Modeling of Vector Magnetic Hysteresis of Soft Magnetic Composite Material


Abstract—Thanks to the unique magnetic properties, soft magnetic composite (SMC) materials and their application in electromagnetic devices have achieved significant development. The typical application example of SMC is the electrical machine with complex structure, such as claw pole and transverse flux machines, in which the magnetic field is basically rotary. To design and analyze such a device, vector magnetic properties of the core material should be properly determined, modeled and applied. This paper presents the modeling of vector magnetic hysteresis of SMC based on a Stoner-Wohlfarth (S-W) elemental operator. A phenomenological mean-field approximation is used to consider the interaction between particles. With the presented model, the magnetization processes of SMC under both alternating and rotating fluxes are numerically simulated. The simulations have been verified by experimental measurements.

Keywords—Vector magnetic hysteresis, modeling, soft magnetic composite.

1. Introduction

Thanks to their many unique properties such as 3-D magnetic isotropy, very low eddy current loss, great design flexibility, and great potential for low cost mass production of electromagnetic devices, SMC (soft magnetic composite) materials and their applications have undergone significant development in the past decade [1]. Typical application examples include claw pole and transverse flux machines in which the flux flows substantially in 3-D space [2,3]. The conventional laminated electrical steel is not suitable for constructing the core of such machines because the flux component perpendicular to the steel plane may cause excessive eddy current loss. The SMC material seems to be an ideal substitute.

The relationship between the magnetic flux density (\(B\)) and the magnetic field strength (\(H\)) is among the basic properties of magnetic materials. When \(B\) and \(H\) are restrained in the same direction, their relation is reduced to the well-known scalar \(B-H\) loop. However, in the 3-D flux machines, \(B\) and \(H\) are not aligned. Furthermore, both \(B\) and \(H\) are rotating and \(B\) lags \(H\) by an angle. In other words, both the magnitudes and directions of the \(B\) and \(H\) may vary, as well as the directional angle difference between the two vectors. Therefore, the vector magnetic properties, such as the vector \(B-H\) relation and core loss, under different vector magnetizations, should also be investigated [3-7]. Owing to the very complex mechanism of the magnetic hysteresis, particularly the vector hysteresis, which is not yet fully understood so far, the development of mathematical models of magnetization process has not been successful, in particular for the engineering practice.

A huge amount of work has been conducted by various researchers for modeling the vector magnetic hysteresis. Among the noticeable work are: (a) the Stoner and Wohlfarth (S-W) model that was postulated based on the rotation of magnetic moments of single domain particles with respect to their easy axes [8]; (b) the vector Presaich model constructed by the superposition of scalar Presaich models [9]; and (c) the combined model that incorporates the vector elemental operator of the S-W model into the Preisach diagram such that the new model has the vector nature of the S-W model while retaining the efficiency of the Preisach model [6,10]. However, the phenomenological modeling of vector hysteresis has long been centered on the classical S-W model [6,7] because of the vector nature of the model.

This paper presents the modeling of vector magnetic hysteresis of SMC based on a Stoner-Wohlfarth (S-W) elemental operator [11]. A phenomenological mean-field approximation is used to consider the interaction between particles [12]. With the presented model, the magnetization processes of SMC under both alternating and rotating fluxes are numerically simulated. The simulations have been verified by experimental measurements.

2. Vector Hysteresis Modeling Based on S-W Model

Vector hysteresis can be defined as vector nonlinearity with the property that past extremum values of input projections along all possible directions may affect future values of the output [9]. Therefore, the mathematical model of vector hysteresis should be able to detect and store past extremes of input projection along all possible directions and choose the appropriate value of the vector output...
according to the accumulated history.

The S-W model assumes that a magnetic material consists of a collection of small particles, each with anisotropy due to stress, crystal structure, or particle shape. Each particle is uniformly magnetized to saturation in the direction of the easy axis, giving a single magnetic domain with moment \( \mathbf{m} \), which is free to rotate in any direction. Such a particle is called the S-W particle. The particle interaction, either due to quantum exchange forces or to magnetic dipole-dipole forces, is not considered.

