Characterization of Rectangular Plates using Complex Natural Resonance

Siyuan Li
School of Information Technology and Electrical Engineering (ITEE),
The University of Queensland,
St. Lucia, QLD4072, Australia
Email: siyuan.li@ uqconnect.edu.au

Chad Owen Hargrave
Sustainable Mining Research Program,
Commonwealth Scientific and Industrial Research Organisation (CSIRO),
Pullenvale, QLD4069, Australia
Email: chad.hargrave@csiro.au

Hoi-Shun Lui
Global Big Data Technologies Centre,
School of Electrical and Data Engineering,
University of Technology Sydney Ultimo, NSW2007, Australia
Email: hoishunantony.lui@uts.edu.au

***Abstract*—The practicality of extracting natural resonant modes from the late-time target responses of conducting rectangular plates with various length-to-width ratios is studied using the matrix pencil method. The extracted resonant modes follow a similar behavior as the results obtained from a theoretical analysis reported in the literature. Our results show that it is feasible for these modes to be excited using a practical system, which opens the opportunities for automated target recognition.**

***Keywords*— Radar Target Recognition, Complex Natural Resonance, Singularity Expansion Method, Rectangular Plate**

I. Introduction

Characterization of definitive radar target using the complex natural resonance (CNR) phenomena has been well studied over the years [1]-[13]. By contrast with traditional narrowband pulses reflected from target that provide us with information such as range and strength of the scatterers, the natural resonant frequencies (NRFs), $s_n = \sigma_n + j\omega_n$, embedded in the late-time (T_L) of the wideband transient signature, which are theoretically target-dependent [1]-[5], enable them to be used as a feature set for classification and subsequently recognition. Mathematically, the late-time of the transient target response can be given by [1]:

$$r(t) = \sum_{n=1}^{M}\left(A_n e^{s_n(t)} + A_n^* e^{s_n^*(t)}\right), t > T_L. \quad (1)$$

Here $\sigma_n (< 0)$ and ω_n are known as the damping factor and resonant frequency, respectively. The superscript * denotes the complex conjugate operation. Under band-limited excitation, it is assumed that only M modes are excited. The target-dependent NRFs can be retrieved from the late-time response using matrix pencil method (MPM) [6]-[8]. The residues A_n, which indicate the strength of the mode, depend on the incident and observing aspect and polarization. A mode may not be able to be retrieved at certain aspect or polarization where its residue is small. To overcome this limitation, one may consider extracting one set of NRFs from multiple target signatures obtained from different aspects [7] or polarization [2]-[5]. Alternatively, NRFs can also be evaluated by zero searching of the moment matrix determinant [1]. This moment matrix is essentially the impedance matrix obtained from the integral equations that describe the scattering problem. The resultant NRFs are certainly target dependant and are considered as a "ground truth". Theoretically, there is an infinite number of NRFs [1]. Modes with similar ω_n but larger σ_n (in magnitude) are higher-order modes [9]. In practice, σ_n of "first-order" modes, are usually smaller in magnitude than those of the "higher-order" modes and thus dominate the response [10]. When NRFs are used as a feature set for target recognition, it is of practical interest to identify these dominant modes. NRF extractions through zero searching of the moment matrix determinant and related formulations have been conducted for wire [9], [10] and rectangular plates [11]. NRF extraction from late-time responses have also been conducted for wire [12] and square plate [13]. To our knowledge, NRF extraction from rectangular planar objects from late-time response, which is of practical interest as rectangular plate is a fundamental geometry to many realistic scatterers, has not been reported. We therefore aim to identify the practically excitable NRFs of rectangular plates in this paper.

II. NRFs of Rectangular Plates

Five perfect electric conductor (PEC) rectangular plates with different width-to-length (W/L) ratios (W ≤ L) range from 0.2 to 1.0 are considered. The ratios of 0.2 and 1.0 correspond to a strip and a square plate, respectively. A physical length of L = 0.3m is chosen. The geometry is modelled in FEKO [14] with the plate lying on the *x*-*y* plane with the length in parallel with the *y*-axis. The target is independently illuminated under parallel polarized plane wave at 18 different elevation angles [$\theta = 0^\circ, 5^\circ, 10^\circ, \ldots, 85^\circ$] along $\varphi = 90^\circ$. Here, $\theta = 0^\circ$ corresponds to the *z* axis and $\varphi = 0^\circ$ corresponds to the *x* axis. In this paper, only monostatic response is considered, which resulted in 18 responses at different elevation angles. The electromagnetic problems are solved in frequency domain using moment method in FEKO [14] from 4.88MHz to 10GHz with 2048 samples. The frequency domain data are then windowed using a Gaussian window followed by an Inverse Fourier Transform [15].

