UNIVERSITY
OF TECHNOLOGY
SYDNEY

Parametric Array Loudspeakers and Applications in Active Noise Control

by Jiaxin Zhong

Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy

under the supervision of [Prof. Ray Kirby, Dr. Mahmoud
Karimi, Prof. Xiaojun Qiu]

University of Technology Sydney
Faculty of Engineering and Information Technology

06/2022

Declaration of Authorship

I, Jiaxin Zhong, declare that this thesis entitled, "Parametric Array Loudspeakers and Applications in Active Noise Control", is submitted in fulfilment of the requirements for the award of Doctorate of Philosophy in the school of Mechanical and Mechatronic Engineering at the University of Technology Sydney. The work presented within is my own. I confirm that:

- This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.
- This document has not been submitted for qualifications at any other academic institution.
- This research is supported by the Australian Government Research Training Program.

Production Note:
Signature: Signature removed prior to publication.

Date: 27 Jan 2022

UNIVERSITY OF TECHNOLOGY SYDNEY

Abstract

Faculty of Engineering and Information Technology
School of Mechanical and Mechatronic Engineering (MME)

Doctor of Philosophy

Parametric Array Loudspeakers and Applications in Active Noise Control

by Jiaxin ZHONG

Parametric array loudspeakers (PALs) are known for their capability of generating highly directional audio sound waves. Owing to this feature, they are used as secondary sources in active noise control (ANC) systems to mitigate the unwanted noise in the target regions whilst at the same time minimizing spillover effects on other areas. The primary aim of this thesis is to investigate the feasibility of using multiple PALs in an ANC system to create a large quiet zone. To achieve this, a partial wave expansion model is proposed first based on the quasilinear solution of both Westervelt and Kuznetsov equations to predict the audio sound generated by a PAL in a free field. The model is then extended to accommodate reflection, transmission, and scattering phenomena, which are common in real applications and can have significant effects on the noise reduction performance of ANC systems. The proposed model is validated by experiments conducted in anechoic rooms, and the validated model incorporated with the multi-channel ANC theory is then used to investigate the quiet zone size controlled by multiple PALs.

It is found the existing prediction models for PALs are either inaccurate or time-consuming, while the proposed model is more than 100 times faster in both near and far fields without any loss of accuracy. It therefore enables reliable and fast simulations for multi-channel ANC systems, which require heavy computations due to large numbers of PALs. A key finding is that the directivity of the audio sound generated by a PAL is severely deteriorated if sound waves are reflected from a non-rigid surface, truncated by a thin partition, or scattered by a sphere (simulating a human head). This implies the sharp directivity for PALs is not guaranteed as expected when they are used in complex acoustic environments. Finally, both simulations and experiments showed that multiple PALs can create a large quiet zone of comparable size when compared to traditional omnidirectional loudspeakers. However, the spillover effects of using PALs on the sound field outside the quiet zone are much smaller, which demonstrates PALs provide a promising alternative as secondary sources in multi-channel ANC systems.

List of Publications

Much of this work has either been published or submitted for publication as journal papers and conference proceedings. The list is as follows:

In journals

1. **Jiaxin Zhong**, Ray Kirby, Mahmoud Karimi, Haishan Zou, and Xiaojun Qiu. "Scattering by a Rigid Sphere of Audio Sound Generated by a Parametric Array Loudspeaker". In: *The Journal of the Acoustical Society of America* 151.3 (2022), pp. 1615–1626.
2. **Jiaxin Zhong**, Tao Zhuang, Ray Kirby, Mahmoud Karimi, Haishan Zou, and Xiaojun Qiu. "Quiet Zone Generation in a Free Field with Multiple Parametric Array Loudspeakers". In: *The Journal of the Acoustical Society of America* 151.2 (2022), pp. 1235–1245.
3. **Jiaxin Zhong**, Ray Kirby, Mahmoud Karimi, and Haishan Zou. "A Cylindrical Expansion of the Audio Sound for a Steerable Parametric Array Loudspeaker". In: *The Journal of the Acoustical Society of America* 150.5 (2021), pp. 3797–3806.
4. **Jiaxin Zhong**, Ray Kirby, and Xiaojun Qiu. "The Near Field, Westervelt Far Field, and Inverse-Law Far Field of the Audio Sound Generated by Parametric Array Loudspeakers". In: *The Journal of the Acoustical Society of America* 149.3 (2021), pp. 1524–1535.
5. **Jiaxin Zhong** and Xiaojun Qiu. "On the Spherical Expansion for Calculating the Sound Radiated by a Baffled Circular Piston". In: *Journal of Theoretical and Computational Acoustics* (2020), p. 2050026.
6. **Jiaxin Zhong**, Shuping Wang, Ray Kirby, and Xiaojun Qiu. "Reflection of Audio Sounds Generated by a Parametric Array Loudspeaker". In: *The Journal of the Acoustical Society of America* 148.4 (2020), pp. 2327–2336.
7. **Jiaxin Zhong**, Shuping Wang, Ray Kirby, and Xiaojun Qiu. "Insertion Loss of a Thin Partition for Audio Sounds Generated by a Parametric Array Loudspeaker". In: *The Journal of the Acoustical Society of America* 148.1 (2020), pp. 226–235.
8. **Jiaxin Zhong**, Ray Kirby, and Xiaojun Qiu. "A Spherical Expansion for Audio Sounds Generated by a Circular Parametric Array Loudspeaker". In: *The Journal of the Acoustical Society of America* 147.5 (2020), pp. 3502–3510.
9. **Jiaxin Zhong**, Ray Kirby, and Xiaojun Qiu. "A Non-Paraxial Model for the Audio Sound behind a Non-Baffled Parametric Array Loudspeaker (L)". In: *The Journal of the Acoustical Society of America* 147.3 (2020), pp. 1577–1580.
10. **Jiaxin Zhong**, Tao Zhuang, Ray Kirby, Mahmoud Karimi, Xiaojun Qiu, Haishan Zou, Jing Lu, "Audio Sound Field Generated by a Focusing Parametric Array Loudspeaker", In: *IEEE Trasactions on Audio, Speech, and Language Processing* Under Review (2022).

