Strategic Game of Energy Hubs in the Joined Energy Markets Applying a Probabilistic-EPEC Approach

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ABSTRACT— The energy hub (EH) is a notion that deals with multi-carrier energy systems. The energy hub operator investigates maximizing profit through strategic behaviors and bidding when participating in joined energy markets containing natural gas and electric power. Energy hubs operators deploy to play strategically through the mathematical models that are most profitable for an economic and competitive environment. Bi-level programming is applied for such conditions separating the complex problem into two levels. In the upper level (UL), each energy hub seeks its profit maximization. In the lower level (LL), market clearing is carried out to maximize social welfare. Game-theoretic approaches like mathematical programs with equilibrium constraints (MPEC) are utilized to solve bi-level programming. Monte-Carlo Simulation is applied to incorporate renewable energy uncertainty. These MPECs form an Equilibrium Constraint with Equilibrium Program (EPEC) to solve the game theory approach. Two case studies are defined to determine the efficacy and correctness of the introduced structure. One example, containing one energy hub as the primary unit and another energy hub as a rival unit, is investigated to verify the joined energy market model. One standard case study is conducted to confirm the proposed model's flexibility in a congested and uncongested network. The proposed model provides the uniqueness and existence of the Nash equilibria, and it is found that the strategic biddings' values in congested mode are higher than uncongested mode values while the players act strategically.

Keywords— Energy Hub, Bi-Level Programming, Joined Energy Market, Monte-Carlo Simulation, Profit Maximization, Strategic Gaming.

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NOMENCLATURE

Sets:

H	EHs
P	Power generated of the EHs
G	Gas wells of the EH
Q	Components inside the EHs
Γ	Efficiency coefficients of EHs components
L	EH demands
T	Time
O_n	Connected buses of np and mp
Z_n	Connected nodes of ng and mg
A_n	Electric power blocks set at np bus
T_n	Natural gas blocks set at ng bus
Δ	Dual variables

Indices:

h	EHs
B	Generation blocks
D	Demands
np	Electrical buses
ng	Electrical buses
mp/mg	Buses and nodes
t	Time
com	Compressors

Parameters:

L_e	Electrical demand of EHs
L_{th}	Heating demand of EHs
L_c	Cooling demand of EHs
η	The efficiency of the EHs components
λ_{hBt}^{P}	The marginal cost of electric power blocks
λ_{ht}^{G}	The marginal cost of natural gas blocks

P_{hBt}^{max} Upper limitation of generated electric power

G_{ht}^{max} Upper limitation of produced natural gas

F_{npmpt}^{max} Upper limitation of transmission line of electric power

F_{ngmgt}^{max} Upper limitation of pipelines of natural gas

β Susceptance of electric power lines

PL Electrical demands of non-EHs

GL Natural gas demands of non-EHs

W Constants of pipelines of natural gas

$Profit_h$ Profit of EHs

Variables:

λ_{npt} Electric locational marginal price (LMP)

λ_{ngt} Natural gas LMP

α_{Pht} Bidding submission of electric power price

α_{Ght} Bidding submission of natural gas price

P_{ht} Bidding submission of the electric power value

G_{ht} Bidding submission of the natural gas value

F_{pt} Electric power of transmission lines

F_{gt} Natural gas flowed in passive pipelines

H_{com} Natural gas of active pipelines

δ_{npt} Voltage angle of np bus

ρ_{ngt} Natural gas pressure at ng node

I/O Input/output of power flow inside EHs

1. INTRODUCTION

1.1. Motivation and Aim

The occurrences of the real world affect power systems as an inseparable compartment of the real world. The EH was introduced as a concept in (Martin Geidl & Andersson, 2007) to propose a new idea about integrating multi-carrier energy systems. Natural gas, a cheap and accessible energy carrier, assists in making the EH concept. Storages and converter components are other properties that are applied inside the EHs. The EH is not isolated from other parts of the system. It may have trade-offs with other players and participants of the energy market: the primary grid, demands, and other EHs. The EH operators require an environment to trade with each other (A. Heidari, S. S. Mortazavi, & R. C. Bansal, 2020), (Heidari & Bansal, 2021) to propose their optimal bidding strategies and bidding—the word "strategic" points to changing the pattern of the market-clearing prices (Ruiz, Conejo, & Smeers, 2012). EHs are seeking profit maximization to reduce their costs. This attitude leads to dealing with other market players: an equilibrium state, which no EHs may enhance the profit by altering its strategies unilaterally. As the EHs' strategies are associated with the market-clearing status, prevailing to all the EHs, the issue serves as multiple leaders and a standard follower system (Leyffer & Munson, 2010). It is formed in the role of a Nash equilibrium expression.

The current research scrutinizes the function of an EH in a joined energy market to maximize profit while satisfying its associated demands. EHs play in a competitive and equilibrium environment. In order to model the strategic method of each player, a bi-level program is applied. In a bi-level program, two levels exist upper level and lower level. At the upper level, profit maximization of each player is obtained. Market clearing of the power system is obtained at a lower level. Game theory approaches are applied to solve the bi-level problem; so, the bi-level problem is converted to mathematical programming with equilibrium constraints (MPEC) and equilibrium programming with equilibrium constraints (EPEC).

1.2. Review of Literature

In the literature, the EH concept was proposed by (Martin Geidl & Andersson, 2007), (M. Geidl et al., 2007) as a new notion for the prospective use of power systems. EH receives multi-carrier energies and delivers to multi demands. Energy storage and converting are carried out within the EHs as a huge node compared to distributed generation (Bansal, 2017), (Adefarati & Bansal, 2016). There are many types of research in EH optimization. In (Hou, Liu, Ma, & Wang, 2020), optimal scheduling and operation of energy hubs using some optimization techniques are investigated. In (A. Heidari, S. Mortazavi, & R. Bansal, 2020) stochastic effect of an EH optimal operation, containing load management and renewable resources, is considered. In (Y. Li, Li, Wen, & Shahidehpour, 2019), a decentral structure for optimal energy flow of the joined multi-carrier system is suggested. This research presents the differences amid the EH concept and a joined energy structure. Hydrogen production plant, as a new source, is introduced in (Moazeni, Miragha, & Defourny, 2019). This paper applies battery and heating storage in the proposed EH. Several probabilistic methods for uncertainty modeling are presented in (Majidi & Zare, 2019). Uncertainty modeling in multi-carrier energy systems are addressed in (Roustai, Rayati, Sheikhi, & Ranjbar, 2018), (Salehpour et al., 2021). The consideration of outage scenario methods in energy hubs is presented in (Faraji, Hashemi-Dezaki, & Ketabi, 2021). The presence of plug-in hybrid electric vehicles in energy hubs, and their optimal operation using renewable energy resources are scrutinized in (Jabarullah, Shabbir, Abbas, Siddiqi, & Berti, 2019), (Emrani-Rahaghi, Hashemi-Dezaki, & Hasankhani, 2021). In (Hemmati, Mehrjerdi, & Nosratabadi, 2021), a microgrid-based energy hub system uses combined heating and power system to decrease its operation cost. The price response program and demand response program are defined in the paper. Different scenarios for dynamic pricing and deficiencies of the central system are explained in (Bahrami, Toulabi, Ranjbar, Moeini-Aghtaie, & Ranjbar, 2018). In (Dolatabadi, Jadidbonab, & Mohammadi-ivatloo, 2019), the difference between risk-averse and risk-seeker for an EH's optimal operation is investigated. This work uses a hybrid method to solve the EH problem. Demand response and stochastic programming with mixed-integer non-linear programming are addressed in (Alipour, Zare, & Abapour, 2018). The flexibility of heating demands is discussed in the paper.

For a business-related notion, game theory is attracted more consideration because of the participation of the power system players in the power market. Every competitor investigates the profit maximization in the presence of other players pointing to equilibrium and competing structure. The goal of the game theory is to reach Nash Equilibrium (NE). The original work (Hobbs, Metzler, & Pang, 2000) introduced the implementation of the game theory in the field of electric power. The action of multi-energy players (MEP) in the electricity market in load collectors is investigated in (Yazdani-Damavandi, Neyestani, Shafie-khah, Contreras, & Catalão, 2018). However, in the current paper, MEP and local energy systems are unified as the primary EH and the rival EHs, and the joined energy market is included. The participation of EH structures in the joined markets is presented in (R. Li, Wei, Mei, Hu, & Wu, 2019). The previous work implements the joined market in a sample distribution system. The equilibrium of the retail market using the game approach is expressed in (Khazeni, Sheikhi, Rayati, Soleymani, & Ranjbar, 2019). The house EH applying NE is presented in (Liang, Wei, & Wang, 2019). The paper designs a distributed algorithm for the EH problem. Load administration using game theory inside EHs is introduced in (Sheikhi, Rayati, Bahrami, & Mohammad Ranjbar, 2015). The structure of market design for optimal management of energy hubs are introduced in (Nasiri et al., 2020), (Jamalzadeh, Hajiseyed Mirzahosseini, Faghihi, & Panahi, 2020). Information transmission and cloud calculations are presented in the paper. An intelligent EH is a result of this cloud transmission. Obtaining Nash Equilibrium in an electric pool market is introduced in (Pozo & Contreras, 2011). Market power exercise and the implication for various energies are introduced (Muhammad Bachtiar Nappu, Bansal, & Saha, 2013), (M. B. Nappu & Bansal, 2011). In (Luo, Liu, Liu, & Liu, 2020), scheduling of energy hubs with Stackelberg game method is considered.

