Parametric study of circular duct breakout transmission loss

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ABSTRACT
Breakout from ducts can be a major source of noise on industrial sites. The emitted noise is harmful at sufficient levels and must be reduced. Reducing noise levels using appropriate methods after the fact can be both costly and time consuming. This can be avoided by correctly predicting duct transmission loss before the duct is installed and choosing a suitable construction. In this paper the breakout from circular ducts will be presented as part of a parametric study. Parameters such as duct radius and wall thickness will be used to allow for a suitable choice of duct construction. In practical systems duct radii can be several metres in diameter and this is reflected by the range of the study. The breakout noise is calculated using a semi analytic finite element method. The first step in this method requires determination of the duct's modal properties which is calculated using a numerical eigen-value analysis. The waves are then propagated along the duct using an analytic expression. A monopole sound source is used to excite the system, simulating the complex acoustic field within HVAC ducts with the transmission loss calculated from the difference between radially propagating sound power to axially propagating sound power.

1 INTRODUCTION

Breakout noise from ducts can be a major source of noise on industrial sites. This noise is generated by sources within the internal fluid and transferred via vibroacoustic coupling through the duct wall and into the environment. Addressing this issue before construction requires knowledge of how the noise will propagate and how changes to the ductwork will affect the vibroacoustic coupling.

Ducts in HVAC and gas turbine intake and exhaust systems are commonly single skinned, axisymmetric cylindrical ducts. This geometry has undergone extensive study (Cummings, 2001) however the majority of the work in this area involves thin plate assumptions. For example, Fuller (Fuller 1986) investigated a cylindrical duct excited by a central monopole source. However the thin plate assumption restricts the frequency range that can be modelled as it does not consider shear displacement. This can limit their usefulness when the ducts have thick walls relative to the radius and also at high frequencies.

With modern computational resources it is possible to represent the finite wall thickness of a duct using the full three dimensional elastic equations. Daneshjou et al. (Daneshjou et al., 2016) developed an analytical model using the three dimensional elastic equations for calculating transmission loss due to an external plane wave. Duan et al. (Duan et al., 2016) presented a semi analytical finite element (SAFE) model for the propagation of vibrations through buried pipes. Using these more detailed models it is possible to predict the vibro-acoustic properties of a waveguide up to high frequencies. This is necessary as noise mitigation is commonly required across the 30 Hz to 8 kHz octave bands.

In this work a SAFE method (Williams et al. 2019, Williams et al. 2020) will be presented to predict the transverse transmission loss of circular ducts. The SAFE method models axial propagation analytically but uses the finite element method to determine the transverse eigensolutions. This gives it an advantage over finite element methods as no mesh is required along the axis and therefore the computational cost is reduced. Sicne the model uses the three dimensional elastic equations it does not have a limited frequency range. It can therefore be used to predict the transmission loss of ducts at high frequencies and where the duct wall is thick. A more comprehensive, and accurate, parametric study of duct performance can then be carried out extending the range of studies that rely upon the thin shell assumption.

The geometry and calculations for sound power are presented in Section 2. A verification of the work by comparison to COMSOL is given in Section 3. Section 4 then shows a selection of transmission losses for infinite length cylindrical ducts excited by a monopole at the centre of the duct.

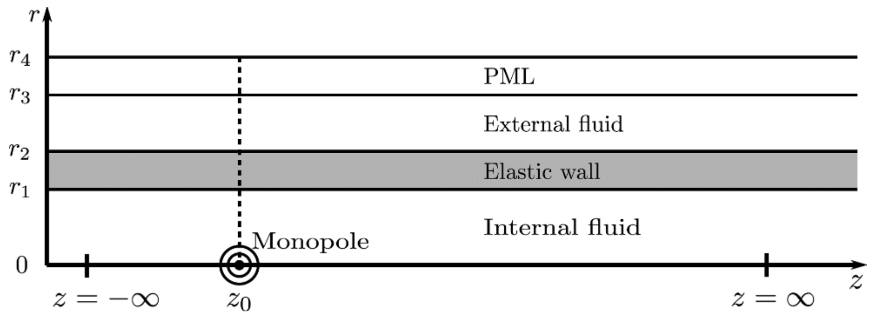

Figure 1: Diagram of the infinite length pipe geometry.