When a magnetic field \( \mathbf{H} \) is applied, the magnetic moment of an S-W particle rotates to the orientation which results in a minimum energy, as shown in Fig. 1.

The total energy of a single domain with moment \( \mathbf{m}_s \) can be expressed as

\[
E(\theta, \mathbf{H}) = K \sin^2 \theta - \mu_0 \mathbf{m}_s \cdot \mathbf{H}
\]  
(1)

where \( K \) is the domain crystal anisotropy constant, and \( \theta \) the angle between \( \mathbf{m}_s \) and the easy axis.

![Fig. 1. Rotation of \( \mathbf{m}_s \) due to an applied field \( \mathbf{H} \).](image)

The positions of minimum energy can be found by solving \( \partial E(\theta, \mathbf{H}) / \partial \theta = 0 \) for \( \partial^2 E(\theta, \mathbf{H}) / \partial \theta^2 > 0 \). It can be shown that there are two energy minima if a small field is applied. As the field strength increases, the positions of these minima change. Initially, these changes are reversible. When the applied field strength exceeds a certain critical value \( H_c \), however, one of the energy minima becomes unstable, and the domain magnetization jumps to the other minimum, which is the global energy minimum. This critical point at which the irreversible domain rotation occurs is the point of minimum energy for which \( \partial^2 E(\theta, \mathbf{H}) / \partial \theta^2 = 0 \). Finally we have

\[
H_{e}^{2/3} + H_{p}^{2/3} = H_{k}^{2/3}
\]  
(2)

where \( H_e = H_s \) (taking the \( x \) axis as the easy axis) and \( H_p = H_s \sin \phi + H_k \cos \phi \) are the components of \( \mathbf{H} \) on the easy axis of the particle and on the axis perpendicular to the easy axis respectively, and \( H_{k} = 2K / (\mu_0 m_s) \).

Fig. 2 plots the rotation of a single S-W particle, where the asteroid boundary, determined by (2), separates the reversible and irreversible domain rotation.

It can be shown that the equilibrium position of \( \mathbf{m}_s \) is on one of the lines tangent to the asteroid and passing through the tip of \( \mathbf{H} \). When \( \mathbf{H} \) is outside the asteroid, two such tangent lines can be drawn, and the equilibrium magnetization is parallel to that making a smaller angle with the easy axis, as shown by \( \mathbf{m}_s^+ \). When \( \mathbf{H} \) is inside the asteroid, four tangent lines can be drawn, and two possible equilibrium magnetic moment are parallel to the two lines making smaller angles with the easy axis, as shown by \( \mathbf{m}_s^- \) and \( \mathbf{m}_s^+ \). Such a response of a single S-W particle can be viewed as a vector elementary hysteresis operator.

![Fig. 2. Asteroid vector elemental hysteresis operator.](image)

The magnetization in a bulk material is the vector sum of the contributions of all of the constituent domains, i.e.

\[
\mathbf{M} = \frac{1}{V} \int_{0}^{2\pi} \int_{0}^{\pi} \mathbf{m}_s(\xi, \psi, \mathbf{H}) \rho(\xi, \psi) \sin(\psi) d\psi d\xi
\]  
(3)

where

\[
\int_{0}^{2\pi} \int_{0}^{\pi} \rho(\xi, \psi) \sin(\psi) d\psi d\xi = 1
\]  
(4)

\( V \) is the sample volume, and \( \rho(\xi, \psi) \) the distribution of the S-W particles in terms of the spherical coordinates (\( \xi, \psi \)).

In the assumptions of the S-W model, the interaction between S-W particles and the pinning effects of domain walls are ignored. In real magnetic materials, however, these effects are important, and should not be neglected. To account for the interaction between domains, a modified S-W model was proposed in Ref. [7] by adding a mean field term, \( \mathbf{H}_{cf} = \mathbf{H} + \alpha \mathbf{M} \), where \( \alpha \) is a constant feedback coefficient. This modifies the energy of an arbitrary particle, and (3) becomes

\[
\mathbf{M} = \frac{1}{V} \int_{0}^{2\pi} \int_{0}^{\pi} \mathbf{m}_s(\xi, \psi, \mathbf{H} + \alpha \mathbf{M}) \rho(\xi, \psi) \sin(\psi) d\psi d\xi
\]  
(5)

This macroscopic mean field interaction is qualitatively correct, but requires further adjustments, in particular, to the easy axis distribution.

