Dissipation of excess pore water pressure against time at various points in the stone column and soft soil regions 94

Figure 4.6a. Surface settlement isochrones against radius for the stone column and soft soil regions 95

Figure 4.6b. Settlement isochrones against radius at time $t = 5\times10^6 s$ for the stone column and soft soil regions with different investigating depths 96

Figure 4.7. Finite element model used for the verification exercise 98

Figure 4.8. Comparison in dissipation of excess pore water pressure between proposed analytical solution and finite element result at points in the stone column and soft soil regions with depth (a) $z = 0.25H$, (b) $z = 0.5H$, and (c) $z = 0.75H$ 101

Figure 4.9. Comparison in average degree of consolidation between proposed analytical solution and finite element result for the stone column and soft soil regions 101

Figure 4.10. Comparison of excess pore water pressure dissipation against time between proposed analytical solution in this study and field measurement at the investigation point ($r = 0.7$ m, $z = 1.5$ m) in soft soil 106

Figure 4.11. Comparison of average surface settlement of soft soil against time between proposed analytical solution in this study and field measurement 107

Figure 4.12. Comparison of average degree of consolidation between proposed analytical solution and Barron's solution 109

Figure 4.13. Influence of permeability ratio k_{ch}/k_{sh} on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 113

Figure 4.14. Influence of radius ratio n_e on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 114

Figure 4.15. Influence of modulus ratio E_c/E_s on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 116

Figure 4.16. Influence of α value on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 118

Figure 4.17. Influence of H/a ratio on (a) average degree of consolidation of stone column and of soft soil, and (b) average differential settlement between stone column and soft soil 120
Figure 5.1. The axisymmetric model of the problem 125
Figure 5.2. Three investigation loading types in the example: (a) Step, (b) Ramp, and (c) Sinusoid 131
Figure 5.3. Dissipation rates of excess pore water pressure at investigation points in (a) stone column and (b) soft soil varying with time duration t_1 of the step loading......... 136
Figure 5.4. Influence of time duration t_1 of the step loading on normalised average surface settlements of (a) stone column and (b) soft soil 138
Figure 5.5. Influence of time duration t_1 of the step loading on the differential average surface settlement between stone column and soft soil 140
Figure 5.6. Dissipation rates of excess pore water pressure at investigation points in (a) stone column and (b) soft soil varying with construction time t_1 of the ramp loading 142
Figure 5.7. Influence of construction time t_1 of the ramp loading on normalised average surface settlements of (a) stone column and (b) soft soil.. 144
Figure 5.8. Influence of construction time t_1 of the ramp loading on the differential average surface settlement between stone column and soft soil 145
Figure 5.9. Variation of excess pore water pressure at investigation points in (a) stone column and (b) soft soil with time considering different angular frequencies φ_B of the sinusoidal loading 149
Figure 5.10. Influence of angular frequency φ_B of the sinusoidal loading on normalised average surface settlements of (a) stone column and (b) soft soil 152

Figure 5.11. Influence of angular frequency φ_B of the sinusoidal loading on the differential average surface settlement between stone column and soft soil 153

Figure 5.12. Verification of excess pore water pressure dissipation rate obtained from the proposed analytical solution and field measurement at an investigation point in soft soil 158

Figure 5.13. Verification of average surface settlement rate of soft soil obtained from the proposed analytical solution and field measurement 159

Figure 6.1. The axisymmetric unit cell model for the consolidation analysis 166

Figure 6.2. Assumed deformation pattern of the composite stone column – soft ground and induced stress components on the column and soil elements at the end of consolidation process 169

Figure 6.3. Distribution of total vertical stresses in (a) stone column and (b) soft soil against depth varying with time 183

Figure 6.4. Variation of total vertical stresses in stone column and soft soil against time at different depths.. 184

Figure 6.5. Excess pore water pressure isochrones against depth at radii (a) $r = 0.5a$ in stone column and (b) $r = 0.5(a + b)$ in soft soil... 187