2 THEORY

In this paper the breakout noise from an infinite length, axisymmetric cylindrical duct with thick pipe walls is investigated. The duct has a single steel wall at a radius of r_1 and a thickness of $t = r_2 - r_1$. The pipe is filled with a fluid with the properties of air where $c_0 = 343$ m s^{-1} is the speed of sound and ρ_0 = 1.2 kg m^{-3} is the density. The pipe is also immersed in air with the same properties between r_2 and r_3. The domain is bounded in the radial direction by a perfectly matched layer (PML) in the region r_3 and r_4 which has the speed of sound $c_0 = \frac{343}{1-0.1i/(40\pi\log_{10}(e^1))}$ m s^{-1}. The dimensions of each duct can be found in Table 1.

The pressure within the fluid, $\hat{p}$, and the displacement within the elastic, $\hat{\boldsymbol{u}}$, is given by the modal expansions (Kirby et al., 2014)

$$\hat{p}(r,z) = \sum_{m=1}^{N} A_m p_m(r)\, e^{-ik_0\gamma_m z}, \tag{1}$$

$$\hat{\boldsymbol{u}}(r,z) = \sum_{m=1}^{N} A_m \boldsymbol{u_m}(r) e^{-ik_0\gamma_m z} \tag{2}$$

where $\boldsymbol{u_m} = [u_r^m, u_z^m], u_r^m$ is the axial displacement, u_z^m is the axial displacement, $k_0 = 2\pi f / c_0$, f is the frequency, A_m is the modal amplitude, p_m and u_m are the transverse eigenfunctions, and γ_m are the eigenvalues of mode m. The eigenfunctions and eigenvalues are calculated numerically through the SAFE method as presented by Duan et a. (Duan et al., 2016) and Nuyen et al. (Nuyen et al., 2016). It will be noted that in eqs. (1) and (2) the propagation in the axial direction is given analytically. The eigenfunctions are then only a function of the cross-section. The cross-section is represented using a 1-dimensional finite element mesh that is constructed over the radius between $r = 0$ and r_4. Each element within the fluid, elastic and PML is a quadratic line element. The final mesh has a total of N nodes.

The charactersitics of a sound source can greatly influence the performance of noise control devices (Williams et al., 2017, Williams et al., 2018). In this investigation the effect of a monopole source is investigated. The modal amplitudes are calculated for a monopole source using the method of Cummings and Astley (Cummings and Astley, 1995)

$$\boldsymbol{A} = \frac{1}{\omega}\boldsymbol{\Gamma}^{-T}\hat{\boldsymbol{p}}^T\boldsymbol{Q} \tag{3}$$

Table 1: Dimensions of the cylindrical ducts.

Duct	A	B	C	D	E	F
Inner radius, r_1 (m)	0.2	1	1	1	0.5	0.5
Thickness, t (mm)	5	2	5	8	1	8
Length, l (m)	3	30	30	30	30	30
Young's modulus, E (GPa)	205	205	205	205	205	205
Density, ρ (kg/m^3)	7850	7850	7850	7850	7850	7850
Poisson's ratio, v	0.28	0.28	0.28	0.28	0.28	0.28

where $\omega = 2\pi f$, $\boldsymbol{Q}$ is a vector of the amplitudes at each node, $\hat{\boldsymbol{p}}$ is a vector of the pressure at each node across the duct, and $\boldsymbol{\Gamma}$ is Scandrett and Frenzen's bi-orthogonality function (Scandrett and Frenzen, 1995). $\boldsymbol{Q}$ is set equal to zero at all nodes except at $r = 0$ and $z = z_0$.