1.3. Contributions and Paper layout

According to the review of literature, there is a gap in the EH's optimal operation in the joined energy market. In the current paper, one EH operator as the primary EH competes with other EHs as rival EHs seeking to maximize their profits. Moreover, the game-theoretic methods are used to consider the synergies between EHs (Y. Li et al., 2019). The whole structure may cast as bi-level programming with two objective functions. The current paper proposes the following statements.

- One EH as the primary unit and another EH as the rival unit is modeled to simultaneously depict the market's economic concepts. Joined energy markets containing natural gas and electric power markets are defined. Weymouth equations and AC power flow are utilized to characterize the non-linear aspects of the scheme.

- Renewable energy resources such as wind speed and solar irradiance are taken into account. Monte-Carlo Simulation is adapted for uncertainty modeling and scenario generation to consider the inherent intermittency of the resources.
- A bi-level program is used to define the joined market structure. Each EH operator as a competitor and a player is modeled in MPEC to seek profit maximization while satisfying their internal constraints and demands. In a recursive mode, the game-theoretic method demonstrates planned players' behavior and seeks Nash Equilibrium. EPEC is used to solve the simultaneous constraints of the MPEC program.

The remainder of the research is detailed as follows. Section II models the proposed structure. Mathematical equations and market structure are expressed in this section. Section III presents the solution methodology. Section IV yields an example to verify the introduced structure and one standard study case to confirm the flexibility of the introduced method. Section V concludes the remarks.

2. MODELING OF EHS

The first step is to define the EHs' structures and their inner components to model the market structure. First, the primary EH scheme is modeled; then, the rival EH is modeled. And finally, the structure of the market and the way of communication among the players are introduced.

2.1. The primary EH model

The primary EH used for the current paper is according to (Ahmad Heidari et al., 2020), (Heidari & Bansal, 2021), and (A Heidari et al., 2020). Fig. 1 shows the structure of the primary EH. The primary EH competes as the primary EH in the joined energy market. It has three kinds of demands, containing electric power, heating, and cooling demands. The EH has two supply sources, containing gas wells and generating power. The EH components are two combined heating and power units (CHP), one transformer, one chiller boiler, one furnace, and one electric heat pump (EHP). The characteristics of the components are according to (Ahmad Heidari et al., 2020), (Heidari & Bansal, 2021).

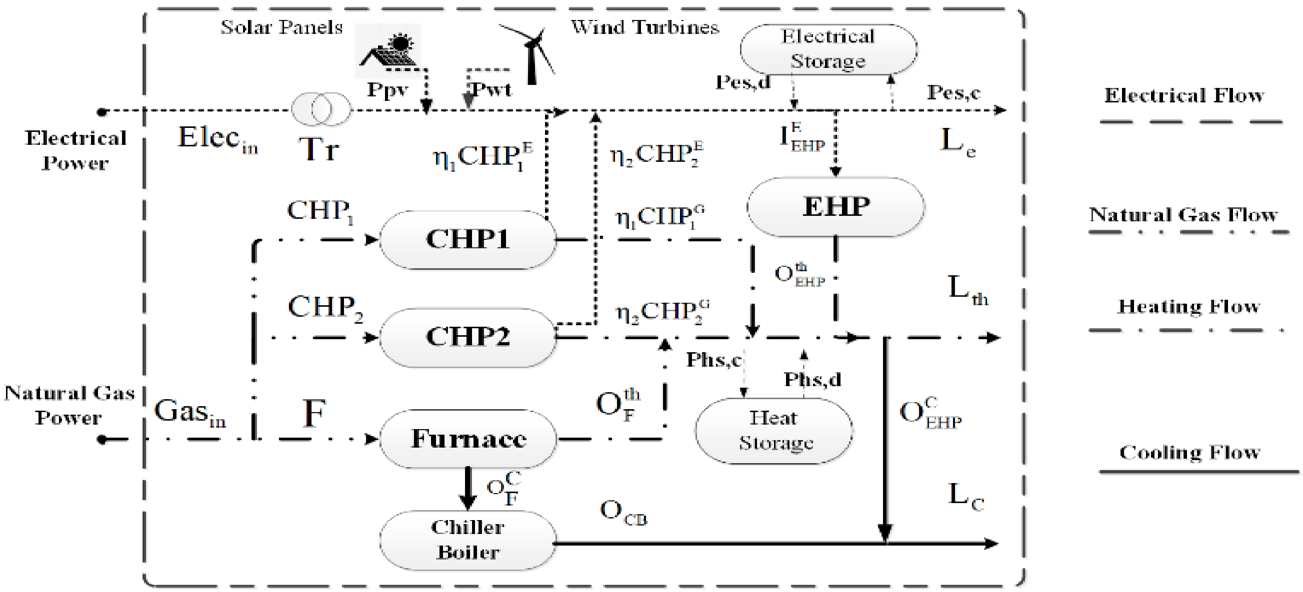

Fig. 1. The primary EH layout.

2.2. The rival EH layout

The rival EH model is according to Fig. 2. The rival EH's inner components are one furnace, one transformer, and one CHP. It owns two different demands, containing electric demands and heating demands. The input sources are supplied by the gas well and generating power owned by the EH.

2.3. Pool Market Model and the Communication

The EHs participate in a pool-based market to satisfy their demands and maximize their profit. The market operator governs the pool market to make transparent what occurs in the energy market. The necessity and tool of this transparency derive from an intelligent and online information transmission structure. Without any attention to the distance of the systems, connection and entry to an intelligent system are required. In (Sheikhi et al., 2015), the topic of cloud systems and various structures for the scheme is expressed.

The structure of the energy market for the current paper is according to Fig. 3. In this Fig., all EHs connect to the market operator via communication systems: Access point name (APN) sim cards, local area network (LAN) systems, wireless, or other systems. Then, the market operator clears the market online. Therefore, a perfect and intact communication system is vital in a total energy market in another rival EHs' presence and market participants.

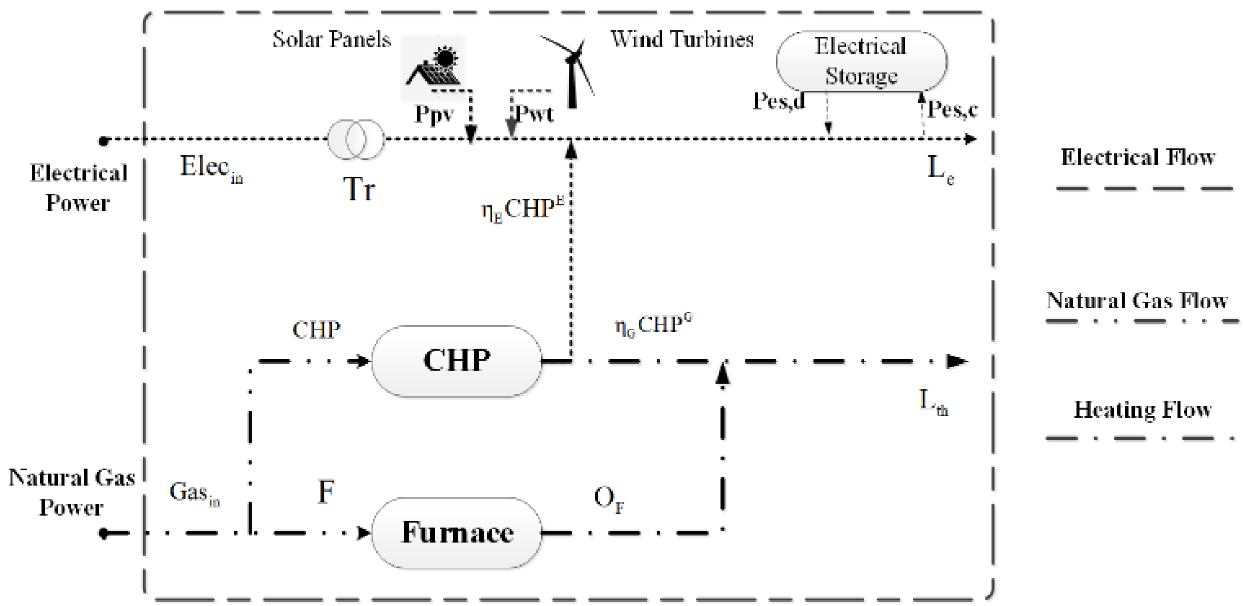

Fig. 2. The rival EH model.

