

Residual Load Capacity and Modelling of Failure Mechanisms of Alkali-Silica Reaction Affected Concrete Structures

by Atila Sarikaya

Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy

under the supervision of
Dr. Nadarajah Gowripalan, principal supervisor
Dr. Jun Li, co-supervisor
Dr. Emre Erkmen, external supervisor.

University of Technology Sydney
Faculty of Engineering and IT

December 2021

Certificate of Authorship/Originality

I, Atila Sarikaya, declare that this thesis is submitted in fulfilment of the requirements for the award of doctor of philosophy, in the Faculty of Engineering and IT at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise reference or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

Signature: Production Note:
Signature removed prior to publication.

Date: 29.11.2021

Residual Load Capacity and Modelling of Failure Mechanisms of Alkali-Silica Reaction Affected Concrete Structures

Abstract

Alkali-silica reaction (ASR) in concrete can impair a structure's essential functions regarding strength and serviceability. Considering the number of structures suffering from ASR, it is of great importance to determine the residual load capacity of ASR-affected concrete structures.

In this study, a phenomenological model is developed to determine the mechanical behaviour of structural elements in the inelastic range. One of the key components of the model is the *coupled elastoplastic-damage constitutive model* (CEPD), which can capture permanent deformations and stiffness degradation that accumulate during inelastic processes. ASR effects, on the other hand, are represented as a combination of ASR-induced residual stresses and eigenstrains. Hence, mechanical stresses associated with the free expansion and ASR-induced internal pressures are considered simultaneously. The model is capable of representing the evolution of plasticity and damage variables under free ASR expansion conditions. In case external stresses are present (at macroscale), they interact with ASR-induced stresses and contribute to the evolution of inelastic variables.

One of the essential contributions in the present work is the *direct-coupling* method in which a prescribed relationship between the strain tensor components is defined. The method allows using a single set of variables. Hence, the number of parameters required for calibration is reduced. Computational efficiency of the method is also shown.

As another contribution, a three-surface composite yield function is developed for concrete. Accordingly, a *transition zone* yield surface defined between compression and tension yield surfaces is proposed. The presence and definition of the transition zone allows one to limit deviatoric stress. Therefore, it provides more accurate results than those obtained from the two-surfaced yield functions based on tension cut-off only. In addition, an explicit return-mapping algorithm is developed for the general multi-surface plasticity, and it is applied to the concrete plasticity computations.

Results obtained from the model are compared with the experimental data. Firstly, stress-strain diagrams for normal concrete (no ASR involved) are compared, and it was shown that results are very close under monotonic and cyclic loads for a large range of concrete strengths. Secondly,

load-displacement diagrams of ASR-affected beams are compared. Under significant ASR expansion conditions, the characteristic features such as degradation in stiffness and loss in load-carrying capacity are observed from the beam model results. On the other hand, for moderate degree of ASR expansion, the load-displacement relations of beams showed no significant difference, which is in agreement with physical experiments.

To my dear wife Cagla, who enlightens me with her love, generosity and passion.

To our lovely children Selim and Elif, the joy of our lives,

And to ‘Grandpa Afer’, who will always live in our hearts.

Acknowledgements

A PhD dissertation is indeed a piece that is formed by collaboration and support of many individuals who directly or indirectly contribute to it. This work is no exception.

I cannot appreciate enough Dr Emre Erkmen's influence on my life. Since we became classmates in our undergraduate studies, he has been one of my best friends. We always enjoyed studying together and having conversations on music, science, mathematics, philosophy and literature after tiresome school days. After graduation, we pursued different ways until our paths crossed again after many years, thanks to the ASR research project opened at UTS. Emre generously accepted being my PhD supervisor, and he provided everything that a PhD candidate needs throughout the research project. I feel privileged that we had one more chance in life to have the pleasure of working together.

