Parametric Array Loudspeakers and Applications in Active Noise Control

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Thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

under the supervision of [ Prof. Ray Kirby, Dr. Mahmoud Karimi, Prof. Xiaojun Qiu ]

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Declaration of Authorship

I, Jiaxin Zhong, declare that this thesis entitled, “Parametric Array Loudspeakers and Applications in Active Noise Control”, is submitted in fulfilment of the requirements for the award of Doctorate of Philosophy in the school of Mechanical and Mechatronic Engineering at the University of Technology Sydney. The work presented within is my own. I confirm that:

- This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Parametric array loudspeakers (PALs) are known for their capability of generating highly directional audio sound waves. Owing to this feature, they are used as secondary sources in active noise control (ANC) systems to mitigate the unwanted noise in the target regions whilst at the same time minimizing spillover effects on other areas. The primary aim of this thesis is to investigate the feasibility of using multiple PALs in an ANC system to create a large quiet zone. To achieve this, a partial wave expansion model is proposed first based on the quasilinear solution of both Westervelt and Kuznetsov equations to predict the audio sound generated by a PAL in a free field. The model is then extended to accommodate reflection, transmission, and scattering phenomena, which are common in real applications and can have significant effects on the noise reduction performance of ANC systems. The proposed model is validated by experiments conducted in anechoic rooms, and the validated model incorporated with the multi-channel ANC theory is then used to investigate the quiet zone size controlled by multiple PALs.

It is found the existing prediction models for PALs are either inaccurate or time-consuming, while the proposed model is more than 100 times faster in both near and far fields without any loss of accuracy. It therefore enables reliable and fast simulations for multi-channel ANC systems, which require heavy computations due to large numbers of PALs. A key finding is that the directivity of the audio sound generated by a PAL is severely deteriorated if sound waves are reflected from a non-rigid surface, truncated by a thin partition, or scattered by a sphere (simulating a human head). This implies the sharp directivity for PALs is not guaranteed as expected when they are used in complex acoustic environments. Finally, both simulations and experiments showed that multiple PALs can create a large quiet zone of comparable size when compared to traditional omnidirectional loudspeakers. However, the spillover effects of using PALs on the sound field outside the quiet zone are much smaller, which demonstrates PALs provide a promising alternative as secondary sources in multi-channel ANC systems.
List of Publications

Much of this work has either been published or submitted for publication as journal papers and conference proceedings. The list is as follows:

In journals


In conference proceedings


The following publications are also outcomes during the PhD candidature, but not included in this thesis:

In journals


In conference proceedings


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<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>ANB</td>
<td>Active Noise Barrier</td>
</tr>
<tr>
<td>ANC</td>
<td>Active Noise Control</td>
</tr>
<tr>
<td>BEM</td>
<td>Boundary Element Method</td>
</tr>
<tr>
<td>CWE</td>
<td>Cylindrical Wave Expansion</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>DFW</td>
<td>Difference Frequency Wave</td>
</tr>
<tr>
<td>DSB</td>
<td>Double Sideband</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite-Difference Time-Domain</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
</tr>
<tr>
<td>FxLMS</td>
<td>Filtered-(x) Least Mean Square</td>
</tr>
<tr>
<td>GBE</td>
<td>Gaussian Beam Expansion</td>
</tr>
<tr>
<td>GHF</td>
<td>Generalized Hypergeometric Function</td>
</tr>
<tr>
<td>IL</td>
<td>Insertion Loss</td>
</tr>
<tr>
<td>KZK</td>
<td>Khokhlov-Zabolotskaya-Kuznetsov</td>
</tr>
<tr>
<td>LDV</td>
<td>Laser Doppler Vibrometer</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Metal-Oxide-Semiconductor Field-Effect Transistor</td>
</tr>
<tr>
<td>NR</td>
<td>Noise Reduction</td>
</tr>
<tr>
<td>PAA</td>
<td>Parametric Acoustic Array</td>
</tr>
<tr>
<td>PAL</td>
<td>Parametric Array Loudspeaker</td>
</tr>
<tr>
<td>PMUT</td>
<td>Piezoelectric Micromachined Ultrasonic Transducer</td>
</tr>
<tr>
<td>PNC</td>
<td>Passive Noise Control</td>
</tr>
<tr>
<td>PWE</td>
<td>Plane Wave Expansion</td>
</tr>
<tr>
<td>RH</td>
<td>Relative Humidity</td>
</tr>
<tr>
<td>SSB</td>
<td>Single Sideband</td>
</tr>
<tr>
<td>SWE</td>
<td>Spherical Wave Expansion</td>
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<tr>
<td>SPL</td>
<td>Sound Pressure Level</td>
</tr>
<tr>
<td>SRT</td>
<td>Square Root</td>
</tr>
<tr>
<td>VSB</td>
<td>Virtual Sound Barrier</td>
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## List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$a$</td>
<td>radius of the circular transducer or PAL</td>
</tr>
<tr>
<td>$A_n$</td>
<td>GBE coefficients</td>
</tr>
<tr>
<td>$B_n$</td>
<td>GBE coefficients</td>
</tr>
<tr>
<td>$B_n(\cdot)$</td>
<td>in Appendix A: a Bessel function $J_n(\cdot)$ or a Hankel function $H_n(\cdot)$</td>
</tr>
<tr>
<td>$d^3r$</td>
<td>the volume element $dxdydz$</td>
</tr>
<tr>
<td>$D(\vartheta, k)$</td>
<td>the directivity at the angle of $\vartheta$ and the wavenumber of $k$</td>
</tr>
<tr>
<td>$\exp(x)$</td>
<td>exponential function</td>
</tr>
<tr>
<td>$f_1, f_2$</td>
<td>higher and lower ultrasonic frequencies, respectively. $f_1 &gt; f_2$</td>
</tr>
<tr>
<td>$f_a$</td>
<td>audio frequency</td>
</tr>
<tr>
<td>$f_u$</td>
<td>average ultrasonic frequency</td>
</tr>
<tr>
<td>$g(r_1, r_2, k)$</td>
<td>the Green’s function in a free field between the points $r_1$ and $r_2$ at a wavenumber of $k$</td>
</tr>
<tr>
<td>$g_{2D}(\rho_1, \rho_2, k)$</td>
<td>the 2D Green’s function in a free field between the points $\rho_1$ and $\rho_2$ at a wavenumber of $k$</td>
</tr>
<tr>
<td>$G(r_1, r_2, k)$</td>
<td>the Green’s function in an arbitrary acoustic environment between the points $r_1$ and $r_2$ at a wavenumber of $k$</td>
</tr>
<tr>
<td>$j_n(z)$</td>
<td>spherical Bessel function of the first kind with an argument of $z$ of order $n$</td>
</tr>
<tr>
<td>$J_n(z)$</td>
<td>Bessel function of the first kind with an argument of $z$ of order $n$</td>
</tr>
<tr>
<td>$J$</td>
<td>cost function for the ANC system</td>
</tr>
<tr>
<td>$k_1, k_2$</td>
<td>the wavenumber of ultrasound at higher and lower ultrasonic frequencies, respectively</td>
</tr>
<tr>
<td>$k_a$</td>
<td>the wavenumber of audio sound at the frequency of $f_a$</td>
</tr>
<tr>
<td>$h_n(z)$</td>
<td>spherical Hankel function of the first kind with an argument of $z$ of order $n$</td>
</tr>
<tr>
<td>$H_n(z)$</td>
<td>Hankel function of the first kind with an argument of $z$ of order $n$</td>
</tr>
<tr>
<td>$H_n(\cdot)$</td>
<td>Struve function with an argument of $z$ of order $n$</td>
</tr>
<tr>
<td>$i$</td>
<td>imaginary unit</td>
</tr>
<tr>
<td>$i_1$</td>
<td>1 or 2 indexing the ultrasound when it is used as the subscript</td>
</tr>
<tr>
<td>$I$</td>
<td>the identity matrix</td>
</tr>
<tr>
<td>$l$</td>
<td>Index distinguishing different modes</td>
</tr>
<tr>
<td>$L$</td>
<td>Lagrangian density</td>
</tr>
<tr>
<td>$m$</td>
<td>Index distinguishing different modes</td>
</tr>
<tr>
<td>$n$</td>
<td>Index distinguishing different modes</td>
</tr>
<tr>
<td>$N_e$</td>
<td>the number of error sensors in ANC systems</td>
</tr>
<tr>
<td>$N_p$</td>
<td>the number of primary sources in ANC systems</td>
</tr>
<tr>
<td>$N_s$</td>
<td>the number of secondary sources in ANC systems</td>
</tr>
<tr>
<td>$O$</td>
<td>origin of coordinate systems</td>
</tr>
<tr>
<td>$p(r, k)$</td>
<td>the sound pressure field at the field point $r$ and the wavenumber $k$</td>
</tr>
<tr>
<td>$P$</td>
<td>ambient pressure</td>
</tr>
</tbody>
</table>
the virtual source density at point \( r \) for audio sound generated by a PAL

field point position; vector from origin to point with coordinates \((x, y, z)\)

position on the transducer surface

\( r_s \)