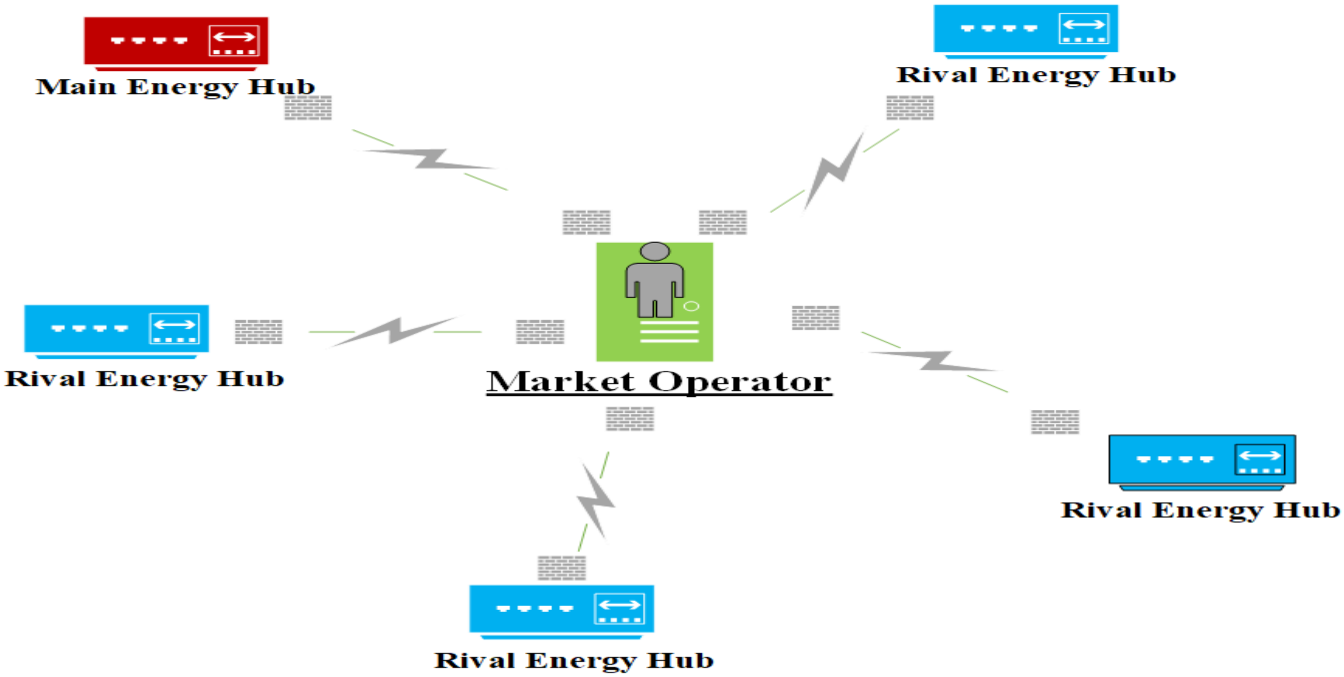

Fig. 3. The market structure and its communication system.

3. SOLUTION METHODOLOGY

A mathematical formulation containing bi-level programming, the MPEC approach, and the EPEC method is introduced in this portion. A flowchart of the suggested model is shown in the last sub-section.

3.1. Mathematical Formulations

The bi-level program is used to define the EHs' formulations in a joined energy market. Two objective functions exist in bi-level programming; The first one is determined in the upper level, and another is defined in the lower level. The upper-level variables are treated as fixed parameters in the lower-level problem. A bi-level programming is converted to MPEC problems according to game-theoretic methods.

3.1.1. Bi-level problem

At first, the bi-level program is modeled to form the constraints.

3.1.1.1 Upper-Level Program

In the bi-level part, (1) to (5) define the upper-level program. The decision variables are: $\{\alpha_{PhBt}, \alpha_{Ght}, P_{hBt}, G_{ht}\}$

Eq. (1) defines the upper-level program's objective function: the negative profit for each participant in the energy market. The notations under the minimize word are the upper-level program's variables: optimal strategic biddings and market clearings for the operator of the objective function. Eqs. (2) and (3) demonstrate the optimal bidding of the power section of the EHs, and (4) represents the optimal bidding of the natural gas section of the EHs.

$$\underset{\alpha_{PhBt}, \alpha_{Ght}, P_{hBt}, G_{ht}, \Delta_{Bi-level}}{Minimize} \sum_{t} \sum_{(h \in H)B} [-\lambda_{(np:h \in \mathrm{A}_n)t} P_{hBt} - \lambda_{(ng:h \in \mathrm{T}_n)t} G_{ht} + \lambda_{hBt}^{P} P_{hBt} + \lambda_{ht}^{G} G_{ht}] \quad (1)$$

Subject to:

$$\alpha_{Ph1t} \geq 0 \quad \forall h \in H, \forall t \in T \quad (2)$$

$$\alpha_{PhBt} \geq \alpha_{Ph(B-1)t} \quad \forall h \in H, \forall B \geq 2, \forall t \in T \quad (3)$$

$$0 \leq \alpha_{Ght} \leq \alpha_{Ght,\max} \quad \forall h \in H, \forall t \in T \quad (4)$$

Eqs. (5-1) to (5-8) present the inner components' equations of the EHs according to Figs 1 and 2. In (5-5), COP stands for coefficient of performance, and CB is abbreviated for chiller boiler.

$$P_{hBt} + G_{ht} + \Gamma_t Q_t = L_t \quad \forall t \quad (5\text{-}1)$$

$$P_{hBt} + \eta_{1t} CHP_{1t}^{E} + \eta_{2t} CHP_{2t}^{E} = I_{EHPt}^{E} + L_{et} \quad (5\text{-}2)$$

$$G_{ht} = CHP_{1t} + CHP_{2t} + F_t \quad (5\text{-}3)$$

$$F_t = O_{Ft}^{th} + O_{Ft}^{C} \quad (5\text{-}4)$$

$$O^{C}_{Ft}\, COP_{CB} = C_{CBt} \tag{5-5}$$

$$I^{E}_{EHPt} = O^{C}_{EHPt} + O^{th}_{EHPt} \tag{5-6}$$

$$L_{tht} = \eta_1 CHP^{G}_{1t} + \eta_2 CHP^{G}_{2t} + O^{th}_{Ft} + O^{th}_{EHPt} \tag{5-7}$$

$$L_{ct} = O_{CBt} + O^{C}_{EHPt} \tag{5-8}$$

3.1.1.2 Lower-Level Program

Eqs. (6) to (13) show the lower level program.

Eq. (6) defines the lower level program's objective function: the negative of social welfare. It is noted that the values of α_{PhBt} and α_{Ght} optimal biddings of power and natural gas of the EH are fixed in the lower level.

$$\underset{P_{hBt}, G_{ht}, \delta_{npt}}{Minimize} \sum_{t} \sum_{hB} [\alpha_{PhBt} P_{hBt} + \alpha_{Ght} G_{ht}] \tag{6}$$

Eq. (7) demonstrates the power flow amid the systems.

$$\sum_{t} [\sum_{(D \in \mathrm{A}_n)} PL_{Dt} - \sum_{(h \in H)B} P_{hBt} + \sum_{mp \in O_n} \beta_{npmpt} (\delta_{npt} - \delta_{mpt})] = 0 \quad : \lambda_{npt} \quad \forall B, \forall t \tag{7}$$

Eqs. (8-1) to (8-5) represent the natural gas system's flow among the EHs. It is noted that (8-2) to (8-5) are the Weymouth equations for the natural gas power (Cong, Shahidehpour, Yong, & Zuyi, 2009). Eq. (8-4) is used for the non-Compressor systems (passive pipelines), and (8-5) is applied for the active pipelines containing the compressor component.

$$\sum_t [\sum_{h \in H} G_{ht} - \sum_{(D \in T_n)} GL_{Dt} + \sum_{mg \in Z_n} F_{gt} + \sum_{com \in Z_n} F_{comt}] = 0 : \lambda_{ngt} \quad \forall t \tag{8-1}$$

$$F_{gt} = sign(\rho_{ng} - \rho_{mg}) W_{ngmg} \cdot \sqrt{|\rho_{ng}^2 - \rho_{mg}^2|} \tag{8-2}$$

$$sign(\rho_{ng} - \rho_{mg}) = \begin{cases} 1, & \rho_{ng} - \rho_{mg} > 0 \\ -1, & \rho_{ng} - \rho_{mg} < 0 \end{cases} \tag{8-3}$$

$$-F_{gt,\max} \le F_{gt} \le F_{gt,\max} \, \nu_{ngt}^{L\max} \quad \forall ng \in Z_n, \forall t \tag{8-4}$$

$$H_{com,\min} \le H_{com} \le H_{com,\max} \tag{8-5}$$

Eqs. (9) - (10) bound the market clearings of the EH values in the energy market. These equations are according to the physical limitations of the generators and gas wells.

$$0 \le P_{hBt} \le P_{hBt}^{\max} \, u_{hBt}^{P\min}, u_{hBt}^{P\max} \quad \forall h, \forall B, \forall t \tag{9}$$

$$0 \le G_{ht} \le G_{ht}^{\max} \, u_{ht}^{G\min}, u_{ht}^{G\max} \quad \forall h, \forall t \tag{10}$$

Eq. (11) limits the power flow in the transmission lines between the EHs.

$$\beta_{npmpt}(\delta_{npt} - \delta_{mpt}) \le F_{npmpt}^{\max} \, \nu_{npmpt}^{L\max} \quad \forall np, \forall mp \in O_n, \forall t \tag{11}$$

Eq. (12) bounds the angle of electrical buses while (13) specifies the slack bus.

$$-\pi \le \delta_{npt} \le \pi : \varepsilon_{npt}^{\min}, \varepsilon_{npt}^{\max} \quad \forall np, \forall t \tag{12}$$

$$\delta_{npt} = 0 : \varepsilon^1 \quad n\,p = 1, \forall t \tag{13}$$

$\Delta_{\text{Bi-level}}$ demonstrates the dual variables in the bi-level problem, expressed by a colon after each constraint.

$$\Delta_{Bi-level}=\left\{\begin{matrix}\lambda_{npt},\lambda_{ngt},\mu_{ht}^{G\min},\mu_{ht}^{G\max},\mu_{hBt}^{P\min},\mu_{hBt}^{P\max}, \\ v_{npmpt}^{L\max},v_{ngmgt}^{L\max},\varepsilon_{npt}^{\min},\varepsilon_{npt}^{\max},\varepsilon^{1}\end{matrix}\right\}$$

