

Developing an Efficiency Relationship for Tapered Pile Groups in Sand Using Analytical and 3D Numerical Analysis

by Amin Shafaghat

Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy (PhD)

under the supervision of A/Prof. Hadi Khabbaz and A/Prof
Behzad Fatahi

University of Technology Sydney
Faculty of engineering and information technology (FEIT)

January 2022

CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Amin Shafaghat, declare that this thesis, is submitted in fulfilment of the requirements for the award of Doctor of Philosophy (PhD), in the School of civil and environmental engineering, faculty of engineering and information technology (FEIT) at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis. This document has not been submitted for qualifications at any other academic institution. This research is supported by the Australian Government Research Training Program.

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Abstract

It is remarkable that a small tapering angle can boost the bearing capacity of a pile foundation notably. It implies that a pile with a small tapering angle, resembling a truncated cone, can approximately accommodate up to 40% more structural loading than its counterpart same volume cylindrical pile.

This study aims to establish an equation for obtaining the optimum tapering angle of bored tapered piles correlated to the pile geometry and sand properties varying with the relative density. The optimum tapering angle corresponds to the maximum axial bearing capacity, while keeping the volume of material in the tapered pile the same as the counterpart straight cylindrical pile.

Firstly, analytical formulations are developed to estimate the axial bearing capacity of bored tapered piles embedded in sand. The proposed governing equations capture the shaft vertical bearing component of the tapered pile, which is unique to tapered piles and varies nonlinearly with the tapering angle. By differentiating the obtained bearing capacity equation with respect to the tapering angle, an optimum tapering angle is achieved. The finite element method, mostly using PLAXIS, is also adopted to conduct the numerical modelling and to calibrate the model parameters of the proposed analytical equation, considering the soil nonlinearities and interaction between the tapered pile and the surrounding soil subjected to axial loading. UBC sand constitutive model is used to simulate the soil response in the vicinity of the tapered pile; and the model parameters are calibrated against laboratory test results for sandy soils with different relative densities. However, due to the complexity of the proposed differentiation and inverse calculation, a numerical solution is used to obtain the results. Consequently, the load-displacement curves of the tapered piles are attained numerically, and the optimum tapering angle, resulting in the maximum axial capacity of the pile, is determined. Results exhibit a good agreement between the analytically determined axial bearing capacity for the tapered pile and the corresponding numerical modelling predictions. Furthermore, a simplified empirical equation is established to select the optimum tapering angle, which can readily be

used by practicing engineers. On the other hand, a new simple equation for prediction of pile group efficiency considering the effect of tapering angle in cohesionless soil under vertical loading condition is developed. Firstly, a simple analytical relationship based on the mathematical definition of the pile group efficiency is developed. However, the effect of tapering angle is captured by defining a new geometry efficiency coefficient related to the shaft vertical bearing component of tapered piles. Thereafter, this new mathematical equation is developed, considering the shaft vertical bearing ratio and the new geometry efficiency coefficient. Furthermore, a numerical analysis is performed for modelling single cylindrical and tapered piles as well as pile groups to validate the proposed mathematical equation. Subsequently, the load-displacement diagrams for a single pile and group of piles are obtained. Then, the bearing capacities of cylindrical and tapered bored piles both as single and group are computed and compared using specific settlement criterion. Besides, the friction resistance ratio and the shaft vertical bearing ratio are separated using existing numerical methods. Having the ratios of various components of bearing capacity, pile group efficiencies can be obtained from both the numerical and mathematical models. The results show that the proposed equation can predict the pile group efficiency by considering the tapering angle as well as other affecting parameters as a simple and novel relationship.

