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Minor or adult? Introducing decision analysis in forensic age estimation

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ABSTRACT

Nowadays, forensic age estimation takes an important role in worldwide forensic and medico-legal institutes that are solicited by judicial or administrative authorities for providing an expert report on the age of individuals. The authorities' ultimate issue of interest is often the probability that the person is younger or older than a given age threshold, which is usually the age of majority. Such information is fundamental for deciding whether a person being judged falls under the legal category of an adult. This is a decision that may have important consequences for the individual, depending on the legal framework in which the decision is made. The aim of this paper is to introduce a normative approach for assisting the authority in the decision-making process given knowledge from available findings reported by means of probabilities. The normative approach proposed here has been acknowledged in the forensic framework, and represents a promising structure for reasoning that can support the decision-making process in forensic age estimation. The paper introduces the fundamental elements of decision theory applied to the specific case of age estimation, and provides some examples to illustrate its practical application.

1. Introduction

This contribution is based on two main standpoints. The first one is that uncertainty on events of forensic interest should be measured by probabilities. The second one is that the age estimation process, intended as the application of medical (scientific) methods for estimating the chronological age of an individual, is a recognized forensic discipline. The conjunction of these two statements leads to a third one, namely, that uncertainty on forensic age estimation findings should logically be measured by probabilities.

The logical foundation for applying probability theory to the evaluation and interpretation of forensic evidence is nowadays generally recognized (see e.g., [1,3,4]), at the point that the guidelines for evaluative reporting published by the ENFSI [5, p. 6] clearly state that "[…]Evaluation of forensic science findings in court uses [probability] as a measure of uncertainty […]". In particular, the Bayesian framework plays a relevant role in the field, since it offers rules to coherently handle the probabilities in the inferential reasoning processes typical of evidence evaluation [6].

Age estimation is a discipline which has relevantly grown in interest in the past few decades [7–9], due to the increased facilitation on world travelling, the professionalization of the criminal organizations involved in human trafficking or smuggling [10]. It has been argued that age estimation examinations should be approached with a holistic perspective, especially in sensitive casework [11]; nonetheless, it is unquestionable that an estimation based on scientific methods offers a fundamental contribution in terms of accurateness and precision of the estimation. In this optic, the recommendations of the Study Group on Forensic Age Diagnostics (AGFAD) provide a coherent framework for practitioners to use [12]. Given that the term "forensic" refers to the application of natural sciences to legal matters [13], the application of medical methods for age estimation is naturally a forensic discipline since such scientific methodology is applied to provide information of legal interest, such as the chronological age[1] of an individual or, most frequently, an evaluation of the possibility that a person of interest is older or younger than a specific age threshold, such as the age of majority [7]. Hence, uncertainty in forensic age estimation should also be assessed by means of probabilities, similarly to other forensic

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[1] The chronological age is defined as the time, measured in days, months and years elapsed from the birth of a person until a specific moment in the timeline (e.g., the time of the examination) [11]. It is not to be confused with the so-called biological age, which refers to a punctual step of the development process of an individual [14].

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disciplines [15]. Nonetheless, this contribution does not cover issues related to how to assign probabilities related to available findings, since this topic has already been largely discussed in the literature (see e.g., [15–21]). Instead, it focuses on how to use these probabilities to make a coherent decision about the possibility that an examined person is an adult or a minor within the meaning of the law. In particular, the aim of this paper is to introduce a normative approach for the decision problem of age estimation. This approach follows the normative theory of decision making based on the principle of maximizing the expected utility (or minimizing the expected loss), which combines, in a mathematical function, both the quantification of the desirability (or undesirability) of decision outcomes, expressed in terms of utilities (or losses), and the uncertainty about unknown states of nature (e.g. the hypotheses of interest) expressed by probabilities. Following this approach, the decision-maker is able to choose the optimal decision among the available ones. This normative approach is well known in forensic science and in the legal framework [22], and was explored to formalize the decision-making process for a large panoply of decision problems encountered by forensic scientists. These include the opportunity of performing a test in kinship determination [23], the number of loci to be analyzed in a DNA test [24], the individualization of a person of interest [25–27], the individualization of a potential source of a stain through a database search [28], the analysis of low-template DNA specimen [29,30], or the decision of processing a fingermark [31]. A decision perspective for statistical inference in forensic science applications is presented in Taroni, et al. [2]. This article focuses on the perspective of the mandating authority in order to illustrate how the choice of a normative approach may allow the decision-maker to make a rational decision in declaring an individual adult or minor within the meaning of the law. The validity of the proposed normative approach for judicial or legal decisions has already been widely discussed (see e.g., [32–35]), thus its application in the context of age estimation problems follows logically as a further step. After a brief overview of the theoretical basis of the normative approach to decision-making, the paper discusses some specific decision problems linked with the field of age estimation, and suggests ah-hoc decision models for solving such problems. Note that some introductive issues with regard to the age estimation of living persons decision problem were presented by Sironi, et al. [36]. This paper is structured as follows: Section 2 presents and discusses existing decision criterion proposed in the age estimation domain, whilst Section 3 illustrates the general principles of decision theory. Sections 4 and 5 apply the proposed normative approach to two scenarios related to the problem of age estimation. Some case studies are presented in Section 6. The paper ends with a discussion and a conclusion in Sections 7 and 8, respectively.

2. Decision-making models for age estimation

The aim of this section is to introduce and comments on some of the existing decision criteria for deciding whether an individual is legally an adult or a minor, or whether an individual is older or younger than 18 years old[2].

2.1. Cut-off criteria

The cut-off criteria proposed in the literature are characterized by either a qualitative or quantitative value related to the evidence considered (i.e., the developmental status of a given examined physical indicator of age) [37,38] or a probability value assigned to a given hypothesis for discriminating between adults and minors (or between individuals older than and younger than the relevant age threshold) [21,39]. In the first case, researches seek a given developmental stage or a combination of developmental stages that allows one to reasonably consider an individual as, for instance, older than 18 years old. An intuitive (though unrealistic) example of this kind of cut-off criteria is the following “if all the wisdom teeth of the examined person have completed their development, then the person can reasonably be considered an adult”.

In the second case, suppose a probability value on the event that the examined person is older than, say, 18 years old is assigned. In a cut-off perspective, scientists seek to find a cut-off value for this probability (say, for example, 0.90) above which the person can be designated an adult. The core idea of this cut-off approach is to identify a limit allowing one to minimize the rate of individuals who are erroneously classified as adults or minors. Note that in the age estimation domain, the first kind of error is undoubtedly the worst scenario [40], thus a cut-off that privileges the minimization of false adult conclusions is to be preferred, even at the cost of an increased rate of false minor conclusions [37]. The cut-off is however solely defined on the basis of the available background data, while information at the mandating authority’s disposal is not taken into account. The decision-maker may dispose of other elements that may play a relevant role in the inferential and decisional processes. Being a standard value for all cases, a fixed cut-off does not allow the decision-maker to evaluate the specificities of the individual case at hand. This represents a limitation, as the cut-off is defined by the scientist on the sole basis of experimental studies, while it is the mandating authority who is in charge of the decision-making process. In a judicial framework, however, the decision-maker ought to have the instruments for reasoning about a specific case and interpreting all available evidence or findings in a flexible, although preferably also in a structured, way. A critical discussion on the use of cut-off values in forensic science has been presented by Robertson and Vignaux [41] and Biedermann, et al. [42].

2.2. Descriptive approaches to decision-making

Cunha, et al. [9] and Baccino, et al. [43] described the issues of a study presented by Polo Grillo, et al. [44] conducted on 47 judicial cases in which the decision-makers were judges. All considered cases involved young migrants lacking valid ID documents and who were therefore subject to an age estimation appraisal to evaluate whether they had reached the age threshold of 18. The analysis of the reports during the study showed that the judges appreciated when uncertainty on the conclusions was expressed in terms of probabilities, because they felt it was easier to understand the risk they were taking when, for instance, declaring a person younger than 18 an adult. Interestingly, results of the study indicate that the judges felt “confident” to pronounce an “adult” verdict when the probability that the examined individual is older than 18 was over 0.70.

This study promotes a so-called descriptive approach to decision-making that seeks to decrypt by empirical observations the mechanism that underlines a decision-making process [45]. However, such empirical studies do not actually provide elements for establishing a decision theory *per se*, since they solely focus on observing the behaviour of individuals who make decisions without taking into account the foundational elements of decision theory. In view of the forensic and judicial framework, a normative approach is more appropriate for setting a reference standard that on the one hand, can assist the decision-maker in the relevant decision-making process, and, on the other hand, makes it possible to analyse and compare conclusions in similar cases.

3. A normative approach to decision-making

According to a large number of scholars, the normative approach to decision-making is probabilistic in nature and focuses on the principle

[2] We make a clear distinction between the fact that a person is older or younger than 18, and the declaration of him or her being an adult or a minor according to the law. In our view, the first one refers to a measure of the quantity known as chronological age, whilst the second one pertains to a judicial judgement. This aspect is discussed in more detail in section 4.

of maximizing the expected utility (e.g. [32,46–48]). Lindley [49] states that this principle is the only one that ensures coherence (or rationality) in decision-making.

The key element of the normative approach to decision making are described in what follows.

3.1. Basic concepts and notation

Any decision-making problem can be decrypted by defining three main components that interact with each other. These components are:

- A collection of n *states of nature*, also referred to as *events*, about which the decision maker is uncertain. The states of nature represent all the real world states that are at the basis of the decision making process; they can represent the hypotheses of interest in a given scenario. They will be denoted as $\theta_1, \theta_2, \ldots, \theta_n$, and the space of all possible states of nature or events will be denoted by Θ. This list is exhaustive, and the uncertainty on these events is measured by means of probabilities, therefore, a set of probabilities $\Pr(\theta_1)$, $\Pr(\theta_2)$, …, $\Pr(\theta_n)$ must be assigned, with $\sum_{j=1}^{n} \Pr(\theta_j) = 1$.
- A collection of m available *decisions*, also referred to as *courses of action*. Decisions are usually denoted by $d_1, d_2, \ldots, d_m$, while D denotes the set of all available decisions. The collection of decisions must be exclusive and exhaustive: this means that the decision-maker must choose one and only one of the listed decisions, and that every possible decision must be included in D. It follows that if a given scenario includes the possibility that the decision maker decides to not decide, the "no decision" possibility must be one of the m available courses of action [50].
- A collection $\mathscr{C}$ of $m \times n$ possible consequences, denoted as $C(d_i, \theta_j)$ or simply C_{ij}, $i = 1, \cdots m$ and $j = 1, \cdots, n$. The consequence C_{ij} occurs when decision d_i is made and state of nature θ_j holds.

To each consequence C_{ij}, one can associate an expression of its desirability. This desirability is generally measured in terms of *utility* and is denoted as $u(d_i, \theta_j) = u(C_{ij})$ for $i = 1, \cdots m$ and $j = 1, \cdots, n$. The utility $u(C_{ij})$ quantifies the desirability of incurring a particular consequence when decision d_i is taken and the state of nature θ_j holds. In some scenarios, it may be preferable to reason about the undesirability of a given consequence. This can be measured in terms of loss, and is denoted by $l(d_i, \theta_j) = l(C_{ij})$. The loss $l(C_{ij})$ can be interpreted analogously.

3.2. Measuring the desirability of decision consequences

The basic notions of decision theory summarized in the current section and in the following one are practice-oriented: the theory is therefore presented in a simplified form. A more extensive discussion about utility theory can be found in Lindley [50], Berger [51], Gittelson [48] and Taroni, et al. [2] with specific reference to forensic science applications.

