Resonance-based Radar Target Classification using the Matrix Pencil Method and the Cauchy Method

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***Abstract*—A comparison of the Matrix Pencil Method and the Cauchy Method is studied. The two methods both serve as natural resonant frequency extraction algorithm but is based on information in different domains. Their performances are analyzed using synthesized data and full-wave simulated data. Both methods present the capability to extract NRFs from targets. Numerical results are presented to show the similarity and differences between the two methods.**

Keywords— Radar Target Recognition, Complex Natural Resonance, Singularity Expansion Method, Matrix Pencil Method, Cauchy Method.

I. Introduction

A wide range of techniques have long been applied for radar target identification. Among the many types, ultra-wideband (UWB) radar is distinctive for its use of time-domain impulses to illuminate targets. The Singularity Expansion Method (SEM), a well-established theory employed by UWB radar, describes the late-time (LT) period of the scattered signal as a sum of damped exponentials. These damped exponentials are target-dependent natural resonant frequencies (NRFs) and can be used as a unique signature for target classification [1]-[2]. These NRFs can be retrieved using the Matrix Pencil Method (MPM) [3]-[4] and the Cauchy method [5]-[6]. MPM operates with time-domain data which takes only the LT response as input. It transforms the time-domain data series to a linear eigensystem by first formulating the data to a Hankel matrix. The NRFs can then be solved as complex eigenvalues [3]-[4]. To employ MPM, the early-time/late-time (ET/LT) boundary needs to be known a priori as inclusion of the ET data will severely degrade the accuracies of the extracted NRFs. In practice, this boundary is not usually available. Attempts have been made to identify this boundary from the response without knowing the target and its orientation [7]-[8].

Table 1: Ground Truth and extracted NRFs for Synthetic Response

Mode	Ground Truth	MPM	Cauchy	Cauchy (QR)
1	-0.10 ± j2.00	-0.10 ± j2.00	-0.15 ± j2.01	-0.10 ± j2.00
2	-0.20 ± j3.00	-0.20 ± j3.00		-0.20 ± j3.00
3	-0.40 ± j5.00	-0.40 ± j5.00	-0.44 ± j5.04	-0.40 ± j5.00
4	-0.30 ± j7.00	-0.30 ± j7.00	-0.34 ± j6.95	-0.30 ± j7.00
5	-0.10 ± j9.00	-0.10 ± j9.00	-0.10 ± j9.00	-0.10 ± j9.00
6	-0.20 ± j12.00	-0.20 ± j12.00	-0.20 ± j12.00	-0.20 ± j12.00
7	-0.30 ± j15.00	-0.30 ± j15.00	-0.30 ± j15.00	-0.30 ± j15.00
8	-0.40 ± j19.00	-0.40 ± j19.00	-0.40 ± j19.00	-0.40 ± j19.00

The Cauchy method, on the other hand, operates with the frequency-domain data. It begins with approximating the target response in the frequency-domain as a transfer function. Following the steps of the algorithm, the polynomials of the transfer function are solved and the NRFs are found via a partial-fraction decomposition [5]-[6]. Unlike MPM, the Cauchy method does not require the knowledge of the ET/LT boundary, which could be an advantage over the MPM provided that the two methods result in a similar level of accuracies. It is therefore important to evaluate the accuracies of the extracted NRFs using the Cauchy method as compared to the ones extracted using MPM and the ground truth.

In this paper, a synthetic target response that contains eight NRFs is first used to study the accuracies of the extracted NRF using both methods. This allow us to fairly compare the two methods as the ground truth is known. To test the algorithms in a more realistic scenario, a 30cm wire and a 30cm × 30cm square plate are selected. To accurately model the ET and LT scattering phenomena, the targets are illuminated using plane wave excitation with $\theta_i = 45°$. The monostatic responses are solved in the frequency domain with 2048 samples from 4.88MHz to 10GHz using moment method in FEKO [9]. We then obtain the corresponding time-domain responses using a Gaussian window and an inverse Fourier Transform [10].