In conference proceedings

1. **Jiaxin Zhong**, Tong Xiao, Benjamin Halkon, Ray Kirby, and Xiaojun Qiu. “An Experimental Study on the Active Noise Control Using a Parametric Array Loudspeaker”. In: *InterNoise 2020*. Seoul, Korea, 2020.

The following publications are also outcomes during the PhD candidature, but not included in this thesis:

In journals

1. **Jiaxin Zhong**, Baicun Chen, Jianchen Tao, and Xiaojun Qiu. “The Performance of Active Noise Control Systems on Ground with Two Parallel Reflecting Surfaces”. In: *The Journal of the Acoustical Society of America* 147.5 (2020), pp. 3397–3407.
2. **Jiaxin Zhong**, Jiancheng Tao, and Xiaojun Qiu. “Increasing the Performance of Active Noise Control Systems on Ground with Two Vertical Reflecting Surfaces with an Included Angle”. In: *The Journal of the Acoustical Society of America* 146.6 (2019), pp. 4075–4085.
3. Shuping Wang, **Jiaxin Zhong**, Xiaojun Qiu, and Ian Burnett. “A Note on Using Panel Diffusers to Improve Sound Field Diffusivity in Reverberation Rooms below 100 Hz”. In: *Applied Acoustics* 169 (2020), p. 107471.

In conference proceedings

1. **Jiaxin Zhong**, Jiancheng Tao, and Xiaojun Qiu. “A Numerical Study on Active Noise Radiation Control Systems between Two Parallel Reflecting Surfaces”. In: *The 18th Asia-Pacific Vibration Conference*. Sydney, Australia, 2019.
2. Xiaojun Qiu, Qiaoxi Zhu, Shuping Wang, and **Jiaxin Zhong**. “A Case Study on the New Reverberation Room Built in University of Technology Sydney”. In: *Proceedings of the 23rd International Congress on Acoustics*. Aachen, Germany, 2019.

Acknowledgements

Throughout the preparing and writing of this thesis, I have received a great deal of support and assistance.

First of all, I would like to thank my principal supervisor, Prof. Ray Kirby, and my co-supervisor, Dr. Mahmoud Karimi, whose expertise is invaluable in supervising my research and writing academic articles. Your insightful feedback and comments pushed me to sharpen my thinking and brought my work to a high level. I would also like to express my sincere gratitude to my previous principal supervisor, Prof. Xiaojun Qiu, for his guidance, encouragement, and support during my PhD candidature. I am grateful to my teammates at UTS, Dr. Benjamin Halkon, Dr. Sipei Zhao, Dr. Shuping Wang, Dr. Qiaoxi Zhu, Dr. Somanath Pradhan, Mr. Tong Xiao, Mr. Mitchell Cullen, for their valuable assistance and suggestions on my research work. Thanks to all staff members of the School of Mechanical and Mechatronic Engineering for always being friendly and helpful. The financial support from the Australian Research Council through the Linkage Project (LP160100616) is acknowledged.

Furthermore, I would like to thank Mr. Tao Zhuang, Mr. Kangkang Wang, Dr. Chengbo Hu, Dr. Haishan Zou, Dr. Kai Chen, Dr. Zhibin Lin, A/Prof. Jianchen Tao, and Prof. Jing Lu in Nanjing University for their kind support when I was stuck in China during the COVID-19 pandemic. I thank you for giving the access to acoustics labs in Nanjing University. Without your support, I could not conduct experiments to finish my thesis.