I am also grateful to my principal supervisor Dr Nadarajah Gowripalan for providing an excellent research environment in which I could conduct my research freely while he was helping me keep my course in the right direction. The support and encouragement he provided throughout my studies is invaluable to me. I have also enjoyed our conversations on science, technology and history, which I find indispensable constituents of a PhD research. I am also thankful to my co-supervisor Dr Jun Li, who has always welcomed me and offered his help.

I would like to express my special thanks and gratitude to Prof Vute Sirivivatnanon, one of the founders of our 'ASR Research Team' in UTS for being our mentor, leading us researchers in difficult times, pursuing new opportunities for us and generously offering his support. It is a privilege for me to work with this distinguished person. Also, I would like to thank to Prof Jianchun Li who inspired me with his sincere ambition and rigour in research, showed a strong interest in our projects, followed our progress and provided valuable feedback throughout the last four years. I also take this opportunity to thank Prof Shami Nejadi and Kirk Vessalas for sharing their experience and knowledge with us.

Special thanks to the peers who conducted research with me in the ASR Research Hub, particularly to Dr Nhu Thuc Nguyen and Dr Jinsong Cao for their kindness and friendship they offered during this journey.

My sincere thanks to the PhD candidature assessment committee members, Dr Behzad Fatahi, Dr Sardar Malek, Dr Sanjay Nimbalkar, and Dr Mina Mortazavi for their feedback and support. I would also like to thank our kind school academic officer Van Le, who helped me whenever I needed. Also, special thanks to the thesis examiners who examined and prepared a detailed report that helped me identify ambiguities in my thesis and inspired me for perfection.

I would like to acknowledge the financial support of Australian Research Council Hub for Nanoscience Based Construction Materials Manufacturing (ARC-NANOCOMM Hub) and Cement, Concrete & Aggregates Australia (CCAA) for their financial support (IH150100006).

I am deeply grateful to my family and friends who encouraged and supported me prior to and during this journey.

My special thanks to Dr Serra-Cimilli Erkmen and Dr Emre Erkmen, who welcomed us in the warmest way; and shared their home, friendship and joy with us. Our kids grew up together which was lovely to witness. With you, it always felt like we're home.

My dear friend Ozgur Ozcikrikci, it would not be possible for me to step up for this journey without your encouragement and support. I am grateful for having a friend like you.

I would like to express my gratitude to my father Ekrem, my mother Bedriye, my brothers Nejdret and Osman, and my sister Ayse, who raised me in their best way. They supported me in every stage of my life and encouraged me to pursue my dreams.

Finally, to my dear wife Cagla and our lovely children Selim and Elif... It would not be possible for me to accomplish this challenge without your love. I am grateful to you for being so considerate during this endeavour. Your genuine kindness is a gift.

Atila Sarikaya

Sydney, Australia

List of Publications

Journal Paper

J-1. **Sarikaya, A.** & Erkmen, R.E. 2019, 'A plastic-damage model for concrete under compression', *International Journal of Mechanical Sciences*, vol. 150, pp. 584-93.
<https://doi.org/10.1016/j.ijmecsci.2018.10.042>

Conference Papers

C-1. **Sarikaya, A.**, Erkmen, R.E., Gowripalan, N., Sirivivatnanon, V. & South, W. 2019, 'A plastic-damage model for concrete under cyclic loads', paper presented to the Concrete 2019, Sydney, Australia, 2019/09/11.

C-2. **Sarikaya, A.**, Erkmen, R.E., Gowripalan, N., Sirivivatnanon, V. 'A plastic-damage model for concrete affected by alkali-silica reaction', 16th International Conference on Alkali-Aggregate Reaction in Concrete (ICAAR), Lisboa, Portugal, 2020-2022.

Please see Appendices G, H and I for details.

