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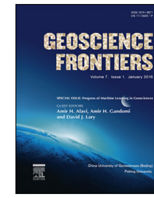

Research paper

Imperialistic Competitive Algorithm: A metaheuristic algorithm for locating the critical slip surface in 2-Dimensional soil slopes

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A B S T R A C T

In this study, Imperialistic Competitive Algorithm (ICA) is utilized for locating the critical failure surface and computing the factor of safety (FOS) in a slope stability analysis based on the limit equilibrium approach. The factor of safety relating to each trial slip surface is calculated using a simplified algorithm of the Morgenstern-Price method, which satisfies both the force and the moment equilibriums. General slip surface is considered non-circular in this study that is constituted by linking random straight lines. To explore the performance of the proposed algorithm, four benchmark test problems are analyzed. The results demonstrate that the present techniques can provide reliable, accurate and efficient solutions for locating the critical failure surface and relating FOS. Moreover, in contrast with previous studies the present algorithm could reach the lower value of FOS and reached more exact solutions.

1. Introduction

Slope stability analysis and determination of critical slip surface are significant problems in embankment dams or road besides slopes. For the slope stability analysis, a wide variety of variables such as external forces (Earthquake loads and existing of structures above slope), pore water pressure, soil parameters and site condition shall be considered.

The calculation of safety factor is the first step in the slope stability analysis. A lot of methods may be applied in order to calculate the factor of safety (FOS), i.e. limit equilibrium, finite element and finite difference methods. The limit equilibrium methods are common practice for this way. This procedure follows as:

- Slippery mass divides into the specific numbers of slices;
- Then, the effective forces acting on each slice are computed;
- Finally, the moment and force equilibrium equation will be extracted.

Because of indeterminacy of these equations, a lot of researchers attempt to solve it by considering some simplifying assumptions (Khajehzadeh et al. 2012a). For example, Fellenius (1936) and Bishop (1955) applied circular slip surface, Janbu (1973), Spencer (1967) and Morgenstern-Price (1965) utilized both circular and non-circular slip surface. As a unique aspect of the Morgenstern-Price method, both the moment and the force equilibriums are satisfied. The use of fewer simplifying assumption is the other feature of this technique. Due to the non-linear nature of the resulted equations from the Morgenstern-Price, a particular technique is needed to solve the mentioned equations simultaneously, i.e. the classical optimization techniques (e.g. Newton-Raphson method) and the metaheuristic optimization techniques (e.g. the method utilized in Zolfaghari et al. (2005)).

However, the survey of literature reveals that this problem also has been solved in direct way without using any optimization techniques. Zhu et al. (2005) proposed an effective, step by step procedure which leads to reasonable results in comparison with the complex optimization based techniques. This adopting classical optimization techniques on locating the critical slip surface may be found in Baker and Garber (1978) variation method, Celestino and Duncan (1981) and Li and White (1987) the alternating variable method, Baker (1980) dynamic programming. The inconvenient nature of these non-circular slip surface approaches has been pointed by priors (i.e. Chen and Shao, 1983) demonstrated the applied objective functions have a lot of local minimums within solution domain. Therefore classical optimization algorithms face

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some troubles to converge to a valid solution unless a good initial solution is selected.

In the recent years, by developing the meta-heuristic optimization methods, these kinds of problems are solved very well (Gandomi and Alavi, 2012). These algorithms are also applied in water, geotechnical, transportation, structural and earthquake engineering (i.e. Gandomi et al., 2013; Yang et al., 2013; Fister et al., 2014; Gandomi, 2014). The metaheuristic approaches have been also employed in slope stability analysis. As examples, the Monte-Carlo techniques by Greco (1996), harmony search, tabu search and fish swarm optimization by Cheng et al. (2007a, 2008), simulate annealing by Cheng (2003), the genetic algorithm (GA) by Zolfaghari et al. (2005), ant colony by Kahatadeniya et al. (2009) and Rezaeean et al. (2011), GSA by Khajehzadeh et al. (2012a) and particle swarm optimization (PSO) by Cheng et al. (2007b), Khajehzadeh et al. (2012b) and three swarm intelligence techniques (including PSO, firefly algorithm, cuckoo search and levy-flight krill herd) by Gandomi et al. (2014).