3.1.2. MPEC model

The MPEC defines the bi-level problem into a single-level structure. The MPEC programming is according to optimality conditions and dual variables of the bi-level problem. Optimality conditions are used to the lower level of the bi-level program. Two ways exist to model an MPEC: primal-dual and Karush-Kuhn-Tucker (KKT) optimality conditions (Gabriel, Conejo, Fuller, Hobbs, & Ruiz, 2012). In KKT optimality conditions, complementarity constraints of the form $0 \le a \perp c \ge 0$ are applied that cause the system complex.

$$\underset{\alpha_{PhBt},\alpha_{Ght},P_{hBt},G_{ht},\delta_{npt},\Delta_{Bi-level}}{Minimize} \sum_{t}\sum_{(h\in H)B}[-\lambda_{(np:h\in A_n)t}P_{hBt}-\lambda_{(ng:h\in T_n)t}G_{ht}+\lambda_{hBt}^{P}P_{hBt}+\lambda_{ht}^{G}G_{ht}] \tag{14}$$

$$\alpha_{Ph1t}\ge 0 : b_{Ph1t}^{\alpha p} \quad \forall h\in H,\forall t\in T \tag{15}$$

$$\alpha_{PhBt}\ge\alpha_{Ph(B-1)t} : b_{PhBt}^{\alpha p} \quad \forall h\in H,\ \forall B\ge 2,\forall t\in T \tag{16}$$

$$0\le\alpha_{Ght}\le\alpha_{Ght,\max} : b_{Ght}^{\alpha g,\max},b_{Ght}^{\alpha g,\min} \quad \forall h\in H,\forall t\in T \tag{17}$$

$$\sum_{t}\{\sum_{(D\in A_n)}PL_{Dt}-\sum_{(h\in H)B}P_{hBt}+\sum_{mp\in O_n}\beta_{npmpt}(\delta_{npt}-\delta_{mpt})\}=0 : b_{Pht}^{ED},\forall B,\forall t \tag{18}$$

$$\sum_{t}\{\sum_{h\in H}G_{ht}-\sum_{(D\in T_n)}GL_{Dt}+\sum_{mg\in Z_n}F_{gt}+\sum_{com\in Z_n}F_{comt}\}=0 : b_{Ght}^{ED} \quad \forall t \tag{19}$$

$$P_{hBt}+G_{hBt}+\Gamma_t Q_t=L_t \quad \forall t \tag{20}$$

$$0\le P_{hBt}\le P_{hBt}^{\max} : b_{hBt}^{P\min},b_{hBt}^{P\max} \quad \forall h,\forall B,\forall t \tag{21}$$

$$0 \le G_{ht} \le G_{ht}^{\max} : b_{ht}^{G\min}, b_{ht}^{G\max} \quad \forall h, \forall t \tag{22}$$

$$\beta_{npmpt}(\delta_{npt} - \delta_{mpt}) \le F_{npmpt}^{\max} : b_{npmpt}^{L\max} \quad \forall np, \forall mp \in \mathrm{O}_n, \forall t \tag{23}$$

$$-F_{ngmgt}^{\max} \le F_{gt} \le F_{ngmgt}^{\max} : b_{ngmgt}^{L\max} \quad \forall ng, mg \in \mathrm{Z}_n, \forall t \tag{24}$$

$$-\pi \le \delta_{npt} \le \pi : b_{npt}^{\delta\min}, b_{npt}^{\delta\max} \quad \forall np, \forall t \tag{25}$$

$$\delta_{npt} = 0 : b_{npt}^{\delta 1} \quad np = 1, \forall t \tag{26}$$

$$\sum_t \{ \sum_{PhB} \alpha_{PhBt} P_{hBt} + \sum_{Gh} \alpha_{Ght} G_{ht} + \sum_{PhB} u_{PhBt}^{P\max} P_{PhBt}^{\max} + \sum_{Gh} u_{Ght}^{G\max} G_{Ght}^{\max} + \sum_{np(mp \in \mathrm{O}_n)} v_{npmpt}^{L\max} F_{npmpt}^{\max} + \sum_{ng(mg \in Z_n)} v_{ngmgt}^{L\max} F_{ngmgt}^{\max} + \sum_{np} \varepsilon_{npt}^{\max} \pi + \sum_{np} \varepsilon_{npt}^{\min} \pi \} = 0 : \gamma_{ht}^{SDT} \tag{27}$$

$$\alpha_{PhBt} + \alpha_{Ght} - \lambda_{(np:h \in \mathrm{A}_n)t} - \lambda_{(ng:h \in \mathrm{T}_n)t} + u_{PhBt}^{P\max} - u_{PhBt}^{P\min} + u_{Ght}^{G\max} - u_{Ght}^{G\min} = 0 \quad : b_{hBt}^{\alpha\lambda} \quad \forall h, \forall B, \forall t \tag{28}$$

$$\sum_t \sum_{mp \in \mathrm{O}_n} \beta_{npmpt}(\lambda_{npt} - \lambda_{mpt}) + \sum_t \sum_{mp \in \mathrm{O}_n} \beta_{npmpt}(v_{npmpt}^{L\max} - v_{mpnpt}^{L\max}) + \varepsilon_{npt}^{\max} - \varepsilon_{npt}^{\min} + (\varepsilon^1)_{np=1,t} = 0 \quad : b_{hnpt}^{\delta} \quad \forall np, \forall h, \forall t \tag{29}$$

$$u_{PhBt}^{P\min}, u_{PhBt}^{P\max} \ge 0 : b_{PhBt}^{P\min}, b_{PhBt}^{P\max} \quad \forall h, \forall B, \forall t \tag{30}$$

$$u_{Ght}^{G\min}, u_{Ght}^{G\max} \ge 0 : b_{Ght}^{G\min}, b_{Ght}^{G\max} \quad \forall h, \forall t \tag{31}$$

$$v_{npmpt}^{L\max} \ge 0 : b_{hnpmpt}^{v\max} \quad \forall np, \forall mp, \forall h, \forall t \tag{32}$$

$$v_{ngmgt}^{L\max} \ge 0 : b_{hngmgt}^{v\max} \quad \forall ng, \forall mg, \forall h, \forall t \tag{33}$$

$$\varepsilon_{npt}^{\min}, \varepsilon_{npt}^{\max} \ge 0 : b_{hnpt}^{\varepsilon\min}, b_{hnpt}^{\varepsilon\max} \quad \forall np, \forall h, \forall t \tag{34}$$

It is noted that (27) demonstrates a strong duality theorem, which defines the primal and dual variables' equality in the optimal conditions.

3.1.3. Solving MPEC Model: EPEC

Since the problem is the integrated solution of MPECs of all competitors in the joined markets, all MPEC programs constitute an Equilibrium Constraint with Equilibrium Program (EPEC) model.

There are two models for the EPEC formula. The model I incorporates all equations in MPEC ((14) to (34)) and $\frac{\partial L_h}{\partial \Delta_{Bi-level}}$ Lagrangian derivative concerning each $\Delta_{Bi-level}$ variables. The EPEC model II is obtained according to the MPEC model in which each inequality forms a complementarity condition with its related dual variable of the inequality. It is noted that complementarity conditions are defined as follows.

$$0 \le a - b \perp c \ge 0 \tag{35}$$

$$a - b \le M.d \tag{36}$$

$$c \le M.(1-d) \tag{37}$$

M is a significant number (10^6 in the current paper) to determine the conditions of the complementarity relations. M values are chosen according to trial and error methods.

There are several methods of EPEC solution according to the literature review. (Pozo & Contreras, 2011) uses a MILP programming approach to tackle the EPEC problem. (Haghighat & Kennedy, 2012) solves the EPEC model by a non-linear complementarity method. This paper realizes the diagonalization method depending on the non-linear Gauss-Seidel approach. In (Wogrin, Barquin, & Centeno, 2013), the presented EPEC model is converted to a MILP, and equilibrium points are obtained applying a diagonalization method. In (Ruiz et al., 2012), The equivalent MILP relation of EPEC acquires the abundance of stationary points. Then, the paper offers mixed-integer linear programming that gives valid equilibrium points. EPEC problems are complex problems to handle that may have several equilibrium points or none because of fundamental non-linearity and non-convexity (complementarity conditions). So, the focus of the (Wogrin, Centeno, & Barquin, 2013) is according to the diagonalization method. In (Kazempour & Zareipour, 2014), The optimality conditions of the

EPEC is achieved by changing each single-level MPEC with its optimality conditions associated with its KKT conditions. In (Moiseeva, Hesamzadeh, & Biggar, 2015), some heuristic and reformulation methods are proposed to solve EPEC, containing disjunctive constraints, SOS1-based approach, and strong duality. With a comparison among the introduced methods, strong duality outperforms two other techniques. In (Dai & Qiao, 2017), the EPEC model is solved by applying game theory and the diagonalization method.

It is burdensome to verify a solution for EPEC, basically, since the possible set of the leader's (Nash competitor's) MPEC is non-convex. No pure solution for EPEC may be presented, while marginally altering the parameter quantities may lead to a solution. There is rich research on the study of EPEC and its solution methodology (Gabriel et al., 2012). Therefore, to find a solution for the EPEC model, the most used approach, which is MILP, is adopted in this paper.

3.2. Flowchart of the represented model

In the diagram of the solution methodology depicted in Fig. 4, the upper-level and lower-level programs are (1)-(13). 1000 scenarios are generated with the Monte-Carlo simulation, each has the same and random probability to model the uncertainty. SCENRED2/GAMS is applied to lower the quantities of the scenarios to 10 [25]. As depicted in Figs. 5 and 6, wind speed's trend is according to Weibull probability distribution function (PDF), and Beta PDF is used for solar irradiance.