Contents

1. Introduction	1
1.1 Background	3
1.1.1 Alkali-Silica Reaction	3
1.1.2 Modelling of ASR-affected Concrete	6
1.2 Research Objectives, Significance and Scope of Works	8
1.2.1 Motivation	8
1.2.2 Objectives	9
1.2.3 Research Outcomes and its Significance	11
1.2.4 Scope of Works	13
1.3 Organisation of the Thesis	15
2. Literature Review	18
2.1 Introduction	19
2.2 Alkali-Silica Reaction in Concrete and Modelling Approaches	20
2.2.1 Main Factors that Influence ASR in Concrete	20
2.2.2 Degradation of Concrete due to ASR	25
2.2.3 Effects of ASR on Structures	29
2.2.4 Modelling of ASR	33
2.3 Macroscale Modelling of ASR Effects	35
2.3.1 Parametric Models	35
2.3.2 Chemomechanical Coupled Models	39
2.4 Coupled Plastic-Damage Constitutive Modelling of Concrete	59
2.4.1 Rate-Independent Plasticity – Preliminaries	60
2.4.2 Computational Plasticity	63
2.4.3 Plastic-Damage Coupling	85
2.4.4 Concrete Modelling	94
2.5 Conclusions Based on the Literature Review	108
2.5.1 Summary	108
2.5.2 Discussion on Research Gaps and the Proposed Methodology	109
3. Coupled Plastic-Damage Model	111
3.1 Introduction	112
3.2 Directly Coupled Plastic-Damage Model	112
3.2.1 Preliminaries: Continuum Mechanics and Linear Elasticity	112
3.2.2 Isotropic Continuum Damage Mechanics: Effective Stress Concept	117

3.2.3	Basic Hypotheses of Coupled Plastic-Damage Framework	119
3.2.4	Thermodynamic Considerations	123
3.2.5	Plasticity Model	125
3.2.6	Damage Model	129
3.2.7	Plastic-Damage Coupling.....	132
3.2.8	Direct Coupling.....	133
3.2.9	Remarks on the Coupled Plastic-Isotropic Damage Model	134
3.2.10	Integration Algorithm for Coupled Plasticity-Damage.....	136
3.3	Rate-Independent Multi-Surface Plasticity	150
3.3.1	Introduction.....	150
3.3.2	Koiter's Rule and the Linear Complementarity Problem	151
3.3.3	Plastic Multipliers and Uniqueness Conditions in Multi-Surface Plasticity 154	
3.3.4	Explicit Integration Algorithm for Multi-surface Plasticity.....	157
3.3.5	Plastic Multipliers and Return Mapping Zone Boundaries.....	162
3.3.6	Multi-Surface Return Mapping Algorithm Case Studies.....	166
3.3.7	Remarks on the Multi-Surface Plasticity Algorithm.....	188
3.4	Specifics Adopted for the Concrete Model	189
3.4.1	Characterisation of Compressive Response	190
3.4.2	Characterisation of Tensile Response	193
3.4.3	Compression-Tension Transition	194
3.4.4	A Discussion on Fracture Properties of Concrete	195
3.5	Validation of the Concrete Constitutive Model.....	196
3.5.1	Interpretation and Calibration of the Concrete Constitutive Model.....	196
3.5.2	Validation Examples	200
3.6	Specifics Adopted for the Reinforcing Steel Model.....	200
4.	Modelling of ASR-affected Concrete	202
4.1	Introduction	203
4.2	Constitutive model	203
4.3	Analysis Procedure.....	205
4.3.1	Stage 1: ASR Only	205
4.3.2	Stage 2: Application of External Loads	208
4.3.3	Interpretation of the Analysis Procedure.....	208
4.4	Validation of the Model with Experimental Data	209
4.4.1	Example 1: Material Behaviour	210

