A Hierarchical Game Model for OFDM Integrated Radar and Communication Systems

Huy T. Nguyen, Dinh Thai Hoang, Nguyen Cong Luong, Dusit Niyato, and Dong In Kim

Abstract—**This paper studies the spectrum allocation problem between spectrum service providers (SSPs) and terminals equipped with orthogonal frequency division multiplexing (OFDM) integrated radar and communication (IRC) systems. In particular, IRC-equipped terminals such as autonomous vehicles need to buy spectrum for their radar functions, e.g., sensing and detecting distant vehicles, and communication functions, e.g., transmitting sensing data to road-side units. The terminals determine their spectrum demands from the SSPs subject to their IRC performance requirements, while the SSPs compete with each other on the service prices to attract terminals. Taking into account the complicated interactions, a hierarchical Stackelberg game is proposed to reconcile the spectrum demand and service price, where the SSPs are the leaders and the terminals are the followers. Due to the spectrum constraints of the SSPs, we model the lower-layer subgame among the terminals as a generalized Nash equilibrium problem. An iterative searching algorithm is then developed that guarantees the convergence to the Stackelberg equilibrium. Numerical results demonstrate the effectiveness of our proposed scheme in terms of social welfare compared to baseline schemes.**

Index Terms—**Integrated radar and communication, spectrum allocation, Stackelberg game, incentive mechanism, pricing.**

I. Introduction

Automation in the logistic by using autonomous terminals, e.g., autonomous vehicles or robotic cars, has gained the global market over the last decade. The market is expected to reach a staggering \$81 and \$290 billion in 2030 and 2040, respectively [1]. One major advantage of the autonomous vehicles is able to navigate and move efficiently and safely in diverse and complex environments with little or without human intervention. For this, the autonomous terminals need to be equipped with integrated radar and communication (IRC) systems. The radar function is to sense and detect surrounding circumstances, e.g., distant vehicles, and the communication function is to transmit sensing data to aggregation units, e.g., road-side units, for further processing.

The radar function is typically based on wideband spectrum techniques, and the sensing data transmitted by the communication function has a large size. Therefore, the IRC-equipped terminals require a huge amount of spectrum from spectrum service providers (SSPs) to simultaneously perform the radar function and the communication function. To motivate the SSPs and the IRC-equipped terminals to participate in the spectrum market, incentive mechanisms need to be designed for the spectrum trading to improve the utility of both the SSPs and the terminals. Although there are several incentive mechanisms proposed for the spectrum trading, they are not designed for the spectrum market with IRC-equipped terminals to guarantee the performance of both radar and communication functions. For example, an incentive mechanism based on the Stackelberg game was proposed in [2], but it is for the spectrum trading between the SSPs and mobile users in 5G networks. Other examples are the use of the non-cooperative game for the spectrum trading between the SSPs and regular vehicles, i.e., vehicles are equipped with communication function [3] or for an opportunistic spectrum access among regular vehicles [4]. Recently, a pricing-based incentive mechanism has been proposed in [5] for a joint radar and communication system, but it mainly focuses on addressing the interference issues for joint radar and data communications.

In this paper, we study the spectrum trading problem between the SSPs and IRC-equipped terminals such as autonomous vehicles. In the problem, the SSPs compete with each other by optimizing their service prices to maximize their own utility and attract more terminals. Given the prices offered by the SSPs, the terminals determine their optimal spectrum demands that maximize the terminals' utility while guaranteeing their IRC service requirements, i.e., radar and communication performance. The interactions between SSPs and terminals are modeled by using the Stackelberg game to find the optimal set of prices/spectrum demands, i.e., equilibrium among entities. Stackelberg game has been widely applied to address many issues in wireless networks such as interference control and resource management [6], [7]. To maximize the utility of both the SSPs and the terminals, we propose a hierarchical Stackelberg game to model the interactions among the SSPs and terminals. The SSPs offer the spectrum resources and set the resource prices first, and then the terminals determine their spectrum demands. Thus, in the game, the SSPs are the leaders, and the terminals are the followers. In particular, the SSPs compete with each other in the upper-layer subgame by determining their optimal spectrum prices, and the terminals compete with each other in the lower-layer subgame by determining their optimal spectrum demands. Given the spectrum constraint of each SSP, it may be difficult to find the Nash equilibrium (NE) of the lower-layer subgame among the terminals. For this, we reformulate the lower-layer subgame as a generalized Nash equilibrium (GNE) problem. Due to the unobtainable closed-form solution in the lower-layer subgame and the linearity in the upper-layer subgame, we develop an iterative SE searching algorithm that efficiently determines the optimal solution, i.e., the Stackelberg equilibrium (SE), of the game. Numerical results demonstrate the effectiveness of our proposed scheme in terms of social welfare compared to baseline schemes.