4. NUMERICAL RESULTS

Two study cases are considered to show the efficacy and correctness of the demonstrated structure. The GAMS optimization environment is used to code the optimization problem (Rosenthal, 2013), and CPLEX solves the complex problem.

4.1. First Case Study

In this section, a simplified case of two EHs is introduced to verify the proposed model. Fig. 7. shows this network. EH 1's layout is according to Fig. 1, and EH 2's structure is shown in Fig. 2. These two EHs are rivals and competing with each other. Characteristics of the components of the EHs are adapted from (Ahmad Heidari et al., 2020), (Heidari & Bansal, 2021). The demand quantities of the EH 1 are according to Fig. 8. The demand values of EH 2 are defined as 0.75 times the EH 1's quantities. Table I gives the data for gas wells and generators. Susceptance of the power line is 10 p.u. the transmission capacity is 100 MW. The maximum value of the pipeline's capacity is supposed to be 200 MW.

4.1.1. Problem solution

Three cases are investigated to explain the problem:

- Case 0. Both EHs play non-strategically in the joined energy market.
- Case 1. The primary EH (EH 1) plays strategically in the joined energy market.
- Case 2. Both EHs play strategically in the joined energy market.

Two modes are considered: uncongested mode and congested mode, with the power lines and pipeline values, are divided by 100. Tables II and III display the results of the example in uncongested and congested modes, respectively. In these Tables, N stands for the non-strategic player, and S stands for the strategic player. It is noted that the Tables specific time is at hour 10 to show the difference between congested and uncongested modes. The profit column gives the profit of EHs in each case for all periods (24 hours).

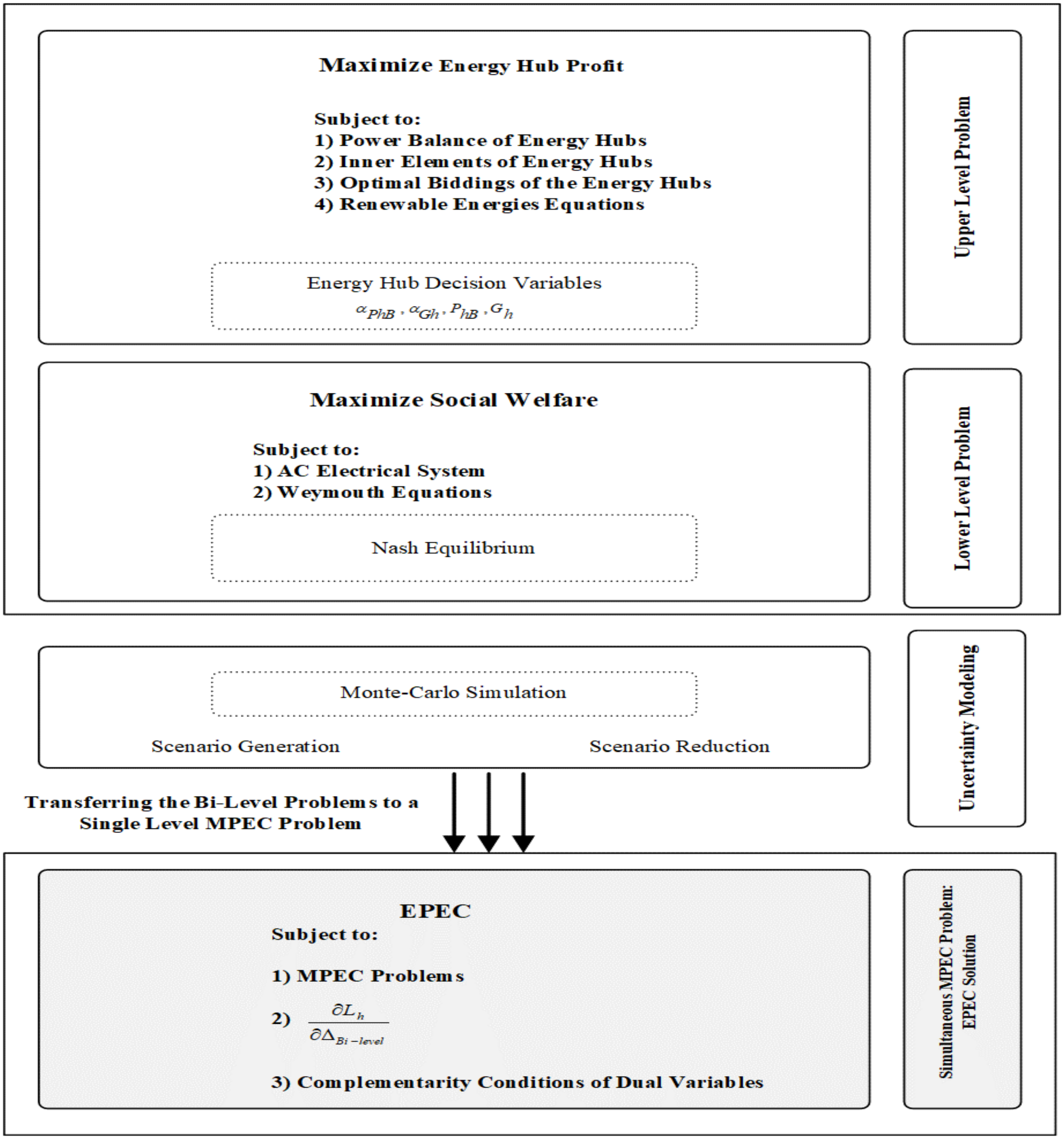

Fig. 4. The flowchart of the solution methodology.

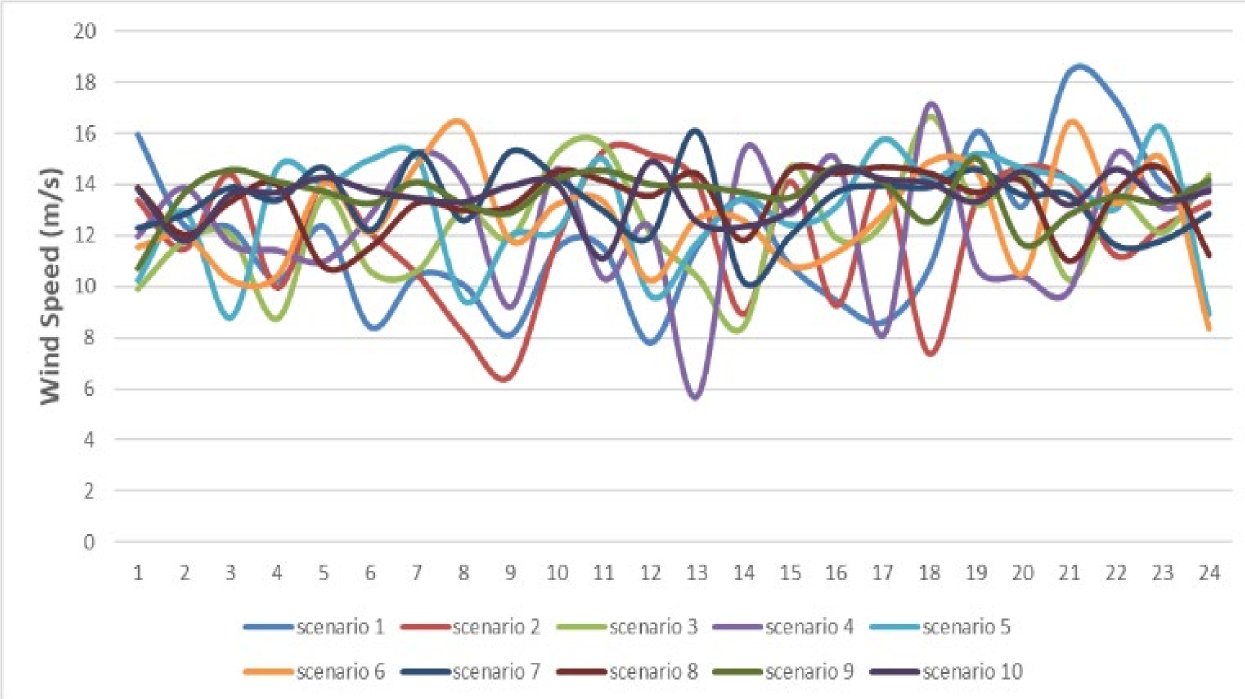

Fig. 5. Wind speed reduced scenarios.