4.4.2	Example 2: Structural level (Swamy & Al-Asali 1989)	216
4.4.3	Example 3: Structural level (Fan & Hanson 1998).....	219
4.4.4	Example 4: Structural level (Allard et al. 2016)	221
4.4.5	Remarks based on the Structural Models.....	222
4.4.6	Some observations on the calibration process of control parameters	223
5.	Conclusions and Recommendations	225
5.1	Summary	226
5.2	Major Conclusions and Contributions to the Field	229
5.2.1	Concrete Modelling.....	229
5.2.2	ASR Modelling	230
5.3	Recommendations for Future Work	231
	References	232
A.	Plasticity Integration Algorithms	250
A.1	Closest-Point Projection Algorithm	251
A.1.1	Algorithmic Elastoplastic Tangent Modulus	253
A.2	Cutting-Plane Algorithm	255
B.	Haigh – Westergaard Stress Space and Stress Invariants	257
B.1	Introduction	258
B.2	Invariants of the Stress Tensor	258
B.3	Linear Elastic Stress-Strain Relations and Decomposition of Stress and Strain Tensors	261
B.4	Derivatives of a Scalar-Valued Function with respect to Lode Invariants.....	262
C.	Multi-Surface Plasticity Equations in Haigh-Westergaard Space	266
C.1	Introduction	267
C.2	Revision of the Linear Complementary Problem (LCP).....	267
C.3	LCP Equations in terms of Lode Invariants	267
C.3.1	A Matrix Entries.....	267
C.3.2	b Vector Entries	269
C.4	Explicit Return Mapping Algorithm in Haigh-Westergaard Space	270
D.	Yield, Plastic Potential and Hardening Function Derivatives with respect to Stress Tensor.....	272
D.1	Introduction	273
D.2	Concrete – Compression Zone Derivatives	273
D.2.1	Yield Function Derivatives	273
D.2.2	Plastic potential function derivatives	274

D.2.3	Hardening Function-Related Derivatives.....	275
D.3	Concrete – Transition Zone Derivatives.....	276
D.3.1	Yield Function Derivatives	276
D.3.2	Plastic Potential Derivatives	277
D.3.3	Hardening Function-Related Derivatives.....	277
D.4	Concrete – Tension Cut-off Zone Derivatives	277
D.4.1	Yield Function Derivatives	277
D.4.2	Plastic Potential Derivatives	278
D.4.3	Hardening Function-Related Derivatives.....	278
E.	Concrete Model Validation Examples	279
E.1	Introduction	280
E.2	Compression Model	280
E.2.1	Comparisons with Karsan & Jirsa (1969)	280
E.2.2	Comparisons with Dahl (1992)	283
E.2.3	Comparisons with Kupfer, Hilsdorf & Rusch (1969)	284
E.2.4	Comparisons with Imran (1994)	286
E.3	Tension Model.....	288
E.4	Compression-Tension Cycles.....	289
E.5	Structural Models	291
E.5.1	Comparisons with Fan & Hanson (1998) – Flexural Failure	291
E.5.2	Comparisons with Allard et al. (2016) – Shear Failure	293
F.	Finite Element Method.....	295
F.1	Introduction	296
F.2	Finite Element Method in Linear Elasticity: A Brief Review	296
F.2.1	Engineering Notation	296
F.2.2	Total Potential Energy Functional and its Minimisation	297
F.2.3	Spatial Discretisation and Discrete Equilibrium Equations	299
F.3	Nonlinear Analysis	301
F.3.1	Linearization	302
F.3.2	Newton-Raphson Procedure	303
F.3.3	Iterative Load Parameter	304
F.3.4	Load Control Method	306
F.3.5	Displacement Control Method	307
F.4	Linearization and Consistent Elastoplastic Tangent Stiffness	308
F.5	Finite Element Types Adopted for Concrete and Steel Modelling	309

G.	Published Paper 1: “A plastic-damage model for concrete under compression”..	310
H.	Published Paper 2: “A plastic-damage model for concrete under cyclic loads”...	321
I.	Published Paper 3: “A plastic-damage model for concrete affected by alkali-silica reaction”	329

