

UNIVERSITY OF TECHNOLOGY SYDNEY
Faculty of Science

**The Coupled Task Scheduling Problem:
Models and Solution Methods**

by

Mostafa Khatami

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

Sydney, Australia

2022

Certificate of Authorship/Originality

I, Mostafa Khatami, certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degree except as fully acknowledged within the text.

I also certify that this thesis has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis.

This research is supported by an Australian Government Research Training Program.

© Copyright May, 2022, Mostafa Khatami

Production Note:
Signature removed
prior to publication.

ABSTRACT

The Coupled Task Scheduling Problem: Models and Solution Methods

by

Mostafa Khatami

The coupled task scheduling problem (CTSP) is studied in this thesis. The problem consists of scheduling a set of jobs on one or a set of machines, where each job consists of at least two tasks. The main characteristic of the problem is a fixed time-lag between the process of each two consecutive tasks of the same job, where its duration is fixed, i.e., the succeeding task cannot be started earlier or later than the time-lag is passed. The fixed time-lags were introduced to model radar tracking systems, and later extended to formulate problems in chemistry manufacturing systems and robotic cells. The motivation for studying the CTSP in this thesis is to model certain problems in healthcare scheduling with the same characteristics. One example is the scheduling of patients in a chemotherapy clinic, where each patient must undergo a number of consecutive treatments with time-lags in between. Meeting the fixed delays between the treatments of a patient is an important factor in gaining the best outcomes for them. To study the CTSP, a literature review is first conducted, followed by studying the problem in different scheduling environments, including the single-machine, parallel-machine, open-shop and flow-shop settings, where we propose several new complexity results and solution algorithms for different variants of the problem.

Regarding the single-machine coupled task problem, a new mathematical formulation and two matheuristic algorithms are proposed for the classical problem, as well as a dynamic programming algorithm for a variant of the problem with time-dependent processing times.

With regard to the parallel-machine environment, we first explore the complexity of the problem and propose NP -hardness proofs for certain cases, followed by approximation bounds for the two-machine problem. The latter result is then extended to the open-shop scheduling environment.

The problem in the flow-shop environment is then extensively investigated under the permutation setting, and also under the case of ordered processing times. A set of publicly available hard data set and state-of-the-art algorithms are proposed for the ordered flow-shops. Then, flow-shop problem with coupled tasks is studied and polynomial-time algorithms are proposed for various settings of the problem, including the ordered processing times.

Dissertation directed by Dr Amir Salehipour
School of Mathematical and Physical Sciences

Dedication

I would have never accomplished this degree without your wholehearted love, *Mahboubeh*. To you, my beloved wife, and our little *Ali*.

Acknowledgements

I would like to thank my principal supervisor Dr Amir Salehipour for his great and continued support during my degree. He has always been available to help me, and his strong support has been essential for my achievements.

I also thank my co-supervisors Professor Daniel Oron (The University of Sydney Business School) and Dr Hanyu Gu (UTS), and my former co-supervisor Dr Feng-Jang Hwang (UTS), for their helps and supports throughout my degree.

I would like to thank Professor Lance Lesley (UTS) and Professor Murray Elder (UTS), for their kind smiles and nice chats during my time at UTS. I also thank Mrs Julia Memar (UTS) for her great supports.

I thank UTS Graduate Research School for providing me with PhD scholarships, and UTS Faculty of Science for assisting me with conference grants.

Finally, I thank my parents for their support, and my wife for her all time love.

Mostafa Khatami
Sydney, Australia, 2022.

List of Publications

Journal Papers

- J-1. Khatami, M., Salehipour, A., and Hwang, F. J. (2019). “Makespan minimization for the m -machine ordered flow shop scheduling problem”. *Computers and Operations Research* 111, 400–414.
- J-2. Khatami, M., Salehipour, A., and Cheng, T. C. E. (2020). “Coupled task scheduling with exact delays: Literature review and models”. *European Journal of Operational Research* 282(1), 19–39.
- J-3. Khatami, M. and Salehipour, A. (2021a). “A binary search algorithm for the general coupled task scheduling problem”. *JOR* 19(4), 593–611.
- J-4. Khatami, M. and Salehipour, A. (2021b). “Coupled task scheduling with time-dependent processing times”. *Journal of Scheduling* 24, 223–236.

Conference Papers

- C-1. Khatami, M., Salehipour, A., and Hwang, F. J. (2018). “Single-machine coupled task scheduling with time-dependent processing times”. *ASOR 2018*. Melbourne, Australia.
- C-2. Khatami, M. and Salehipour, A. (2019). “A simple heuristic for the coupled task scheduling problem”. *MODSIM 2019*. Canberra, Australia.
- C-3. Khatami, M. and Salehipour, A. (2020). “A relax-and-solve algorithm for the ordered flow-shop scheduling problem”. *IEEE IEEM 2020*. Singapore.