Finally, step-tapered piles (those with a larger top diameter, and a smaller diameter at lower sections) are analysed numerically. In this study the behaviour of step-tapered piles having only one step under axial loading condition is investigated. Three series of piles embedded in sand are examined numerically using the three-dimensional finite element method. Each set consists of five piles, including one reference straight sided wall pile and four step-tapered piles having the same volume. Different internal friction angles (to represent loose, medium and dense sands) and corresponding elastic modulus and lateral earth pressure coefficients are considered to observe their effect on the bearing capacity and settlement of piles. The load-displacement diagram of each pile is obtained, and accordingly, the frictional and end bearing resistances are calculated. Some MATLAB codes are developed to get the numerical data and carry out the calculations. Moreover, the normal and shear stress states, plastic points, and deformations around the step and toe of piles are

computed and compared. According to the results, the advantages of step-tapered piles over their counterpart cylindrical ones in terms of bearing capacity and settlement are discussed. Finally, the optimum stepped length of the pile is determined.

To my late father,

My lovely mother,

My wise brother

And

*My supportive fiancé whose love and encouragements have always been with me
throughout this journey.*

Acknowledgement

During this challenging and pleasant journey, I learned many things beyond my specific area of expertise from some nice people around me. Therefore, I would like to mention some of them below and to appreciate them as I definitely owe them this achievement.

I am very grateful to my very knowledgeable supervisors, Associate Professor Hadi Khabbaz and Associate Professor Behzad Fatahi, who guided me throughout this journey. Unquestionably, I will always be appreciable to them and owe both of them a very big thank you. Thanks to their supervision during my research at UTS, I could develop my research professionally in several good journal papers as well as prestigious conferences and book chapters. On the other hand, now that I am comparing myself to where I was standing before starting the PhD at UTS, I can see how I have been developed myself as well. Therefore, I believe this approximately four years of my life that I dedicated to my research at UTS as a PhD student, was a very right time and right place that I was standing in. I also thank Miss Van Le, the school academic officer, who dedicatedly supported me during my research at UTS.

I should confess that my mother and my brother, Amirhosein, played a significant role in my success to reach to this current point. Although, spending about four years far away from them as an overseas PhD student made this research journey rather difficult, they both covered me from long distance and encouraged me with their positive attitude and constructive advices. I am very grateful to them both and will never take their existence in my life as granted, particularly after losing my late father.

My best ever gift in my life, my lovely fiancé Sahar who supported me every day and every moment. I truly don't know which word I should use to describe her significant role on my success. Having a supportive, kind, understanding, energetic, funny, generous, creative, loyal and diligent person by my side, made it much easier for me

to overcome the difficulties of this journey. I am grateful for having her in my life and do appreciate her support during this time.

Moreover, my friends whose existence around me was full of positive energy and motivation, particularly in tough situations. I am grateful to Danon and Sasan as two of my best friends who encouraged me during this journey. Having you lovely friends around me is one of the reasons for where I am standing.

I am also grateful to Australian Government research program and university of technology Sydney for supporting my PhD studies with a full scholarship. Many thanks to the Graduate Research School (GRS) and Faculty of Engineering and Information Technology (FEIT) for providing financial assistance for me to attend conferences and to be able to present my work for my peers from all over the world.

Finally, this thesis has been completed during the pandemic of Covid-19, which was a big challenge for humankind. Hence, as a researcher, I appreciate all the scientists who are working on novel methods to manufacture effective vaccines and medications to break off the pandemic.

List of publications extracted from this thesis

Journal papers:

Shafaghat, A., & Khabbaz, H. (2020). Recent advances and past discoveries on tapered pile foundations: a review. *Geomechanics and Geoengineering*, 1-30.

Shafaghat, A., Khabbaz, H., Fatahi, B. (2021). Analytical and Numerical Approaches to Attain the Optimum Tapering Angle for Axially Loaded Bored Piles in Sandy Soils. *International Journal of Geomechanics*, ASCE, DOI: 10.1061/(ASCE)GM.1943-5622.0002056

Shafaghat, A., Khabbaz, H., Fatahi, B. (2021). Axial and Lateral Efficiency of Tapered Pile Groups in Sand Using Mathematical and Three-Dimensional Numerical Analyses, Published in the journal of *Performance of Constructed Facilities*, ASCE

Book chapters and conference papers:

Shafaghat, A. and Khabbaz, H., 2021. Numerical Evaluation of Bearing Capacity of Step-Tapered Piles Using P-Y Curves Analysis. In *Advancements in Geotechnical Engineering* (pp. 200-212). Springer, Cham.