List of Figures

Figure 1.1: Photograph of a thin section of concrete undergoing ASR (field of view ~ 5 mm) (Thomas, Fournier & Folliard 2013)	4
Figure 1.2: A road over-pass pier impacted by ASR (Blight & Alexander 2011).....	5
Figure 2.1. Normalised ASR expansion vs temperature (Saouma 2014)	21
Figure 2.2. ASR expansion vs moisture (Comi, Kirchmayr & Pignatelli 2012)	22
Figure 2.3. The effect of applied stresses to ASR expansion (Hobbs 1988)	22
Figure 2.4. Acceleration of lateral ASR expansion by applied load (Dunant & Scrivener 2012b)	23
Figure 2.5. Normalized static/dynamic elastic modulus - expansion data and fitting curve (Esposito et al. 2016).....	25
Figure 2.6. Normalized compression strength - expansion data and fitting curve (Esposito et al. 2016).....	26
Figure 2.7. Normalized tensile strengths (splitting, modulus of rupture, direct) - expansion data and fitting curves (Esposito et al. 2016).....	27
Figure 2.8. An interpretation of AAR-creep interaction (Saouma 2014)	28
Figure 2.9. Influence of reinforcement on ASR cracking (IStructE 1992).....	30
Figure 2.10. Example - Cracking follows the dominant stress routes (Blight & Alexander 2011)	31
Figure 2.11. Map cracking in massive column foundation block of a bridge structure (Thomas, Fournier & Folliard 2013).....	31
Figure 2.12. Fontana Dam - downstream view (left) and horizontal crack reaching through the inspection gallery (right) (Ingraffea 1990).....	32

Figure 2.13. Normalised AAR expansion vs AAR factors (Léger, Côté & Tinawi 1996)	37
Figure 2.14. CTMR-AAR simulation model (Léger, Côté & Tinawi 1996)	37
Figure 2.15 Mesoscopic mechanism of ASR swelling. (Ulm et al. 2000)	43
Figure 2.16. Chemoelastic pressure-spring device of ASR swelling. (Ulm et al. 2000)	44
Figure 2.17. Definition of latency time and characteristic time in normalized isothermal expansion curve. (Ulm et al. 2000)	45
Figure 2.18: Principal plastic strain vectors on a dam. Surface pop-outs. (Ulm et al. 2000)	46
Figure 2.19. (a) ASR extent (%) and, (b) plastic strains ($\mu\text{m}/\text{m}$) after 2.5 years. (Ulm et al. 2000)	47
Figure 2.20. a) Swelling pressure in the pore space. b) Constitutive model. (Ulm, Peterson & Lemarchand 2002)	48
Figure 2.21. ‘Graphical representation of normalized retardation parameters Γt and Γc ’ (Saouma 2014)	51
Figure 2.22. ‘Weight of volumetric AAR redistribution in selected cases’ (Saouma & Perotti 2006)	52
Figure 2.23. ‘Rheological scheme of the model’ (Morenon et al. 2017)	56
Figure 2.24. ‘Plastic ASR hardening law’ (Morenon et al. 2017)	57
Figure 2.25. Damage activation functions and elastic domain in the principal effective stress space. (Comi, Fedele & Perego 2009)	59
Figure 2.26. ‘Hyperbolic approximation to Mohr-Coulomb yield function’ (Abbo & Sloan 1995)	77
Figure 2.27. ‘Three-surface yield criterion’ (Krabbenhoft et al. 2007)	84