As mentioned by Cheng et al. (2012), the cited metaheuristic methods may be trapped on local minimum and don't converge to a proper solution. This phenomenon occurs due to the presence of a band of weak soil layer. The objective function is non-convex and discontinuous on solution domain and there are several strong local minima in the search space. Therefore it is necessary to find a robust algorithm that can evade from these local minimum and converge to the global minimum. In this way, Atashpaz-Gargari and Lucas (2007) developed Imperialistic Competitive Algorithm (ICA) optimization algorithm based on imperialistic competition. This algorithm starts from a random initial solution to reach the final solution. The population divides into two major classifications named: colonies and imperialist. Over times each imperialist try to expand its empire based on some defined rules. Among all the new developed optimization algorithms, the ICA is selected for optimization of slope stability because of good performance on preliminary benchmark tests (Atashpaz-Gargari and Lucas, 2007; Atashpaz-Gargari et al., 2008). This algorithm is also adopted very well on the structural problems (Kaveh and Talatahari, 2010a,b; Talatahari et al., 2012).

The main objective of this paper is to apply the ICA on the soil slopes which contains a band of weak layer. According to the above mentioned benefits of this algorithm it is expected to find satisfactory results for this complex slope stability problem. It is worth to emphasize that the Morgenstern-Price method for evaluating the FOS and the Cheng's (2003) method for producing the trial non-circular slip surface are selected. Finally, Zhu et al.'s (2005) approach is used to simplify the Morgenstern-Price method.

2. Generation of trial slip surface

A trial slip surface generation algorithm is required to find the position of critical failure surface. In this paper, it is used a method that proposed by Cheng (2003). The procedure of producing the trial slip surface is shown in Fig. 1. In this figure, $y = T(x)$ presents mathematical function of the slope geometry and $y = R(x)$ describes the bedrock line.

The first step to produce slip surface is dividing the failure soil mass into n-vertical slices. The failure surface is represented by vertices as follows:

$$V = [x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_{n+1}, y_{n+1}] \tag{1}$$

In order to reduce the number of variables, the width of all the slices is considered to be equal that is obtained using the following equation:

$$x_{i+1} = x_i + \frac{x_{n+1} - x_1}{n} \times (i-1) \tag{2}$$

To determine the y-coordinates of each point the upper and lower bounds ($y_{i,max}$ and $y_{i,min}$) that are slope geometry and bedrock orderly are considered.

Trial slip surfaces have to be concave upward; this fact is formulated as follows:

$$\alpha_1 \le \alpha_2 \le \cdots \le \alpha_i \tag{3}$$

where α_i is the inclination of the base of the slice i.

For more details, refer to Cheng (2003).

3. Calculation of safety factor

The Morgenstern-Price method is one of the limit equilibrium methods that satisfied both the moment and the force equilibrium for any shape of slip surface. In this method, slippery mass is

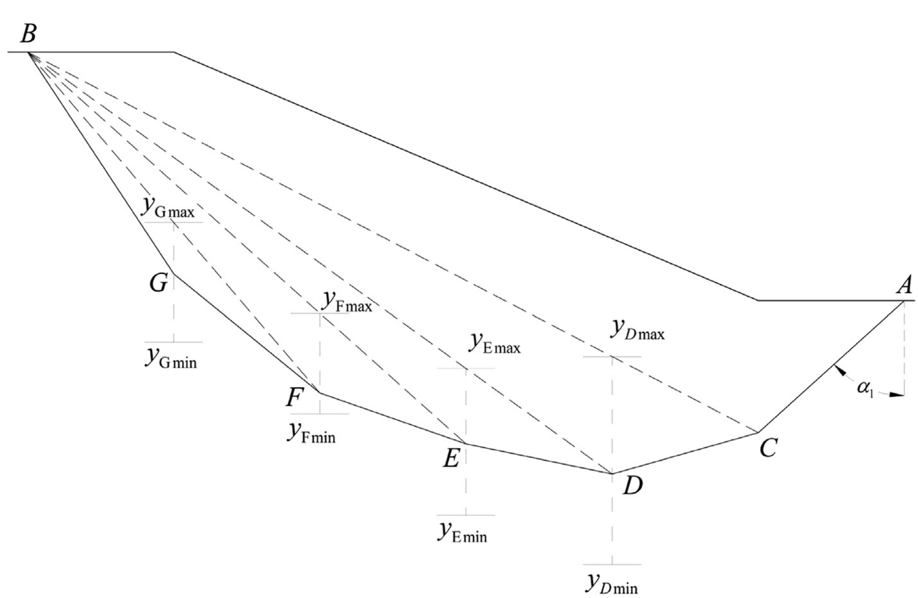

Figure 1. Generation of non-circular slip surface.

divided into the number of vertical slices. The forces applied to each slice are shown in Fig. 2.