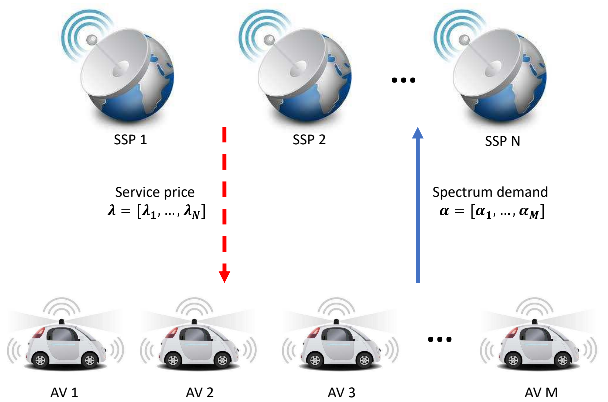

Fig. 1. An example of hierarchical Stackelberg game in the presence of multiple SSPs and multiple AVs.

II. System Model and Game Formulation

A. System Model

We consider a spectrum market that consists of a set of M IRC-equipped terminals, denoted by $\mathcal{M} \triangleq \{1, \ldots, m, \ldots, M\}$, and a set of N SSPs, denoted by $\mathcal{N} \triangleq \{1, \ldots, n, \ldots, N\}$. Each SSP n has L_n orthogonal subcarriers for trading to the terminals. The orthogonal subcarriers can be used by the terminals to modulate orthogonal frequency-division multiplexing (OFDM) symbols. The subcarrier spacing Δf between two consecutive subcarriers is equal in all SSPs' spectrum, where $\Delta f = 1/T$ with T being the duration of OFDM symbol. The entire bandwidth of SSP n is thus equal to $\mathcal{B}_n = \Delta f L_n$. Denote $b_{n,m}$ as a set of subcarriers that SSP n allocates to IRC-equipped terminal m for activating both radar and communication functions. Thus, $|b_{n,m}|$ is then equal to the proportion of $0 \leq |b_{n,m}|/L_n \leq 1$ the entire bandwidth of SSP n. Denote $\alpha_{n,m}$ as the amount of bandwidth allocated to terminal m by SSP n. Thus, we have $\alpha_{n,m} = (|b_{n,m}|/L_n)\mathcal{B}_n$, where $|.|$ is the cardinality operator.

In the IRC, the transmit signal of S consecutive integrated OFDM symbols of terminal m is given as [8]

$$x_m(t) = e^{j2\pi f_m^c t} \sum_{h=0}^{S-1} \sum_{k=0}^{|b_{n,m}|-1} a_m^k c_m^{k,h} e^{j2\pi k \Delta f (t - hT_s)} \times \mathrm{rect}[(t - hT_s)/T_s],$$

where f_m^c is the center frequency allocated to the terminal m, a_m^k is the amplitude of subcarrier k in terminal m, $c_m^{k,h}$ is the phase code of subcarrier k and OFDM symbol h of terminal m. T_s is the duration of each completed OFDM symbol, which satisfies $T_s = T + T_g$ with the cycle prefix duration T_g. rect$[t/T_s]$ is rectangle function, which is equal to one for $0 \leq t \leq T_s$, otherwise, it is zero.

Let $g_m(t)$ denote the impulse response of an extended target at terminal m. The received signal of terminal m can be expressed as

$$y_m(t) = \int_{-\infty}^{+\infty} g_m(\tau) x_m(t - \tau) d\tau + n_m(t),$$

where $n_m(t)$ is complex additive white Gaussian noise (AWGN) with zero mean and power spectral density $N_m(f)$.

For radar object detection and identification, the conditional mutual information (MI) between the received signal and the target impulse response is an important metric. The estimation accuracy of the target impulse response increases as the conditional MI increases because it can improve the radar detection [9]. Let $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_M]^T$ denote the spectrum demand vector, where $\boldsymbol{\alpha}_m = [\alpha_{1,m}, \ldots, \alpha_{N,m}]$ is the spectrum demand profile that terminal m requests to N SSPs. The conditional MI of terminal m is given by [9] [1]

$$\begin{aligned} I_m(\boldsymbol{\alpha}_m) &= \mathbb{E}\left\{ \frac{1}{2} \Delta f S T_s \sum_{n=1}^{N} \sum_{k=1}^{|b_{n,m}|} \log(1 + p_m^k v_m^k) \right\}, \\ &= \frac{1}{2} S T_s \sum_{n=1}^{N} \alpha_{n,m} \, \mathbb{E}\left\{ \log(1 + p_m^k v_m^k) \right\}, \end{aligned} \tag{1}$$

where $p_m^k = |a_m^k|^2$ is the transmit power of subcarrier k, $T_p = ST_s$ is the total transmit duration of OFDM signal. $v_m^k = ST_s^2 |G_m(f_m^k)|^2 / (N_m(f_m^k) T_p)$ is the target-to-noise ratio (TNR) in subcarrier k, where $G_m(f_m^k)$ is the Fourier transform of $g_m(t)$, $f_m^k = f_m^c + k\Delta f$ is the frequency of subcarrier k. We assume that the covariance of the target impulse response vector can be obtained at each terminal.