As aforementioned, utility measures the desirability of a consequence according to the decision maker's personal objectives and preferences. Let $u(\cdot)$ be a measurable function mapping decision consequences $\mathscr{C}$ into the set of real numbers $\mathbb{R}$, $u: \mathscr{C} \to \mathbb{R}$. The construction of such a function requires a supplementary effort, and different strategies can be implemented. Taroni, et al. [2] provide a wide description with reference to forensic science applications. In this paper we consider an approach that goes from general to specific considerations, starting by reasoning from a qualitative perspective and, in a second step, focusing on the quantitative assignments of loss values [26]. Let us consider three consequences, say C_1, C_2, C_3 among the possible consequences in space $\mathscr{C}$ for a hypothetical scenario. Suppose that the decision maker is able to order the three consequences from the least desirable (say, C_1) to the most desirable (say, C_3) according to his or her own preferences. These two consequences can therefore be defined respectively as the worst and the best consequences, according to the decision-maker's preferences. Consequence C_2 is an intermediate consequence. Formally, this preference ordering can be expressed as follows:

$$C_3 \succ C_2 \succ C_1, \tag{1}$$

where the symbol $\succ$ indicates that the consequence on the left is strictly preferred over the consequence on the right.

This preference ordering will represent the starting point for the construction of the utility function that must reflect the decision maker's preferences among possible decision outcomes. In particular, it may be shown that a utility function reflecting the preference ordering in Eq. (1) must be such that

$$u(C_3) > u(C_2) > u(C_1) \tag{2}$$

that is, the utility values for such consequences must reflect the same order when arranged from the largest to the smallest [23].

Various strategies can be implemented for the construction of the utility function. Following ideas discussed e.g. by [50–52], and afterwards suggested by Taroni, et al. [23,24] with reference to forensic scenarios, one strategy consists in interpreting the utility as a probability. In particular, a [0,1] scale can be considered, where the value 0 is assigned to the worst consequence and the value 1 is assigned to the best consequence. Formally, for the hypothetical scenario under discussion, it results $u(C_1) = 0$ and $u(C_3) = 1$. In order to assign the utility for the intermediate consequence C_2, it can be favorably compared with the best consequence C_3 and unfavourably compared with the worst consequence C_1. Based on this qualitative comparison, it suffices to find a unique value, say $u(C_2) \in (0, 1)$, such that C_2 is just as desirable as obtaining C_3 with probability $u(C_2)$ and C_1 with probability $1 - u(C_2)$ [23,24][3]. Note that this strategy can be generalized for any intermediate consequence C_{ij} [27]: the utility $u(C_{ij})$ can be expressed as the probability of obtaining the best consequence given a specific decision d_i and a specific state of nature θ_j, formally $\Pr(C_3 | d_i, \theta_j) = u(C_{ij})$ [23]. For a more extended discussion on the theoretical basis of the approach of interpreting utility as probability see DeGroot [53].

The next step is to assign the value of this probability $\Pr(C_3 | d_i, \theta_j)$. Building a utility function can be a difficult task. Here again, various strategies are available for the decision-maker. An interesting discussion is provided by Biedermann, et al. [26]. For the purpose of this article, this aspect will not be inspected further; however, an intuitive strategy for quantifying the desirability of an intermediate consequence (in terms of losses) is presented in Section 4.5. It is solely noted that the assignment of the utilities for the intermediate consequences is a subjective (meaning personal, not arbitrary) choice of the decision-maker. In fact, it is extremely important that all the assigned utility values must be coherent with each other. A detailed discussion and examples are presented in Berger [51] and Biedermann et al. [27].

3.3. Measuring the undesirability of decision consequences

In some situations it would be preferable to evaluate the consequences of a decision in terms of losses rather than utilities or gains [50]. Formally, the loss of consequence C_{ij} can be defined as the difference between the utility of the most favorable consequence and the utility of consequence C_{ij} that is actually incurred, thus [51]:

$$l(C_{ij}) = \max_j [u(C_{ij})] - u(C_{ij}). \tag{3}$$

Note that this is equivalent to assigning a loss equal to 0 to the most favourable consequence (e.g., C_3), and a loss equal to 1 to the least

[3] For instance, if the decision-maker assigns the value of 0.75 to $u(C_2)$, this means that he or she is indifferent between obtaining the intermediate consequence C_2 fur sure and obtaining the best consequence C_3 with probability 0.75 and the worst consequence C_1 with probability 0.25.

favourable consequence (e.g., C_1). Since losses and utilities are strictly related, the strategy described above for assessing the desirability of the intermediate consequence (e.g., C_2) is still valid. In this case, the decision-maker has to look for the value $l(C_2) \in (0, 1)$ such that C_2 is just as desirable as obtaining the best consequence C_3 with probability $1 - l(C_2)$ and the worst consequence C_1 with probability $l(C_2)$. Again, this strategy can be applied for any intermediate consequence C_{ij}. As mentioned above, Section 4.5 presents an alternative way to assign the loss value for an intermediate consequence.

3.4. Criterion for a rational decision

Under the normative approach considered in this paper, the decision maker is asked to choose one of the possible decisions $d_1, d_2, \ldots, d_m$ based on all available knowledge on the possible states of nature $\theta_1, \theta_2, \ldots, \theta_n$, as well as on the desirability/undesirability of the decision consequences C_{ij} that is expressed in terms of utilities/losses. All these elements can be combined mathematically in a function called expected utility for every decision d_i:

$$\bar{u}(d_i) = \sum_{j=1}^{n} u(C_{ij}) \times \Pr(\theta_j), \tag{4}$$

or expected loss:

$$\bar{l}(d_i) = \sum_{j=1}^{n} l(C_{ij}) \times \Pr(\theta_j). \tag{5}$$

The optimal (most rational) decision is the one that maximizes the expected utility, or minimizes the expected loss, respectively. Formally, this is written as

$$\underset{i}{argmax}\ \bar{u}(d_i) = \underset{i}{argmax} \sum_{j=1}^{n} u(C_{ij}) \times \Pr(\theta_j) \tag{6}$$

or

$$\underset{i}{argmin}\ \bar{l}(d_i) = \underset{i}{argmin} \sum_{j=1}^{n} l(C_{ij}) \times \Pr(\theta_j) \tag{7}$$

respectively. This is often referred to as the *Bayes decision* or *Bayes action.*

4. A rational decision for age estimation

4.1. Basic components

The scope of this paper is to introduce and promote the use of a normative approach to assist the decision-maker when asked to decide whether an examined person is legally an adult or a child.

The three main components characterizing the decision problem described in Section 3.1, are now explained for the scenario under consideration. First, one must list the states of nature that refer to the events that the examined person would be older or younger than the legal threshold of interest, namely:

- θ_1: the examined person is 18 years old or older;
- θ_2: the examined person is younger than 18 years of age.

Note that in this scenario, the probabilities of these events, i.e., $\Pr(\theta_1)$ and $\Pr(\theta_2)$, are those reported in the hypothetical expert's report. Note also that the two states of nature are mutually exhaustive and exclusive, and so the two probabilities of interest are complementary, that is $\Pr(\theta_2) = 1 - \Pr(\theta_1)$.

Second, one must list the available decisions, say the choice of declaring an individual an adult or a minor, on the basis of the legal definition. The decisions can be formulated as follows:

Table 1
A decision matrix for the age estimation problem described in Section 4.1, including the decisions, the states of nature and the associated consequences.

	State of nature	Age ≥ 18 (θ_1)	Age < 18 (θ_2)
Decision	Adult (d_1)	Correct adult (C_{11})	Incorrect adult (C_{12})
	Minor (d_2)	Incorrect minor (C_{21})	Correct minor (C_{22})

- d_1: to declare the examined person an adult within the context of the law;
- d_2: to declare the examined person a minor within the context of the law.

Third, the combination of the two states of nature and the two decisions leads to four possible consequences, namely:

- $C(d_1, \theta_1) = C_{11}$: the examined person is correctly declared an adult within the context of the law;
- $C(d_1, \theta_2) = C_{12}$: the examined person is erroneously declared an adult within the context of the law;
- $C(d_2, \theta_1) = C_{21}$: the examined person is erroneously declared a minor within the context of the law;
- $C(d_2, \theta_2) = C_{22}$: the examined person is correctly declared a minor within the context of the law.

All the main components characterizing the decision problem for the age estimation scenario are summarized in Table 1.

4.2. Building a loss function for the age estimation scenario

The first step for measuring the desirability of the possible consequences is to order them from the most desirable to the least desirable. In the present scenario, the best (most desirable) consequence is logically the one of a correct classification of the examined person, no matter whether this amounts to correctly declaring an individual an adult or a minor. Thus, consequences C_{11} and C_{22} are both at the rank of the best consequence. The worst (least desirable) consequence is incurred when an individual is erroneously declared an adult (i.e., C_{12}). This one implies that an actual minor is deprived of his or her fundamental childhood rights, which is not admissible, to the point that this kind of error has been defined ethically unacceptable [40]. The last consequence, C_{21}, also implies that a person is attributed an incorrect legal category. However, in general, this generates favourable conditions for the person, since criminal or administrative procedures are generally benevolent for minors. For example, applications of minor asylum seekers must be evaluated under child-sensitive procedural safeguards and special protective measures [11]. Although favourable to the examined person, such special measures also involve social expenses that would not have been incurred if the person had correctly been declared an adult, yet the desirability of this latter consequence can certainly be considered higher than the one characterizing the worst consequence C_{12}. Nonetheless, it is logically less desirable than the best consequences C_{11} and C_{22}. Consequence C_{21} is therefore an intermediate consequence. The following preference ordering is therefore proposed:

$$C_{11} \sim C_{22} \succ C_{21} \succ C_{12}, \tag{8}$$

where the symbol ~ indicates that the two consequences are equally desirable. Quantifying the desirability of alternative or various consequences in terms of losses is felt as more appropriate in the present case. It appears in fact more intuitive to reason about the loss of erroneously declaring a person a minor or an adult, than to think about the gain associated with a correct declaration of a given legal category. As illustrated in Section 3.2, a [0,1] scale is adopted for the loss function. Thus, a value equal to 1 is assigned to the loss associated with the worst

Table 2
Loss function for the two-action age estimation problem described in Table 1.

	State of nature	Age ≥ 18 (θ_1)	Age < 18 (θ_2)
Decision	Adult (d_1)	$l(C_{11}) = 0$	$l(C_{12}) = 1$
	Minor (d_2)	$l(C_{21}) = l_{21}$	$l(C_{22}) = 0$

consequence C_{12}, i.e., $l(C_{12}) = 1$, whilst the value of 0 is assigned to the losses associated with the preferred consequences, i.e., $l(C_{11}) = l(C_{22}) = 0$. This reflects the view that no loss is incurred when the examined individual is classified in the correct legal category (from the perspective of the decision-maker). The loss for the intermediate consequence C_{21} must take a value between 0 and 1, say $l(C_{21}) = l_{21}$. The loss function for the current scenario is sketched out in Table 2. Note that this particular loss function is generally referred to as a « $0 - l_{ij}$ » loss function [2], and it is particularly suitable for a two-action decision problem, like the one at hand.

The value of l_{21} can be assigned following the strategy described in Section 3.3, or in Section 4.5.

4.3. Minimizing the expected loss

Once the main elements characterizing the decision problem have been defined and the losses assigned, the expected losses for all the possible decisions can be formulated as shown in Eq. (5). The expected losses of possible decisions can be quantified as

$$\bar{l}(d_1) = \sum_{j=1}^{2} l(C_{1j}) \times \Pr(\theta_j) = \underbrace{l(C_{11})}_{0} \times \Pr(\theta_1) + \underbrace{l(C_{12})}_{1} \times \Pr(\theta_2) = \Pr(\theta_2) = 1 - \Pr(\theta_1) \quad (9)$$

for d_1, and

$$\bar{l}(d_2) = \sum_{j=1}^{2} l(C_{2j}) \times \Pr(\theta_j) = \underbrace{l(C_{21})}_{l_{21}} \times \Pr(\theta_1) + \underbrace{l(C_{22})}_{0} \times \Pr(\theta_2) = l_{21} \times \Pr(\theta_1) \quad (10)$$

for d_2.