The original formulation of Morgenstern-Price method is very complicated; hence a new approach of this method used in this paper proposed by Zhu et al. (2005), and Khajehzadeh et al. (2012a,b) implemented this method very well for calculating the factor of safety in the slope stability analysis.

The procedure of this method is briefly described below.

All the effective forces on each slice including the self-weight, the seismic force, shear and normal inter-slice forces and the water pore pressure are shown in Fig. 2.

In Fig. 2, W_i is the weight of slice i; N_i is the effective normal force at the base of slice i; S_i is the mobilized shear strength at the base of slice i ($S_i = (N_i \tan\varphi_i + c_i b_i \sec\alpha_i)/F_s$); c_i is the effective cohesion intercept at the base of slice i; φ_i is the effective internal friction angle at the base of slice i; l_i is the length of the base of slice i; U_i is the pore water pressure at the base of slice i; α_i is the inclination of the slice base; b_i is the width of slice i; h_i is the average height of slice i; α_h is the horizontal seismic coefficient; h_a is the height of the center of the slice; ω_i is inclination of surcharge load and β_i is the inclination of the slice at the top.

i. Compute R_i and T_i using following equations:

$$R_i = [W_i \cos\alpha_i - W_i\alpha_h \sin\alpha_i + Q_i \cos(\omega_i - \alpha_i) - U_i] \times \tan\phi'_i + c'_i b_i \sec\alpha_i \quad (4)$$

$$T_i = W_i \sin\alpha_i + W_i\alpha_h \cos\alpha_i - Q_i \sin(\omega_i - \alpha_i) \quad (5)$$

ii. Select inter slices forces function. In this study a constant function is opt($f(x) = 1$).

iii. Choose initial values of F_s (FOS) and λ (scaling factor) seen following criteria:

$$F_s > -\frac{\sin\alpha_i - \lambda f_i \cos\alpha_i}{\cos\alpha_i + \lambda f_i \sin\alpha_i}\tan\phi' \quad (6)$$

iv. Calculate ϕ_i and ψ_{i-1} using Eqs. (7) and (8).

$$\Phi_i = (\sin\alpha_i - \lambda f_i \cos\alpha_i)\tan\phi'_i + (\cos\alpha_i + \lambda f_i \sin\alpha_i)F_s \quad (7)$$

$$\Psi_{i-1} = \left[(\sin\alpha_i - \lambda f_i \cos\alpha_i)\tan\phi'_i + (\cos\alpha_i + \lambda f_i \sin\alpha_i)F_s\right]/\Phi_{i-1} \quad (8)$$

v. Compute F_s using Eq. (9), then determine ϕ_i, ψ_{i-1} by repeating step iv. Then compute F_s again.

$$F_s = \frac{\sum_{i=1}^{n-1}\left(R_i \prod_{j=i}^{n-1}\Psi_j\right) + R_n}{\sum_{i=1}^{n-1}\left(T_i \prod_{j=i}^{n-1}\Psi_j\right) + T_n} \quad (9)$$

vi. Use Eqs. (10) and (11) to reach E_i and λ respectively.

$$E_i\Phi_i = \Psi_{i-1}E_{i-1}\Phi_{i-1} + F_sT_i - R_i \quad (10)$$

$$\lambda = \frac{\sum[b_i(E_i + E_{i-1})\tan\alpha_i + W_i\alpha_h h_i + 2Q\sin\omega_i h_i]}{\sum[b_i(f_iE_i + f_{i-1}E_{i-1})]} \quad (11)$$

Iterate above procedure to the difference between the computed F_s and λ values in each iteration be within an acceptable tolerance limit.