Shafaghat, A., Khabbaz, H., Fatahi, B. (2021). Developing an Efficiency Equation for Tapered Pile Groups in Sand Using Mathematical and Numerical Analyses (Sustainable Civil Infrastructures)

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List of Notations

English letters

A_b : Pile toe area;

A_{bn} : Projected area of the ledge of a segment;

A_{p-bot} : Step-tapered pile toe area;

A_{p-top} : Step-tapered pile top cross-section area;

A_s : Pile lateral surface area;

A_{sn} : Lateral surface area of an element;

D_{av} : Average diameter of a tapered pile;

D_b : Base diameter of a tapered pile;

D_{top} : Top diameter of a tapered pile;

E_{50} : Young's modulus (The secant modulus in drained triaxial test);

E_{oed} : Young's modulus (Tangent modulus for primary oedometer loading);

E_{ur} : Young's modulus (Unloading-Reloading);

K : Lateral earth pressure coefficient;

K_0 : At rest lateral earth pressure coefficient;

K_t : Taper factor for the shaft resistance of tapered piles;

L : Length of pile;

m : Number of piles in each row;

n : Number of piles in each column;

N_t : Bearing capacity factor for tapered piles;

P_g : Perimeter of a group of piles;

P_s : Perimeter of a single of piles;

Q_b : Base bearing capacity of a single pile;

Q_f : Frictional bearing capacity of a single pile;

Q_g : Total bearing capacity of a pile group;

Q_s : Total bearing capacity of a single pile;

Q_{sv} : Shaft Vertical bearing capacity of a single pile;

r : Radius of the pile's nth segment;

R_f : Failure ratio;

R_{inter} : Interface coefficient;
 S : Spacing between piles in a group;
 S_g : Settlement of a pile group;
 S_s : Settlement of a single pile;
 V_c : Volume of a cylindrical pile;
 V_t : Volume of a tapered pile;
 q_{bt} : Toe resistance of a single pile;
 q_{st} : Shaft resistance of a single pile;
 q_{sv} : Vertical bearing resistance stemming from the body of a tapered pile;
 r_b : Bottom radius of the pile;
 r_c : Radius of the counterpart same volume cylindrical pile;
 r_t : Top radius of the pile;
 η'_s : Geometric efficiency coefficient (Shaft resistance);
 η'_{sv} : Geometric efficiency coefficient (Shaft vertical resistance);
 ϕ_i : Internal friction angle of the interface;

Greek letters

α : Tapering angle;
 α_{max} : Maximum tapering angle for a pile with constant volume;
 α_{opt} : Optimum tapering angle;
 α_r : Ratio of optimum tapering angle to the maximum tapering angle;
 β : Correlation coefficient;
 γ : Soil unit weight;
 γ^p : Plastic shear strain increment;
 η_f : Stress ratio at failure;
 ν_i : Poisson's ratio of the interface;
 σ' : Mean stress in the plane of loading;
 σ'_b : Effective stresses at pile toe and mid-length of the pile;
 σ'_{bn} : Effective normal stress (at the middle of the nth pile element);
 σ'_m : Effective stresses at mid-length of the pile;
 ϕ_i : Internal friction angle of the interface;
 $d\varepsilon_v^p$: Increment of plastic volumetric strain;

$d\eta$: Change in shear stress ratio;

δ : Pile-soil interface friction angle;

ζ : A correlation coefficient controls the approaching rates of assumed functions for parameters;

η : Shear stress ratio;

η_g : Pile group efficiency;

λ : Correlation coefficient;

μ : A coefficient corresponding to the portion of mobilized Passive earth pressure coefficient;

τ : Shear stress;

ψ : A coefficient in bearing capacity factor suggested by Janbu corresponding to relative density;

ψ' : Dilation angle of soil;

ϕ : Internal friction angle of soil;

ϕ' : Effective soil friction angle;