Preprints under review

- P-1. Khatami, M. and Salehipour, A. (2021c). “The coupled task scheduling problem: An improved mathematical program and a new solution algorithm”. *Submitted to International Transactions in Operational Research*.
- P-2. Khatami, M., Salehipour, A., and Cheng, T. C. E. (2021b). “Flow-shop scheduling with exact delays to minimize makespan”. *Submitted to Computers & Industrial Engineering*.
- P-3. Khatami, M., Oron, D., and Salehipour, A. (2021a). “Scheduling coupled tasks on parallel identical machines”. *Submitted to Annals of Operations Research*.

Contents

Certificate	ii
Abstract	iii
Dedication	v
Acknowledgments	vi
List of Publications	vii
List of Figures	xi
List of Tables	xiii
1 Introduction	1
1.1 Definitions, scope and classification	1
1.2 Research aims	3
1.3 Thesis Organisation	3
2 Literature review and applications	5
2.1 Literature review	5
2.1.1 The single-machine scheduling problem	6
2.1.2 The shop scheduling problem	12
2.2 Applications	20
3 Benchmarks and mathematical models	23
3.1 Benchmark instances	23
3.1.1 Previous instance generation schemes	23
3.1.2 The single-machine CTSP	24
3.1.3 The shop CTSP	25
3.2 Mathematical models	26
3.2.1 The single-machine models	27
3.2.2 The flow-shop models	32
3.3 Performance evaluation of models	37

4	Coupled tasks on a single-machine	42
4.1	Time-dependent scheduling	42
4.1.1	Problem definition	43
4.1.2	Minimising the makespan	44
4.1.3	Lower bound	52
4.1.4	Computational experiments	53
4.2	Binary search algorithm	57
4.2.1	Lower bound	57
4.2.2	Upper bound	60
4.2.3	The feasibility problem	60
4.2.4	Computational experiments	62
4.3	Proposed new formulation	64
4.3.1	Removing existing constraints	64
4.3.2	Introducing new constraints	65
4.3.3	The enhanced mixed-integer program	67
4.4	The relax-and-solve algorithm	69
4.4.1	Solution representation	70
4.4.2	The initial sequence	70
4.4.3	Pre-processing	70
4.4.4	Relax and solve operations	70
4.5	Computational experiments	71
5	Coupled tasks on parallel machines	77
5.1	<i>NP</i> -hardness proof	77
5.2	Approximation results	81
5.3	Optimal schedule for $Pm (a, L, b) C_{\max}$ and $Pm (p, L, p) C_{\max}$	83
5.3.1	Problem $Pm (a, L, b) C_{\max}$	83
5.3.2	Problem $Pm (p, L, p) C_{\max}$	85
6	Coupled tasks on flow-shops	87
6.1	Ordered flow-shops	87
6.1.1	Problem definition and formulation	88
6.1.2	Proposed solution methods	90

6.1.3	Computational experiments	95
6.2	A relax-and-solve algorithm	106
6.2.1	The proposed relax-and-solve method	106
6.2.2	Neighborhoods	107
6.2.3	Computational experiments	109
6.3	Flow-shops with coupled tasks	111
6.3.1	Properties of the problem	111
6.3.2	Distinct delays	113
6.3.3	Ordered delays	115
7	Concluding remarks	125
7.1	Limitations and Obtained results	125
7.2	Future research directions	126
	References	128

List of Figures

1.1	A coupled-task job.	1
1.2	Single-machine with flexible delays and single-machine coupled task scheduling.	3
1.3	Shop coupled task scheduling and no-wait shop scheduling.	3
2.1	Number of papers published in different time periods (last updated on 01/08/2021).	5
2.2	Interleaving jobs j and j' (a) and nesting jobs j and j' (b).	7
2.3	Schematic of the m -machine flow-shop CTSP.	13
2.4	An example showing that an optimal schedule for $F2 L_j C_{\max}$ is not necessarily a permutation one.	14
2.5	Schematic of the two-machine flow-shop problem with coupled tasks on the first machine, and a single task on the second machine.	18
2.6	The two-machine chain re-entrant flow-shop CTSP.	19
3.1	Interleaving jobs j and j' (a) and nesting jobs j and j' (b).	29
3.2	Calculation of (a) $r_{jj'4}$ and (b) $l_{jj'4}$ for two jobs j (1, 4, 3) and j' (1, 10, 2).	29
3.3	An example to clarify the correct definition of idle times.	34
4.1	Contribution of an interleaving pair of jobs (a), and a single job (b) to the makespan.	44
4.2	Two possible schedules for a two-job instance: (l, k) , where interleaving occurs, and (k, l) , where interleaving is not possible.	46
4.3	Counter example for generalising the result of Theorem 2.	47
4.4	A three-job example showing that appending job 2 (a) leads to a smaller makespan than interleaving jobs 1 and 2 (b).	49
4.5	The schedule for a four-job instance delivered by Algorithm 1.	52