The radiated sound power at the outer surface of the elastic wall over a distance $l = z_1 - z_0$ is given as

$$W_r = 2\pi r_2 \int_{z_0}^{z_1} \hat{p}(r_2, z)\hat{U}_r^*(r_2, z)\, dz \tag{4}$$

where $\hat{U}_r^*$ is the radial displacement of the fluid. The transverse transmission loss is defined as the difference between the incident sound power from the source and the radiated sound power so that

$$TTL = 10 \log_{10}\left(\frac{W_i}{W_r}\right) \tag{5}$$

where the incident soud power (Kirby et al., 2014) is

$$W_i = \sum_{m=1}^{N} |A_m|^2 \frac{\pi k_0}{\omega \rho_0} Re[\gamma_m] \int_0^{r_1} |p_m|^2 r dr. \tag{6}$$

3 VALIDATION

The SAFE method is validated against results from COMSOL's FE method in this section. The comparison is carried out using duct A which has a relatively thick wall compared to the internal radius.

COMSOL was used to develop a 2D axisymmetric finite element model of the coupled vibro-acoustic system. The finite element mesh was created across the radial-axial plane while being rotationally symmetric around the axis. The domain was bounded at the edges by a PML to remove reflections and approximate an infinite domain.

Figure 2a shows the pressure close to the source for the SAFE and FE methods. The data overlays at most frequencies. Comparing the radial acoustic velocity on the outer suface of the plate also shows the data overlays at most frequencies, see Figure 2b. This demonstrates that the models are in agreement.

The main advatange of using the SAFE method instead of the finite element method is that the propagation along the axis is analytical. This allows for a simpler 1-dimensional mesh across the radius rather than the 2-dimensional mesh that is used by the finite element method. A consequence of this is that a PML is only needed along the radius and is not needed at the extremes of the z-axis. This change also improves computational time as the matrix has a much smaller size when considering long uniform pipe lengths.

4 TRANSMISSION LOSS PREDICTIONS

In this section the transmission losses of ducts will be presented. The frequency range will extend above the critical frequency. At this frequency higher order modes are able to propagate within the duct wall. The duct wall must therefore be represented using the full 3-dimensional elastic equations.

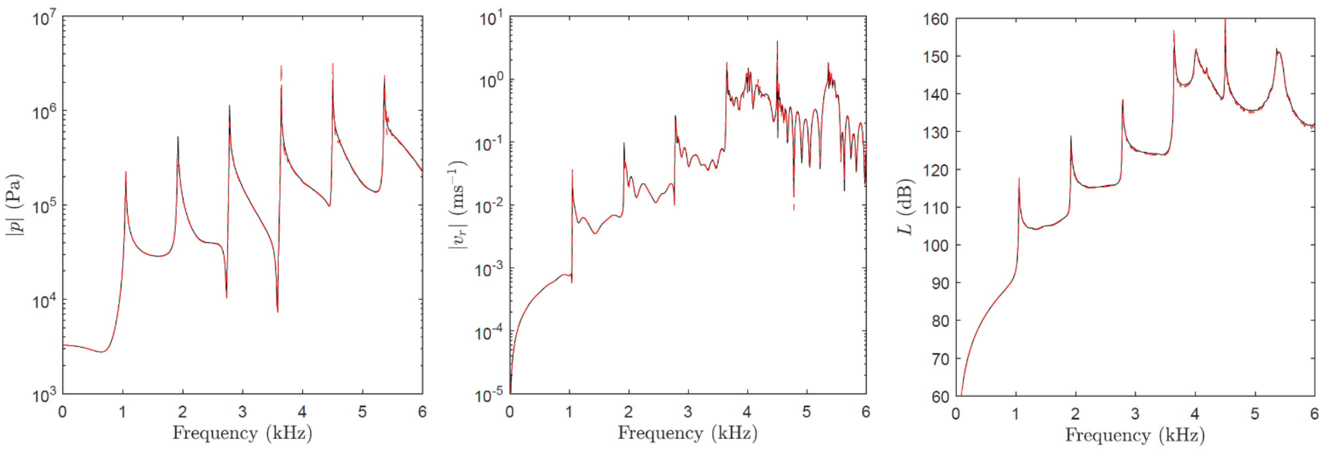

Figure 2: Comparison between the SAFE method (——) and the equivalent FE solution in Comsol (– – – –). (a) Real pressure on the axis 200 mm from the source; (b) Radial acoustic velocity on the wall's outer surface 1.5 m from the source; (c) Radiated sound power over 1.5 m.