Table 1. Normalized NRF ($s_n L/c$) for rectangular planar object with different Length/Width (L/W) ratios

	(a) to (f) NRF extracted from 18 target responses						(g) W/L = 1.0 (NRF extracted from one response)		
	(a) W/L = 0.2	(b) W/L = 0.4	(c) W/L = 0.6	(d) W/L = 0.8	(e) W/L = 1.0	(f) Wire	$\theta = 15^\circ$	$\theta = 45^\circ$	$\theta = 75^\circ$
1	-0.45±j2.49 -0.88±j2.66^	-0.57±j2.29 -1.08±j2.48^	-0.64±j2.17 -1.19±j2.51^	-0.79±j2.20	-0.84±j2.09	-0.219±j2.97	-0.89±j2.06	-0.88±j2.06	-0.86±j2.08
2	-0.82±j5.48 -1.52±j6.44^	-1.12±j5.25	-1.38±j5.09	-1.65±j5.23	-1.24±j5.59	-2.28±j6.13	-1.43±j5.77	-1.33±j5.55	-1.20±j5.67
3	-1.04±j8.39	-1.49±j8.23	-2.20±j9.12	-1.58±j8.51	-1.85±j8.32 -2.76±j8.52^	-0.53±j9.01^ -0.26±j9.25	-1.25±j8.56	-1.36±j7.97	-1.66±j8.45
4	-1.26±j11.56	-1.93±j11.55	-1.65±j11.59	-2.17±j11.56	-1.36±j12.66	-0.32±j12.40	-1.59±j11.87		-1.37±j12.79
5	-1.43±j14.63	-1.61±j15.14	-1.98±j14.60	-2.14±j15.49	-2.16±j14.94	-0.34±j15.53	-1.54±j13.48	-1.31±j13.81	-1.64±j14.97
6	-1.91±j17.98	-1.77±j17.99	-1.97±j17.99	-1.78±j18.07	-1.86±j18.34	-0.36±j18.68	-2.05±j18.34	-1.67±j17.72	-1.51±j18.68
7	-2.61±j21.74	-2.47±j20.82	-2.12±j21.01	-2.49±j20.85	-2.28±j20.78	-0.38±j21.83	-2.37±j20.92	-1.69±j21.95	-1.95±j21.29
8		-2.04±j24.94	-2.03±j24.19	-1.75±j24.48	-1.64±j24.61	-0.39±j24.98	-1.54±j24.41		-1.56±j25.03
9	-2.80±j28.15	-2.17±j29.12	-2.02±j27.46	-1.70±j27.59	-1.81±j27.84	-0.41±j28.13	-3.12±j27.57	-1.51±j27.40	-1.99±j27.94
10	-4.24±j33.09	-3.08±j34.26^	-1.87±j31.61^	-2.39±j31.20	-2.05±j31.22	-0.42±j31.28	-2.50±j30.27	-1.41±j30.61	-1.45±j31.09

A. NRFs of Rectangular Plates

The normalized NRFs ($s_n L/c$, where $c = 3 \times 10^8 m/s$) of 4 rectangular plates [(a) to (d)] are listed in Table 1. For each target, the late-time period of all 18 responses are input to the modified MPM algorithm [7] resulted in one set of NRF. By incorporating the 18 responses, aspect dependency has been taken into account such that the resultant NRFs can be regarded as "ground truth". We found that the resonant frequencies (ω_n) of the first 10 modes are fairly similar for all the 5 targets, which agrees with the findings reported in Fig. 2 of [11] where ω_ns are fairly stable as W/L varies. Modes 8 is not found for rectangular plate (a). The NRFs denoted by ^ are the ones with similar ω_n but higher σ_n, which could be higher-order modes [9].

B. NRFs of Square Plate and Wire

The results for a square plate [(e) W/L = 1.0] are listed in Table 1 with all the 10 modes identified when 18 responses are used. To evaluate how the aspect dependency would impact on the NRF extraction, columns 7 to 10 [(g)] show the extracted NRF based on a single monostatic response at different aspect angle using the original MPM [6]. Only 8 modes are found at 45° and all 10 modes are extracted at 15° and 75°. Comparing with the normalized NRFs of a wire in [2] and [12], all the first 10 modes can be found at $\theta = 15^\circ$. 9 and 8 modes are found at 45° and 75°, respectively (Note that θ is measured from the wire in [2] and [12], which corresponds to $\theta = 90^\circ$ here. In other words, $\theta = 15^\circ$, 45° and 75° in [2] and [12] correspond to $\theta = 75^\circ$, 45° and 15° in this example). As a sanity check, we revisit the CNR phenomena of a 30cm wire target (f) and extract one set of NRF from 18 target responses [$\theta = 0^\circ$ to 85°]. The results show good agreement with the ground truth in [10], and agree with Fig. 2 in [11] where ω_n of the wires and plates are fairly similar.