According to the principle of minimizing the expected loss, decision d_1 turns out to be preferred to decision d_2 when $\bar{l}(d_1) < \bar{l}(d_2)$, thus when

$$1 - \Pr(\theta_1) < l_{21} \times \Pr(\theta_1) \Longleftrightarrow \Pr(\theta_1) > \frac{1}{l_{21} + 1}. \quad (11)$$

If the inequality in Eq. (11) does not hold, that is if $\bar{l}(d_1) > \bar{l}(d_2)$, the preferred decision will be d_2.

To better understand the criterion formalized in Eq. (11), Fig. 1 illustrates the expected losses of decision d_1 (solid line) and decision d_2 (dashed line) for values of $\Pr(\theta_1)$ ranging from 0 to 1, and for $l_{21} = 0.25$ (Fig. 1a) and $l_{21} = 0.50$ (Fig. 1b). The optimal decision is the one for which, for a given value of $\Pr(\theta_1)$ and a given choice of l_{21}, the expected loss is the lowest. This is highlighted in Fig. 1 by a bold line. Note that, from Eq. (11), the threshold value of $\Pr(\theta_1)$ that switches the preference for one decision to the other is 0.80 when $l_{21} = 0.25$ and 0.67 when $l_{21} = 0.50$.

Fig. 1 perfectly illustrates how lowering the loss value l_{21} increases the range of values of $\Pr(\theta_1)$ that make d_2 the optimal decision. This is entirely justifiable according to the proposed decision model and its preference structure: when the undesirability of the intermediate consequence is considerably lower than that of the worst one, then the probability that the examined individual is an adult, $\Pr(\theta_1)$, must be rather high in order to declare d_1 as the optimal decision. Conversely, when the undesirability of the intermediate consequence is close to that of the worst consequence, a more moderate value of $\Pr(\theta_1)$ suffices for d_1 to be the optimal decision. In fact, if the decision-maker considers a wrong declaration for an actual adult as almost equally detrimental as a wrong declaration for an actual minor, then the decision model shows that the risks provided by choosing either decision over the other are similar.

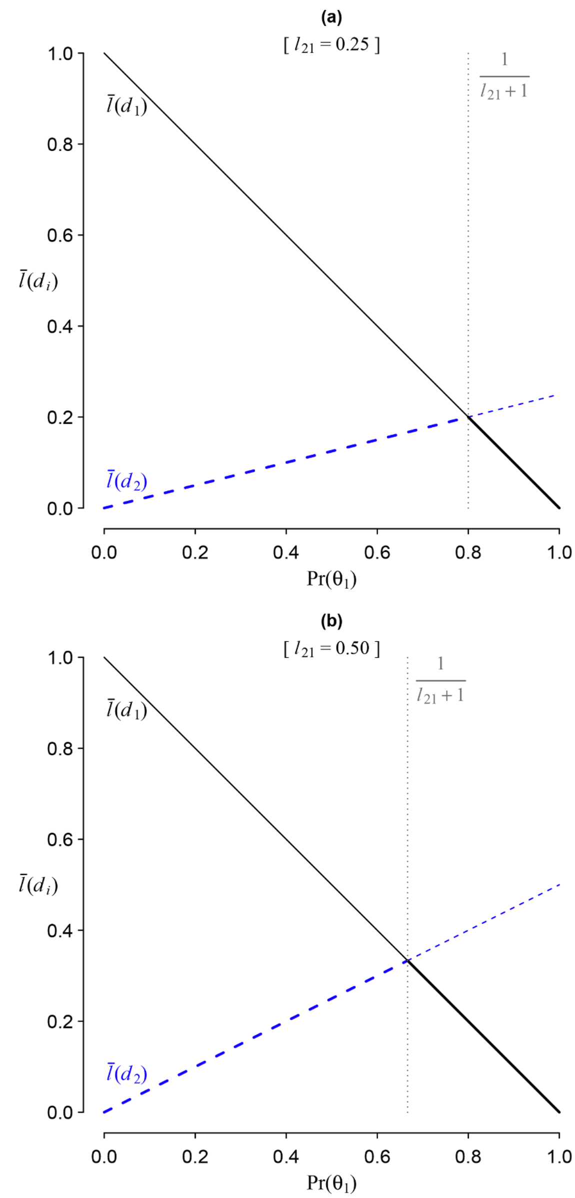

Fig. 1. Expected losses $\bar{l}(d_1)$ (solid line) and $\bar{l}(d_2)$ (dashed line) for values of $\Pr(\theta_1)$ ranging from 0 to 1, with $l_{21} = 0.25$ (a) and $l_{21} = 0.50$ (b). The dotted vertical segments indicate the threshold value of $\Pr(\theta_1)$ that switches the preference between the two decisions, respectively $\Pr(\theta_1) = 0.80$ (a) and $\Pr(\theta_1) \approx 0.67$ (b). A bold line (either solid or dashed) is used to highlight values of $\Pr(\theta_1)$ for which decisions (d_1 or d_2) are to be preferred.

The inequality expressed in Eq. (11) also implies that if $\Pr(\theta_1) \le 0.50$, then the optimal decision is always d_2, since the lower bound of $1/(l_{21} + 1)$ is equal to 0.50. This is a direct consequence of the decision model proposed for the scenario at hand, where the worst scenario occurs when erroneously declaring a minor an adult, and a [0,1] loss system is chosen. This means that if the intermediate consequence is perceived as almost equally undesirable as the worst consequence, say $l_{21} = 0.99$ (and thus $1/(l_{21} + 1) \approx 0.50$), the ranges of $\Pr(\theta_1)$ that make one decision preferable over the other one are also almost equivalent.

4.4. A question of classification

Attributing an examined individual to a specific cohort has often been treated as a classification problem in the age estimation domain [21,39,54,55]. From a normative decision perspective, the « $0 - l_{ij}$ » loss function can be used to obtain a coherent decision criterion for classification. As mentioned above, the optimal decision is d_1 when $\bar{l}(d_1) < \bar{l}(d_2)$, thus, from Eqs. (9) and (10) it follows that decision d_1 is to be preferred to decision d_2 when:

$$l(C_{11}) \times \Pr(\theta_1) + l(C_{12}) \times \Pr(\theta_2) < l(C_{21}) \times \Pr(\theta_1) + l(C_{22}) \times \Pr(\theta_2). \tag{12}$$

Recalling the loss function summarized in Table 2, where $l(C_{11}) = l(C_{22}) = 0$, the inequality in Eq. (12) can be rearranged and the decision criterion in favour of d_1 becomes [26]:

$$\frac{\Pr(\theta_1)}{\Pr(\theta_2)} > \frac{l(C_{12})}{l(C_{21})}, \tag{13}$$

where the ratio on the left side represents the odds[4] in favour of θ_1. The inequality in Eq. (13) states that decision d_1 is to be preferred to decision d_2 if and only if the odds in favour of θ_1 are greater than the ratio between the loss associated with the outcome resulting from a wrong adult declaration C_{12} (i.e., to declare a person younger than 18 an adult) and the loss associated with the outcome resulting from a wrong minor declaration C_{21} (i.e., to declare a person who is 18 or older a minor). Given the loss function adopted for the current scenario (Table 2), the decision criterion in favour of decision d_1 in Eq. (13) simplifies to:

$$\frac{\Pr(\theta_1)}{\Pr(\theta_2)} > \frac{1}{l_{21}} \tag{14}$$

Hence, the threshold value for the odds in favour of θ_1 is given by $1/l_{21}$.

A simple manipulation of Eq. (14), where $\Pr(\theta_2)$ is replaced by $1 - \Pr(\theta_1)$, allows us to rearrange the decision criterion in favor of decision d_1 to:

$$\Pr(\theta_1) > \frac{1}{l_{21} + 1} \tag{15}$$

Decision d_1 is to be preferred to decision d_2 for values of $\Pr(\theta_1)$ greater than $1/(l_{21} + 1)$, as also obtained in Eq. (11).

Although this ratio can be seen as a cut-off as discussed in Section 2.1, there is a fundamental difference. In fact, this ratio is personally chosen by the decision-maker, in accordance with his or her preference ordering and loss scale, defined by his or her assignment of the value for l_{21}. That is, the preferred decision depends on the decision-maker's quantification of the undesirability of an erroneous minor declaration in the case at hand, and not on a general value fixed outside and independently of this decision-making process.

4.5. An alternative strategy for quantifying the loss for the intermediate consequence

The decision criterion expressed by the inequality in Eq. (13) offers a valuable starting point for formalizing a feasible strategy that can be implemented to assign a loss value to the intermediate consequence, here $l(C_{21}) = l_{21}$ [26]. The key idea is to perform a comparative analysis between the desirability of the intermediate consequence and the desirability of the worst consequence. The decision-maker has thus to evaluate, based on the specific knowledge on the case at hand, how much he or she feels that the worst consequence is less preferable compared to the intermediate consequence at hand. In fact, the comparison formalized by the inequality in Eq. (13) implies that what really matters is the ratio between the losses associated with erroneous declarations. The magnitude of the loss ratio can be represented by a single factor, say x, that states how many times lower the loss value for one consequence is compared to the other. Since the undesirability of the worst consequence is quantified with a loss value of 1, i.e., $l(C_{12}) = 1$, the previous statement can be formalized as follows:

$$l(C_{12}) = xl(C_{21}) \Longleftrightarrow 1 = xl_{21}, \quad \text{for } x > 1 \tag{16}$$

Following the results in Eq. (16), the loss of the intermediate consequence C_{21}, i.e., l_{21}, can be assigned as follows. The decision-maker simply needs to consider how much worse he or she considers erroneously declaring a minor an adult compared to erroneously declaring an adult a minor [26]. If the worst consequence is perceived as x times worse than the intermediate consequence, then the loss of this latter consequence can logically be computed as $1/x$. Suppose for instance that in a particular case the decision-maker considers that it is two times worse to erroneously declare a minor an adult than the opposite. This makes $x = 2$ and $l_{21} = 1/2 = 0.50$. Suppose the decision-maker considers consequence C_{12} much worse than C_{21}; in this case, he or she may prefer to choose $x = 10$, and then $l_{21} = 1/10 = 0.10$.

Note that this relationship also implies that, when the desirability of the intermediate consequence is not perceived as considerably higher than that of the worst consequence (thus x takes a low value), then a small increase of x implies an important variation in l_{21}. Conversely, when the intermediate consequence is considered as considerably more desirable than the worst one (thus x takes a high value), different choices for x generate a limited variation in l_{21}. For instance, for $x = 3$, $l_{21} \approx 0.33$, whilst for $x = 11$, $l_{21} \approx 0.09$. This is a direct consequence of the decision model: when the desirability of the intermediate consequence is considerably higher than that of the worst one, then the model tends to make decision d_2 the optimal one. Conversely, when the desirability of the intermediate consequence is close to that of the worst consequence, both decisions could be the optimal one (Fig. 1), and thus a little variation of the desirability may have a relevant influence in the decision-making process. Note that this qualitative strategy can be employed for assigning the loss value for any intermediate consequence C_{ij}.