Figure 6.6. Dissipation rates of excess pore water pressure at various investigation points in stone column and soft soil.. 188

Figure 6.7. Surface settlement isochrones for stone column and soft soil against radius 190

Figure 6.8. Settlements of stone column and soft soil against radius at time $t = 10^6 s$ for different investigation depths.. 192

Figure 6.9. Isochrones of the shear stress in soft soil against radius at depth $z = 0.1H$ 193
Figure 6.10. Variation of the shear stress in soft soil against radius at time $t = 10^6 s$ for different investigation depths 194
Figure 6.11. Isochrones of the shear stress in soft soil against depth at radius $r = 0.5(a+b)$ 195
Figure 6.12. Variation of the shear stress in soft soil against depth at time $t = 2\times10^6 s$ for different investigation radii 196
Figure 6.13. Finite element model for the verification exercise 198
Figure 6.14. Verification on the distribution of final total vertical stresses in stone column and soft soil against depth between proposed analytical solution and finite element result 199
Figure 6.15. Verification on the change of total vertical stresses in stone column and soft soil against time at depth $z = 0.5H$ between proposed analytical solution and finite element result 200
Figure 6.16. Verification on the dissipation rate of excess pore water pressures at various points in stone column and soft soil with depths (a) $z = 0.25H$, (b) $z = 0.5H$, and (c) $z = 0.75H$ between proposed analytical solution and finite element result 202
Figure 6.17. Verification on the average degree of consolidation of stone column and of soft soil between proposed analytical solution and finite element result 204
Figure 6.18. Verification on the surface settlement of stone column and soft soil at various time t between proposed analytical solution and finite element result 205
Figure 6.19. Verification on the distribution of final shear stress in soft soil against depth at two different radii between proposed analytical solution and finite element result .. 206

Figure 6.20. Verification on surface settlements of soil-cement deep mixing column and surrounding soft soil between proposed analytical solution and field measurement....208

LIST OF NOTATIONS

Latin Notations

a	Radius or half width of stone column
b	Radius or half width of unit cell
A	Rate of the ramp loading
B	Parameter controlling the amplitude of the sinusoidal loading
$A_{2mn}^{(i)}$, $B_{2mn}^{(i)}$	Constants for eigenfunctions $\Psi_{2mn}^{(i)}$
$A_{2mn}^{(ii)}$, $B_{2mn}^{(ii)}$	Constants for eigenfunctions $\Psi_{2mn}^{(ii)}$
$A_{cmn}^{(i)}$, $B_{cmn}^{(i)}$	Constants to be determined for eigenfunctions $R_{cmn}^{(i)}$
$A_{smn}^{(i)}$, $B_{smn}^{(i)}$	Constants to be determined for eigenfunctions $R_{smn}^{(i)}$
$A_{cmn}^{(ii)}$, $B_{cmn}^{(ii)}$	Constants to be determined for eigenfunctions $R_{cmn}^{(ii)}$
$A_{smn}^{(ii)}$, $B_{smn}^{(ii)}$	Constants to be determined for eigenfunctions $R_{smn}^{(ii)}$
c	Transferring rate of total vertical stress from soft soil to stone column
c_{1h}, c_{1v}	Horizontal and vertical consolidation coefficients of stone column, respectively
c_{2h}, c_{2v}	Horizontal and vertical consolidation coefficients of soft soil, respectively
c_{ch}, c_{cv}	Horizontal and vertical consolidation coefficients of stone column, respectively
c_{sh}, c_{sv}	Horizontal and vertical consolidation coefficients of soft soil, respectively