III. CONCLUSION

Target-dependent NRFs are successfully extracted from late-time responses of various rectangular plates. These NRFs show that the damping factor varies as the width of the plate. This observation agrees with the findings reported in [11], where the NRFs were obtained in a theoretical manner. Our findings indicate that the NRFs of rectangular plates can be practically retrieved from late-time responses in an accurate manner, which opens the opportunity for target recognition applications.

REFERENCES

[1] C. E. Baum, "The Singularity Expansion Method" in Transient Electromagnetic Fields, L.B. Felsen Ed., New York Springer-Verlag, pp. 129-179, 1976.

[2] H. S. Lui, F. Aldhubaib, S. Crozier and N. V. Shuley, "Ultra Wideband Transient Scattering and its Applications to Automated Target Recognition," Antennas and Wave Propagation, pp. 143-166, IntechOpen, 22 Sep. 2018.

[3] H. S. Lui and N. V. Shuley, "Resonance Based Radar Target Detection with Multiple Polarisations", Proc. IEEE Antenna Propag. Society Int. Symp., Albuquerque, NM, 9–14 Jul., 2006, pp. 3259-3262.

[4] H. S. Lui and N. V. Shuley, "On the Polarization Response Using Resonances for Target Recognition," Asia Pacific Microw. Conf., Yokohama, Japan, 7-10 Dec., 2010, pp. 1813-1816.

[5] H. S. Lui, "Characterization of Radar Target Using Multiple Transient Responses", IEEE Antennas Wireless Propag. Lett., Vol. 14, pp. 1750-1753, Sep. 2015

[6] T. K. Sarkar and O. Pereira, "Using the Matrix Pencil Method To Estimate the Parameters of a sum of Complex Exponentials", IEEE Antennas Propag. Mag., Vol. 37, No.1, pp 48-55, Feb., 1995.

[7] T. K. Sarkar, S. Park, J. Koh and S.M. Rao, "Application of the Matrix pencil method for Estimating the SEM (Singularity Expansion Method) Poles of Source-Free transient Responses from Multiple look Directions", IEEE Trans. Antennas Propag, Vol. 48, No. 4, pp 612-618, Apr., 2000.

[8] S. Li, C. O. Hargrave and H. S. Lui, "Resonance-based Radar Target Classification using the Matrix Pencil Method and the Cauchy Method," Submitted to IEEE Antenna Propag. Society Int. Symp., Singapore, 4–10 Dec., 2021.

[9] F. M. Tesche, "On the Analysis of Scattering and Antenna Problems Using the Singularity Expansion Technique" IEEE Trans. Antennas Propag., Vol. 21, No.1, pp. 53-62, Jan., 1973.

[10] E. J. Rothwell, K.M. Chen and D.P. Nyquist, "Approximate Natural Response of an Arbitrarily Shaped Thin Wire Scatterer", IEEE Trans. Antennas Propag., Vol. 39, No. 10, pp 1457-1462, Oct. 1991.

[11] W. M. Sun, K. M. Chen, D. P. Nyquist, and E. J. Rothwell, "Determination of the Natural Modes for a Rectangular Plate," IEEE Trans. Antennas Propag., Vol. 38, No. 5, pp. 643-652, May 1990.

[12] H. S. Lui and N. Shuley "On the Analysis of Electromagnetic Transients from Radar Targets using Smooth Pseudo Wigner-Ville Distribution (SPWVD)", Proc. IEEE Antenna Propag. Society Int. Symp., Honolulu, HI, 10-15 Jun., 2007, pp. 5701-5704.

[13] C. O. Hargrave, I. V. Clarkson and H. S. Lui, "Late-Time Estimation for Resonance-Based Radar Target Identification," IEEE Trans. Antennas Propag., Vol. 62, No. 11, pp. 5865–5871, Nov. 2014.

[14] Altair Engineering Inc., 2021 www.altair.com/feko/, Accessed May 2021

[15] H. S. Lui and N. V. Shuley, "On the modelling of Transient Scattering under Ultra Wideband Sources", Asia Pacific Symp. Electromagn. Compat., Beijing, China, 12-16 Apr. 2010, pp. 854-857.