5. An alternative model for the optimal decision in age estimation

The two-action decision problem presented in the previous sections configures a scenario where a decision-maker must decide whether declaring an individual whose chronological age is unknown an adult or a minor. Nonetheless, in some situations, the decision-maker may perceive the need to make no decision or to suspend the decision. This may be due to a feeling of insufficient evidence for making an informed decision. Consider for instance the protocol suggested by the EASO [11]: it is structured in multiple steps organized in a hierarchical sequence that consider the examinations for age estimation ordered on the basis of increasing intrusiveness and discrimination power. Thus, the decision-maker could feel doubtful about declaring a person adult or minor after conducting the examinations included in one step, and suspend the decision by asking for further examinations, such as those included in the subsequent steps envisaged by the protocol. From another perspective, there may be situations where the decision-maker might feel it to be inappropriate, for humanitarian or ethical reasons, to attribute an examined individual to a formal category, and he or she may prefer to suspend the decision and impose special measures for that individual. In all of these scenarios, it may be appropriate to extend the proposed decision model to encompass a third possible decision where the decision is suspended in order to, for instance, let the decision-maker collect new items of evidence.

A similar approach has already been explored in the age estimation domain. Corradi, et al. [39] suggested defining a "zone of indifference" (ZOI) across the age threshold of 18. The ZOI is defined as an interval in

[4] Note that in the current decision problem there are only two states of nature, so θ_1 is the complement of θ_2, i.e., $\Pr(\theta_1) = 1 - \Pr(\theta_2)$.

months or years around the age of 18 years, whose amplitude is context-related (civil or criminal cases). For example, the ZOI may consider ages ranging from 17 to 18 years old. If the probability (informed by the observed evidence) that the chronological age falls into this interval is greater than the probability of its complement, then no decision is made. From a normative decision-theoretic perspective, the ZOI is a third state of nature. We suggest considering the possibility where the decision is suspended as being one of the options among the available decisions rather than one of the options of the expert's results; it is a task for the decision-maker to consider this option.

5.1. Basic components

In addition to the decision list described in Section 4.1, a third decision should be considered, and this third option can be defined as a "suspended" decision, denoted d_3. This leads to the definition of two further consequences, namely C_{31} and C_{32}, which refer to a suspension of the decision when the person is actually older (C_{31}) or younger (C_{32}) than 18. Similarly to what was suggested by Biedermann, et al. [27] in the field of forensic individualization, these consequences can be interpreted as "neutral", a term that indicates that it is assumed that the decision outcome does not have any immediate consequence (either positive or negative) for the examined person. Table 3 summarizes all the basic components of the decision problem considered in the current scenario.

Table 3
A decision matrix for the age estimation problem described in Section 5.1, including the available decisions, the states of nature and the associated consequences.

	State of nature	Age ≥ 18 (θ_1)	Age < 18 (θ_2)
Decision	Adult (d_1)	Correct adult (C_{11})	Incorrect adult (C_{12})
	Minor (d_2)	Incorrect minor (C_{21})	Correct minor (C_{22})
	Suspended (d_3)	Neutral (C_{31})	Neutral (C_{32})

5.2. Building a loss function for the "three actions" age estimation scenario

Recall the preference ordering presented in Eq. (8). It is now necessary to revise it in order to include the new consequences (Table 3).

Whatever the personal structuring of preferences is, it can reasonably be assumed that it is preferable to "suspend" the decision rather than make an erroneous declaration (both for an adult and a minor). At the same time, the consequences of a correct declaration are definitely the most desirable[5]. Thus, the preference ordering described in Eq. (8) can be extended to:

$$C_{11} \sim C_{22} \succ C_{31} \sim C_{32} \succ C_{21} \succ C_{12}, \tag{17}$$

if the two "neutral" consequences are considered as equally desirable, and to:

$$C_{11} \sim C_{22} \succ C_{32} \succ C_{31} \succ C_{21} \succ C_{12}, \tag{18}$$

if consequence C_{31} is considered as less desirable than consequence C_{32}.

The comparison of the desirability of the two "neutral" consequences deserves further considerations. On one hand, there may be circumstances where the decision-maker may reasonably assume that a "suspended" decision does not affect a person younger than 18 differently from a person older than 18, and in this case the (un-)desirability of these two consequences can be considered as equivalent (Eq. (17)). This perspective assumes that the declaration of the examined person as a minor or an adult is only postponed until the required additional information becomes available. On the other hand, in other cases it may be more reasonable to assign the (un-)desirability of the two "neutral" consequences different values. Suppose, for instance, that the decision-maker is acting in a legal framework in which a person of unknown age is considered as a minor until a formal decision is taken. Suppose also that in this legal framework, special measures are implemented for minors, such as ensuring a reinforced assistance and support (this may be the case for unaccompanied minor asylum seekers). In this case, "suspending" the decision when the examined person is 18 years old or older implies that the specific measures would unnecessarily be implemented, which generates additional costs for society that would otherwise have been avoided. Therefore, in this scenario, the decision-maker may consider consequence C_{31} (to "suspend" the decision when the examined person is 18 or older) as less desirable compared to consequence C_{32} (to "suspend" the decision when the examined person is younger than 18), as presented in Eq. (18).

When the two "neutral" consequences C_{31} and C_{32} are treated as equivalent (Eq. (17)), their associated losses, $l(C_{31}) = l_{31}$ and $l(C_{32}) = l_{32}$, are equal (say l_3). In the other case, the loss associated with consequence C_{31} has to be greater than the loss associated with the other "neutral" consequence C_{32}, formally, $l_{31} > l_{32}$, in order to reflect the preference ordering defined in Eq. (18). While the losses associated with the best and worst outcomes are set equal to 0 and 1, respectively, the other loss values (i.e., l_{21}, l_{31} and l_{32}) can be assigned using the strategy described in Sections 3.2 and 4.5. This procedure is however more complex, as several comparisons must be performed to ensure the overall coherence of the preference structure [51]. The loss function is summarized in Table 4.

5.3. Minimizing the expected loss

In this section we'll apply the decision criterion of expected loss minimization to the case where the two "neutral" consequences are equally desirable (Section 5.3.1), and to the alternative case where the two "neutral" consequences are not considered equally desirable (Section 5.3.2).

5.3.1. Equally desirable neutral consequences

In this case, the expected losses of decisions d_1 and d_2 are those formulated in Eqs. (9) and (10). For d_3, the expected loss can be obtained as follows:

$$\begin{aligned}\bar{l}\,(d_3) &= \sum_{j=1}^{2} l(C_{3j}) \times \Pr(\theta_j) = \underbrace{l(C_{31})}_{l_3} \times \Pr(\theta_1) + \underbrace{l(C_{32})}_{l_3} \times \Pr(\theta_2) \\ &= l_3 \times \underbrace{[\Pr(\theta_1) + \Pr(\theta_2)]}_{1} = l_3\end{aligned} \tag{19}$$

According to the principle of minimizing the expected loss, the optimal decision is the one for which the expected loss is the lowest. Again, the graphical representation of the problem eases the interpretation (Figs. 2 and 3). It is worth recalling that according to the preference ordering expressed in Eq. (17), the loss value associated with consequences C_{31} and C_{32} must be lower than the one for consequence C_{21}, formally $l_3 < l_{21}$.

Basically, the decision-maker can be faced with two scenarios [27]: a first one (Scenario 1) where only decisions d_1 and d_2 can be optimal according to the principle of expected loss minimization (Fig. 2), and a second one (Scenario 2) where all the available decisions can be optimal. Some considerations on the two hypothetical scenarios are discussed in the following paragraphs (Fig. 3).

Scenario 1. This scenario occurs when the value assigned to l_3 is higher than the value of the y-coordinate of the intersection between $\bar{l}\,(d_1)$ and $\bar{l}\,(d_2)$, formally when

$$l_3 > \frac{l_{21}}{l_{21}+1} \tag{20}$$

[5] Note that this preference ordering is provided under the point of view of the decision maker, here the mandating authority.

Table 4
Loss function for the three-action decision model (or age estimation model) described in Table 3. Note that if the two "neutral" consequences C_{31} and C_{32} are considered equivalent, then $l_{31} = l_{32} = l_3$.

	State of nature	Age $\geq$ 18 (θ_1)	Age $<$ 18 (θ_2)
Decision	Adult (d_1)	$l(C_{11}) = 0$	$l(C_{12}) = 1$
	Minor (d_2)	$l(C_{21}) = l_{21}$	$l(C_{22}) = 0$
	Suspended (d_3)	$l(C_{31}) = l_{31}$	$l(C_{32}) = l_{32}$

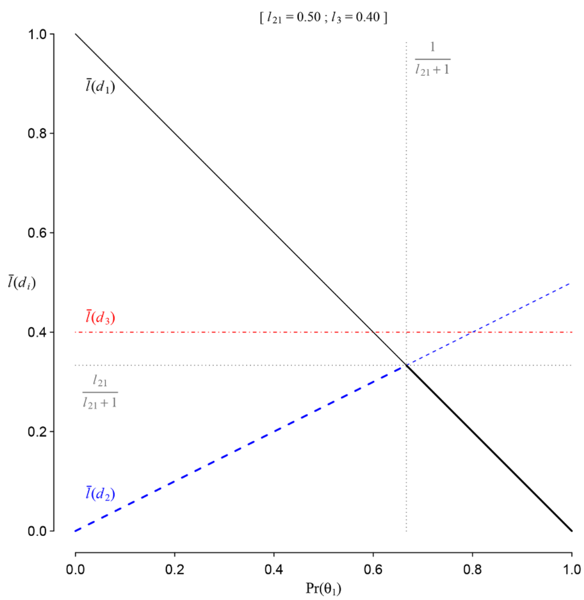

Fig. 2. Expected losses $\bar{l}\,(d_1)$ (solid line), $\bar{l}\,(d_2)$ (dot-dashed line), and $\bar{l}\,(d_3)$ (dash-dotted line) for values of Pr(θ_1) ranging from 0 to 1, with $l_{21} = 0.50$, and $l_3 = 0.40$. The dotted vertical line indicates the threshold value of Pr(θ_1) that switches the preference between decisions d_1 and d_2, Pr(θ_1)= $1/(l_{21} + 1) \approx 0.\ 67$. The dotted horizontal line indicates the threshold value $l_{21}/(l_{21} + 1) \approx 0.\ 33$: if l_3 is higher than this value, then only decisions d_1 and d_2 can be the optimal decision, as shown here. The bold lines highlight the values of Pr(θ_1) for which d_1 (solid line) and d_2 (dashed line) are to be preferred.

For any value of l_3 greater than $l_{21}/(l_{21} + 1)$, the expected loss of d_3, $\bar{l}\,(d_3)$, is always greater than the expected loss of d_1 or d_2, so decision d_3 can never be the optimal one. This is illustrated in Fig. 2, with $l_{21} = 0.50$ and $l_3 = 0.40$. The y-coordinate in Eq. (20) becomes equal to 0. 33, while the expected loss $\bar{l}\,(d_3)$ is constant and equal to 0.40, making decision d_3 always less preferable to d_1 or d_2. The inequality in Eq. (20) implies that for high values of l_{21}, l_3 must also be pretty high for decision d_3 never to be the optimal decision. For instance, for $l_{21} = 0.90$, this occurs only if $l_3 > 0.47$, whilst for $l_{21} = 0.10$ it occurs only if $l_3 > 0.09$, that is a value close to that assigned to l_{21}. From a decision-theoretic point of view, this condition suggests that when the decision-maker believes that the "neutral" consequences are relatively little desirable, nearly as much as a wrong "minor" declaration (and thus high values are assigned for both l_3 and l_{21}), then a "suspended" decision (d_3) is never optimal. This seems logical, considering that this decision does not provide a substantial added value to the decision-maker if its outcomes are perceived as negative as the outcomes originating from a wrong "minor" declaration (C_{21}). Conversely, if the decision-maker perceives this latter consequence (C_{21}) as considerably less desirable than a "neutral" consequence (i.e., giving rise to a remarkable difference between losses l_{21} and l_3), then the "suspended" decision (d_3) may logically also be the optimal decision.