For communication, an important performance metric is the data information rate (DIR). We assume that the communication channel is slowly time-variant and frequency selective. The average DIR of terminal m is given by [12]

$$\begin{aligned} R_m(\boldsymbol{\alpha}_m) &= \mathbb{E}\left\{ \Delta f \sum_{n=1}^{N} \sum_{k=1}^{|b_{n,m}|} \log(1 + p_m^k w_m^k) \right\}, \\ &= \sum_{n=1}^{N} \alpha_{n,m} \, \mathbb{E}\left\{ \log(1 + p_m^k w_m^k) \right\}, \end{aligned} \tag{2}$$

where $w_m^k = |g_m^k|^2/\sigma^2$ is the signal-to-noise (SNR) in subcarrier k with g_m^k being the channel gain of subcarrier k, and σ^2 is the noise power of the communication receiver. We assume that the transmitter channel-state-information (CSI) can be obtained at the terminals.

B. Game Formulation

Observing from (1) and (2), each terminal desires to be allocated more subcarriers to improve the performance of radar and communication services. Therefore, the achievable revenue of terminal m over SSP n can be regarded as its own satisfaction utility. The satisfaction utility of terminal m when associated with SSP n is formulated by

$$\mathcal{U}_{n,m}(\alpha_{n,m}) = \frac{\log(1 + \theta_m \alpha_{n,m})}{\log(1 + \theta_m \mathcal{B}_n)}, \tag{3}$$

where θ_m is the slope of the curve that varies over terminals. This satisfaction utility of the terminal is a strictly concave

[1] The MI metric has been widely considered in the literature as an important radar performance criterion [8]–[10]. Especially, the application of MI in vehicular networks has been investigated in [11].

and increasing function [13]. From (3), we can observe that the satisfaction utility satisfies $\lim_{\alpha_{n,m}\to 0} \mathcal{U}_{n,m}(\alpha_{n,m}) = 0$ and $\lim_{\alpha_{n,m}\to \mathcal{B}_n} \mathcal{U}_{n,m}(\alpha_{n,m}) = 1$. The utility function of terminals implies that their satisfaction will be increased if they are allocated more spectrum resource from the SSPs.

In practice, the SSPs can offer different subscription levels to terminals. To attract more IRC-equipped terminals, the SSPs can reduce their offer prices. In addition, depending on the offered prices and service quality of the SSPs, the terminals can choose their appropriate subscription levels. Let denote $\boldsymbol{\lambda} = [\boldsymbol{\lambda}_1, \dots, \boldsymbol{\lambda}_N]^T$ as the price vector, where $\boldsymbol{\lambda}_n = [\lambda_{n,1}, \dots, \lambda_{n,M}]$ is the price per MHz bandwidth unit that SSP n offers to M terminals. The subscription probability between SSP n and terminal m can be determined by [14]

$$\psi_{n,m} = \frac{\gamma_{n,m}}{\sum_{n=1}^{N} \gamma_{n,m}}, \tag{4}$$

where the subscription parameter of SSP n to terminal m is given by $\gamma_{n,m} = \frac{\lambda_n^{max} - \lambda_{n,m}}{p_n}$ with p_n being the attraction level of SSP n. The attraction level of an SSP is given based on its service quality. We observe that if the service price is relatively small or the attraction level of SSP n is small, then the parameter $\gamma_{n,m}$ is large. Thus, terminal m is more likely to subscribe to SSP n. In contrast, the subscription probability of terminal m on SSP n will be low if the service price offered by SSP n is high. Therefore, in order to attract more terminals to buy the spectrum resource, the SSPs can reduce their prices to increase the subscription probability. It is natural that the terminals tend to buy the service from the SSPs who have lower prices or higher quality. However, the SSP cannot set a very low price that results in a low revenue. As a result, the incentive design guarantees the fairness among SSPs, where the SSPs should set their prices in the feasible region and obtain revenues corresponding to their resource trading.

From the SSP's perspective, each SSP wants to maximize its own revenue by selling spectrum to all terminals. The utility of SSP n is defined as the revenue received from all the terminals, which is given by

$$\mathcal{L}_n(\boldsymbol{\lambda}_n|\boldsymbol{\psi}, \boldsymbol{\alpha}, \boldsymbol{\lambda}_{-n}) \triangleq \sum_{m=1}^{M} \psi_{n,m}\lambda_{n,m}\alpha_{n,m}, \tag{5}$$

where $\boldsymbol{\lambda}_{-n}$ is the pricing strategies observed by other SSPs. Denote $\boldsymbol{\psi} = [\boldsymbol{\psi}_1, \dots, \boldsymbol{\psi}_N]$ as the subscription probability vector of all the SSPs, where $\boldsymbol{\psi}_n = [\boldsymbol{\psi}_{n,1}, \dots, \boldsymbol{\psi}_{n,M}]$ is the subscription probability vector of SSP n to M terminals. Since the SSPs offer their service prices, the subscription probability ψ and the terminal demand $\boldsymbol{\alpha}$ are also altered.