4. Imperialistic Competitive Algorithm's review

Imperialistic Competitive Algorithm (ICA) is one of the recent evolutionary algorithms (Atashpaz-Gargari and Lucas, 2007). This algorithm mimics the imperialistic competition process. ICA works by seven following steps:

i. *Initializing empires*

Like the other optimization algorithms, an array containing of variables will be made as solution. For an N_{var}-dimensional optimization problem a $1 \times N_{var}$ dimensional vector considered as a solution that is named country as Eq. (12).

$$country = [P_1, P_2, P_3, \ldots, P_{N_{var}}] \quad (12)$$

In this step N_{pop} country will be generated, N_{imp} of the best solution will be selected as the imperialist and the others will

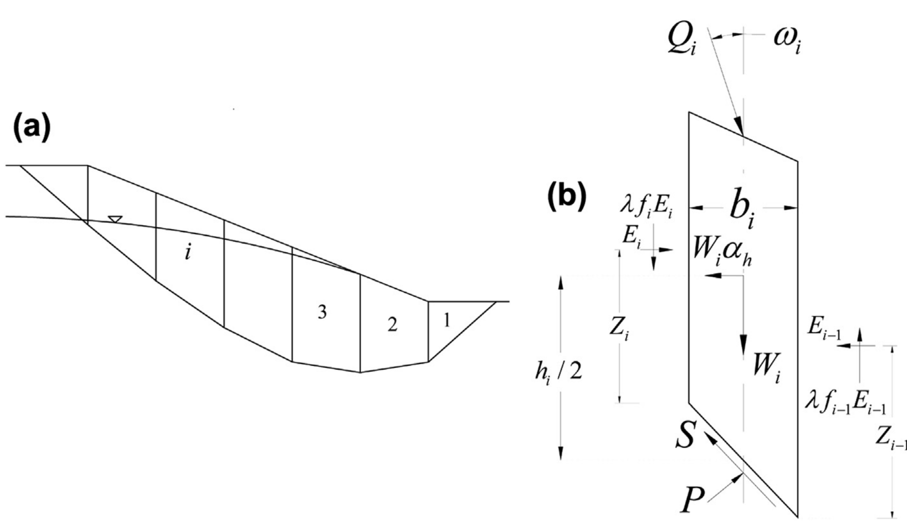

Figure 2. Slope geometry and ith slice effective forces. (a) General failure surface; (b) inter-slice forces in slice number i.

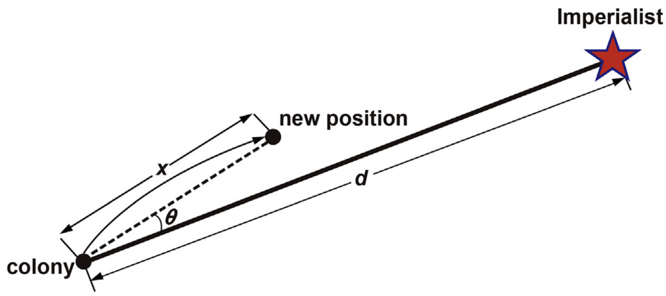

Figure 3. Colony movement direction.

form the colonies. The colonies divide among empires and the country with better solution has greater portion of the colonies.

ii. *Imperialist moving the colonies toward the imperialist*

Each colony will be made improve by means of its imperialist. ICA simulates this fact by moving the colonies toward imperialists along a vector from colony to imperialist. Fig. 3 shows movement direction with the distance equal to x as follows:

$$x \sim U(0, \beta \times d) \tag{13}$$

where β is a number greater than 1, d is the distance between colony and imperialist and x is a uniform random number.

In this algorithm θ is defined to specify new direction inclination respect to original direction. θ is a random number with uniform distribution as follows:

$$\theta \sim U(-\gamma, \gamma) \tag{14}$$

where γ is a parameter to adjust the deviation value. β and θ force algorithm to search the solution space around imperialist.

iii. *Exchanging position of the imperialist and a colony*

Similar to other optimization techniques, the best solution should be selected to guide algorithm converge to the global minimum. Through moving the colonies toward imperialist a better solution may be found. In this case the colony and the imperialist will exchange their position.

iv. *Total power of an empires*

As mentioned the stronger imperialist has greater empires. Total power of empires is relating to the power of imperialist and the power of colonies. In the ICA, $T.C._n$ is defined to index this fact by the following equation:

$$T.C._n = Cost(imperialist_n) + \xi\ mean\{Cost(colonies\ of\ empire_n)\} \tag{15}$$

in which, $T.C._n$ is the total cost of nth empire and ξ is a positive number which is considered to be less than 1.