\( r_s,\lt \)  \( \min(r, r_s) \)

\( r_s,\gt \)  \( \max(r, r_s) \)

\( r_v,\lt \)  \( \min(r, r_v) \)

\( r_v,\gt \)  \( \max(r, r_v) \)

\( \mathcal{R} \)  the radial component for audio sound generated by a PAL

\( \mathcal{R} \)  Rayleigh distance; pressure-amplitude reflection coefficient

\( S \)  the radiation surface of a planar source

\( t \)  time

\( \mathcal{J} \)  pressure amplitude transmission coefficient

\( \mathbf{v}(r, k) \)  the velocity field (also known as acoustic particle velocity vector) at the field point \( r \) and the wavenumber \( k \)

\( v_x, v_y, v_z \)  the components of the velocity field \( \mathbf{v} \) in \( x, y, \) and \( z \) directions, respectively

\( v_r, v_\theta, v_\varphi \)  the components of the velocity field \( \mathbf{v} \) in radial \( r \), zenithal \( \theta \), and azimuthal \( \varphi \) directions, respectively

\( v_\rho \)  the component of the velocity field \( \mathbf{v} \) in polar radial \( \rho \) direction

\( V \)  the volume of the virtual audio sound source

\( (x, y, z) \)  rectangular (also known as Cartesian) coordinates

### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \alpha )</td>
<td>pure-tone sound absorption coefficient for atmospheric absorption, describing amplitude decay with distance, Np/m</td>
</tr>
<tr>
<td>( \delta )</td>
<td>the sound diffusivity parameter</td>
</tr>
<tr>
<td>( \delta_{mn} )</td>
<td>Kronecker delta function; the value is 0 if ( m \neq n ), and 1 if ( m = n )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>error function</td>
</tr>
<tr>
<td>( \varepsilon_n )</td>
<td>Neumann factor</td>
</tr>
<tr>
<td>( \theta )</td>
<td>zenith (also known as polar) angle in spherical coordinates</td>
</tr>
<tr>
<td>( \Gamma(\cdot) )</td>
<td>Gamma function</td>
</tr>
<tr>
<td>( \rho )</td>
<td>the polar radius in cylindrical coordinates ((\rho, \varphi, z))</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>linear ambient density of air</td>
</tr>
<tr>
<td>( \tilde{\rho} )</td>
<td>the fluid density</td>
</tr>
<tr>
<td>( \rho )</td>
<td>the transverse coordinate vector ((x, y))</td>
</tr>
<tr>
<td>( \tau )</td>
<td>retarded time</td>
</tr>
<tr>
<td>( \Phi(r, k) )</td>
<td>the velocity potential field at the field point ( r ) and the wavenumber ( k )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>angular frequency</td>
</tr>
<tr>
<td>( \omega_1, \omega_2 )</td>
<td>the angular frequency of ultrasound</td>
</tr>
<tr>
<td>( \omega_a )</td>
<td>the angular frequency of audio sound</td>
</tr>
</tbody>
</table>
**Other symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>∇</td>
<td>Gradient operator</td>
</tr>
<tr>
<td>∇^2</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>∇^2⊥</td>
<td>the transverse Laplace operator</td>
</tr>
<tr>
<td>n!</td>
<td>factorial of n</td>
</tr>
<tr>
<td>(\begin{pmatrix} a &amp; b &amp; c \ d &amp; e &amp; f \end{pmatrix})</td>
<td>Wigner 3j symbol</td>
</tr>
</tbody>
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**Constants**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>(c_0 = 343, \text{m/s})</td>
<td>linear speed of sound</td>
</tr>
<tr>
<td>(\rho_0 = 1.21, \text{kg/s}^3)</td>
<td>linear ambient density of air</td>
</tr>
<tr>
<td>(\beta = 1.2)</td>
<td>the nonlinear coefficient in air</td>
</tr>
<tr>
<td>(\pi = 3.1415926)</td>
<td>Archimedes’s constant</td>
</tr>
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