Figure 2.28. ‘A general yield surface consisting of multi regular yield surfaces and the critical regions predicting the rough position of updated stress’ (Meng et al. 2018).....	85
Figure 2.29. ‘Unloading response of elastic damage (a), elastic plastic (b) and elastic plastic damage (c) models’ (Jason et al. 2006)	86
Figure 2.30. ‘Schematic representation of the smeared crack model as an elastic unit coupled in series to a crack unit’ (Hofstetter & Meschke 2011).....	87
Figure 2.31. Decomposition of strains into elastic and plastic parts in the classical 1D plasticity	88
Figure 2.32. A coupled plasticity-damage model having a mechanism for ‘damage strains’	88
Figure 2.33. ‘Sketch of a uniaxial tension test of a material exhibiting elastic, plastic and damage strains, the latter consisting of the addition of two damage mechanisms, say the formation of cracks and voids’ (Armero & Oller 2000a).	89
Figure 2.34. ‘Elliptic function r ’ (Menetrey & Willam 1995)	97
Figure 2.35: ‘Three deviatoric sections of three-parameter concrete criterion for eccentricities: (a) $e = 0.5$; (b) $e = 0.6$ ’ (Menetrey & Willam 1995).....	97
Figure 2.36: ‘Split of the hardening function into a hardening and softening part’ (Grassl, Lundgren & Gylltoft 2002)	101
Figure 2.37: A stress-strain response comparison of CDPM1 and CDPM2 under tension-compression cyclic loading (Grassl et al. 2013)	103
Figure 2.38: Uniaxial stress-strain diagram of Lee & Fenves (1998) model.....	104
Figure 2.39: Intersection of Rankine and Menetrey–Willam failure surfaces (Červenka & Papanikolaou 2008).....	105
Figure 2.40: Triaxial stress-strain curves (Ahmed, Voyiadjis & Park 2020) – Comparison with Sarikaya & Erkmen (2019)	108
Figure 3.1: General three-dimensional solid body.....	113

Figure 3.2: Undeformed and deformed configurations.....	114
Figure 3.3: Uniaxially loaded solid: (a) damaged and (b) fictitious undamaged states.	117
Figure 3.4: Uniaxial stress-strain relations based on the strain equivalence hypothesis	118
Figure 3.5: Composition of strains – uniaxial case.....	120
Figure 3.6: Depiction of elastic and damage energy terms on a uniaxially loaded plastic- damage material	122
Figure 3.7: Interpretation of damage strain ε_d and effective stiffness E for the uniaxial case.....	135
Figure 3.8: Elastic domain in the stress space, bounded by a number of smooth yield surfaces.....	150
Figure 3.9: Possible cases in an elastoplastic setting – associative case.....	154
Figure 3.10: Corner zone, its boundaries, and single-surface zones.....	163
Figure 3.11. Constant yield surfaces with no hardening case.....	167
Figure 3.12. Single-step stress return (single-surface plasticity)	167
Figure 3.13. Single-step stress return to the corner.....	168
Figure 3.14. Linear yield surfaces and corner zone boundaries.....	168
Figure 3.15. A case where only one surface is active whilst both yield surfaces are positive.	169
Figure 3.16. Single-step stress return to the corner.....	170
Figure 3.17. Linear hardening – initial stage	171
Figure 3.18. Linear hardening – final stage	171

Figure 3.19. Linear hardening – stress return to corner – final stage.....	172
Figure 3.20. Linear softening – stress return to corner – final stage.....	172
Figure 3.21. Linear hardening – stress return to corner – initial stage.....	173
Figure 3.22. Linear hardening – stress return to corner – final stage.....	173
Figure 3.23. Narrowing down of corner zone due to softening	174
Figure 3.24. Nonlinear hardening – single-surface stress return: Error due to overshooting.....	174
Figure 3.25. Nonlinear hardening – multi-surface stress return: Error due to overshooting	176
Figure 3.26. Nonlinear hardening – multi-surface stress return – modified scheme	176
Figure 3.27. Non-isotropic hardening – Boundaries and zones at initial stage.....	177
Figure 3.28. Non-isotropic hardening – single-surface stress return: Error due to overshooting.....	178
Figure 3.29. Non-isotropic hardening – Evolution of elastic zone and corner zone.....	178
Figure 3.30. Non-isotropic hardening – multi-surface stress return: Error due to overshooting.....	179
Figure 3.31. Non-isotropic hardening – modified multi-surface stress return.....	179
Figure 3.32. Yield functions, elastic zone, corner zone and corner zone boundaries ...	180
Figure 3.33. Single-surface stress return onto the nonlinear yield function – no hardening	180
Figure 3.34. Single-surface stress return onto the linear yield function – no hardening	181
Figure 3.35. Multi-surface stress return – no hardening	181