### 3. Numerical Implementation

In numerical implementation, the magnetization is obtained by the vector sum of the magnetic moments \( \mathbf{m}_s \) of an assembly of \( N_p \) magnetic particles, as expressed below

\[
\mathbf{M} = \sum_{i=1}^{N_p} \mathbf{m}_s(H + \alpha \mathbf{M})
\]  
(6)

#### 3.1 1D Alternating Hysteresis
In the simulation, 1800 S-W pseudo-particles are uniformly distributed in a plane. The vector magnetization is computed under an alternating magnetic field excitation. Fig. 3 illustrates the hysteresis loop of $M_{sh}$-$H$, where $M_{sh}$ is the component of magnetization aligned with the external field. It can be seen that the hysteresis loops can be modified by adjusting the dimensionless parameter $\alpha$ in the interaction mean field. Although these loops are not identical, they are very similar in character.

Since the particles are uniformly distributed, all other components of $\textbf{M}$ are zero, and the vector magnetization $\textbf{M}$ is equal to $M_{sh}$, in this symmetrical case. This means that $\textbf{H}$ and $\textbf{M}$ are collinear and the components may be treated as scalars. Fig. 4 shows the angular positions of the applied field $\textbf{H}$ and the resultant magnetization $\textbf{M}$ in the Cartesian coordinates.

In a magnetically anisotropic material, the S-W particles are not uniformly distributed, and $\textbf{M}$ will not be aligned with the applied $\textbf{H}$. The position of $\textbf{M}$ is then determined by both $\textbf{H}$ and the particle distribution function.

### 3.2 2D Rotational Hysteresis

When a rotating field $\textbf{H}$ is applied, the magnetization $\textbf{M}$ can also be calculated by adding the magnetic moments $\textbf{m}$ of all particles. The simulation results corresponding to various magnitudes of $\textbf{H}$ are shown in Fig. 5.

In an isotropic magnetic material, because of the uniform particle distribution, $\textbf{M}$ lags the applied $\textbf{H}$ vector a constant angle for a given magnitude of $\textbf{H}$. The lag angle of the $\textbf{M}$ versus the $\textbf{H}$ magnitude is illustrated in Fig. 6. As expected, the lag angle of $\textbf{M}$ is nearly zero in the low field region where all the changes of magnetic momentum of single domain particles are reversible, and the lag angle increases when $\textbf{H}$ increases. When $\textbf{H}$ reaches about half of the anisotropy field $H_K$, the lag angle increases very quickly to the maximum, which is nearly 24°, and then decreases with the increase of $\textbf{H}$ magnitude. It can also be seen in Fig. 6 that the lag angle approaches zero when the $\textbf{H}$ magnitude is larger than the anisotropy field $H_K$. 

![Rotating Field](image)
3.3 3D Rotational Hysteresis

When a specified rotating excitation field is applied in 3D space, the magnetization \( M \) in the magnetic material will also rotate in space. The computing algorithm of magnetization for a bulk magnetic material is also based on the model of an S-W particle.

Fig. 7 illustrates the loci of the rotating \( H \) (marked by +) and the magnetization \( M \) (marked by * ) of the isotropic material. A circular rotating field is applied in the plane that is inclined 30° from the horizontal plane. According to the calculation, the \( M \) component perpendicular to the circular rotating field plane is cancelled due to the uniform distribution of the particles. Thus, the magnetic magnetization \( M \) is in the same plane with the field strength \( H \). The calculation shows that for a normalized field \( H \) of 0.6, the magnitude of the normalized \( M \) is 0.82, and it lags the excitation field for about 10°.

4. Conclusions

In this paper, the vector magnetization of soft magnetic composite material under either an alternating excitation field or a rotating excitation field is evaluated by using a modified S-W model which incorporates a mean interaction field into the classical S-W model. The analysis results would be helpful for deep understanding and modeling of the vector magnetization process of soft magnetic composite material.

References

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