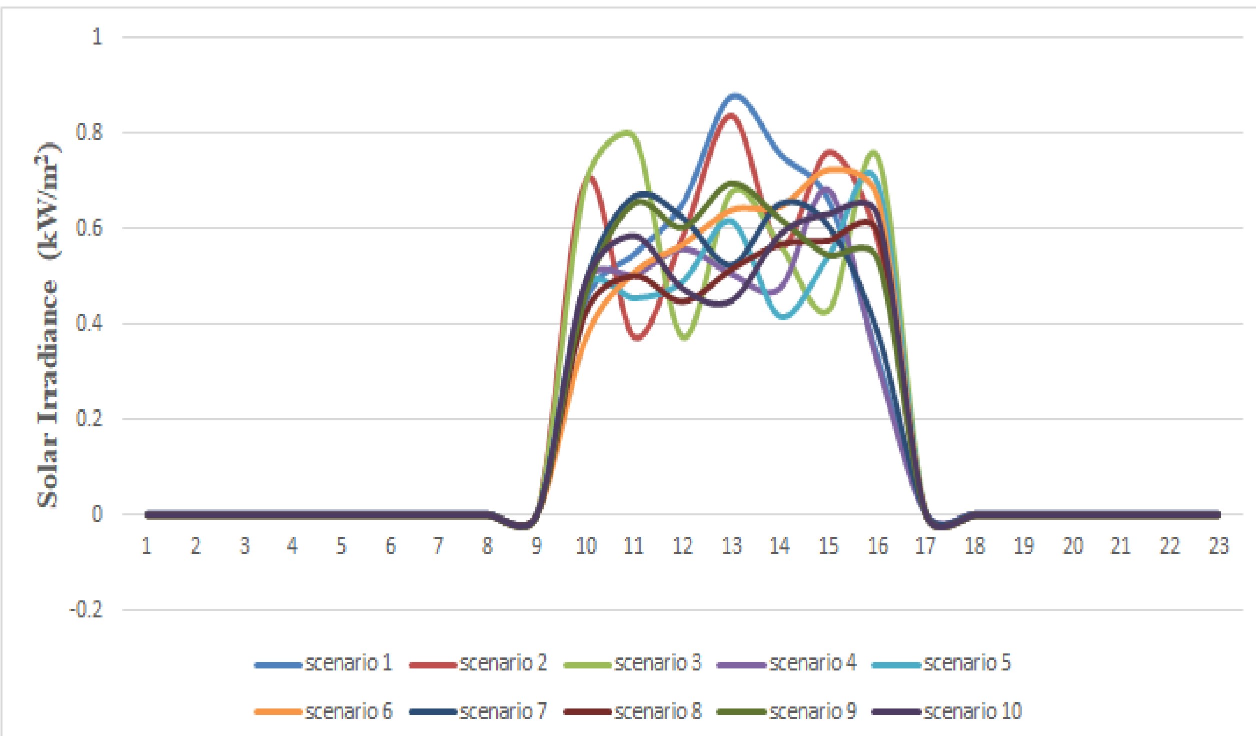

Fig. 6. Solar irradiance reduced scenarios.

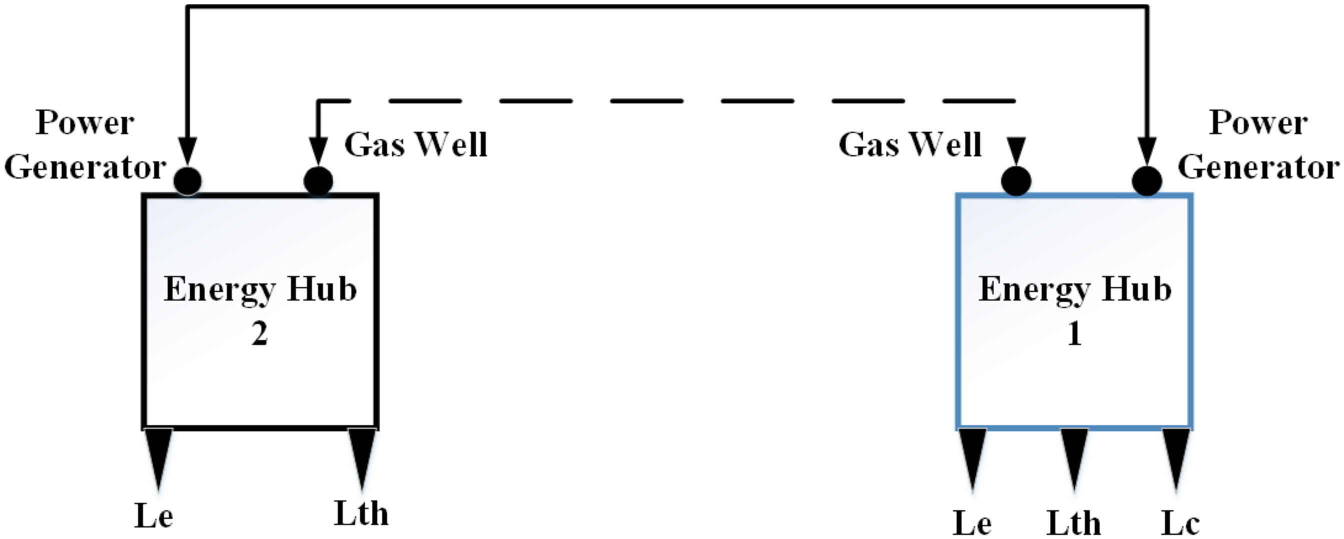

Fig. 7. The layout of two EHs.

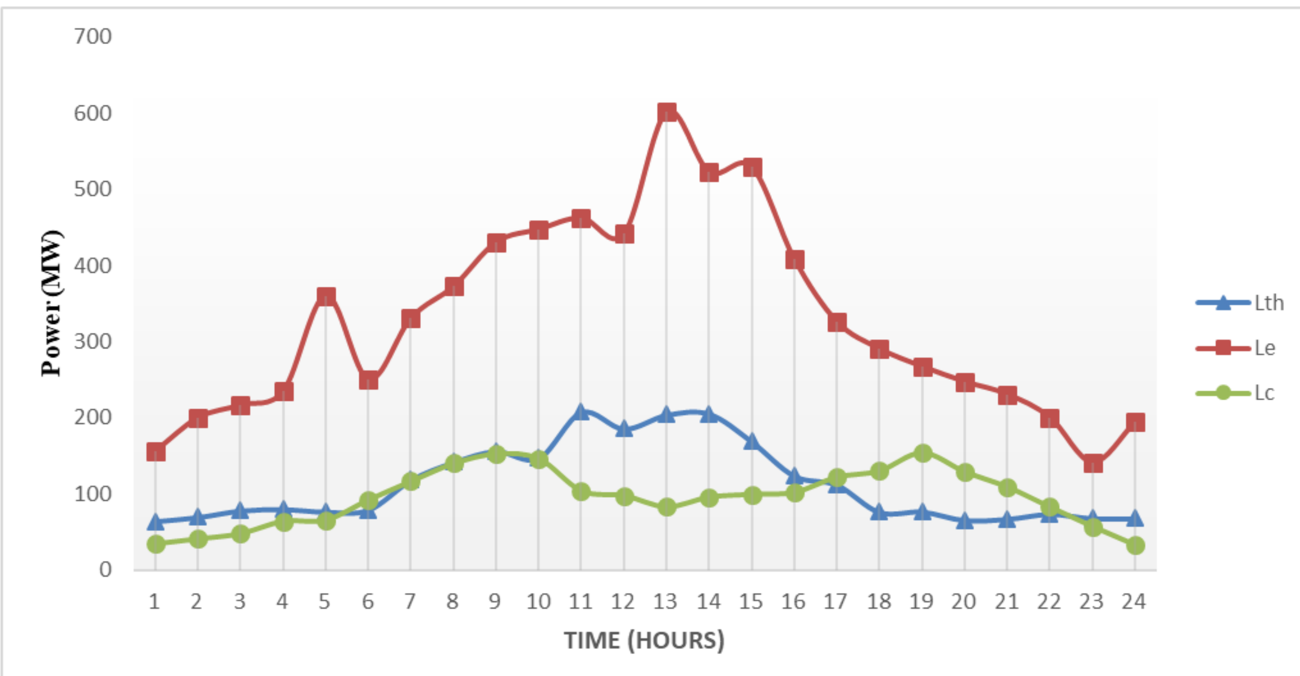

Fig. 8. The demand values of EH 1 in MW.

TABLE 1

CHARACTERISTICS OF EHS' POWER GENERATORS AND GAS WELLS

Players	Max. Power		Generation Cost		Gas Well Bounds		Gas Well Cost
	Block 1 (MW)	Block 2 (MW)	Block 1 ($/MWh)	Block 2 ($/MWh)	Max (MW)	Min (MW)	Cost ($/MWh)
EH 1	250	200	11.34	13.66	800	100	5.23
EH 2	150	100	19.56	25.44	600	100	6.19

TABLE 2

RESULTS OF THE EXAMPLE IN UNCONGESTED MODE

Cases	$(\alpha_P)_{H1}$	$(\alpha_G)_{H1}$	$(\alpha_P)_{H2}$	$(\alpha_G)_{H2}$	P_{H1}	G_{H1}	P_{H2}	G_{H2}	$Profit_{H1}$	$Profit_{H2}$	F_p	F_g
N-N	-	-	-	-	450	800	43.55	16.27	34392	882	167.97	260.09
S-N	18.39	10	-	-	343.55	216.27	150	600	93051.768	38100	61.52	-323.64
S-S	30	10	24.78	10	343.55	216.27	150	600	143301.57	50727.589	61.52	-323.64

TABLE 3

RESULTS OF THE EXAMPLE IN CONGESTED MODE

Cases	$(\alpha_P)_{H1}$	$(\alpha_G)_{H1}$	$(\alpha_P)_{H2}$	$(\alpha_G)_{H2}$	P_{H1}	G_{H1}	P_{H2}	G_{H2}	$Profit_{H1}$	$Profit_{H2}$	F_p	F_g
N-N	-	-	-	-	382.03	739.91	111.52	76.36	18045	1764	100	200
S-N	25.44	10	-	-	243.55	339.91	250	476.36	94665.191	19554	-38.48	-200
S-S	30	10	30	10	382.03	339.91	111.52	476.36	154827.75	49265.866	100	-200

As shown in Table II in Case 0, the second row, they submit no biddings since both EHs play non-strategically. Columns 6-9 show market clearings of the EHs. The primary EH (unit 1) is cheaper than unit 2, producing much more than unit 2. Columns 10-11 give the profit of each EH. Each EH unit's profit is the summation of its power profit and its natural gas profit, according to Eq. (1). The last two columns give the power flow and natural gas flow through the lines. In Case 1, the primary EH just bids strategically, and the column 2-3 give its strategic biddings according to its generation cost and gas well cost in Table I. The values for the market clearings decrease for unit 1, and increase for unit 2 compared to Case 0 due to the strategic play of EH 1. The profits of both EHs increase; however, the increment of EH 2's is higher due to its more generation than EH 1 and compared to Case 0. In Case 2, both units bid strategically, leading to an increase in optimal biddings of the EH 1. The EH 1's profit increases 54% compared to Case 1, while unit 2's profit increment is 33% since unit 1 bids higher than unit 2 in Case 2 and compared to Case 1. As evident from the last column, the direction of natural gas flow is from unit 2 to unit 1 (minus sign).