From the terminals' side, they want to purchase the spectrum resources to guarantee the IRC service requirements while maximizing their utilities with the lowest costs. The utility of terminal m is defined as the total profit minus the costs paid to all SSPs, which is given by

$$\begin{aligned}&\mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m}) \\ &\triangleq \sum_{n=1}^{N} \psi_{n,m}\tau_m \mathcal{U}_{n,m}(\alpha_{n,m}) - \sum_{n=1}^{N} \psi_{n,m}\lambda_{n,m}\alpha_{n,m},\end{aligned} \tag{6}$$

where $\boldsymbol{\alpha}_{-m}$ is the resource demands observed from the other terminals, and τ_m is the positive revenue coefficient that captures the intrinsic value of terminal m's model.

Even though the SSPs want to maximize their revenues by selling as much spectrum resources to the terminals with high service prices, they have to compete with each other in the market to attract more customers. Given the prices offered by the SSPs, the terminals select their resource demands from all the SSPs to maximize their utilities while guaranteeing the IRC service requirements. The optimal strategies of SSPs and terminals can be determined as the solution of a hierarchical Stackelberg game, where the SSPs act as the leaders and the IRC-equipped terminals are the followers. In particular, the SSPs will first determine the pricing strategies, and then the terminals decide their spectrum demands.

The upper-layer optimization problem is formulated as

$$\max_{\boldsymbol{\lambda}_n} \mathcal{L}_n(\boldsymbol{\lambda}_n|\boldsymbol{\psi}, \boldsymbol{\alpha}, \boldsymbol{\lambda}_{-n}), \forall n \in \mathcal{N}, \tag{7a}$$
$$\text{s.t} \quad 0 \le \lambda_{n,m}, \forall m \in \mathcal{M}. \tag{7b}$$

The lower-layer optimization problem is formulated as

$$\max_{\boldsymbol{\alpha}_m} \mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m}), \forall m \in \mathcal{M}, \tag{8a}$$
$$\text{s.t} \quad 0 \le \alpha_{n,m}, \forall n \in \mathcal{N}, \tag{8b}$$
$$0 \le I_m(\boldsymbol{\alpha}_m) - I_m^{min}, \tag{8c}$$
$$0 \le R_m(\boldsymbol{\alpha}_m) - R_m^{min}, \tag{8d}$$
$$0 \le \mathcal{B}_n - \sum_{m=1}^{M} \alpha_{n,m}, \forall n \in \mathcal{N}, \tag{8e}$$

where constraints in (8c) and (8d) are the IRC service thresholds for radar I_m^{min} and communication R_m^{min}, respectively. The constraints in (8e) ensure that the spectrum sold to the terminals from each SSP cannot exceed its available spectrum.

III. Game Analysis

The solution of the above hierarchical game can be obtained by determining the SE of the game. In this game, each SSP first offers the price per bandwidth unit to all terminals. Based on the prices set by all SSPs, each terminal determines the spectrum demands $\boldsymbol{\alpha}_m, \forall m \in \mathcal{M}$ and the corresponding SSPs based on the subscription probability $\boldsymbol{\psi}$. Thus, we adopt the backward induction method to analyze the interactions between the SSPs and the terminals. In particular, the lower-layer subgame is first analyzed to find the NE among the terminals given the service prices. Then, the solution of the game is determined based on the observation of the followers' responses and the leaders' strategies.

In the multi-leader multi-follower scenario, each SSP can associate with multiple terminals to maximize its revenue, while each terminal can subscribe to multiple SSPs from various offered prices in the market. In the following, we provide the definitions of the NE of the lower-layer subgame and the SE of the hierarchical game.

Definition 1. *Given the SSPs' strategies, the solution of the lower-layer subgame is an NE if the following condition holds*

$$\mathcal{F}_m(\boldsymbol{\alpha}_m^*|\boldsymbol{\lambda}^*, \boldsymbol{\alpha}_{-m}^*) \ge \mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}^*, \boldsymbol{\alpha}_{-m}^*), \forall m \in \mathcal{M}. \tag{9}$$

Definition 2. *The solution $(\boldsymbol{\lambda}^*, \boldsymbol{\alpha}^*)$ is an SE if the following condition is satisfied*

$$\mathcal{L}_n(\boldsymbol{\lambda}_n^*|\boldsymbol{\psi}, \boldsymbol{\alpha}^*, \boldsymbol{\lambda}_{-n}^*) \geq \mathcal{L}_n(\boldsymbol{\lambda}_n|\boldsymbol{\psi}, \boldsymbol{\alpha}^*, \boldsymbol{\lambda}_{-n}^*), \forall n \in \mathcal{N}. \quad (10)$$

A. Lower-layer Subgame

In general, finding the NE at the lower-layer subgame is challenging due to the presence of the shared constraints among terminals, i.e., the constraint in (8e). Thus, the optimal spectrum demands $\boldsymbol{\alpha}_m^*$ depend on the joint adversary's strategies of the other terminals $\boldsymbol{\alpha}_{-m}^*$. Therefore, the traditional approach based on the derivative method is not suitable to find the best response from the followers [6]. Thus, in this paper, we propose to use the GNE method [15] which can achieve a joint optimal solution for the followers by solving the GNE problem. The key idea of this method is to reformulate the original problem into an equivalent better known problem, and then analyzing the existence and uniqueness of the NE.