Finally, the considerations expressed in Section 4.3 are logically also valid in this specific case allowing for three courses of action.

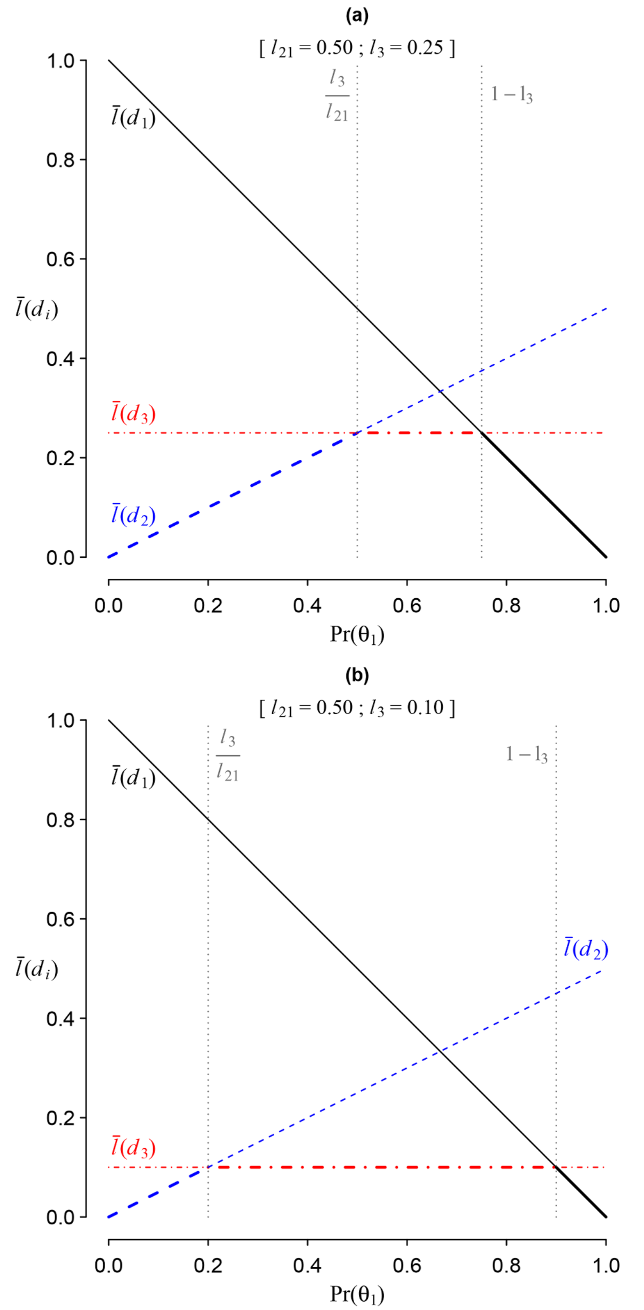

Fig. 3. Expected losses $\bar{l}\,(d_1)$ (solid line), $\bar{l}\,(d_2)$ (dashed line), and $\bar{l}\,(d_3)$ (dash-dotted horizontal line) for values of Pr(θ_1) ranging from 0 to 1, with $l_{21} = 0.50$, and $l_3 = 0.25$ (a) and $l_3 = 0.10$ (b). The dotted vertical lines indicate the threshold values of Pr(θ_1) that switch the preference among the available decisions: Pr(θ_1)= l_3/l_{21} for d_2 and d_3, and Pr(θ_1)= $1 - l_3$for $d_3$and d_1. These values are equal to Pr(θ_1)= 0.50 and Pr(θ_1)= 0.75 in (a), and Pr(θ_1)= 0.20 and Pr(θ_1) = 0.90 in (b). The bold lines highlight the values of Pr(θ_1) for which decision d_1 (solid line), d_2 (dashed line) and d_3 (dash-dotted line) are to be preferred.

Scenario 2. The relationship formalized by the inequality in Eq. (20) implies that decision d_3 can only be the favorite one with respect to decisions d_1 and d_2 when l_3 is strictly lower than $l_{21}/(l_{21} + 1)$. For instance, if l_{21} is set equal to 0.50, this amounts to an upper bound for l_3 equal to 0.33. This is illustrated in Fig. 3, with $l_3 = 0.25$ (Fig. 3a), and $l_3 = 0.10$ (Fig. 3b). In both of these cases, there are some values of Pr(θ_1) for which decision d_3 has the lowest expected loss and is therefore to be preferred to d_1 and d_2.

The x-coordinates corresponding to the intersections between the three expected losses are highlighted by the vertical segments in Fig. 3.

They correspond to the threshold values of $\Pr(\theta_1)$ at which the decision with the lowest expected loss (and therefore also the optimal decision) switches. Fig. 3 shows that, given the preference structure and the assigned loss values (i.e., $l_{21} = 0.50$ and $l_3 < 0.\overline{33}$), all available decisions can be optimal, for different probability values of the state of nature θ_1. The only things that matter are the intersections between $\bar{l}(d_1)$ and $\bar{l}(d_3)$, and between $\bar{l}(d_2)$ and $\bar{l}(d_3)$. The values of $\Pr(\theta_1)$ corresponding to these two points can be computed by equating the expected losses, that is

$$\bar{l}(d_1) = \bar{l}(d_3) \Longleftrightarrow 1 - \Pr(\theta_1) = l_3 \Longleftrightarrow \Pr(\theta_1) = 1 - l_3 \tag{21}$$

and

$$\bar{l}(d_2) = \bar{l}(d_3) \Longleftrightarrow l_{21} \times \Pr(\theta_1) = l_3 \Longleftrightarrow \Pr(\theta_1) = l_3/l_{21} \tag{22}$$

Thus, decision d_1 is to be preferred when the probability of θ_1 is greater than $1 - l_3$. Conversely, decision d_2 is to be preferred when the probability of θ_1 is smaller than l_3/l_{21}. In all other intermediate cases, the optimal decision is d_3.

From Eq. (21) it follows that the lower the value assigned to l_3, the more restricted the range of possible values of $\Pr(\theta_1)$ is that makes decision d_1 optimal (see Fig. 3). Conversely, Eq. (22) illustrates that the threshold value of $\Pr(\theta_1)$ that defines the optimal decision between d_2 and d_3 depends on both of the losses associated with the intermediate consequences: the key factor is the magnitude of the difference between l_3 and l_{21}. A greater difference corresponds to a larger portion of $\Pr(\theta_1)$ values that make d_3 the optimal decision. For instance, look at the examples in Fig. 3 for $l_{21} = 0.50$: when $l_3 = 0.25$, $\Pr(\theta_1)$ must be lower than 0.50 for d_2 to be the optimal decision, and greater than 0.75 for d_1 to be the optimal one; and when $l_3 = 0.10$, these two threshold values become 0.20 and 0.90, respectively. This numerical relationship reflects the preference structure behind this decision model. That is, if the decision-maker believes that a "neutral" consequence does not correspond to a highly undesirable outcome with severe or important losses, then it may be preferable to decide to suspend the decision so as to avoid incurring an erroneous "minor" or "adult" declaration which would be perceived as considerably worse.

Note that for extremely low values of l_3, say for example 0.01, d_1 and d_2 are preferred only for an extremely restricted range of values of $\Pr(\theta_1)$, independently of the value assigned to l_{21}. For instance, for $l_{21} = 0.50$ and $l_3 = 0.01$, decision d_1 is preferred only when $\Pr(\theta_1)$ is higher than 0.99, and decision d_2 is preferred only when $\Pr(\theta_1)$ is lower than 0.02. However, from a decision-theoretic point of view, such a low value for l_3 seems hard to justify, since it means that the decision-maker considers the desirability of the consequences of a "suspended" decision similar to that of the best consequence.

5.3.2. *Not-equally desirable neutral consequences*

Let us now consider a case where one neutral consequence is more "severe" than the other. This is expressed by the preference ordering in Eq. (18), where the consequence implied by the decision to "suspend" the decision whenever the examined person is 18 or older (C_{31}) is perceived as less desirable than a "suspended" decision in the case of a person younger than 18 (C_{32}). Again, the expected losses of decisions d_1 and d_2 are those formulated in Eqs. (9) and (10), whilst the expected loss for decision d_3 can be obtained as follows:

$$\begin{aligned}\bar{l}(d_3) &= \sum_{j=1}^{2} l(C_{3j}) \times \Pr(\theta_j) = \underbrace{l(C_{31})}_{l_{31}} \times \Pr(\theta_1) + \underbrace{l(C_{32})}_{l_{32}} \times \Pr(\theta_2) \\ &= l_{31} \times \Pr(\theta_1) + l_{32} \times [1 - \Pr(\theta_1)] = \Pr(\theta_1) \times (l_{31} - l_{32}) + l_{32}\end{aligned} \tag{23}$$

Note that according to the preference ordering expressed in Eq. (18), the losses for the three intermediate consequences C_{21}, C_{31} and C_{32} must respect this order: $l_{32} < l_{31} < l_{21}$.

Similar to what we discussed in Section 5.3.1, the decision-maker can be faced with two scenarios: in the first one, only decisions d_1 and d_2 can be optimal according to the principle of expected loss minimization (Scenario 1), whilst in the second one, all three available decisions can be optimal (Scenario 2). Let us examine the specific qualities of each of these two hypothetical scenarios.

Scenario 1: From Eqs. (10) and (23), we know that the expected loss for decisions d_2 and d_3, $\bar{l}(d_2)$ and $\bar{l}(d_3)$, are monotonically increasing functions, whilst from Eq. (9) the expected loss for the third decision, d_1, is monotonically decreasing. Hence, Scenario 1 occurs if and only if $\bar{l}(d_3)$ is greater than $\bar{l}(d_2)$ for all values of $\Pr(\theta_1)$ lower than $1/(l_{21} + 1)$, formally

$$\bar{l}(d_3) > \bar{l}(d_2), \quad \forall \ \Pr(\theta_1) < \frac{1}{l_{21} + 1} \tag{24}$$

Remember that this ratio is the *x*-coordinate of the intersection point between $\bar{l}(d_1)$ and $\bar{l}(d_2)$ that switches the preference from d_2 to d_1 (see Section 4.3 and Eq. (11)). Using Eqs. (10) and (23), it is possible to rearrange the inequality of Eq. (24) to show that Scenario 1 occurs when:

$$\Pr(\theta_1) < \frac{l_{32}}{l_{21} + l_{32} - l_{31}}, \quad \forall \ \Pr(\theta_1) < \frac{1}{l_{21} + 1}. \tag{25}$$

Thus, after some reformulation of Eq. (25) we obtain

$$l_{21} < \frac{l_{31}}{1 - l_{32}}. \tag{26}$$

Eq. (26) indicates that Scenario 1 occurs when the loss for an incorrect "minor" declaration (l_{21}) is smaller than the ratio $l_{31}/(1 - l_{32})$. Hence, the "suspended" decision (d_3) would never be the optimal decision when the losses for consequences C_{21} and C_{31} are very similar and there is an important difference between the losses of the two "neutral" consequences. Again, this observation supports the coherence of the decision model: decision d_3 is never optimal if one of the two "neutral" consequences is perceived as being as undesirable as the consequences leading to a wrong declaration (either as a minor or an adult).