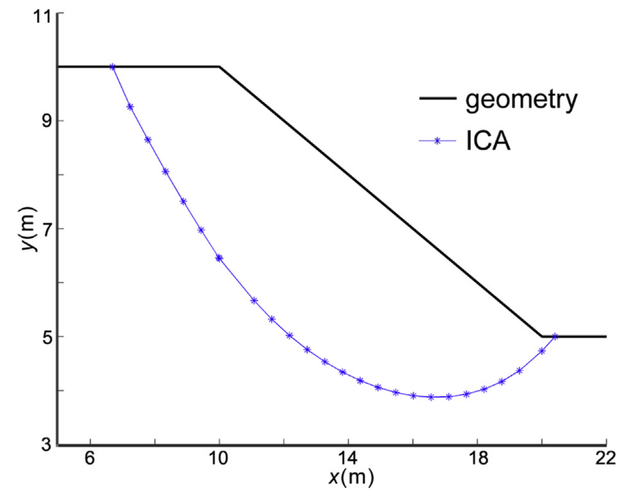

Figure 4. Slope geometry and critical slip surface for Example 1.

v. *Imperialist competition*

In fact all empires try to develop their colonies and take control of the colonies of other empires. It is equal to increase in the power of more powerful empires and decrease in the power of weaker ones. ICA models this rule by selecting some of the weakest colonies of the weakest empires and dividing among other empires. Each empire takes possession of these colonies by a specific probability relating to their total power.

vi. *Eliminating the powerless empires*

In this step powerless empires will collapse in the imperialistic competition. In ICA losing all of the colonies are considered as a collapse criteria.

vii. *Convergence*

By doing mentioned steps continuously, the most powerful imperialist will take possession of all the colonies. Finally all the colonies will have the same position as imperialist. In this situation the algorithm should be terminated and it can be defined as termination criteria.

For more detail Atashpaz-Gargari and Lucas (2007) is referable.

5. Numerical examples

To investigate effectiveness and validity of ICA on slope stability optimization problems, four benchmark examples are analyzed and their results are reported. In each example location of critical slip surface and relating FOS are reported, moreover all the previous studies on slope stability analysis are reported in the tables and compared to the present study. Because of chaotic performance of heuristic algorithms, each examples 20 times ran and results reported based on mean, best and standard deviation. The ICA parameters used in this study are exposed in Table 1.

Table 1
ICA parameters adjustment.

Parameters name	Number of countries	Number of imperialists	Number of decades	Revolution rate	β	γ	ζ	Damp ratio	Uniting threshold	*Zarib*	α
Value	300	8	500	0.3	2	0.5	0.02	0.99	0.02	1.05	0.1

Table 2
Values of safety factor for Example 1.

Optimization algorithm	FOS
Yamagami and Ueta (1988) BFGS method	1.338
Yamagami and Ueta (1988) DFP method	1.338
Greco (1996) pattern search	1.326–1.330
Greco (1996) Monte Carlo	1.327–1.333
Malkawi et al. (2001) ordinary method of slice	1.238
Solati and Habibagahi (2006) janbu	1.380
Jianping et al. (2008) spline	1.321
Jianping et al. (2008) line	1.324
Kahatadeniya et al. (2009)	1.311
ICA (current study)	1.3206

Example 1. The first example is a homogenous slope with an effective friction angle φ of 10°, an effective cohesion intercept c of 9.8 kPa, a unit weight γ of 17.64 kN/m^3 taken out from Yamagami and Ueta (1988). Fig. 4 shows the geometry and the most critical slip surface of the slope. In addition to Yamagami and Ueta (1988), this example analyzed in the works of Greco (1996) by pattern search and the Monte-Carlo methods and Kahatadeniya et al. (2009) used the ant colony optimization (ACO). A comparative summary of previous studies is tabulated as Table 2. The obtained result by ICA (FOS of 1.3206 with SD of 0.00692) is found to be a relatively good and satisfactory solution.