Figure 3.36. Multi-surface stress return – no hardening: Stress returns onto the linear surface at first iteration.....	182
Figure 3.37. Multi-surface stress return –return vector components with different convergence rates	183
Figure 3.38. Multi-surface stress return – no hardening: Modified scheme	183
Figure 3.39. Yield functions, elastic zone, corner zone and corner zone boundaries ...	184
Figure 3.40. Single-surface stress return (first surface) – no hardening	184
Figure 3.41. Single-surface stress return (second surface) – no hardening	185
Figure 3.42. Multi-surface stress return – no hardening	185
Figure 3.43. Multi-surface stress return – no hardening: Modified scheme	186
Figure 3.44. Multi-surface general case –Zones and zone boundaries: Initial state	186
Figure 3.45. Multi-surface general case – Stress return with overshooting.....	187
Figure 3.46. Multi-surface general case – Modified stress return.....	187
Figure 3.47. Multi-surface general case –Zones and zone boundaries: Initial and final states.....	187
Figure 3.48: Three-surface composite yield surface – A Rendulic plane	189
Figure 3.49: Hardening and softening functions (compression zone)	192
Figure 3.50. Softening of the tension cut-off yield surface and accompanying transition zone yield surface.....	194
Figure 3.51: Compression model parameters on uniaxial compression stress-strain curve	197
Figure 3.52: Tension model parameters on uniaxial stress-strain curve.....	199
Figure 4.1: Rheological model with mesoscale and macroscale – uniaxial stress case	204

Figure 4.2: Interpretation of coupled elastoplastic-damage model – uniaxial case	204
Figure 4.3: Uniaxial model with no ASR: The effective stress is equal to the applied stress.	209
Figure 4.4: Uniaxial model with no external load (free ASR expansion).....	209
Figure 4.5: Evolution of (a) axial stress, (b) plasticity parameter and (c) damage variable under uniaxial compression loads (no-ASR)	211
Figure 4.6: Stage-1 (free ASR expansion) stress-strain curve: no stress is generated..	212
Figure 4.7: Stage-1 analysis hardening variable evolutions – (a) no coupling, (b) coupled.	213
Figure 4.8: Stage-1 analysis damage variable evolutions – (a) no coupling, (b) coupled	213
Figure 4.9: Stress-strain curves of coupled and uncoupled cases (Stage-2) No-ASR case	214
Figure 4.10: Compression hardening variable of coupled and uncoupled cases (Stage-2) compared with the No-ASR case	214
Figure 4.11: Compression damage variable of coupled and uncoupled cases (Stage-2) compared with the No-ASR case	215
Figure 4.12: Effect of ASR on the uniaxial compressive stress-strain behaviour of concrete	216
Figure 4.13: Beam experiment details (Swamy & Al-Asali 1989).....	217
Figure 4.14: Example 2 – Stage-1 deflections – hogging under ASR expansion (half beam).....	217
Figure 4.15: Stage-2 deflections – sagging due to external loads (half beam)	218
Figure 4.16: Load-deflection response of the beam: Comparison with Swamy & Al-Asali (1989).....	218

Figure 4.17: Beam experiment details (Fan & Hanson 1998)	219
Figure 4.18: Stage-1 deflections – hogging due to ASR expansion (half beam).....	220
Figure 4.19: Load-deflection response of the beam: Comparison with Fan & Hanson (1998)	220
Figure 4.20: Beam experiment details (Allard et al. 2016).....	221
Figure 4.21: Load-deflection response of the beam: Comparison with Allard et al. (2016)	222
Figure B.1: Unified coordinates of Haigh-Westergaard stress space	260
Figure B.2: Unit vectors in deviatoric and volumetric planes.....	264
Figure E.1: Monotonic uniaxial compressive loading – Experiments: Karsan & Jirsa (1969).....	281
Figure E.2: Cyclic uniaxial compressive loading – Experiments: Karsan & Jirsa (1969)	282
Figure E.3: Damage density evolution – cyclic loading experiment of Karsan & Jirsa (1969) (model 2)	282
Figure E.4: Uniaxial compressive behaviour of normal and high strength concretes – Experiments: Dahl (1992).....	283
Figure E.5: Lateral and axial strains under uniaxial compressive loading – Experiments: (Kupfer, Hilsdorf & Rusch 1969)	284
Figure E.6: Lateral and axial strains under equibiaxial compressive loading – Experiments: (Kupfer, Hilsdorf & Rusch 1969).....	285
Figure E.7: Lateral and axial strains under equibiaxial compressive loading – Experiments: (Kupfer, Hilsdorf & Rusch 1969) – Loss of accuracy with linear potential function	286

