# Element-Rotated Linear, Planar, and Conformal Arrays Radiating Shaped Patterns 

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#### Abstract

Array element rotation could serve as the fifth dimension that can be modulated to synthesize desired array patterns except for the excitation amplitude, phase, element position, and time sequence. A method of synthesizing shaped patterns for linear, planar, and cylindrical conformal arrays by optimizing element rotations and excitation phases considering mutual coupling (MC) is presented. The rotation of an element in its local coordinate system (LCS) is approximately described by rotating its vectorial active element pattern (VAEP). Then, the rotated patterns are transformed to the global coordinate system (GCS) by coordinate transformation and scattered interpolation to calculate the array pattern. In the end, a refined strategy is adopted to reduce the error introduced in the rotated VAEP approximation. Numerical examples and measurements are presented to validate the effectiveness of the proposed method.


## I. Introduction

Shaped-beam array has received considerable attention for their significant applications in radar and wireless communications [1]. To obtain a desired shaped pattern with good mainlobe and sidelobe level (SLL) control, most of the existing methods optimize both the excitation amplitudes and phases, especially for the conformal array [2]. Consequently, many unequal power dividers (PDs) are required in the feeding network. The element rotation technique has drawn increased attention recently [3]-[5]. However, the existing element rotation techniques deal with linear or planar arrays only. In this work, we first present to synthesize shaped patterns for conformal arrays based on the element rotation. The rotation of an element in its own local coordinate system (LCS) is approximately described by rotating its vectorial active element pattern (VAEP). Then, coordinate transformation and scattered interpolation are applied to express the rotated VAEPs in the global coordinate system (GCS) to calculate the array pattern. To reduce the inaccuracy caused by antenna deformation and mutual coupling (MC) change, a refined strategy is employed. Numerical examples and measurements are provided to show the favorable effects of the proposed method.

## II. Formulations

Consider a cylindrical array with $N$ elements in the global coordinate system (GCS) $x y z$ as shown in Fig. 1. To facilitate the formulation, a local coordinate system (LCS) $x_{n}^{\prime} y_{n}^{\prime} z_{n}^{\prime}$ is built at the $n$th element. The VAEPs $E_{\theta_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime} ; 0\right)$ and $E_{\phi_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime} ; 0\right)$ of the $n$th element without rotation in the LCS can be easily obtained from full-wave simulation. Now,


Fig. 1. The element-rotated cylindrical conformal array.
assume the $n$th element is rotated by an angle $\xi_{n}$ around $z_{n}^{\prime}$ axis, its electrical pattern in the LCS can be approximately obtained by mathematically rotating $E_{\theta_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime} ; 0\right)$ and $E_{\phi_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime} ; 0\right)$ by assuming the rotation does not change the antenna curvature and MC much

$$
\begin{align*}
\vec{E}_{n}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime} ; \xi_{n}\right) & \approx E_{\theta_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime}-\xi_{n} ; 0\right){\overrightarrow{\theta^{\prime}}}_{n} \\
& +E_{\phi_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime}-\xi_{n} ; 0\right){\overrightarrow{\phi^{\prime}}}^{\prime} \tag{1}
\end{align*}
$$

where $m=1,2, \ldots, M$ are discrete sampling points. Then, since the elements are rotated around different normal directions, rotational and translational coordinate transformations are employed to transform the rotated VAEPs in different LCSs to the common GCS as

$$
\begin{align*}
\vec{E}_{n}\left(\theta_{n, m}, \phi_{n, m} ; \xi_{n}\right) & \approx E_{n, \theta}\left(\theta_{n, m}, \phi_{n, m} ; \xi_{n}\right){\overrightarrow{\theta^{\prime}}}_{n} \\
& +E_{n, \phi}\left(\theta_{n, m}, \phi_{n, m} ; \xi_{n}\right){\overrightarrow{\phi^{\prime}}}_{n} \tag{2}
\end{align*}
$$

where $E_{n, \theta}\left(\theta_{n, m}, \phi_{n, m} ; \xi_{n}\right)$ and $E_{n, \phi}\left(\theta_{n, m}, \phi_{n, m} ; \xi_{n}\right)$ in the GCS are obtained by using coordinate transformation from $E_{\theta_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime}-\xi_{n} ; 0\right)$ and $E_{\phi_{n}^{\prime}}\left(\phi_{n, m}^{\prime}, \phi_{n, m}^{\prime}-\xi_{n} ; 0\right)$, respectively. The unite vectors ${\overrightarrow{\theta^{\prime}}}_{n}$ and ${\overrightarrow{\phi^{\prime}}}_{n}$ are also expressed in the GCS. Then, with the co-polarization (CoP) $\vec{p}_{c o}$ and crosspolarization (XP) $\vec{p}_{X}$ defined in [2], the CoP and XP patterns could be obtained by simply projecting ${\overrightarrow{\theta^{\prime}}}_{n}, \vec{\phi}^{\prime}{ }_{n}$ to $\vec{p}_{c o}, \vec{p}_{X}$.