Example 2. This example is from the work by Arai and Tagyo (1986). In this case, a layer of weak soil is sat between two stronger soil layers. The geometrical characteristics and the critical slip surfaces of the slope are shown in Fig. 5. Table 3 collected soil parameters for each layer. Arai and Tagyo (1986) solved this problem using Janbu's simplified method in combination with the conjugate gradient method. Sridevi and Deep (1992) and Malkawi et al. (2001) utilized the random search technique (RST-2) and Monte Carlo method, respectively. Khajehzadeh et al. (2012b) explored the efficiency of PSO and MPSO optimization algorithms. In the present study this problem is also solved using the ICA optimization technique. Table 4 shows a comparison among the resultant minimum FOS obtained from the current study with previous studies. Minimum FOS by ICA is equal to 0.392 with SD of 0.00133 that shows an appropriate performance on the Example 2. However the achieved FOS by ICA is not less than the one by MPSO, since the SD is not reported in the work by Khajehzadeh et al. (2012b). It is not possible to compare these algorithms proficiency on present example exactly.

Table 3
Soil layers parameters for Example 2.

Layer	γ (kN/m^3)	c (kPa)	φ (°)
1	18.82	29.4	12
2	18.82	9.8	5
3	18.82	294.0	40

Table 4
Values of safety factor for Example 2.

Optimization algorithm	FOS
Conjugate gradient (Arai and Tagyo, 1986)	0.405
RST-2 (Sridevi and Deep, 1992)	0.401
Monte Carlo (Malkawi et al., 2001)	0.401
PSO (Khajehzadeh et al., 2012b)	0.393
MPSO (Khajehzadeh et al., 2012b)	0.391
ICA (current study)	0.392

Example 3. This example is proposed by Fredlund and Krahn (1977), which contained a soil slope consisting of a weak layer sandwiched between two other strong ones. Some studies devoted to analyze this example, such as Baker (1980), Kim and Salgado (2002), and Zhu et al. (2003). The slope geometry, location of slip surface is shown in Fig. 6. Moreover soil parameters are shown in Table 5. Obtained FOS by ICA is equal to 1.3625 with SD of 0.05102 that shows using ICA is the most efficient method among all the studies (summarized in Table 6).

Example 4. For the last complicated geotechnical, the example proposed by Zolfaghari et al. (2005) is considered. The soil parameters and slope geometry are shown in Table 7 and Fig. 7, respectively. This problem is analyzed in the works done by Zolfaghari et al. (2005) using genetic algorithm, Cheng et al. (2008) using the artificial fish swarm algorithm (AFSA), Kahatadeniya et al. (2009) using the ant-colony method and Cheng et al. (2012) using HSPSO. Table 8 provides a comparative study on the current example based on previous studies. Because of presence of thin weak soil layer between two strong ones multiple strong local minima have occurred and ACO and GA failed to converge to a very good solution. The computed FOS by ICA (FOS of 1.0642 with SD of 0.08475) shows the present study provides a good solution in this example that is able to compete with other proposed method.

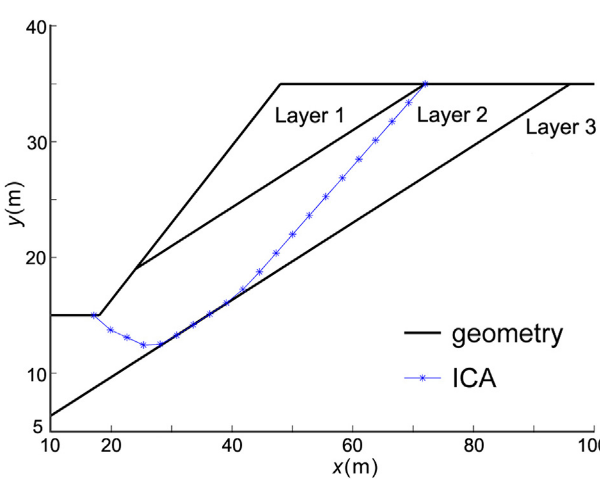

Figure 5. Slope geometry and critical slip surface for Example 2.