Theorem 1. *Given the objective in* (8a) *and constraints in* (8b)-(8e) *in the lower-layer optimization problem, the following properties hold*

- ***P1.*** *The followers' spectrum demands set $\Theta_m = \{\boldsymbol{\alpha}_m : \alpha_{n,m} \geq 0, I_m(\boldsymbol{\alpha}_m) \geq I_m^{min}, R_m(\boldsymbol{\alpha}_m) \geq R_m^{min}, \mathcal{B}_n \geq \sum_{m=1}^{M} \alpha_{n,m}, \forall n \in \mathcal{N}\}$ is convex and compact for any non-empty set $\boldsymbol{\alpha}_{-m}, \forall m \in \mathcal{M}$.*
- ***P2.*** *The objective function $\mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m})$ is a twice continuous differentiable (C^2) concave function w.r.t. $\boldsymbol{\alpha}_m, \forall m \in \mathcal{M}$.*

Proof. It is straightforward to prove **P1** by observing the formulation problem in (8). Also, it is easy to prove **P2** by showing that the Hessian matrix $\nabla^2_{\boldsymbol{\alpha}_m^2} \mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m})$ is negative semidefinite. Due to the space limitation, the detail proof is omitted. □

From Theorem 1, the local optimization problem (8) is a concave programming problem, which can be resorted to the mathematical tool of Quasi-Variational Inequalities (QVI) [15] to clarify the existence of the GNE in the lower-layer subgame.

Definition 3. *It is true that $\Theta \in \mathcal{R}^n$ is the closed convex set, $\Theta \overset{\triangle}{=} \prod_{m=1}^{M} \Theta_m$ and $f(\boldsymbol{\alpha}) = (-\nabla_{\boldsymbol{\alpha}_m} \mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m}))_{m=1}^{M}$ is the gradient based mapping. The variational inequality (VI) problem, i.e., $VI(\Theta, f(\boldsymbol{\alpha}))$, is finding a vector $\boldsymbol{\alpha}^* \in \Theta$ such that [16]:*

$$(\Xi - \boldsymbol{\alpha}^*)^T f(\boldsymbol{\alpha}^*) \geq 0, \quad \forall \Xi \in \Theta. \quad (11)$$

Lemma 1. *A joint solution for followers $\boldsymbol{\alpha}^*$ is a GNE in the lower-layer subgame if and only if $\boldsymbol{\alpha}^*$ is a solution of the QVI problem $VI(\Theta, f(\boldsymbol{\alpha}))$.*

Proof. Given **P1** and **P2** in Theorem 1, Lemma 1is achieved following from [15, Th. 3.3]. □

To verify the existence of the NE in the lower-layer subgame, it is necessary to show that the solution set of QVI problem $VI(\Theta, f(\boldsymbol{\alpha}))$ is non-empty.

Theorem 2. *Given the feasible SSPs' strategies $\boldsymbol{\lambda}$, the lower-layer subgame admits at least one GNE.*

Proof. By using **P2** in Theorem 1, $\mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m})$ is a (C^2) concave function w.r.t. $\boldsymbol{\alpha}_m$. Thus, we can construct the Jacobian of $f(\boldsymbol{\alpha})$, i.e., $\nabla_{\boldsymbol{\alpha}} f(\boldsymbol{\alpha}) = \left[\nabla^2_{\boldsymbol{\alpha}_m, \boldsymbol{\alpha}_{m'}} \mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m})\right]_{m,m'=1}^{M}$. We can verify that $\nabla_{\boldsymbol{\alpha}} f(\boldsymbol{\alpha})$ is positive semidefinite which guarantees a nonempty, closed, and convex set of GNE in the lower-layer subgame [15]. □

Since the optimization problem in (8) is a concave programming problem, the GNE of the lower-layer subgame can be obtained using Karush-Kuhn-Tucker (KKT) conditions. The KKT conditions of follower m are given as follows

$$\nabla_{\boldsymbol{\alpha}_m} \mathcal{F}_m(\boldsymbol{\alpha}_m|\boldsymbol{\lambda}, \boldsymbol{\alpha}_{-m}) + \sum_{i=1}^{4} \boldsymbol{\mu}_m^i (\nabla_{\boldsymbol{\alpha}_m} \mathcal{F}_m^i(\boldsymbol{\alpha}_m)) = 0, \quad (12a)$$

$$0 \leq \boldsymbol{\mu}_m^i \perp \mathcal{F}_m^i(\boldsymbol{\alpha}_m) \geq 0, \quad (12b)$$

where $\boldsymbol{\mu}_m^i$ and $\mathcal{F}_m^i, (i = \{1, 2, 3, 4\})$ are the KKT multipliers and right-hand side expressions corresponding to the constraints in (8b)-(8e), respectively. Expressions in (12b) are the complementary conditions, and the operator $\perp$ represents component-wise orthogonality.