Table III presents the results of Table II with the power line and natural gas pipeline flows fixed to values of 100 and 200, respectively. As shown in Table III compared to Table II, the primary EH increases optimal biddings in Case 1 and Case 2 compared to uncongested mode, where it bids more due to congestion in electric power lines and natural gas pipelines. As evident, the electric power line and natural gas pipeline values are bound to their set values except in Case 1. The point that is worth mentioning in Table III compared to Table II is in profit columns. Like Table II, with the addition of each strategic player, the profit of the player increases. Congestion leads to a profit increase for EH 1 except in Case 0, in which unit 1 plays non-strategically.

Fig. 9. Depicts optimal power and natural gas flow of both EHs plus power flow through the power line and natural gas flow through the pipeline in Case 2-uncongested mode. Hour 10 is circulated with dashes to verify the resultant TABLES. It is noted that at hours 11 and 14, the values of the F_g are negative, indicating that the flow direction is from unit 1 to unit 2.

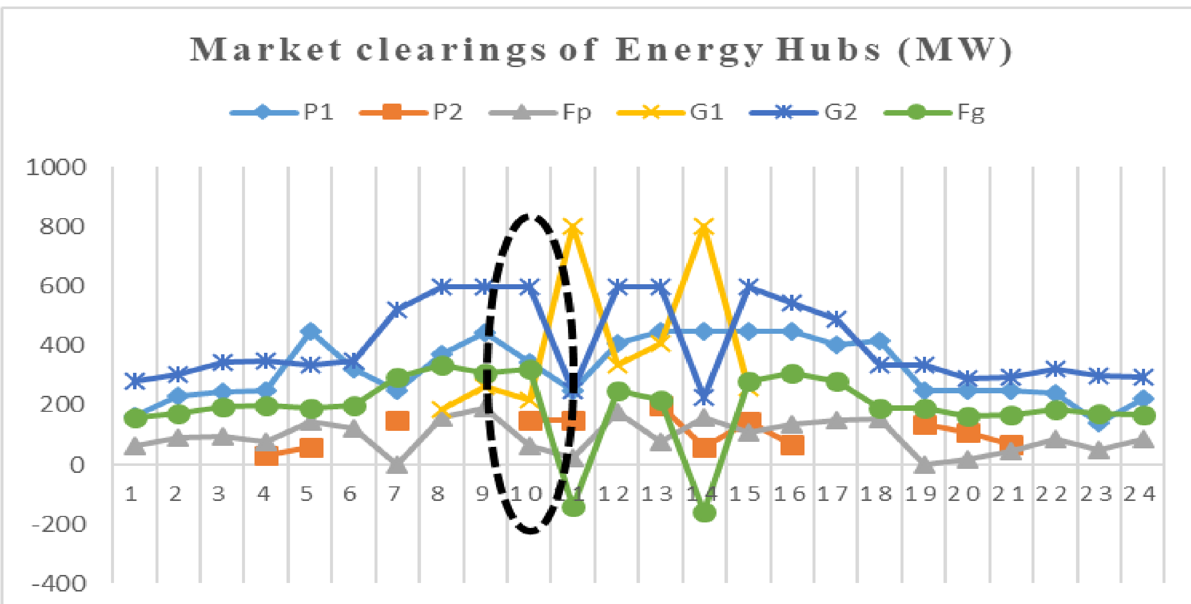

Fig. 9. Market clearings of EHs in example section, case 2- uncongested mode

4.1.2. Discussions on answers: Have Nash Equilibrium points been found?

In this sub-section, the uniqueness and existence of the Nash equilibria are discussed for the above example. If the payoff function (minus energy cost) is strictly concave, then it is N-person concave game. In (Sheikhi et al., 2015), the existence and uniqueness of Nash equilibrium are proved for the game. For clarification, one state is considered: strategic electric biddings of the primary EH in the uncongested mode in Case 1. Fig. 10 depicts a 3-D diagram of the $(\alpha_P)_{H1}$ concerning generation cost changes of the primary EH plants in Case 1. In Fig. 10, the left vertical axis shows the changes of the generation cost of the EH 1 in steps of five. The right axis shows the optimal electrical biddings' tracking changes ($(\alpha_P)_{H1}$), and the horizontal line depicts the periods. As shown in the 3-D diagram, there is a flat surface in $(\alpha_P)_{H1}$ values (here 30) with generation blocks of 25-30 to end. This result shows that the model will converge with an upper bound for optimal electrical biddings values; however, the model depends on the initial values. Nevertheless, the uniqueness and existence of each Nash equilibrium of the model are guaranteed.

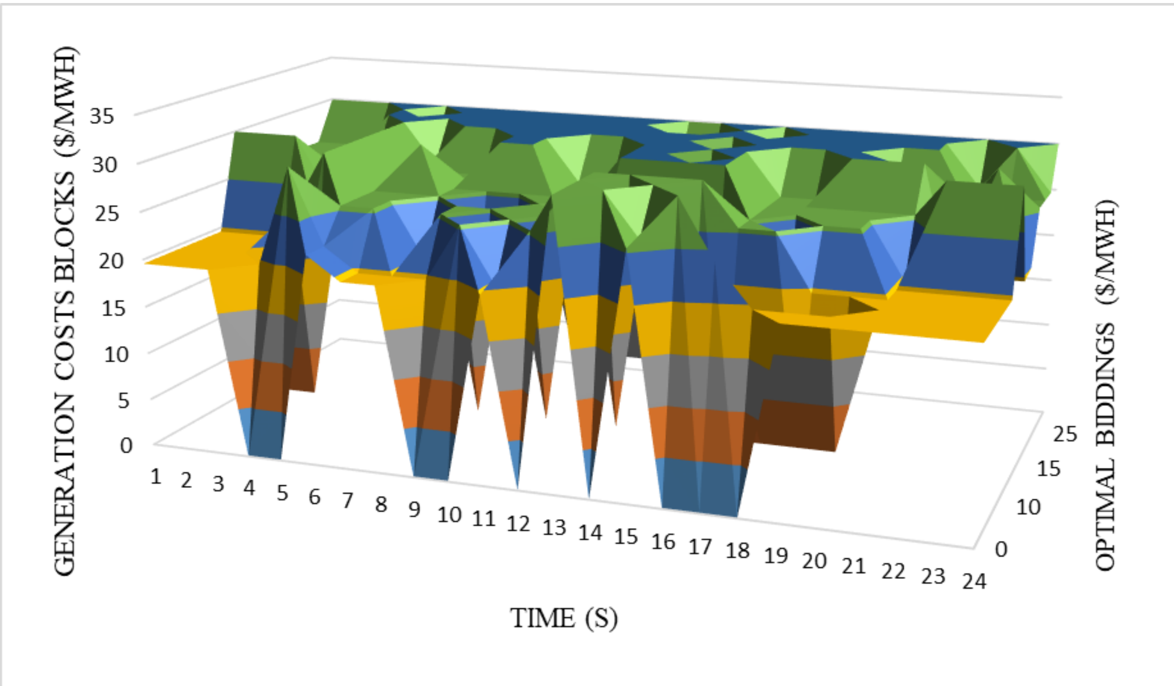

Fig. 10. 3-D diagram of optimal biddings of the primary EH in Case 1-uncongested mode.

4.2.Second Case Study

One standard case study is adapted to verify the flexibility and correctness of the proposed model. The data for the EHs are as those from Table I.

4.2.1. Six-bus power system and 7-node natural gas system

The second study case is shown as follows. In Fig. 11, EH 1 belongs to player 1 in the energy market, and its layout is according to Fig. 1. EH 2 and EH 3 belong to player 2, and their layouts are according to Fig. 2. There are three gas demands GL1=GL2=GL3=10 MW and three power demand PL1=PL2=PL3=5 MW. A compressor component is located between node 2 and node 4 with a capacity of 200 MW. Bus 4 is considered a Slack bus.