Finally, I would like to express my endless gratitude to my parents, Mr. Ruiliang Zhong and Ms. Weiguo Liao, for their wise counsel, sympathetic ear, and unconditional support. You are always there for me.

Contents

Titlepage **ii**

Declaration of Authorship **iii**

Abstract **v**

List of Publications **vii**

Acknowledgements **ix**

Contents **xiv**

List of Figures **xxii**

List of Tables **xxiii**

List of Abbreviations **xxv**

List of Symbols **xxvii**

1 Introduction **1**
- 1.1 Background and motivation 1
- 1.2 Objectives 5
- 1.3 Thesis outline 6
- 1.4 Contributions 7

2 Literature Review **9**
- 2.1 Parametric array loudspeakers (PALs) 9
  - 2.1.1 Physical mechanisms 9
  - 2.1.2 Prediction models 11
  - 2.1.3 Implementation and applications 15
- 2.2 Active noise control (ANC) 17
  - 2.2.1 Approaches and algorithms 18
  - 2.2.2 Quiet zone generation 19
  - 2.2.3 ANC using directional loudspeakers 20
  - 2.2.4 ANC using PALs 21
- 2.3 Summary 25

3 Sound Fields Generated by a PAL **27**
3.1 Governing equations . . . 27
3.2 Quasilinear solution . . . 28
3.2.1 Three-dimensional model . . . 29
3.2.2 Two-dimensional model . . . 31
3.3 The sound field on front side of a baffled PAL . . . 33
3.3.1 The near field, Westervelt far field, and inverse-law far field . . . 33
3.3.2 Gaussian beam expansion . . . 35
3.3.3 Convolution model . . . 37
3.4 The sound field on back side of a non-baffled PAL . . . 38
3.4.1 Theory . . . 38
3.4.2 Results and discussions . . . 39
3.5 Summary . . . 43

4 Improved Prediction Models for PALs **45**
4.1 Spherical wave expansion (SWE) for a circular source . . . 45
4.1.1 Theory . . . 46
4.1.2 Calculation of the radial component . . . 48
4.1.2.1 Existing methods . . . 49
4.1.2.2 Proposed closed-form solution . . . 50
4.1.3 Results and discussions . . . 51
4.2 SWE for a circular PAL . . . 53
4.2.1 Quasilinear solution of Westervelt equation . . . 53
4.2.2 Quasilinear solution of Kuznetsov equation . . . 55
4.2.3 Approximation in the inverse-law far field . . . 56
4.2.4 Numerical simulations . . . 57
4.2.4.1 Validation of the SWE . . . 57
4.2.4.2 The transition distance from the near field to the Westervelt far field . . . 59
4.2.4.3 The transition distance from the Westervelt far field to the inverse-law far field . . . 63
4.3 Cylindrical wave expansion (CWE) for a phased array source . . . 65
4.3.1 Theory . . . 65
4.3.2 The radial component . . . 67
4.4 CWE for a phased array PAL . . . 68
4.4.1 Quasilinear solution of Westervelt equation . . . 68
4.4.2 Quasilinear soultion of Kuznetsov equation . . . 69
4.4.3 Approximation in the inverse-law far field . . . 70
4.4.4 Velocity profiles for a steerable PAL . . . 70
4.4.5 Results and discussions . . . 71
4.4.5.1 Conventional PAL with a uniform excitation . . . 72
4.4.5.2 Steerable PAL generating one beam . . . 74

4.4.5.3 Steerable PAL generating dual beams 75
4.5 Summary 76

5 Physical Properties for Audio Sound Generated by a PAL 79

5.1 Reflection from an infinitely large surface 79
5.1.1 Theory 80
5.1.2 Numerical simulations 82
5.1.3 Experiments 86
5.2 Transmission through a thin partition 89
5.2.1 Theory 90
5.2.1.1 Transmission of audio sound generated by incident ultrasound . 91
5.2.1.2 Audio sound generated by transmitted ultrasound 93
5.2.1.3 Insertion loss of the partition 94
5.2.2 Simulations and discussions 94
5.2.3 Experiments 98
5.3 Scattering by a rigid sphere 101
5.3.1 Theory 103
5.3.1.1 Ultrasound field 103
5.3.1.2 Audio sound field 105
5.3.2 Numerical simulations 105
5.3.2.1 The accuracy of the Westervelt equation 105
5.3.2.2 Scattering effects by a rigid sphere 108
5.3.3 Experiments 109
5.4 Summary 113

6 ANC using PALs 117

6.1 Single channel ANC system using a PAL 117
6.1.1 Experimental setup 117
6.1.2 Results and discussions 118
6.2 Multi-channel ANC system using multiple PALs 120
6.2.1 Theory 121
6.2.2 Simulations 123
6.2.2.1 2D secondary source array 124
6.2.2.2 3D secondary source array 128
6.2.3 Experiments 130
6.3 Summary 132