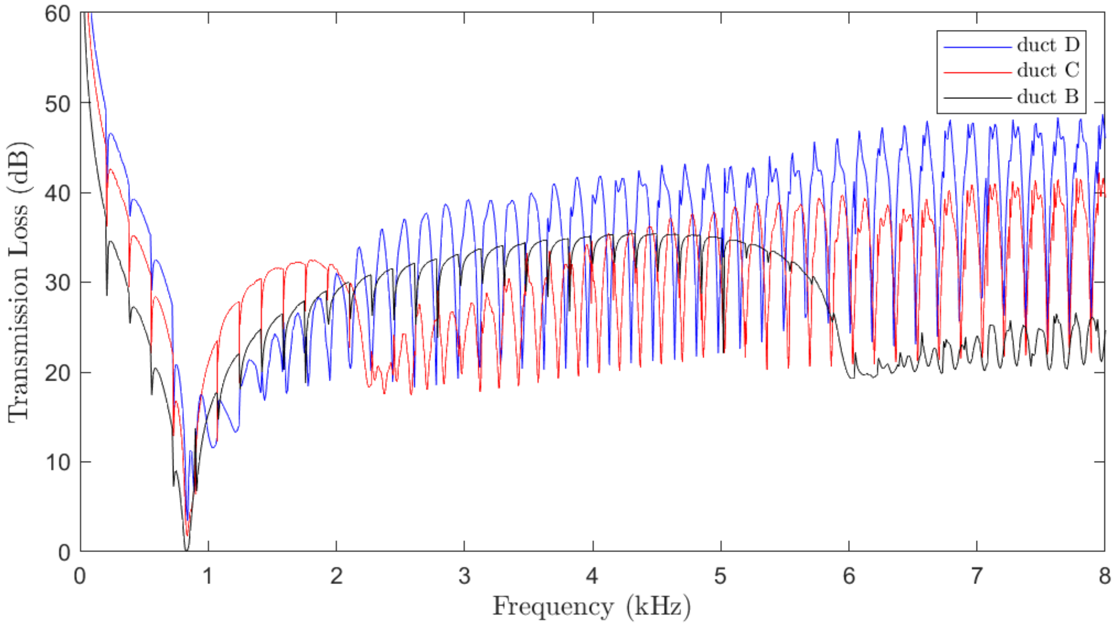

Figure 3: Transverse transmission loss of ducts B, C, and D.

When investigating cylindrical ducts there are three frequency ranges of interest. At low frequencies $(f < f_r)$ the transmission loss is dependant upon the stiffness, at mid frequencies $(f_r < f < f_c)$ the transmission loss is mass dependant, and at high frequencies $(f_c < f)$ the transmission loss is dependant upon the angle of incidence. The boundary between the regions is defined by the ring frequency

$$f_r = \frac{1}{2\pi(r_1+t/2)}\sqrt{\frac{E}{\rho}} = 830 \text{ Hz} \tag{7}$$

and critical frequency

$$f_c = \frac{c_0}{2\pi t}\sqrt{\frac{12\rho\,(1-\nu^2)}{E}} \tag{8}$$

where the Young's modulus is E =2.05 x 10^{11} Pa, the Poisson ratio is $\nu = 0.28$ and $\rho = 7850$ kg m^{-3} is the density of steel. Above f_c it is possible for higher order modes to propagate in the finite width elastic wall.

The first 3 ducts (ducts B, C, and D) investigated have equal internal radius and radiating length so that $r_1 = 1.0$ m and radiating length l = 30 m. Wall thickness varies between ducts so that t = 2 mm, 5 mm, and 8 mm. The duct thicknesses have been chosen as they are common steel thicknesses used in duct construction. Given these dimensions f_r = 830 Hz and the respective critical frequencies are 5.9 kHz, 2.4 kHz, and 1.5 kHz. It is common for noise control specifications to require a noise level to be met between the 30 Hz and 8 k Hz octave bands. For each of the above common duct designs the high frequency region therefore covers a significant potion of the relevant frequency range.