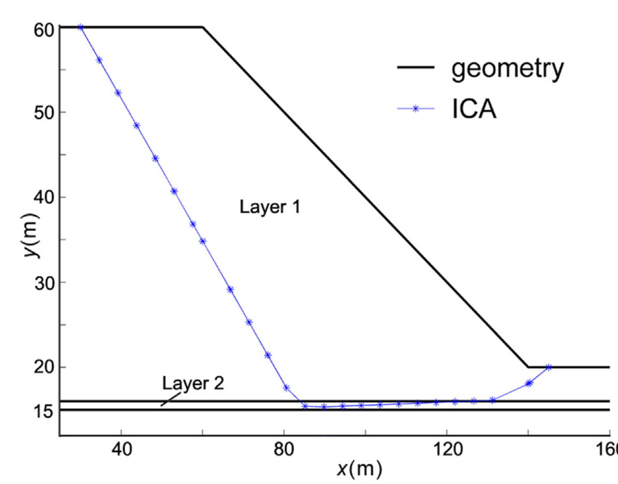

Figure 6. Slope geometry and critical slip surface for Example 3.

Table 5
Soil layers parameters for Example 3.

Layer	γ (kN/m^3)	c (kPa)	φ (°)
1	19.22	28.73	20
2	19.22	0	10

Table 6
Values of safety factor for Example 3.

Optimization algorithm	FOS
Fredlund and Krahn Zhu	1.373
Lee and Jiang	1.381
ICA (current study)	1.3625

Table 7
Soil layers parameters for Example 4.

Layer	γ (kN/m^3)	c (kPa)	φ (°)
1	19.00	15.0	20
2	19.00	17.0	21
3	19.00	5.00	10
4	19.00	35.0	28

6. Conclusion

In this study locating the most critical slip surface of soil slopes containing a weak soil layer is performed. An appropriate slip surface, kinematically acceptable, should be concave upward. Thus, some constraints should be considered to prevent unrealistic slip surface such as zigzag curves. Cheng (2003) proposed method is considered to produce trial non-circular slip surface. In this problem the objective function is considered non-polynomial hard type (Cheng et al. 2012). Even though many new optimization algorithms have developed, they have not been applied to geotechnical problems yet. Therefore, a computer program based on the ICA and the Morgenstern-Price method is written in MATLAB R2012a in order to automate locating the critical failure surface. Four illustrative problems that are proposed in the previous studies by some researchers are examined here again to benchmark the ICA algorithm performance. The achieved results in this study are compared with some reported results in the previous literature such as using Monte Carlo random-walk type, GA, PSO, MPSO, AFSA, and HSPSO. As a result, mentioned algorithms enhanced the accuracy of FOS values and location of critical slip surface. This study proves the ICA algorithm is the most proficient algorithm among the others because of smaller FOS with low standard deviation.

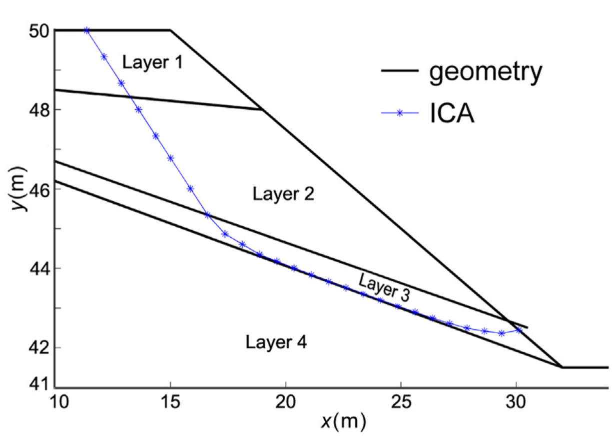

Figure 7. Slope geometry and critical slip surface for Example 4.

Table 8
Values of safety factor for Example 4.

Optimization algorithm	FOS
Genetic algorithm (Zolfaghari et al., 2005)	1.24
Simulated annealing (Cheng et al., 2007)	1.2813
Genetic algorithm (Cheng et al., 2007)	1.1440
Particle swarm optimization (Cheng et al., 2007)	1.1095
Simple harmony search (Cheng et al., 2007)	1.2068
Modified harmony search (Cheng et al., 2007)	1.1385
Tabu search (Cheng et al., 2007)	1.4650
Ant colony optimization (Cheng et al., 2007)	1.5817
Gravitational search algorithm (Khajehzadeh et al., 2011)	1.0785
ICA (current study)	1.0642

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