MAKING SENSE OF NUMBER

Improving Personal Numeracy

Many adults feel that they lack the necessary foundational knowledge in mathematics required to confidently use mathematics in daily life and in their careers. *Making Sense of Number* is a concise introduction to personal and professional numeracy skills, helping readers to become more mathematically competent. It includes relevant content to assist pre-service teachers to improve numeracy for the classroom or to prepare for the LANTITE, as well as support for practising teachers to develop their understanding and skills in numeracy.

Making Sense of Number focuses on number sense as a conceptual framework for understanding mathematics, covering foundational areas of mathematics that often cause concern such as multiplication, fractions, ratio, rate and scale. The authors use real-world examples to explain mathematical concepts in an accessible and engaging way. Learning activities throughout the book help readers self-assess their understanding of the mathematical concepts discussed, and answers to activities are included.

Written by authors with over 30 years' experience teaching mathematics at primary, secondary and tertiary levels, *Making Sense of Number* is an essential guide for both pre-service teachers and those looking to improve their understanding of numeracy.

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PREFACE

Rather than being a 'how to do' or 'how to teach' mathematics guide, this text specifically aims to assist people who self-identify as needing some additional support in becoming more mathematically confident and competent (although at times we will mention the importance of certain aspects for teachers). The approach taken is to focus on a few foundational concepts that we feel, from our experience, cause common mathematical difficulties for people. The reader is asked to reflect on and self-assess in relation to some core content knowledge and also their personal attitudes to mathematics. From that point, the text is intended for the reader to engage in areas where it is felt assistance is needed. Broadly the focus is to assist the reader to take some steps to improve their number sense. As we progress through the chapters, we will provide learning activities to help clarify ideas and to give readers a chance to apply the ideas. When relevant, the answers to the questions are provided in the Appendix.

ABOUT NUMBER SENSE

Number sense can be many things to many people, so we have no intention of trying to present the myriad possibilities. We have chosen topics (as reflected in the contents page) that we feel are at the core of mathematical understanding and being numerate but which we find are the most common areas of concern. Because mathematics can be inherently complex and abstract, understanding foundational elements that develop personal number sense is vital. Whenever possible, authentic scenarios are used to help the reader contextualise the mathematics concepts and give insight into how they are used and why they are so important.

ABOUT THE AUTHORS

Annette and Geoff each have over 20 years' experience teaching mathematics in secondary and primary schools respectively. They have taught mathematics education courses at tertiary level for the last 15 years. During this time, they have been researching with and working with in-service teachers in the field of mathematics. In more recent years, they have also had considerable experience helping pre-service education students prepare for or re-sit their LANTITE (Literacy and Numeracy Test for Initial Teacher Education), a process during which some common issues have become apparent. These many years of experience across the spectrum of mathematics education have informed the focus of this text.