Since the optimization problem in (8) can be solved by KKT conditions, $\boldsymbol{\alpha}^*$ is a GNE of the lower-layer subgame, which thus provides the unique NE in the form of $(\boldsymbol{\alpha}^*, \boldsymbol{\mu}^{1,*}, \boldsymbol{\mu}^{2,*}, \boldsymbol{\mu}^{3,*}, \boldsymbol{\mu}^{4,*})$. To determine the GNE of the lower-layer subgame in a distributed manner, the synchronous GNE searching algorithm can be developed based on the Gauss-Seidel-type method. The main idea of this algorithm is to iteratively solve each follower's problem based on the synchronous responses from other followers, that finally reaches the GNE of the lower-layer subgame [15, Alg. 5.2].

B. Upper-layer Subgame

Due to the unobtainable closed-form solution in the lower-layer subgame and the linearity of the utility function in the upper-layer subgame, it is impossible to apply the conventional approach using the gradient-based optimization method to find the solution of Stackelberg game. In this paper, we develop an iterative SE searching algorithm based on sub-gradient approach, which can guarantee the convergence of the optimal solution, i.e., SE. The SE searching approach has been widely applied in Stackelberg game for game formulation in which the closed-form solution in the lower-layer is impossible to be obtained, e.g., [7]. The key idea of the iterative approach is that each SSP's strategy will be affected by the reactions of the other SSPs as well as the responses of the terminals. In the upper-layer subgame, the SSP can increase its service price to maximize its own revenue. However, the terminals will change their behaviors, which are unlikely to associate with the SSP if other SSPs have lower service prices. Observing from (4), there is a tradeoff between the service price and the subscription probability. Therefore, the SSPs cannot increase their prices selfishly in order to attract more terminals. As a result, the strategies of the SSPs will be regulated to the optimal solution.

At the initialization of the algorithm, each SSP assumes that there is no competition in the market and sets its service price to the maximum value. Then, the SSP explores that decreasing its service price can receive greater revenue, and there exist competitors in the market. In particular, if the SSP decreases its price, while other SSPs' strategies keep unchanged, the terminals are likely to subscribe to the SSP and the SSP's revenue can be increased. In contrast, if the SSP increases its service price, the terminals will subscribe to other SSPs that have higher subscription probabilities, and the SSP's revenue decreases. Therefore, the SSP needs to determine its own strategy to attract more terminals while observing the strategies of other SSPs. Basically, the SSP will increase or decrease the current price with a small step δ and calculate the achievable utility with the prediction of followers' demand at each iteration. In this way, the SSPs can make the adjustment on their service prices in the next iteration to increase their utilities. Otherwise, the service prices will be kept unchanged. After the finite number of iterations, the strategies of the SSPs will converge to the equilibrium without any further change.

IV. Numerical Results

In this section, we provide numerical results to evaluate the performance of our proposed scheme. Each SSP has 128 subcarriers for trading to all terminals. The power is equally allocated to each subcarrier. The additive white Gaussian noise (AWGN) follows a zero-mean normal distribution, and the target frequency response and the frequency response of the communication channels follow an exponential distribution with unit mean. The slope of the curve in the IRC-equipped terminals is randomly selected in the range of $(0, 1]$. The maximum service price and revenue coefficient are set as $\lambda_n^{max} = 10^{-1}, \forall n \in \mathcal{N}$, and $\tau_m = \tau = 1, \forall m \in \mathcal{M}$. Other IRC parameters are set as follows: $S = 10$, $T_s = 5 \times 10^{-6}$ s, $T = 10^{-4}$ s, $\delta = 10^{-3}$, and $\epsilon = 10^{-4}$ [8]. The radar and communication thresholds are set as 100 bit and 1 Mbit/s, respectively. To evaluate the proposed scheme, we introduce the Greedy scheme and Random scheme as baseline schemes. In the Greedy scheme, the SSPs set service prices to the maximum values, then the terminals determine their spectrum demands given the prices set by the SSPs. In the Random scheme, the SSPs randomly select the service prices.

The proposed scheme aims to maximize the utility of both the SSPs and the terminals. Thus, we first compare the social welfare, i.e., the total utility of the SSPs and the terminals with the fixed $N = 4$ and $p_n = 0.5, \forall n \in \mathcal{N}$. As shown in Fig. 2, the social welfare obtained by the proposed scheme is much higher than those obtained by the baseline schemes regardless the number of terminals in the market. The reason is explained as follows. With the proposed scheme, the SSPs set the service prices accounting for the reactions, i.e., the resource demands, of the terminals to maximize their utility. With the Greedy scheme, the SSPs always set high prices, and the terminals may be not willing to buy the spectrum resource. This results in decreasing the utility of the SSPs, the utility of the terminals, and the social welfare.

Next, it is worth discussing the impact of the market scalability on the utility of each SSP. Without loss of generality, we discuss how the utility of SSP 1 changes as the number of SSPs and terminals in the market varies. As shown in Fig. 3, given the number of terminals and the attraction level of SSP 1, i.e., p_1, the utility of SSP 1 decreases as the number of SSPs increases. The reason is that more SSPs joining in the market increase the competition among them. To attract the terminals, SSP 1 needs to reduce its service price that results in decreasing its utility. As seen, given the number of SSPs in the market, the utility of SSP 1 increases with the increase of terminals. The reason is that as the number of terminals in the market increases, there may be more terminals that are willing to buy the spectrum from SSP 1. This results in improving the utility of SSP 1. Note that the utility of SSP 1 decreases as its attraction level, i.e., p_1, increases. The reason is that as p_1 increases, the service price offered by SSP 1 increases and the terminals are not likely to purchase the spectrum from SSP 1. This also verifies the theoretical analysis as in (4).