Figure E.8: Lateral and axial strains under triaxial compressive loading with constant confinement pressures – Experiments: Imran (1994), normal concrete ($w/c=0.55$).....	287
Figure E.9: Lateral and axial strains under cyclic compressive loading with constant confinement pressure – Experiments: Imran (1994), normal concrete ($w/c=0.55$)	287
Figure E.10: Monotonic uniaxial tensile loading – Experiments: Gopalaratnam & Shah (1985).....	288
Figure E.11: Cyclic uniaxial tensile loading – Experiments: Gopalaratnam & Shah (1985)	289
Figure E.12: Uniaxial tension-compression cyclic loading – comparison with Lee & Fenves (1998).....	290
Figure E.13: A closer view of the tension-compression transition	290
Figure E.14: Beam experiment details (Fan & Hanson 1998).....	292
Figure E.15: Comparison of model results with the experiments of Fan & Hanson (1998)	292
Figure E.16: Beam experiment details (Allard et al. 2016)	293
Figure E.17: Comparison of model results with the experiments of Allard et al. (2016)	294
Figure F.1: A three-dimensional body and its discretisation via finite elements.....	296
Figure F.2: Load control algorithm	307
Figure F.3: Displacement control algorithm	308

List of Tables

Table 2.1. Lower bound residual mechanical properties as percentage of values for unaffected concrete at 28-day (IStructE 1992)	29
Table 2.2. Overview of modelling approaches for ASR in concrete (Esposito & Hendriks 2017)	34
Table 2.3: Boundary Value Problem (BVP) of Rate-Independent Plasticity – Local Form	63
Table 2.4: Incremental solution of BVP – Fully implicit case.....	69
Table 2.5: Fully implicit stress return mapping algorithm.....	71
Table 3.1: Rate and incremental forms of LCP terms.....	162
Table 4.1: Example 1 concrete parameters	210
Table 4.2: Example 2 – Concrete parameters	217
Table 4.3: Example 2 – Steel parameters.....	217
Table 4.4: Example 3 – Concrete parameters	219
Table 4.5: Example 3 – Steel parameters.....	219
Table 4.6: Example 4 – Concrete model parameters adjusted for structural model comparisons	221
Table 4.7: Example 4 – Steel model parameters adjusted for structural model comparisons	221
Table E.1: Adjusted model parameters for Karsan & Jirsa (1969) experiments	281
Table E.2: Adjusted model parameters for Dahl (1992) experiments	283
Table E.3: Adjusted model parameters for Kupfer, Hilsdorf & Rusch (1969) experiments	284

Table E.4: Adjusted model parameters for Imran (1994) experiments.....	286
Table E.5: Adjusted model parameters for Dahl (1992) experiments	288
Table E.6: Adjusted model parameters for (Lee & Fenves 1998) comparisons	289
Table E.7: Concrete model parameters adjusted for structural model comparisons.....	291
Table E.8: Steel model parameters adjusted for structural model comparisons	291

Abbreviations

AAR	Alkali-Aggregate Reaction
ASR	Alkali-Silica Reaction
BVP	Boundary Value Problem
CEPD	Coupled Elasto-Plastic-Damage (model)
CP	Complementarity Problem
CPA	Generalised Cutting Plane Algorithm
CPPA	Closest Point Projection Algorithm
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
KKT	Karush-Kuhn-Tucker (conditions)
LCP	Linear complementarity problem
NR	Newton-Raphson (technique)