$C_{mn}^{(i)}$, $C_{mn}^{(ii)}$	Fourier-Bessel coefficients
$C_{mn}^{T(*)}$	Fourier-Bessel coefficients
$C_{mn}^{(*)}$	Alternative notation of the coefficients $C_{mn}^{(i)}$ and $C_{mn}^{(ii)}$
d	Parameter regulating the change of $\sigma_{1f}(z)$ with depth, $d = 25/(H\,n_e)$
E_1, E_2	Young's moduli of stone column and soft soil, respectively
E_c, E_s	Young's modulus of stone column and soft soil, respectively
f_{cz}, f_{sz}	Variation of total vertical stress against depth for stone column and soft soil, respectively
G_2	Shear modulus of soft soil
$G_{ij}^{(*)}$	Green's function
H	Thickness of soft soil stratum
i, j	Subscripts denote regions of the unit cell; $i, j = 1$ for stone column region; $i, j = 2$ for soft soil region
(i), (ii)	Superscripts denote different real eigenvalues pairs to be considered
I_0, I_1	Modified Bessel functions of the first kind of order zero and one, respectively
J_0, J_1	Bessel functions of the first kind of order zero and one, respectively
k_{1h}, k_{1v}	Horizontal and vertical permeability coefficients of stone column, respectively
k_{2h}, k_{2v}	Horizontal and vertical permeability coefficients of soft soil, respectively

k_{ch}, k_{cv}	Horizontal and vertical permeability coefficients of stone column, respectively
k_{sh}, k_{sv}	Horizontal and vertical permeability coefficients of soft soil, respectively
m	Integer index corresponding to z-domain
M_1, M_2	Constrained moduli of stone column and soft soil, respectively
n	Integer index corresponding to r-domain
n_e	Radius ratio of unit cell to stone column, $n_e = b/a$
n_{scr}	Stress concentration ratio
N_k	Horizontal permeability ratio of stone column to soft soil, $N_k = k_{1h}/k_{2h}$ or $N_k = k_{ch} / k_{sh}$
$N_{mn}^{(i)}$, $N_{mn}^{(ii)}$	Norms to determine $C_{mn}^{(i)}$ and $C_{mn}^{(ii)}$, respectively
$N_{mn}^{(*)}$	Constant to determine the coefficient $C_{mn}^{(*)}$, $C_{mn}^{T(*)}$
q	External loading
q_0	Uniform external loading or initial surcharge
$q_{\max}$	Maximum of step and ramp loadings on the composite ground surface
r, z	Cylindrical coordinates
r', z'	Integration variables corresponding to the cylindrical coordinates
$R_{cmn}^{(i)}$, $R_{smn}^{(i)}$	Eigenfunctions of eigenvalues $\nu_{cmn}^{(i)}$ and $\nu_{smn}^{(i)}$, respectively
$R_{cmn}^{(ii)}$, $R_{smn}^{(ii)}$	Eigenfunctions of eigenvalues $\nu_{cmn}^{(ii)}$ and $\nu_{smn}^{(ii)}$, respectively
$\bar{S}_1$, $\bar{S}_2$	Average surface settlements of stone column and soft soil, respectively (Chapter 5)

$\overline{S}_{1ref}$, $\overline{S}_{2ref}$ Referenced average surface settlements of stone column and soft soil, respectively (Chapter 5)

$\overline{S}_1^*$, $\overline{S}_2^*$ Normalised average surface settlements of stone column and soft soil, respectively (Chapter 5)

$\overline{S}_{\sigma 01}$, $\overline{S}_{\sigma 02}$ Average surface settlements of stone column and soft soil due to the total vertical stress values σ_{01} and σ_{02}, respectively (Chapter 5)

S_1, S_2 Settlements of stone column and soft soil, respectively (Chapter 6)

$\overline{S}_2$ Average settlement of soft soil against radius (Chapter 6)

S_{1f}, S_{2f} Final settlements of stone column and soft soil, respectively (Chapter 6)

S_c, S_s Settlement at points with depth $z = z_0$ in stone column and soft soil, respectively

$\overline{S}_c$, $\overline{S}_s$ Average surface settlement of stone column and soft soil, respectively

t Elapsed time

t' Integration variable corresponding to time

t_1 Duration for the first step loading or construction time for the ramp loading

t_f Ending time of the consolidation in soft soil, corresponding to the homogeneous consolidation formulation