4.6	An instance to illustrate calculation of the lower bound for model S3-T.	53
4.7	The main effects plot for constraints (3.25) and (4.24).	68
4.8	The interactions plot for constraints (3.25) and (4.24).	68
4.9	The solution representation in the proposed R&S algorithm.	70
5.1	An interleaving pair in the constructed schedule for CTP1 instance.	79
5.2	The constructed schedule for CTP3 instance.	82
5.3	The constructed schedule for problem $O2 (a_j, L_j, b_j) C_{\max}$	83
5.4	The optimal schedule for a three-job instance of $P2 (a, L, b) C_{\max}$	84
5.5	The optimal schedule for the three-job instance of $P2 (p, L, p) C_{\max}$	85
6.1	Comparison between the instances of benchmark T and the arbitrary instances (denoted as R in the figure).	99
6.2	<i>RPDs</i> of the heuristic algorithms on three benchmark sets T, S and L.	100
6.3	<i>RPDs</i> of the ILS, IGA_{BR} and the solver CPLEX.	105
6.4	The solution representation in the proposed R&S algorithm.	107
6.5	Selection of π'_1 and π'_2 in the first sub-problem of N_1	108
6.6	Selection of π'_1 and π'_2 in the first sub-problem of N_2	109
6.7	Selection of π'_1 and π'_2 in the first sub-problem of N_3	109
6.8	The optimal permutation (a) and non-permutation (b) schedules for problem $I_{2 \times 2}$	112
6.9	The equivalent no-wait schedule to the optimal schedule for case P.	114
6.10	The optimal schedule for problem $I_{3 \times 3}$	116

List of Tables

2.1	Distribution of papers by journals and conference proceedings.	6
3.1	Naming convention for instances of the single-machine CTSP.	26
3.2	Numbers of decision variables and constraints in the models.	37
3.3	Comparison of the performance of the studied mathematical models.	38
3.4	Detailed performance of the single-machine weighted models (a “-” denotes that the model cannot produce an outcome within the time limit).	40
3.5	Detailed performance of the single-machine makespan models (a “-” denotes that the model cannot produce an outcome within the time limit).	41
3.6	Detailed performance of the flow-shop models.	41
4.1	The operation of Algorithm 1 for a four-job instance.	51
4.2	Number of feasible and optimal solutions delivered by Heur _{cons} and Gurobi.	54
4.3	Gap (in %) and the computation time for Heur _{cons} and Gurobi.	55
4.4	Overall results for Heur _{cons} and Gurobi.	55
4.5	Gap to the lower bound for Heur _{cons} and Gurobi.	56
4.6	Assessing the performance of Heur _{cons} , Heur _{LPT} and Heur _{SPT}	56
4.7	The number of feasible solutions obtained by the binary search and Gurobi.	63
4.8	The number of optimal solutions obtained by the binary search and Gurobi.	64
4.9	The number of best solutions obtained by the binary search and Gurobi. .	65
4.10	The gap (in %) from the best solution.	66
4.11	An overview of the outcomes of the binary search heuristic and Gurobi. . .	67
4.12	Computational results for methods MIP_S5 and MIP_S3.	72
4.13	Computational results for methods BS_S5 and BS_S3.	73
4.14	Computational results for R&S.	74
4.15	Gap from the lower bound for MIP_S5, BS_S5, and R&S.	75
4.16	Improvement (in % of total) gained in the three steps of the R&S algorithm.	76

6.1	An instance of a 5-job 6-machine ordered flow-shop scheduling problem (Panwalkar and Woollam, 1980).	88
6.2	Summary of metrics N_{Best} , $ARPD$ and $ARPT$ for the four heuristic algorithms on the three benchmark sets.	100
6.3	Wilcoxon signed-rank test for $RPDs$ of the heuristic algorithms.	101
6.4	Value of the parameters used in the ILS algorithm.	102
6.5	Detailed comparison of the methods on the benchmark T.	102
6.6	Detailed comparison of the methods on the benchmark S.	103
6.7	Detailed comparison of the methods on the benchmark L.	104
6.8	Metrics N_{Best} , $ARPD$ and $ARPT$ for ILS, IGA_{BR} , and CPLEX.	105
6.9	Wilcoxon signed-rank test for $RPDs$ of ILS, IGA_{BR} and CPLEX.	105
6.10	Summary of the outcomes of different solution methods.	110
6.11	Summary of the outcomes over different instance sizes.	110
6.12	Data for problem $I_{2 \times 2}$	111
6.13	The distance matrix of the TSP for the coupled task flow-shop problem. . .	113
6.14	The data for problem $I_{3 \times 3}$	116
6.15	The reduced distance matrix of TSP for problem $I_{3 \times 3}$	116