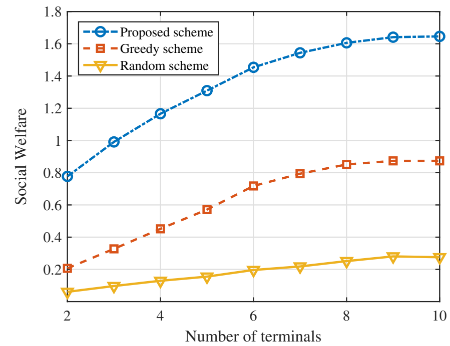

Fig. 2. Social welfare versus the number of terminals.

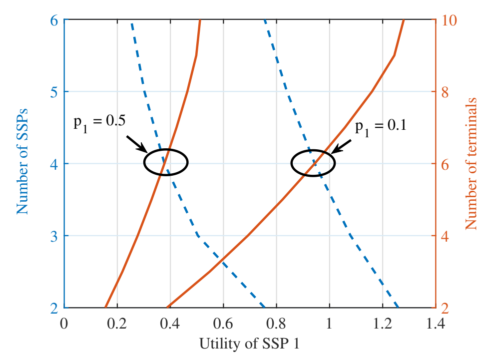

Fig. 3. The impacts of the number of SSPs on SSP 1's utility, i.e., the solid lines, and of the number of terminals on SSP 1's utility, i.e., the dash lines.

It is important to show the NE of the subgame among the SSPs. For the presentation purpose, we consider a market including two SSPs, i.e., SSP 1 and SSP 2, and three terminals. The attraction levels of the SSPs are the same, i.e., $p_1 = p_2 = 0.5$. Fig. 4 shows the NE of the subgame among the SSPs for terminal 1 with different revenue coefficients. As seen, given the revenue coefficient, e.g., $\tau = 1$, the pricing strategies of the two SSPs offering to terminal 1 intersect at a point, which is called a NE of the subgame. At this point, each SSP's pricing strategy is optimal given the pricing strategies of other SSPs. In other words, no SSP has an incentive to change

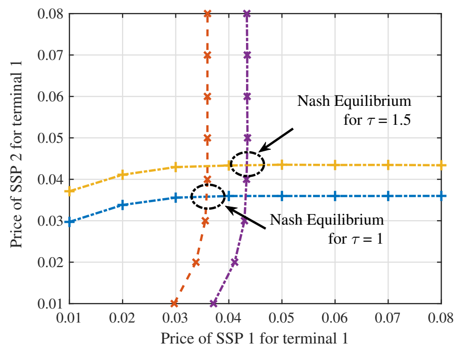

Fig. 4. The NE of the subgame of SSP 1 and SSP 2.

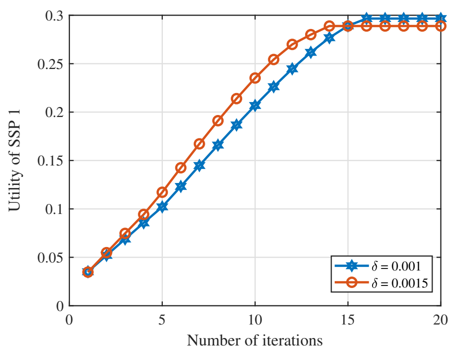

Fig. 5. Convergence of the iterative SE searching algorithm.

its price given the prices offered by other SSPs. This result also shows the fairness among SSPs, where each SSP must set an appropriate price to obtain the revenue corresponding to its capacity. As the revenue coefficient τ increases, the terminals are more willing to buy the spectrum resources and the SSPs can increase the service prices to improve their utility.

Finally, we show the convergence of the proposed scheme. Given the step size, our proposed algorithm is able to converge within the finite iterations [17]. However, the convergence speed depends on the step size. As shown in Fig. 4, with a larger step size, the convergence speed of the algorithm is faster, but the performance, i.e., the utility of the SSP, obtained by the algorithm is lower. In each iteration, given the resource prices offered from N SSPs, the proposed algorithm solves the lower-layer problem in (8) of M terminals that has the computational complexity of $\mathcal{O}\big(n_c^{2.5}(n_v^2 + n_c)\big)$, which involves $n_v = MN$ decision variables and $n_c = MN + 2M + N$ constraints [18, p.4]. Given the spectrum demands of M terminals, the algorithm solves the upper-layer problem in (7) of the SSPs by updating MN prices for N SSPs that has the computational complexity of $\mathcal{O}(NM^2)$. As such, the computational complexity of the proposed algorithm is $\mathcal{O}(n_c^{2.5}(n_v^2 + n_c) + NM^2)$.