Scenario 2: As mentioned in the previous section, the key elements to consider here are the two intersections between the three expected losses. Again, the values of $\Pr(\theta_1)$ corresponding to these two points are computed by equating the expected losses that intersect each other:

$$\begin{aligned}\bar{l}(d_1) = \bar{l}(d_3) \Longleftrightarrow 1 - \Pr(\theta_1) = \Pr(\theta_1) \times (l_{31} - l_{32}) + l_{32} \Longleftrightarrow \Pr(\theta_1) \\ = \frac{1 - l_{32}}{1 + l_{31} - l_{32}},\end{aligned} \tag{27}$$

and

$$\begin{aligned}\bar{l}(d_2) = \bar{l}(d_3) \Longleftrightarrow l_{21} \times \Pr(\theta_1) = \Pr(\theta_1) \times (l_{31} - l_{32}) + l_{32} \Longleftrightarrow \Pr(\theta_1) \\ = \frac{l_{32}}{l_{21} + l_{32} - l_{31}}.\end{aligned} \tag{28}$$

Hence, decision d_1 is preferred when $\Pr(\theta_1)$ is greater than $(1 - l_{32})/(1 - l_{31} + l_{32})$, decision d_2 is preferred when $\Pr(\theta_1)$ is smaller than $l_{32}/(l_{21} + l_{32} - l_{31})$, and in-between, the optimal decision is d_3.

Eq. (23) shows that the loss value associated with the "neutral" consequence for an actual minor, i.e., l_{32}, determines the *y*-intercept of the expected loss function $\bar{l}(d_3)$. That is, for a given value of l_{21}, the lower the value of l_{32}, the larger the interval of $\Pr(\theta_1)$ values that make d_3 the optimal decision. The equation also shows that the difference between the losses for the two "neutral" consequences defines the slope of the function: the greater the difference, the greater the slope and the smaller the range of $\Pr(\theta_1)$ values that make d_3 the optimal decision. Fig. 4a and b illustrate how for a fixed value of l_{21}, a smaller difference leads to a greater range of $\Pr(\theta_1)$ values that make d_3 the optimal decision. However, for most cases, a large difference between l_{31} and l_{32} seems unreasonable, since there is no reason to believe that consequence C_{31} (i.e., a "suspended" decision for a person 18 years old or older) is much less desirable than consequence C_{32} (i.e., a "suspended" decision for a person younger than 18). Therefore, in most cases, the

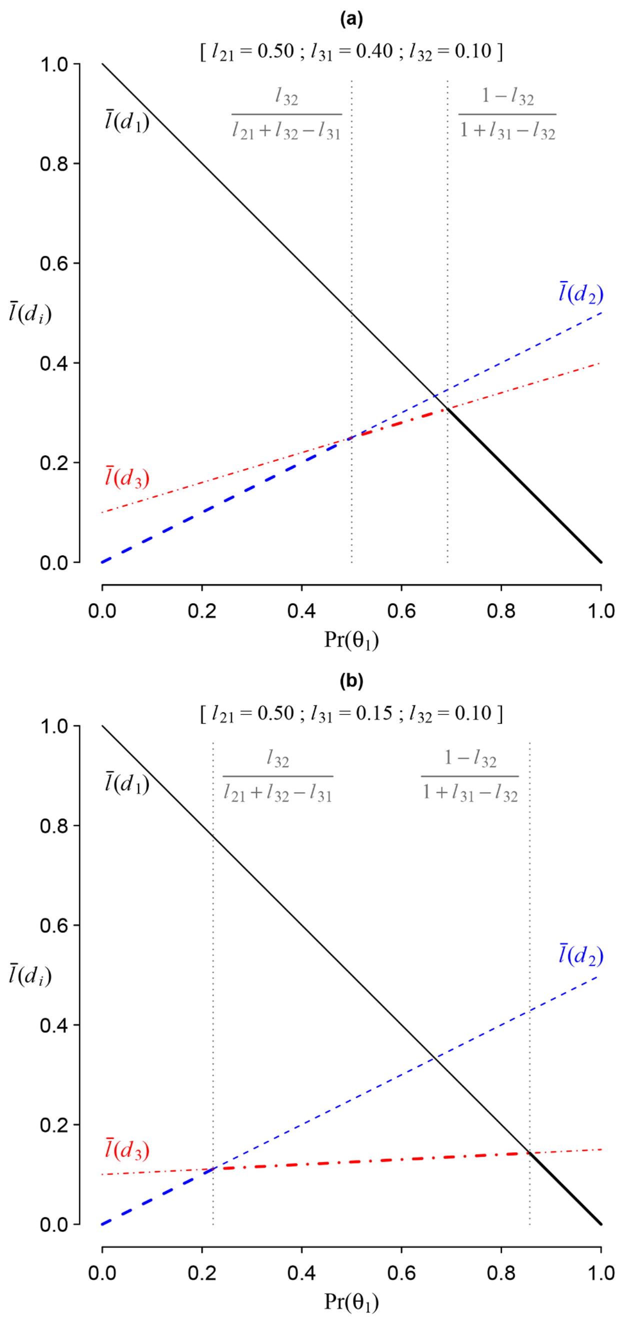

Fig. 4. Expected losses $\bar{l}(d_1)$ (solid line), $\bar{l}(d_2)$ (dashed line), and $\bar{l}(d_3)$ (dash-dotted line) for values of $\Pr(\theta_1)$ ranging from 0 to 1, with $l_{21} = 0.50$, $l_{31} = 0.40$ and $l_{32} = 0.10$ (a) and $l_{21} = 0.50$, $l_{31} = 0.15$ and $l_{32} = 0.10$ (b). The dotted vertical lines indicate the threshold values of $\Pr(\theta_1)$ that switch the preference among the available decisions: $\Pr(\theta_1) = l_{32}/(l_{21} + l_{32} - l_{31})$ and $\Pr(\theta_1) = (1 - l_{32})/(1 + l_{31} - l_{32})$. These values are $\Pr(\theta_1) = 0.50$ and $\Pr(\theta_1) \approx 0.69$ in (a), and $\Pr(\theta_1) \approx 0.22$ and $\Pr(\theta_1) \approx 0.86$ in (b). The bold lines highlight the values of $\Pr(\theta_1)$ for which decision d_1 (solid line), d_2 (dashed line) and d_3 (dash-dotted line) are to be preferred.

values of the two losses l_{31} and l_{32} should be similar, i.e., $l_{31} \approx l_{32}$, making the current scenario analogous to the second scenario discussed in Section 5.3.1.

6. Case example

Suppose that a mandating authority requests an expert to evaluate if a person lacking valid ID documents is younger or older than 18 years old. Suppose that the appraisal is conducted following the AGFAD recommendations [12]. Suppose also that the expert's report states a probability of 0.70 that the person is at least 18 years old. So, recalling the notation used throughout this paper, $\Pr(\theta_1) = 0.70$ and $\Pr(\theta_2) = 0.30$. The mandating authority takes the role of the decision-maker to decide whether the person has to be declared a minor or an adult within the context of the law.

6.1. Two-action decision model

Case 1. Suppose that the decision-maker believes that erroneously declaring a person younger than 18 an adult (C_{12}) is two times worse than erroneously declaring an adult a minor (C_{21}). In this case, according to Eq. (16), $l_{21} = 0.50$. Then, given Eqs. (9) and (10), the expected losses of the two possible decisions are $\bar{l}(d_1) = 0.30$ and $\bar{l}(d_2) = 0.35$, respectively. Thus $\bar{l}(d_1) < \bar{l}(d_2)$ and according to the principle of minimizing the expected loss, the optimal decision is to declare the person an adult (d_1).

Case 2. Suppose that the decision-maker believes that an incorrect "adult" declaration is considerably worse than an incorrect "minor" declaration, say 10 times worse. Following the same line of reasoning, the decision-maker assigns a value of 0.10 to l_{21}. While the expected loss associated with decision d_1, $\bar{l}(d_1)$, remains unchanged, the expected loss associated with decision d_2, $\bar{l}(d_2)$,is now 0.07. Hence, $\bar{l}(d_2) < \bar{l}(d_1)$ and the optimal decision is to declare the person a minor (d_2).

6.2. Three-action decision model

Case 3. Suppose that the decision-maker perceives an incorrect "adult" declaration two times worse than an incorrect "minor" declaration. Then, as highlighted in *Case 1*, l_{21} is equal to 0.50. Suppose also that he or she believes that the desirability of the two "neutral" consequences are equivalent and that they are not very desirable, since they involve elevated social expenses and time-consuming procedures. Thus, the value of l_3 is assigned close to l_{21}, say $l_3 = 0.40$. The expected losses can be quantified according to Eqs. (9), (10) and (22), and give $\bar{l}(d_1) = 0.30$, $\bar{l}(d_2) = 0.35$ and $\bar{l}(d_3) = 0.40$, respectively. Hence, $\bar{l}(d_1) < \bar{l}(d_2) < \bar{l}(d_3)$, and the optimal decision is to declare the examined person an adult (d_1). Note that in this case, $l_3 > l_{21}/(l_{21} + 1)$ and thus only decisions d_1 and d_2 can be the optimal decision.

Case 4. Suppose now that the decision-maker is still going to assign a loss value of 0.50 to C_{21}, $l_{21} = 0.50$, but that he or she believes the "neutral" consequences (still considered as equivalent) to be considerably less undesirable than an incorrect "adult" declaration, say in the order of ten times. Hence, a value of 0.10 is assigned to l_3. The expected losses associated to d_1 and to d_2, $\bar{l}(d_1)$ and $\bar{l}(d_2)$, are unvaried, whilst the expected loss associated to d_3, $\bar{l}(d_3)$, is now equal to 0.10. Therefore, it follows $\bar{l}(d_3) < \bar{l}(d_1) < \bar{l}(d_2)$, and the decision to be preferred is to suspend the decision (d_3).

Case 5. Suppose that the decision-maker feels that an incorrect adult declaration is ten times worse than an incorrect minor declaration. This will result in a loss equal to 0.10 associated to an erroneous minor declaration, $l_{21} = 0.10$. Suppose also that the decision-maker believes that the loss associated to a "suspended" decision outcome is similar to that associated to an incorrect minor declaration, and that this lead him or her to assign to the loss of a neutral consequence a value equal to 0.09, $l_3 = 0.09$. Hence, the expected losses associated to the three available decisions are now quantified as $\bar{l}(d_1) = 0.30$, $\bar{l}(d_2) = 0.07$ and $\bar{l}(d_3) = 0.09$, respectively. Therefore, $\bar{l}(d_2) < \bar{l}(d_3) < \bar{l}(d_1)$, and the optimal decision is to declare the examined person a minor (d_2).

Case 6. Finally, consider the scenario described in *Case 5*, with the sole difference that the loss value associated to the "neutral" consequences is now equal to 0.05, $l_3 = 0.05$. In this case, $\bar{l}(d_1)$ and $\bar{l}(d_2)$ remain unchanged, whilst $\bar{l}(d_3)$ is now 0.05. Therefore $\bar{l}(d_3) < \bar{l}(d_2) < \bar{l}(d_1)$, and the "suspended" decision (d_3) is optimal one.

The last two examples allow to highlight the impact that a small variation in the quantification of the undesirability of a "neutral" consequence might have in terms of optimal decision when the undesirability of an erroneous minor declaration is rather modest.

Note that no examples are presented for the model that considers the two "neutral" consequences having different loss values, since, as discussed in Section 5.4, the values of these two losses should be similar in the majority of cases. Thus these results would be analogous to those presented above for Cases 3 to 6.

7. Discussion

7.1. Who is to decide?

As pointed out in Section 1, the decision-making problem is discussed from the perspective of the mandating authority. That is, a clear distinction is made between the role of the forensic expert and the decision-maker, who is, in this perspective, the one who makes the decision of interest. The practical implementation of this decision model requires many elements to be considered, including the definition of the list of decisions, the choice of the structure of the preferences among the decisions' consequences, and the assignment of their associated loss values. In our opinion, the mandating authority (either lawyers, judges, or administrative authorities) is in the best entity to coherently cope with the task of building the loss function and assigning loss values, since it possesses the necessary knowledge about the legal framework in which the age estimation examination is requested. It is true that the probabilities of the states of nature, here $\Pr(\theta_1)$ and $\Pr(\theta_2)$, are assigned by the expert and they should be included in the expert's report. We note, however, that there are situations where it is advisable that the expert also take on the role of the decision-maker, depending on the framework of the circumstances of a particular case and the internal organization of the participants of a legal or administrative procedure.