$T_{mn}^{(i)}$, $T_{mn}^{(ii)}$ Time-dependent functions of eigenvalues $\beta_{mn}^{(i)}$ and $\beta_{mn}^{(ii)}$, respectively

u_1, u_2 Excess pore water pressures at any point in stone column and soft soil, respectively

$u_i^{(i)}$ | Excess pore water pressure at any point in the foundation corresponding to the contribution of eigenvalues pair $(\nu_{1mn}^{(i)}, \nu_{2mn}^{(i)})$

$u_i^{(ii)}$ | Excess pore water pressure at any point in the foundation corresponding to the contribution of eigenvalues pair $(\nu_{1mn}^{(ii)}, \nu_{2mn}^{(ii)})$

$u_i^{(*)}$ | Alternative notation of $u_i^{(i)}$ and $u_i^{(ii)}$

$\bar{u}_1$, $\bar{u}_2$ | Average excess pore water pressures within stone column and soft soil, respectively (Chapter 5)

$\bar{u}_1$, $\bar{u}_2$ | Average excess pore water pressures against radius of stone column and soft soil, respectively (Chapter 6)

u_c, u_s | Excess pore water pressure at any point in stone column and soft soil, respectively

$u_c^{(i)}$, $u_s^{(i)}$ | Excess pore water pressure at any point in stone column and soft soil due to the contribution of eigenvalues pair $(\nu_{cmn}^{(i)}, \nu_{smn}^{(i)})$, respectively

$u_c^{(ii)}$, $u_s^{(ii)}$ | Excess pore water pressure at any point in stone column and soft soil due to the contribution of eigenvalues pair $(\nu_{cmn}^{(ii)}, \nu_{smn}^{(ii)})$, respectively

$\bar{u}_c$, $\bar{u}_s$ | Average excess pore water pressure within stone column and soft soil, respectively

$\bar{U}_1$, $\bar{U}_2$ | Average degree of consolidation for stone column and for soft soil, respectively

$\bar{U}_c$, $\bar{U}_s$ | Average degree of consolidation of stone column and of soft soil, respectively

Y_0, Y_1 | Bessel functions of the second kind of order zero and one, respectively

z_0 | Depth of equal strain plane (i.e. no differential settlement)

Z_m	Eigenfunctions of eigenvalues λ_m

Greek Notations

α	Ratio of bottom to top vertical stress of stone column and soft soil
α_1	Settlement parameter against z-domain at time t
α_{1f}	Final settlement parameter against z-domain
β_f	Settlement parameter against r-domain
$\beta_{mn}^{(i)}$, $\beta_{mn}^{(ii)}$	Eigenvalues corresponding to pairs $(\nu_{1mn}^{(i)}, \nu_{2mn}^{(i)})$ and $(\nu_{1mn}^{(ii)}, \nu_{2mn}^{(ii)})$, respectively
γ	Shear strain in soft soil
γ_f	Final shear strain in soft soil
γ_w	Unit weight of water
$\overline{\varepsilon}_{vc}$, $\overline{\varepsilon}_{vs}$	Average volumetric strain of stone column and soft soil, respectively
κ_c, κ_s	Square root of vertical to horizontal permeability ratio of stone column and soft soil, respectively
$\Delta_{mn}^{(i)}$, $\Delta_{mn}^{(ii)}$	Temporary variables
$\Delta\overline{S}$	Average differential settlement between soft soil and stone column
$\Theta_i^{(i)}$	Excess pore water pressure at any point in the foundation corresponding to the contribution of eigenvalues pair $(\nu_{1mn}^{(i)}, \nu_{2mn}^{(i)})$ for the homogeneous consolidation formulation