Figure 3 shows the TTL of ducts B, C and D. For ducts B and C the breakout is observed to be low at f_r and at f_c, as expected. In the case of duct D the f_r and f_c are similar and it is not possible to differentiate the effect of each. Above f_c the TTL of each duct is dominated by coincidence radiation, where the radial wavenumber of a mode equals the wall's circumferential wavenumber. As a result of this form of breakout it is possible for a duct to have a worse performance than another duct despite having a thicker duct wall. For example, duct C performs poorly compared to duct B between 2 kHz and 5 kHz depite having a much thicker wall. Similarly duct D performs similarly to duct B between 1 kHz and 2 kHz despite have a wall that is four times as thick. This poses a risk when designing noise control solutions and demonstrates that an accurate model is needed which can account for the non-planar modes in the elastic.

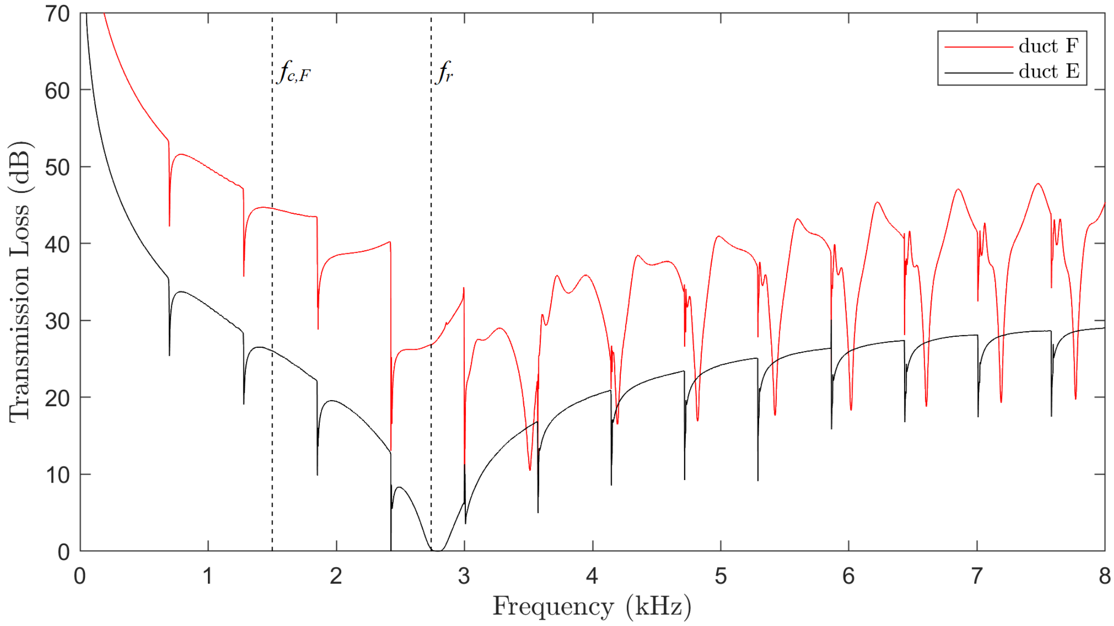

Figure 4: Transverse transmission loss of ducts E and F.

Ducts B, C and D were designed so that $f_r < f_c$. This is defined as an acoustically thin duct wall. However eqs. (7) and (8) show that, by choosing a specific set of values, a duct may be designed where $f_c < f_r$. These ducts are defined as having acoustically thick duct walls. The dimensions of duct F are set so that f_r = 2.7 kHz and f_c = 1.5 kHz and it therefore has an acoustically thick duct wall. Duct E is included for comparison to duct F. Duct E has the same internal radius but has an acoustically thin duct wall so that f_c = 12 kHz. For acoustically thin walls it is expected that there will be a decrease to transmission loss at the critical frequency. This was demonstrated for ducts B and C, see Figure 3. However for acoustically thick walls no sudden drop in performance is observed around the critical frequency, see Figure 4.