Nonetheless, this paper focuses on *how* to decide in forensic age estimation, not on *who* is entitled to make this decision. That is, the theoretical framework promoted in this paper to support decision-making is valid no matter who the actor is taking the role of decision-maker (which can be the mandating authority, the expert or even a third actor in the legal or administrative procedure).

7.2. A question of probability

One of the standpoints of this paper is that probability is the standard measure for uncertainty [56], and therefore uncertainty in forensic proceedings should be expressed by means of probabilities [5]. In the normative decision-theoretic approach illustrated in this paper, the probabilities of the events of interest (or states of nature) represent a key ingredient of the decision-making process. Throughout the whole paper these probabilities have been denoted as unconditional probabilities, that is unaffected by other events. This choice was made for the sake of simplicity, but it does not reflect reality, since conditioning on one's available knowledge is always implicitly considered when a probability is assigned. In fact, a probability - especially when interpreted in the subjective form - is always conditioned by one's background knowledge at a given time [57]; hence, the extended notation for the probability of the states of nature is $\Pr(\theta_j|I)$, where I represents the knowledge at the disposal (at a given time) of the person who assigns the probability. Note that given the perspective described in the previous section, this person is the forensic expert. In the age estimation framework, such knowledge may refer to the sex of the examined person, their ethnic origin, their social and financial conditions while growing up, as well as the expert's personal knowledge. Furthermore, in the age estimation discipline, the probability of the states of nature should also be informed by all the other available items of (age-related) evidence, such as the developmental stage reached by the physical age indicators collected during the medical examination of the questioned individual (for example the physical attributes to be examined following the AGFAD recommendations) [12][6]. Therefore, these probabilities should be denoted $\Pr(\theta_j|E, I)$, where E represents all available (age-related) evidence. In a coherent reasoning structure, these probabilities are derived through the inferential reasoning process that accompanies the expert's evaluation of the evidence. This confirms the view according to which the Bayesian approach provides an ideal solution for this kind of inference problem [6]. Note also that if the probability is conditioned by age-related evidence E and available knowledge I, so is the expected loss function, and the correct extended notation for the loss function would be[7] $\bar{l}(d_i|E, I)$

7.3. Wrong decisions do not exist (in a normative framework)

This is a crucial aspect to understand. As noted by Lindley [50, p. 22]

> "[…] it will not be possible to say that a decision is right but only that these decisions cohere, or not. It is the relationships between events or decisions that matter, not the individual events or decision […]".

That is, it appears more transparent to refer to the coherence of a decision than to its correctness. Nonetheless, even if a decision is coherent as a result of a normative decision-theoretic approach, the outcomes might be unfavourable, as is the case of a wrong "minor" declaration of an actual adult. This means that applying a normative approach to decision making should not be seen as a way to avoid false "adult" or false "minor" declarations (although this may be an indirect consequence), but rather a framework to ensure coherence and transparency in the decision-making process. When the adopted decision leads to an incorrect or erroneous declaration, the decision-maker can prove that the choice was the most rational one given all the elements at his or her disposal. Obviously, the strong statement expressed in the title of this subsection is valid only if the decision-maker correctly follows the rules and the principles provided or dictated by decision theory.

7.4. The decision-making process must be independent from the reported results

Lindley [50, p. 157] pointed out that

> "[…] the value inserted for the utilities and probabilities are in no sense correct and any other values wrong. They represent the decision-maker's individual preferences and may be modified by him. The only inviolate feature of them is their coherence. […] The utility values must cohere with related quantities in other decision problems. A decision problem in isolation can have any values for utilities and probabilities. It is the coherence with other problems that constrains their values […]"

This means that for the age estimation problem, the decision-maker should provide their own qualification and quantification of the undesirability of decision outcomes in terms of losses. Like probabilities, there does not exist correct or wrong loss values: the only requirement is that they must be coherent with each other (i.e., they must reflect the actual preference structure). The construction of the loss (or utility)

[6] Other items of evidence may be a falsified ID document or incoherent statements provided by the examined person on his or her age.

[7] Note that when normative decision theory is applied to a decision that follows an inferential problem approached from a Bayesian perspective, it is commonly referred to as "Bayesian decision theory".

function must be based strictly on the consideration of the individual case and other similar cases, and never on the uncertainty of the states of nature. Thus, for instance, the decision-maker should not care about the reported probability of the state of nature θ_j when characterizing their preference structure. For example, the line of reasoning that a high probability for the state of nature θ_1 allows one to ignore the assignments of the loss values, since the high value of $\Pr(\theta_1)$ dominates that of the loss values in the decision-making process, is incoherent in the decision approach presented here.

7.5. Decision theory is an instrument to help the decision-maker

As highlighted by Biedermann, et al. [26], the implementation of normative theory should not be seen as an attempt to deprive the decision-maker of his or her responsibility in making the ultimate decision. Decision theory should not be seen as an independent, standalone algorithm for making decisions, but rather "[t]he theoretical framework merely intends to equip practical decision-makers with a powerful analytical and logical instrument to help them deal with the various factors thought to have a bearing on the decision problem they face […]" [26, p. 37]. Faced with the decision to declare an examined person an adult or a minor, every decision-maker has to weigh the consequences of every possible decision based on the specific framework in which the decision is made (that defines the preference ordering of all of the possible consequences) and on all the available information (e.g., the probabilities of the states of nature). Generally, such a weighting is performed intuitively by the decision-maker. Thus, from this perspective, a normative decision model provides the instruments for formalizing this intuition into a transparent and reputable process. A disturbing factor against the application of a normative approach might be the quantification of some concepts that are usually weighted intuitively, such as the desirability of a consequence. However, this paper presents useful approaches and strategies a decision-maker can refer to Section 4.5 provides a discussion, while Section 7.7 will add some further considerations. Hence, the practical implementation of the suggested decision models does not imply a massive change in the decisional-paradigm for the decision-maker, but only overcomes a potential reticence to normative approaches.

7.6. Why is decision theory important?

We are not pretending that a normative approach to decision-making is unavoidable for any kind of decision problem. However, some decisions deserve special attention because of their vital consequences on an individual's life. The decision-making process for age estimation discussed in this paper is, in our opinion, one of these. The decision to declare a person an adult or a minor in the context of the law may have severe implications on the judged person's life. In asylum law, for example, minors can benefit from favourable conditions and special protection. Moreover, in a legal system where the adult criminal law is stricter than the juvenile one, being declared a minor is beneficial. Hence, as mentioned above, the decision-maker needs to dispose of all the tools that can aid and support him or her in the decision-making process, not only to make sure that the decision is rational, but also to guarantee transparency, which is fundamental for challenging decisions.

7.7. Alternative approach for assigning the loss function

Throughout this paper, the loss values for the possible consequences have been confined to [0,1]. This choice guarantees some conceptual and practical advantages, such as being able to assign the loss value for an intermediate consequence through a qualitative comparison with the best and worst consequences[8]. However, other scales may be employed. For instance, in other areas, such as in economy, a "money strategy" can also be applied [51,52]. In this case, the desirability of a consequence is expressed in terms of monetary losses (or gains): such an approach can also be applied to age estimation. Let us consider the two-action decision model and suppose that, in a given legal system, an individual younger than 18 is declared an adult. During the appeal, the person demonstrates his or her actual age (for example by presenting some authentic ID documents that were not available before), and thus the court awards the compensation quantified in a monetary value of 100,000 (i.e., in a given monetary unit). Thus, one may use this value to assign the loss for the consequence of an incorrect adult declaration (following the nomenclature presented in this paper, this is C_{12}). Suppose also that in the same legal system, the cost for taking care of a minor can be quantified in a monetary amount of 50000. Then, this value can be used to assign the loss for the intermediate consequence C_{21}, since it may be assumed that declaring a person older than 18 a minor generates unnecessary costs in the order of such an amount. Hence, assuming that no losses are generated by providing a correct declaration (consequences C_{11}, C_{22}), the loss function can be assigned as shown in Table 5 for the two-action decision model in this scenario.

Table 5
Monetary approach to the loss function assignment for the two-action age estimation problem described in Table 1.

	State of nature	Age ≥ 18 (θ_1)	Age < 18 (θ_2)
Decision	Adult (d_1)	$l(C_{11}) = 0$	$l(C_{12}) = 100000$
	Minor (d_2)	$l(C_{21}) = 50000$	$l(C_{22}) = 0$

Recall the case example discussed in Section 6. With $\Pr(\theta_1) = 0.70$ and $\Pr(\theta_2) = 0.30$, Eq. (9) gives the expected loss $\bar{l}(d_1) = l(C_{12}) \times \Pr(\theta_2) = 30000$ for decision d_1, whilst Eq. (10) gives the expected loss $\bar{l}(d_2) = l(C_{21}) \times \Pr(\theta_1) = 35000$ for decision d_2. Thus, $\bar{l}(d_1) < \bar{l}(d_2)$, and according to the principle of minimizing the expected loss, the optimal decision is to declare the person an adult (d_1).

Such a monetary amount is likely not available very often; however, the decision-maker might have all the necessary instruments and information to estimate a coherent amount. The decision-maker should use whatever scale puts him or her in the best position to coherently assign the loss function: to reason in terms of money may be easier in some situations. However, it is important to highlight that the theoretical framework presented in this paper does not depend on the scale chosen; that only influences the quantification of the expected losses. Moreover, any linear transformation of a loss function is still a loss function [51], so the decision-theoretic model remains the same.

7.8. Descriptive approach vs normative approach

The example provided by Polo Grillo, et al. [44] and reported by Cunha, et al. [9] and Baccino, et al. [43] indicates that in that particular situation, the judges (i.e., the decision-makers) felt confident in declaring an examined individual an adult when the probability of the individual being at least 18 years old was 0.70 (see Section 2). Actually, the examples provided in Section 6 show that this probability may lead to either an "adult" or a "minor" declaration, depending on the personal choices of the decision-maker in terms of quantifying the undesirability of incorrect declarations. Surely, the choices of the judges studied in the example of Polo Grillo, et al. [44] was based on a global (and personal) appreciation of the specific context in which the decisions were made, however, this appreciation cannot be formalized without knowledge of the rules provided by normative decision theory. That is, in a so-called descriptive perspective it would be difficult to justify why a given expert's probability is sufficient to declare a person an adult in one case, but not enough in another case.

[8] See also Biedermann, et al. [26] for a review of the advantage of this choice.

8. Conclusion

Biedermann, et al. [25, p. 1–2] stated that:

> "[…] there are many day-to-day situations in which a decision must be made and where spending too much time on introspection is neither necessary nor desirable. But there are also other situations in which it is appropriate to formalise intuition – as an integral part of logical reasoning in the face of uncertainty – and devote time to a serious analysis of how to make a decision, so as to guarantee that throughout decision analysis one is able to measure the quality of decisions […]".