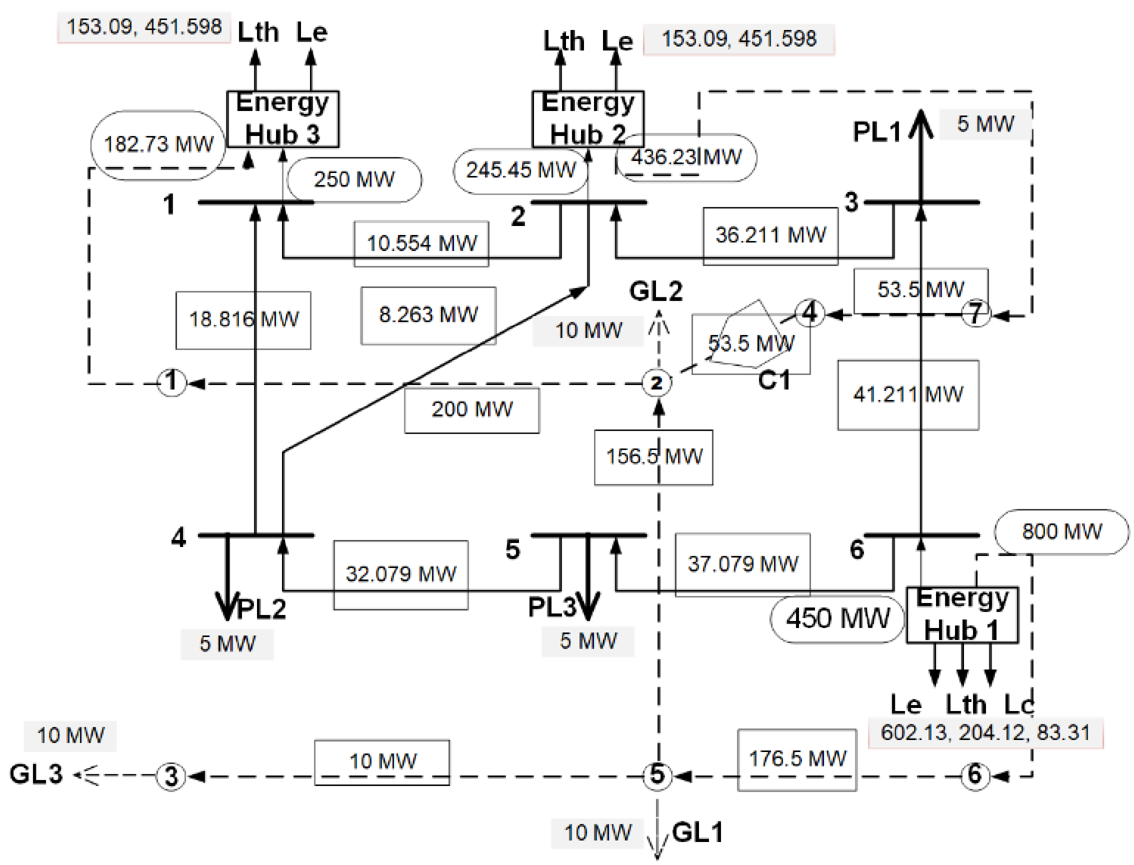

Fig. 11. Market clearings of EHs, Case 2-Uncongested mode.

4.2.2. Problem solution

Three cases are expressed to solve the problem:

- Case 0. Both players act non-strategically in the joined market.
- Case 1. Player 1 (EH 1) plays strategically in the joined market.

- Case 2. Both players act strategically in the joined market.

Two modes are considered: uncongested mode and congested mode with the power lines and natural gas pipelines divided by 100. Tables IV and V represent the case study results in uncongested mode and congested mode, respectively. It is noted that the Tables definite period is at hour 13 (demand peak) to show the difference between congested and uncongested modes. The profit column presents the profit of two players in three cases for all times (24 hours). As apparent in Tables IV and V, the strategic biddings' values in congested mode are higher than uncongested mode values. This relation exists in the S-S state (when two players act strategically) than the S-N state (when one player acts strategically). It is noted that the values of $(\alpha)_H$ shows the difference between marginal costs of the units and optimal biddings resulted from the players' strategic participant. Since player 2 has two EHs, there is a comma sign in its related column to show the values for EH 2 and EH 3, respectively. The last two columns give the profits of each player. As shown in Tables IV and V, there is an increase in profit values for both players. However, in congested mode compared to uncongested mode, there are increases in player 1's profit and decreases in player 2's. Specifically, player 1, owning EH 1, enjoys a 35% increase in its profit when the lines are congested. However, the strategic biddings of player 2 increase in congested mode compared to uncongested mode, and its profit decreases due to a total decrease in produced energies of player 2's units. In Fig. 10, the values in rectangular shapes show the power line and pipeline values. The values in oval shapes present each EH's produced values, and gray shapes give demand values.

TABLE 4

RESULTS OF THE CASE STUDY IN AN UNCONGESTED MODE

Cases	$(\alpha_P)_{H1}$	$(\alpha_G)_{H1}$	$(\alpha_P)_{H2}$	$(\alpha_G)_{H2}$	Player1's Profit	Player 2's Profit
N-N	-	-	-	-	69331	3528
S-N	30	5.23	-	-	123951.796	57192
S-S	30	10	30,30	6.19,6.19	162643.983	86790.23

TABLE 5

RESULTS OF THE CASE STUDY IN A CONGESTED MODE

Cases	$(\alpha_P)_{H1}$	$(\alpha_G)_{H1}$	$(\alpha_P)_{H2}$	$(\alpha_G)_{H2}$	Player1's Profit	Player 2's Profit
N-N	-	-	-	-	41596	3907.286
S-N	30	10	-	-	130411.94	33958.522
S-S	41.34	15.23	30,30	10,10	220741.045	78940.558

Fig. 12 depicts the timing diagram of the Compressor in three cases in uncongested mode. As shown in Fig. 12, the gas flow of the Compressor is identical in Case 0 and Case 1. However, in Case 2, the gas flow reaches maximum flow (200 MW) at some hours: these hours are those in which EH 3 produces no natural gas, and EH 2 produces values higher than half of the values.

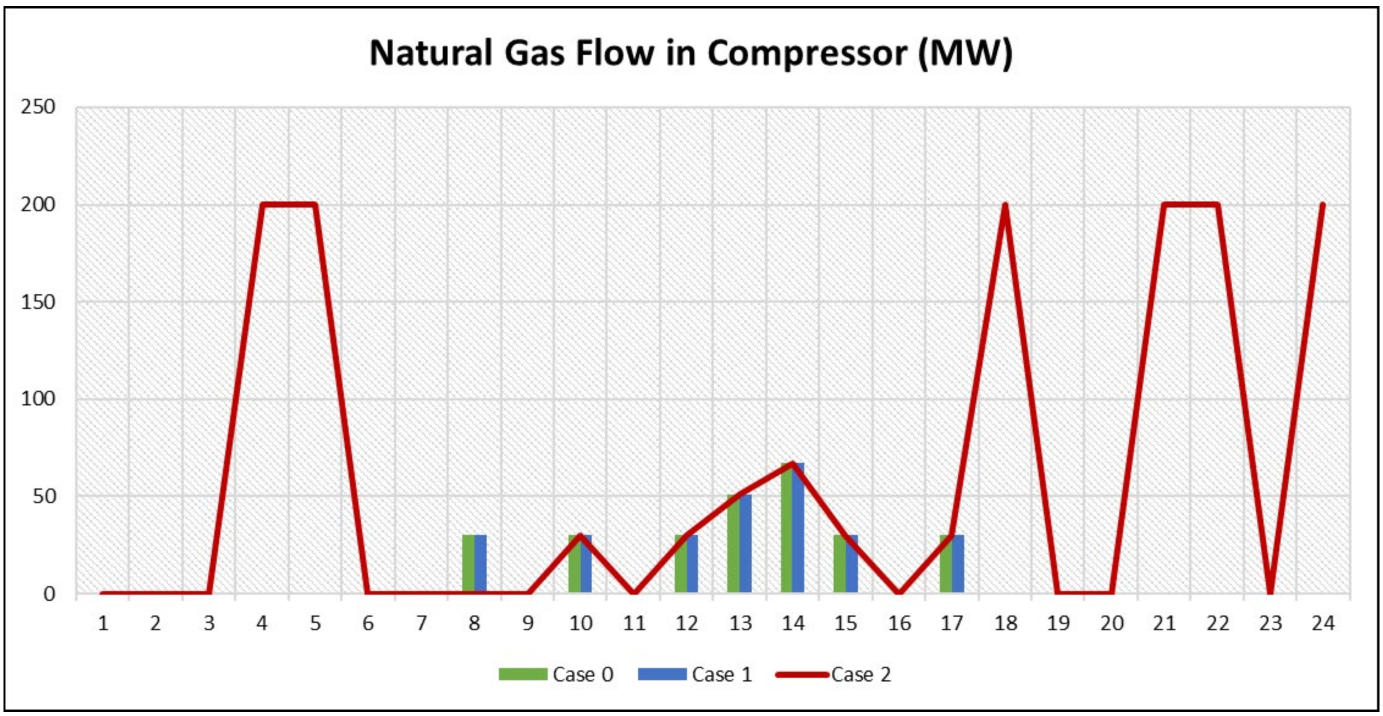

Fig. 12. Gas flow in Compressor in three cases in uncongested mode

5. CONCLUSIONS

In this paper, EHs' strategic behavior in a competitive and joined energy market is investigated. Bi-level programming is used due to the difference between each EH's profits seeking their units' profit maximization and market clearings. Bi-level programming is converted to MPEC to solve the strategic bidding submission of EHs. To solve MPEC, EPEC is utilized in a recursive method. This paper's novelties are categorized using game-theoretic approaches for strategic bidding submission of multi-EHs in the joined energy market. The represented model is verified by one example and one standard case study, which shows its flexibility in dealing with a wide range of joined and complex systems. Three cases and two modes containing uncongested and congested modes are investigated in two case studies. According to the case studies, the uniqueness and existence of the Nash equilibria are found. For the second case study, a multi-EHs system is adapted to verify the flexibility of the proposed joined market. It is found that the strategic biddings' values in congested mode are higher than uncongested mode values. This relation exists when two players act strategically rather than one player acts strategically. Specifically, the primary EH enjoys a 35% increase in its profit when the lines are congested. However, the strategic biddings of player 2 increase in congested mode compared to uncongested mode, and its profit decreases due to a total decrease in produced energies of its units.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

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