In Figure 5 the pressure/radial stress and axial displacement are plotted across the r-z plane for duct F at 4820 Hz. This frequency is above f_c and therefore higher order modes are able to propagate within the elastic wall. Figure 5a shows that the radial stress is fairly planar despite the complex pressure field inside the duct. However propagation of the axial displacement within the elastic wall is not planar, see Figure 5b. This is caused by the strong excitation of a higher order mode within the duct wall. This mode of propagation requires a the 3-dimensional elastic equations to be applied and cannot be predicted when applying thin shell assumptions.

5 CONCLUSIONS

A SAFE method is presented for the cacluation of duct breakout noise. This is applied to an infinite length, cylindrical duct which is excited by an internal monopole on the duct's centreline. The model couples the internal fluid and the external fluid to the elastic duct wall and applies the full three dimensional elastic equations. The SAFE method is validated against a finite element model developed in COMSOL and a good agreement is found between the two.

Models which rely upon thin shell assumptions are limited in their frequency range as they assume no shear displacement. This restrict can be troublesome when dealing with high frequency noise problems and tick duct walls. The SAFE method does not rely upon thin wall assumptions and is therefore not limited by frequency. This is demonstrated using plots of the axial displacement across the radial-axial plane above the critical frequency. Predictions of duct transmission loss are presented at high frequencies showing that the model can be used as part of a larger parametric study into duct breakout noise.

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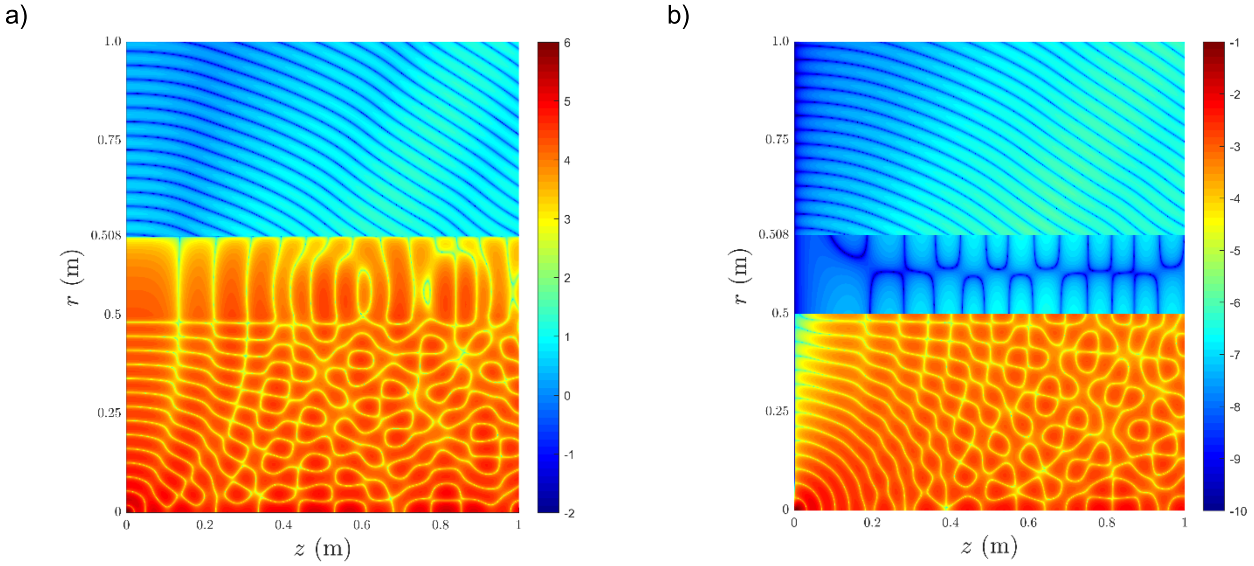

Figure 5: Plot of $\log_{10}(|Re\,[\hat{x}]|)$ for duct F at 4820 Hz, where (a) $\hat{x}$ is pressure and radial stress, (b) $\hat{x}$ is axial displacement.

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