Forensic experts and judicial authorities (lawyers, judges, or administrative authorities) are asked to make decisions that may imply important implications for the individuals targeted by the decision. The age estimation scenario in which an authority has to decide whether an individual is legally an adult or not is undoubtedly one of these cases. The implementation of a normative approach does not protect against unfavorable outcomes, however such a framework allows the decision-maker to make a coherent decision, and to make this decision in a transparent way. With this paper, it is not intended to suggest that a decision made outside of a normative framework is by definition non-coherent or incorrect. Instead, the normative framework represents a structured way of thinking for decision-making.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Joyce, Career story: consultant forensic statistician. Communication with Ian Evett, Significance 2 (2005) 34–37.
[2] F. Taroni, S. Bozza, A. Biedermann, P. Garbolino, C. Aitken, Data Analysis in Forensic Science: A Bayesian decision perspective, John Wiley & Sons, Chichester, 2010.
[3] Comitee of forensic experts, Expressing evaluative opinions: A position statement, Sci. Justice 51 (2011) 1–2.
[4] I.W. Evett, The logical foundations of forensic science: towards reliable knowledge, Phil. Trans. R. Soc. B 370 (2015) 1–10.
[5] European Network of Forensic Scientific Institutes (ENFSI), ENFSI Guideline for Evaluative Reporting in Forensic Science: Strengthening the Evaluation of Forensic Results Across Europe, accessed 15.09.2020, < http://enfsi.eu/wp-content/uploads/2016/09/m1_guideline.pdf > .
[6] F. Taroni, A. Biedermann, Probability and Inference in Forensic Science, in: G. Bruinsma, D. Weisburd (Eds.), Encyclopedia of Criminology and Criminal Justice, Springer Science & Business Media, New York, 2014, pp. 3947–3957.
[7] A. Schmeling, G. Geserick, W. Reisinger, A. Olze, Age estimation, Forensic Sci. Int. 165 (2007) 178–181.
[8] A. Schmeling, W. Reisinger, G. Geserick, A. Olze, Age estimation of unaccompanied minors. Part I. General considerations, Forensic Sci. Int. 159 (2006) S61–S64.
[9] E. Cunha, E. Baccino, L. Martrille, F. Ramsthaler, J. Prieto, Y. Schuliar, N. Lynnerup, C. Cattaneo, The problem of aging human remains and living individuals: A review, Forensic Sci. Int. 193 (2009) 1–13.
[10] H. Law, L. Mensah, S. Bailey, J. Nelki, Immigration, Asylum Seekers and Undocumented Identity, in: S. Black, A. Aggrawal, J. Payne-James (Eds.), Age Estimation in the Living: the Practioner's guide, John Wiley & Sons, Hoboken, 2010, pp. 19–29.
[11] EASO, Practical Guide on Age Assessment, accessed 15.09.2020, < https://www.easo.europa.eu/sites/default/files/easo-practical-guide-on-age-assesment-v3-2018.pdf > .
[12] A. Schmeling, C. Grundman, A. Fuhrmann, H.-J. Kaatsch, B. Knell, F. Ramsthaler, W. Reisinger, T. Riepert, S. Ritz-Timme, F.W. Rösing, K. Rötzscher, G. Geserick, Criteria for age estimation in living individuals, Int. J. Legal Med. 122 (2008) 457–460.
[13] F. Crispino, M. Houck, Principles of forensic science, in: J.A. Siegel, P.J. Saukko (Eds.), Ecyclopedia of Forensic Sciences, Academic Press, Waltham, 2013, pp. 133–138.
[14] A. Kemkes-Grottenthaler, Aging through the ages: Historical perspectives on age indicator methods, in: R.D. Hoppa, J.V. Vaupel (Eds.), Paleodemography. Age Distributions from Skeletal Samples, Cambridge University Press, Cambridge, 2002, pp. 48–72.
[15] E. Sironi, J. Vuille, N. Morling, F. Taroni, On the Bayesian approach to forensic age estimation of living individuals, Forensic Sci. Int. 281 (2017) e24–e29.
[16] D. Lucy, The presentation of results and statistics for legal purposes, in: S. Black, A. Aggrawal, J. Payne-James (Eds.), Age Estimation in the Living: The Practioner's Guide, John Wiley & Sons, Hoboken, 2010, pp. 267–283.
[17] E. Sironi, M. Gallidabino, C. Weyermann, F. Taroni, Probabilistic graphical models to deal with age estimation of living persons, Int. J. Legal Med. 130 (2016) 475–488.
[18] P. Thevissen, S. Fieuws, G. Willems, Human dental age estimation using third molar developmental stages: Does a Bayesian approach outperform regression models to discriminate between juveniles and adults? Int. J. Legal Med. 124 (2010) 35–42.
[19] Ø. Bleka, T. Wisløff, P.S. Dahlberg, V. Rolseth, T. Egeland, Advancing estimation of chronological age by utilizing available evidence based on two radiographical methods, Int. J. Legal Med. 133 (2018) 217–229.
[20] L.W. Konigsberg, S.R. Frankenberg, H.M. Liversidge, Status of mandibular third molar development as evidence in legal age threshold cases, J. Forensic Sci. 64 (2019) 680–697.
[21] F. Corradi, V. Pinchi, I. Barsanti, S. Garatti, Probabilistic classification of age by third molar development: The use of soft evidence, J. Forensic Sci. 58 (2013) 51–59.
[22] A. Biedermann, S. Bozza, F. Taroni, Normative decision analysis in forensic science, Artif. Intell. & Law 28 (2020) 7–25.
[23] F. Taroni, S. Bozza, C. Aitken, Decision analysis in forensic science, J. Forensic Sci. 50 (2005) 894–905.
[24] F. Taroni, S. Bozza, M. Bernard, C. Champod, Value of DNA tests: A decision perspective, J. Forensic Sci. 52 (2007) 31–39.
[25] A. Biedermann, S. Bozza, F. Taroni, Analysing and exemplifying forensic conclusion criteria in terms of Bayesian decision theory, Sci. Justice 58 (2017) 159–165.
[26] A. Biedermann, S. Bozza, F. Taroni, The decisionalization of individualization, Forensic Sci. Int. 266 (2016) 29–38.
[27] A. Biedermann, S. Bozza, F. Taroni, Decision theoretic properties of forensic identification: Underlying logic and argumentative implications, Forensic Sci. Int. 177 (2008) 120–132.
[28] S. Gittelson, A. Biedermann, S. Bozza, F. Taroni, The database search problem: A question of rational decision making, Forensic Sci. Int. 222 (2012) 186–199.
[29] S. Gittelson, C.R. Steffen, M.D. Coble, Low-template DNA: A single DNA analysis or two replicates? Forensic Sci. Int. 264 (2016) 139–145.
[30] S. Gittelson, A. Biedermann, S. Bozza, F. Taroni, Decision analysis for the genotype designation in low-template-DNA profiles, Forensic Sci. Int. Gen. 9 (2014) 118–133.
[31] S. Gittelson, S. Bozza, A. Biedermann, F. Taroni, Decision-theoretic reflections on processing a fingermark, Forensic Sci. Int. 226 (2013) e42–e47.
[32] D.V. Lindley, Probability and the law, J. R. Stat. Soc. Ser. D (The Statistician) 26 (1977) 203–212.
[33] S.E. Fienberg, M.J. Schervish, The relevance of Bayesian inference for the presentation of statistical evidence and for legal decisionmaking, B.U. L. Rev. 66 (1986) 771–798.
[34] B. Robertson, G.A. Vignaux, Probability - The logic of the law, Oxford J. Legal Stud. 13 (1993) 457–478.
[35] M. Redmayne, Expert Evidence and Criminal Justice, Oxford University Press, New York, 2001.
[36] E. Sironi, S. Bozza, F. Taroni, Age estimation of living persons: A coherent approach to inference and decision, in: Z. Obertová, A. Stewart, C. Cattaneo (Eds.) Statistics for Forensic Anthropology, Academic Press, 2020.
[37] R. Cameriere, L. Ferrante, D. De Angelis, F. Scarpino, F. Galli, The comparison between measurement of open apices of third molars and Demirjian stages to test chronological age of over 18 year olds in living subjects, Int. J. Legal Med. 122 (2008) 493–497.
[38] S. De Luca, R. Biagi, G. Begnoni, G. Farronato, M. Cingolani, V. Merelli, L. Ferrante, R. Cameriere, Accuracy of cameriere's cut-off value for third molar in assessing 18 years of age, Forensic Sci. Int. 235 (2014) 102.
[39] F. Corradi, V. Pinchi, I. Barsanti, R. Manca, S. Garatti, Optimal age classification of young individuals based on dental evidence in civil and criminal proceedings, Int. J. Legal Med. 127 (2013) 1157–1164.
[40] P.M. Garamendi, M.I. Landa, J. Ballesteros, M.A. Solanoa, Reliability of the methods applied to assess age minority in living subjects around 18 years old. A survey on a Moroccan origin population, Forensic Sci. Int. 154 (2005) 3–12.
[41] B. Robertson, G.A. Vignaux, Interpreting evidence: Evaluating forensic science in the courtroom, John Wiley & Sons, Chichester, 1995.
[42] A. Biedermann, F. Taroni, S. Bozza, M. Augsburger, C.G.G. Aitken, Critical analysis of forensic cut-offs and legal thresholds: A coherent approach to inference and decision, Forensic Sci. Int. 288 (2018) 72–80.
[43] E. Baccino, E. Cunha, C. Cattaneo, Aging the dead and the living, in: J.A. Siegel, P.J. Saukko (Eds.), Ecyclopedia of Forensic Sciences, Academic Press, Waltham, 2013, pp. 42–48.
[44] B. Polo Grillo, C. Cattaneo, C. Panigalli, M. Grandi, Can a combined hand–wrist method increase accuracy in age determnation of offenders who claim to be under

18 years of age?, in: 16th Meeting of the International Association of Forensic Sciences, Proceedings, Montpellier, France, 2–7 September, 2002.
[45] A. Biedermann, F. Taroni, C. Aitken, Liberties and constraints of the normative approach to evaluation and decision in forensic science: A discussion towards overcoming some common misconceptions, Law Prob. Risk 13 (2014) 181–191.
[46] I.J. Good, Rational decision, J. R. Stat. Soc.Ser. B (Methodological) 14 (1952) 107–114.
[47] W. Edwards, Influence diagrams, Bayesian imperialism, and the Collins case: An appeal to reason. Cardozo Law Rev., 13(1991–1992), 1025–1079.
[48] S. Gittelson, Evolving from Inferences to Decisions in the Interpretation of Scientific Evidence, Ecole des Sciences Criminelles, University of Lausanne, Lausanne, 2013.
[49] D.V. Lindley, The choice of sample size, J. R. Stat. Soc.Ser. D (The Statistician) 46 (1997) 129–138.
[50] D.V. Lindley, Making Decision, John Wiley & Son, Chichester, 1985.
[51] J.O. Berger, Statistical Decision Theory and Bayesian Analysis, 2nd ed., Springer-Verlag, New York, 2010.
[52] D. Koller, N. Friedman, Probabilistic Graphical Models. Principles and Techniques, The MIT Press, Londre, 2009.
[53] M.H. DeGroot, Optimal Statistical Decisions, McGraw-Hill, New York, 1970.
[54] V. Pinchi, F. De Luca, M. Focardi, F. Pradella, G. Vitale, F. Ricciardi, G.-A. Norelli, Combining dental and skeletal evidence in age classification: Pilot study in a sample of Italian sub-adults, Leg. Med. 20 (2016) 75–79.
[55] E. Sironi, F. Taroni, Bayesian Networks for the age classification of living individuals. A study on transition analysis, J. Forensic Leg. Med. 1 (2015) 124–132.
[56] D.V. Lindley, Probability, in: C.G.G. Aitken, D.A. Stoney (Eds.), The Use of Statistics in Forensic Science, Ellis Horwood, Chichester, 1991, pp. 27–50.
[57] F. Taroni, A. Biedermann, S. Bozza, P. Garbolino, C. Aitken, Bayesian Networks for Probabilistic Inference and Decision Analysis in Forensic Science, 2nd ed., John Wiley & Sons, Chichester, 2014.