Quantum state estimation and work extraction processes beyond IID

by

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A thesis submitted in satisfaction of the
requirements for the degree of

Doctor of Philosophy

University of Technology Sydney

Faculty of Engineering & Information Technology

August 2021

Certificate of original authorship

I, Maria Quadeer, declare that this thesis is submitted in fulfilment of the requirements for the award of the degree of Doctor of Philosophy in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis. This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

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Maria Quadeer

To ammu.

Abstract

This thesis explores two specific problems within the broad context of one-shot information theory.

In the first part of this thesis, we investigate Bayesian and minimax estimators for quantum state estimation under general Bregman divergences with single-shot measurements. We also study the problem of covariant state estimation and obtain optimal measurements for the same.

In the second part, we study work extraction processes mediated by finite-time interactions with an ambient bath—*partial thermalizations*—as continuous time Markov processes for two-level systems. We analyze the distribution of work for the case where the energy gap of a two-level system is driven at a constant rate. We also analyse work extraction cycles by modifying the Carnot cycle, incorporating processes involving partial thermalizations.

Acknowledgements

I will be, forever, indebted to Marco for his patience and time in helping me get so far. His democratic approach towards research and his readiness to discuss conceptual issues have made these past years sufferable. His encouragement when it came to doing things differently and his evertimely feedback have enabled me to pursue things my own way. He has tolerated my digressions, sometimes with great enthusiasm, and which I have deeply enjoyed.

None of this would have been possible without Chris, who agreed to fund my PhD, and has been a pillar of great help and support throughout these years. His kindness, empathetic attitude, and a disposition to listen with care made life in Sydney bearable. His silent support is something that I will always cherish.

I would like to thank Kamil for his time and willingness to share his research ideas with me and for teaching me the shrewd skill that is to publish papers.

I have to thank: Desh, for setting a great example as an academic and a person—the time I spent with him and others at RRI in Bangalore has had a lasting impression; Sesh, for letting me do a project with him during my masters and his general help & guidance; Prof. Sibasish, who trusted and supported me during a challenging time, and Manik da for the many nights spent on the rooftop of the hostel at IMSc discussing physics and socialist ideas.

I could not have been who I am without abbu, who taught me some of the most important lessons in life, and who would have been very proud if he were alive today. Bereft of his presence, I wonder if this void can ever be filled. I am beholden to ammu, in perpetuity. She has taken care of me in the most distressing of times and has been a source of great strength

and determination. The family mentions would be heavily incomplete without my nani-ammi and nana-abba, who took care of me when I was little, and who's ideals far exceeded their times. Their memories are an eternal source of love and warmth in my life.

I must say a few words about my middle sister, Iram, who has always fed me amazing dishes and who's *coolness* I cannot mimic, and my little darling sister, Fatima, who has made a gazillion elaborate cards for me. I have missed them deeply over these years.

And now to mention the most current person who has been a constant throughout the highs and lows of life and who has kept me sane—Geet—he prefers not be spoken too much about.

I would also like to thank Lily, Robyn, and Camila for offering generous help with the university paperwork whenever needed.

I must also acknowledge the support of the Sydney Quantum Academy's top-up and extension scholarships that gave me a-much-needed extra time in the end.

Finally, I would like to thank the socialist policies around higher education in India that enabled a nobody.

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