In addition, owing to that the rotated VAEPs obtained in the GCS do not share unified angle samplings after coordinate transformation. A scattered interpolation based on the Delaunay triangulation is applied to unify the angle samplings as

$$
\begin{equation*}
E_{n, \nu}\left(\theta_{n, m}, \phi_{n, m} ; \xi_{n}\right) \stackrel{f_{\text {Int }}}{\longrightarrow} E_{n, \nu}^{\text {Int }}\left(\theta_{m}, \phi_{m} ; \xi_{n}\right) \tag{3}
\end{equation*}
$$

where $E_{n, \nu}$ denotes the CoP $E_{n, c o}$ or the XP $E_{n, X}, f_{\text {Int }}$ is the scattered interpolation function. Then, the array pattern can be calculated in the unified angle sampling grid, and with which, shaped patterns can be synthesized by optimizing element rotations and excitation phases using the particle swarm optimization (PSO). However, the real array pattern might vary much from the synthesized one due to the neglect of antenna deformation and MC change in (1). Hence, a
refined strategy is applied to iteratively perform full-wave simulation of the array with corrected antenna curvature, and re-optimization using decreasing rotation angles and updated VAEPs. The array pattern at the $k$ th $(k=1,2, \ldots, K)$ refine step is given by

$$
\begin{equation*}
F_{\nu}^{(k)}\left(\theta_{m}, \phi_{m}\right)=\sum_{n=1}^{N} E_{n, \nu}^{I n t}\left(\theta_{m}, \phi_{m} ; \xi_{n}^{(k)}\right) e^{j\left[\beta \vec{r}_{n} \cdot \vec{u}\left(\theta_{m}, \phi_{m}\right)+\alpha_{n}\right]} \tag{4}
\end{equation*}
$$

where $F_{\nu}^{(k)}$ denotes $F_{c o}^{(k)}$ or $F_{X}^{(k)}, \beta=2 \pi / \lambda$ is the wavenumber in free space, $\alpha_{n}$ is the excitation phase of the $n$th element located at $\vec{r}_{n}, \vec{u}\left(\theta_{m}, \phi_{m}\right)$ is the propagation vector. $E_{n, c o}^{I n t}\left(\theta_{m}, \phi_{m} ; \xi_{n}^{(k)}\right)$ and $E_{n, X}^{I n t}\left(\theta_{m}, \phi_{m} ; \xi_{n}^{(k)}\right)$ are obtained by using coordinate transformation, projection, and interpolation from $E_{\theta_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime}-\xi_{n}^{(k)} ; 0\right)$ and $E_{\phi_{n}^{\prime}}\left(\theta_{n, m}^{\prime}, \phi_{n, m}^{\prime}-\xi_{n}^{(k)} ; 0\right)$, which are obtained by mathematically rotating the updated VAEPs from the HFSS simulation in the $(k-1)$ refining step. Generally, $K=3$ is enough to obtain a highly accurate result. Note that the above formulation is for the cylindrical conformal arrays. However, it is similar for linear and planar arrays except that only translation transformation is required for the coordinate transformation. Besides, no antenna deformation is involved when rotating the elements in linear or planar arrays.

## III. Numerical and Measurement Results

The proposed method is utilized to synthesize a flat-top pattern for a 24 U -slot patch antenna linear array, a circular flat-top pattern for an $11 \times 11$ cavity-backed circular patch antenna planar array, and a flat-top pattern for a 24 U slot patch antenna cylindrical array. Fig. 2(a) and (e) show the synthesized and simulated patterns for the linear and cylindrical array, whereas Fig. 2(c) shows the simulated pattern for the planar array, all in the initial step. It is seen that, initially, the simulated real patterns deteriorate a lot since the MC change and antenna deformation are not considered in the pattern synthesis. However, as shown in Fig. 2(b), (d), and (f), the real patterns after performing three refining steps are with much better mainlobe shape and SLL performance. Very good agreements are observed between the synthesized and real patterns. Besides, compared to the phase-only method, the SLL is reduced by more than 3 dB when optimizing the element rotations. The linear and cylindrical arrays are fabricated and measured, as shown in Fig. 3. The measured patterns generally match well with the simulated ones.

## IV. Conclusion

A refined joint rotation/phase optimization method is proposed to synthesize shaped patterns for linear, planar, and cylindrical conformal arrays. Though the MC and antenna curvature will be severely altered after rotation, numerical examples have shown the excellent capability of the proposed method in dealing with these challenges and synthesizing desired shaped patterns with reduced SLLs. Good agreements are observed between the measured and simulated results. By using the proposed method, many unequal PDs can be saved, which could result in a much simplified feeding network.


Fig. 2. The patterns obtained in the initial (left column) and 3rd refining step (right column) for the three arrays. (a), (b) for the linear array, (c), (d) simulated patterns for the planar array, and (e), (f) for the cylindrical array.


Fig. 3. Pictures of the fabricated (a) linear and (b) cylindrical array prototypes, and (c), (d) the corresponding simulated and measured patterns of them.

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