$\Theta_i^{(ii)}$	Excess pore water pressure at any point in the foundation corresponding to the contribution of eigenvalues pair $(\nu_{1mn}^{(ii)}, \nu_{2mn}^{(ii)})$ for the homogeneous consolidation formulation
$\Theta_i^{(*)}$	Alternative notation of $\Theta_i^{(i)}$ and $\Theta_i^{(ii)}$
$\bar{\bar{\Theta}}_1$, $\bar{\bar{\Theta}}_2$	Average excess pore water pressures within stone column and soft soil corresponding to the homogeneous consolidation formulation, respectively
λ_m	Eigenvalues in *z*-domain
$\mu_{cmn}^{(i)}$, $\mu_{smn}^{(i)}$	Alternative forms of the eigenvalues $\beta_{mn}^{(i)}$ for stone column and soft soil, respectively
$\mu_{cmn}^{(ii)}$, $\mu_{smn}^{(ii)}$	Alternative forms of the eigenvalues $\beta_{mn}^{(ii)}$ for stone column and soft soil, respectively
$\nu_{1mn}^{(i)}$, $\nu_{2mn}^{(i)}$	Eigenvalues in *r*-domain corresponding to eigenfunctions $\Psi_{1mn}^{(i)}$ and $\Psi_{2mn}^{(i)}$, respectively
$\nu_{1mn}^{(ii)}$, $\nu_{2mn}^{(ii)}$	Eigenvalues in *r*-domain corresponding to eigenfunctions $\Psi_{1mn}^{(ii)}$ and $\Psi_{2mn}^{(ii)}$, respectively
$\nu_{cmn}^{(i)}$, $\nu_{smn}^{(i)}$	Eigenvalues corresponding to $R_{cmn}^{(i)}$ and $R_{smn}^{(i)}$, respectively
$\nu_{cmn}^{(ii)}$, $\nu_{smn}^{(ii)}$	Eigenvalues corresponding to $R_{cmn}^{(ii)}$ and $R_{smn}^{(ii)}$, respectively
ν_{P1}, ν_{P2}	Poisson's ratios for stone column and soft soil, respectively
σ_{01}, σ_{02}	Initial total vertical stresses within stone column and soft soil, respectively

σ_1, σ_2	Total vertical stresses within stone column and soft soil, respectively
$\bar{\sigma}_1$, $\bar{\sigma}_2$	Average total vertical stresses within stone column and soft soil, respectively
$\bar{\sigma}_2$	Average total vertical stress in soft soil against radius (Chapter 6)
$\sigma_{1f}(z)$	Final total vertical stress in stone column at any depth z
$\sigma_{2f}(r,z)$	Final total vertical stress at any coordinate (r,z) in soft soil
$\bar{\sigma}_{2f}$	Average of final total vertical stress in soft soil against radius
σ'_c, σ_c	Effective and total vertical stress in stone column, respectively
σ'_s, σ_s	Effective and total vertical stress in soft soil, respectively
σ_{cr}, σ_{sr}	Total vertical stress distribution on the top of stone column and soft soil under the applied external loading, respectively
$\bar{\sigma}_c$, $\bar{\sigma}_s$	Average total vertical stress within stone column and soft soil, respectively
$\bar{\sigma}_{cr}$, $\bar{\sigma}_{sr}$	Average total vertical stress on top of stone column and soft soil, respectively
τ	Shear stress in soft soil
τ_f	Final shear stress in soft soil
φ_B	Angular frequency of the sinusoidal loading
ω_m	Temporary variable
$\Psi^{(i)}_{1mn}$, $\Psi^{(i)}_{2mn}$	Eigenfunctions corresponding to eigenvalues $\nu^{(i)}_{1mn}$ and $\nu^{(i)}_{2mn}$, respectively

$\Psi_{1mn}^{(ii)}$, $\Psi_{2mn}^{(ii)}$	Eigenfunctions corresponding to eigenvalues $\nu_{1mn}^{(ii)}$ and $\nu_{2mn}^{(ii)}$, respectively
$\Psi_{imn}^{(*)}$	Alternative notation of $\Psi_{imn}^{(i)}$ and $\Psi_{imn}^{(ii)}$
$(*)$	Alternative superscript of (i) and (ii)