V. Conclusions

In this paper, we have investigated the spectrum allocation among multiple SSPs and multiple IRC-equipped terminals. The trading problem has been modeled as the hierarchical Stackelberg game in which the SSPs and the terminals are the leaders and the followers, respectively. The lower-layer subgame has been modeled as the GNE problem under the shared constraints among followers. The iterative SE searching algorithm has been developed to efficiently obtain the solution of the game based on the reactions of the followers and the strategies of the leaders, where the optimal solution leads to improving the utility of both the SSPs and terminals. The numerical results have demonstrated that our proposed scheme outperforms the baseline schemes in terms of social welfare. In the future work, other metrics to evaluate the radar performance can be further explored such as range resolution and velocity resolution.

References

[1] K. Ghaffarzadeh and N. Jiao. Mobile robots, autonomous vehicles, and drones in logistics, warehousing, and delivery 2020-2040. [Online]. Available: https://www.idtechex.com/en/research-report/mobile-robots-autonomous-vehicles-and-drones-in-logistics\ \-warehousing-and-delivery-2020-2040/706

[2] Q. Zhang, B. Fu, Z. Feng, and W. Li, "Utility-maximized two-level game-theoretic approach for bandwidth allocation in heterogeneous radio access networks," *IEEE Trans. Veh. Tech.*, vol. 66, no. 1, pp. 844–854, Mar. 2016.

[3] Z. Xiao, X. Shen, F. Zeng, V. Havyarimana, D. Wang, W. Chen, and K. Li, "Spectrum resource sharing in heterogeneous vehicular networks: A noncooperative game-theoretic approach with correlated equilibrium," *IEEE Trans. Veh. Tech.*, vol. 67, no. 10, pp. 9449–9458, Jul. 2018.

[4] N. Cheng, N. Zhang, N. Lu, X. Shen, J. W. Mark, and F. Liu, "Opportunistic spectrum access for CR-VANETs: A game-theoretic approach," *IEEE Trans. Veh. Tech.*, vol. 63, no. 1, pp. 237–251, Jul. 2013.

[5] C. Shi, F. Wang, S. Salous, and J. Zhou, "A robust Stackelberg game-based power allocation scheme for spectral coexisting multistatic radar and communication systems," in *Proc. IEEE Radar Conf*, Apr. 2019, pp. 1–5.

[6] K. Zhu, E. Hossain, and A. Anpalagan, "Downlink power control in two-tier cellular OFDMA networks under uncertainties: A robust Stackelberg game," *IEEE Trans. Comm.*, vol. 63, no. 2, pp. 520–535, Dec. 2014.

[7] H. Zhang, Y. Xiao, S. Bu, R. Yu, D. Niyato, and Z. Han, "Distributed resource allocation for data center networks: A hierarchical game approach," *IEEE Trans. Cloud Comput.*, Apr. 2018.

[8] Y. Liu, G. Liao, J. Xu, Z. Yang, and Y. Zhang, "Adaptive OFDM integrated radar and communications waveform design based on information theory," *IEEE Commun. Lett.*, vol. 21, no. 10, pp. 2174–2177, Jul. 2017.

[9] Y. Yang and R. S. Blum, "MIMO radar waveform design based on mutual information and minimum mean-square error estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 1, pp. 330–343, May 2007.

[10] M. R. Bell, "Information theory and radar waveform design," *IEEE Trans. Inf. Theory*, vol. 39, no. 5, pp. 1578–1597, Sep. 1993.

[11] Z. Zhang, Z. Du, and W. Yu, "Mutual-information-based OFDM waveform design for integrated radar-communication system in Gaussian mixture clutter," *IEEE Sensors Lett.*, vol. 4, no. 1, pp. 1–4, Oct. 2019.

[12] J. Liu, W. Chen, Y. J. Zhang, and Z. Cao, "A utility maximization framework for fair and efficient multicasting in multicarrier wireless cellular networks," *IEEE/ACM Trans. Networking*, vol. 21, no. 1, pp. 110–120, May 2012.

[13] H. Shajaiah, A. Abdel-Hadi, and C. Clancy, "Utility proportional fairness resource allocation with carrier aggregation in 4G-LTE," in *Military Communications Conference (MILCOM)*, Nov. 2013, pp. 412–417.

[14] Y. Chen, J. Zhang, and Q. Zhang, "Utility-aware refunding framework for hybrid access femtocell network," *IEEE Trans. Wireless Comm.*, vol. 11, no. 5, pp. 1688–1697, Mar. 2012.

[15] F. Facchinei and C. Kanzow, "Generalized nash equilibrium problems," *Ann. Oper. Res.*, vol. 175, no. 1, pp. 177–211, Mar. 2010.

[16] G. Scutari, D. P. Palomar, F. Facchinei, and J.-S. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 35–49, Apr. 2010.

[17] S. Boyd, S. P. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.

[18] D. Peaucelle, D. Henrion, Y. Labit, and K. Taitz, "User's guide for SeDuMi interface 1.04," 2002. [Online]. Available: http://homepages.laas.fr/peaucell/software/sdmguide.pdf