Table 2: NRFs obtained from Monostatic Responses ([a] These modes were extracted from 18 monostatic responses using M-MPM [11])

	(a) 30 cm Wire			(b) 1m Wire	(c) 30 cm × 30 cm Square Plate			
Mode	MPM	Cauchy-QR	Cauchy-QR (LT)	MPM [2]	MPM	Cauchy-QR	Cauchy-QR (LT)	M-MPM [11][a]
1	-0.17±j3.00	-0.18±j3.20	-0.17±j3.06	-0.253±j2.87	-0.88±j2.06	-0.80±j2.75	-0.77±j2.05	-0.84±j2.09
2	-0.22±j6.12	-0.27±j6.38	-0.21±j6.24	-0.370±j5.93	-1.33±j5.55	-1.05±j5.43	- 1.56±j5.96	-1.24±j5.59
3	-0.28±j9.27	-0.32±j9.58	-0.24±j9.38	-0.458±j9.01	-1.36±j7.97		- 2.01±j8.79	-1.85±j8.32
4	-0.33±j12.39	-0.20±j12.66	-0.26±j12.49	-0.512±j12.1		-3.64±j11.77		-1.36±j12.66
5	-0.36±j15.55	-0.29±j15.66	-0.29±j15.65	-0.609±j15.2	-1.31±j13.81	-1.94±j13.49	-1.35±j13.10	-2.16±j14.94
6	-0.46±j18.68	-0.29±j19.18	-0.24±j19.02	-0.833±j18.4	-1.67±j17.72	-2.02±j16.49	-3.04±j16.34	-1.86±j18.34
7				Not found at 45°	-1.69±j21.95	-2.76±j22.97	-2.73±j21.51	-2.28±j20.78
8	-0.47±j24.97	-0.30±j25.00	-0.32±j24.98	-1.043±j24.5		-2.75±j25.15	-1.76±j25.14	-1.64±j24.61
9	-0.45±j28.09	-0.32±j28.13	-0.32±j28.13	-0.732±j27.9	-1.51±j27.40	-2.70±j27.42	-1.85±j27.26	-1.81±j27.84
10	-0.44±j31.24	-0.37±j31.30	-0.29±j31.25	-1.294±j30.9	-1.41±j30.61	-2.67±j31.66	-2.61±j31.73	-2.05±j31.22

II. Comparison of the Two Methods

The eight normalized NRFs ($s_n/L = (\sigma_n \pm j\omega_n)/L$) of the synthetic response and the extracted NRFs are listed in Table 1. Our results show that the NRFs obtained using both methods are well matched with the ground truth. The second NRF is absent when Cauchy is used, but it is recovered by Cauchy-QR where a QR decomposition is applied to a submatrix aiming to reduce computational error [5].

Wires are considered as high-Q object that have small (in magnitude) damping factors (σ_n) in their complex NRFs, which result in easily distinguishable sharp peaks in the frequency response. The results in Table 2 confirm the capability of MPM and Cauchy-QR on realistic signals rather than only the ideally synthesized ones. We are aware that the Cauchy method operates with the frequency domain data, which has taken into account of both the ET and LT responses such that the extracted NRFs are slightly different to the ones obtained from MPM. To better compare the performance of the algorithm, we have applied a Fourier Transform to the LT response of the wire target and extracted the NRF using Cauchy-QR method (Cauchy-QR (LT)). We found that the NRFs are now better matched with the ones obtained from MPM. As a sanity check, the results for a 1m wire presented in an earlier study [2] is included and found that the NRFs follow a similar pattern and that mode 7 was not extracted at this illumination angle.

Metallic planar structures are sometimes classified as low-Q targets. This means that some of the NRFs have a relatively large (in magnitude) damping factor (σ_n). In the time domain, these modes will die out rapidly [1] which could be too short to be captured in the late-time response. Table 1 shows the NRFs extracted using MPM, Cauchy-QR and Cauchy-QR (LT). These NRFs are also compared with the ones presented in [11] where the NRFs are obtained based on 18 monostatic target responses at different aspect angles using a modified MPM (M-MPM) algorithm [4]. It is shown that mode 8 is not found using MPM but is successfully retrieved using Cauchy (QR) and Cauchy (QR, LT). Mode 4 is also not found using MPM and Cauchy (QR, LT). Using Cauchy (QR), mode 4 is retrieved but the magnitude of σ_n (= 3.64) is larger than the one using M-MPM (= 1.36), which could be a higher-order mode. Mode 3 is successfully retrieved using MPM and Cauchy-QR (LT) but not found in Cauchy-QR. This may indicate that the ET component in the frequency domain may corrupt this mode.

III. Conclusion

NRF extraction from target signatures using MPM and Cauchy method are studied. Our results from synthetic signal and high-Q wire scatterer show that both methods can successfully extract NRFs from target response. While the results of both MPM and Cauchy show high similarity for the synthetic signal and high-Q wire scatter, the results from the relatively low-Q square plate appear to be supplementary, where the modes that are not found in MPM can be identified using Cauchy. This opens a door for the two methods to be used as a fusion to obtain more comprehensive data sets for classification.

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