7 Conclusions and Future Work 135

7.1 Conclusions 135
7.2 Future work 137

A Detailed Derivations of Formulae **139**
A.1 Integral of triple Legendre polynomials 139
A.2 Normalized Bessel functions 140
A.3 Normalized spherical Bessel functions 141
A.4 Calculation of the integral $\int J_0(x)\mathrm{d}x$ 142

B Attenuation of Pure-Tone Sound in Air Due to the Atmospheric Absorption **145**
B.1 Attenuation coefficient 145
B.2 Absorption distance of PALs 146

References **149**

List of Figures

1.1 Structure of an ANC system. 2

1.2 Sound pressure level (SPL; dB re 20 μPa) generated by a point source located at $(x, z) = (0, 6\,\mathrm{m})$ at 1 kHz: (a) primary (noise) field; (b) the noise at the $(x, z) = (0, 0)$ is controlled by introducing a secondary source at $(x, z) = (0, 4\,\mathrm{m})$. Circle, primary source; cross, secondary source; square, error point. 3

1.3 SPL (dB re 20 μPa) generated by a point source located at $(x, z) = (0, 6\,\mathrm{m})$ at 1 kHz and the sound at the $(x, z) = (0, 0)$ is controlled by a traditional omnidirectional loudspeaker at: (a) $(x, z) = (0, 0.5\,\mathrm{m})$; and (b) $(x, z) = (0, 2\,\mathrm{m})$. Circle, primary source; cross, secondary source; square, error point. 3

1.4 Sketch of a binaural ANC system using two PALs. From Fig. 6 in [35]. 4

1.5 SPL (dB re 20 μPa) at 1 kHz generated by: (a) a traditional loudspeaker; (b) a PAL located at the origin and of the same radiation surface. 4

1.6 SPL (dB re 20 μPa) generated by a point source located at $(x, z) = (0, 6\,\mathrm{m})$ at 1 kHz and the sound at the $(x, z) = (0, 0)$ is controlled by a PAL at: (a) $(x, z) = (0, 0.5\,\mathrm{m})$; and (b) $(x, z) = (0, 2\,\mathrm{m})$. Circle, primary source; cross, secondary source; square, error point. 5

2.1 Demonstration of the generation of audible sound beam by using the PAA. From Fig. 1 in [76]. 10

2.2 Front view of the first audio spotlight (PAL). From Fig. 2 in [87]. 10

2.3 Contour lines indicate the SPL normalized by each maximum level from −6 to 0 dB in steps of 1 dB. The carrier frequencies for sources 1 and 2 are 77 kHz and 52 kHz, respectively. From Fig. 10 in [90]. 11

2.4 Audio spot creation using two PALs. From Fig. 1 in [95]. 12

2.5 Block diagram of the implementation of a PAL. From Fig. 11 of [76]. 15

2.6 Commercial ultrasonic emitters. 16

2.7 Several commercial products of the PAL. 18

2.8 Block diagram of the FxLMS algorithm. From Fig. 2.3 in [184]. 19

2.9 Sketch of an ANB system. From Fig. 1(a) of [37]. 20

2.10 (a) The proposed directional source; (b) a coherent line source of length l is controlled by 3 secondary sources. From Figs. 1 and 5 in [38]. 21

2.11 Illustration of the concept of a directional control source (PAL, denoted by “Spotlight”) being used in combination with a virtual sensor to allow tracking of a local quiet zone, without spillover in the rest of the field. The shaded area shows the region that is affected by the secondary source. From Fig. 1 of [29]. 22

2.12 Sound fields steered by 30°. Left column, numerical results; right column, experimental results; top row, before control; middle row, after control using a traditional loudspeaker; bottom row, after control using a PAL. Cross, noise source; Square, secondary source; Circle, error point. From Figs. 11 and 12 in [30]. . . . 23
2.13 The laboratory made focused PAL in [40]. 23
2.14 Simulation (left column) and measurement (right column) audio sound pressure of locally global noise control at 1 kHz: (top row) before control, (middle row) after control with the focused PAL, and (bottom) after control with an ideal point source. Cross mark, primary source; square mark, secondary source; circle mark, error point. From Figs. 3 and 8 in [40]. 24
2.15 Experimental setup of an ANC system using a commercial PAL in a room. From Fig. 5 in [203]. 24
2.16 Experiment results of the dual-channel ANC system that achieves 10.4 dB and 11.6 dB noise reductions at the left and right ears, respectively. From Fig. 8 in [35]. 25
2.17 A concentrically-nested length-limited PAL made for ANC. From Fig. 2 in [205]. 25

3.1 Sketch of a PAL and the geometric description of rectangular and spherical coordinate systems. 30
3.2 Sketch of a phased array PAL and the geometrical description of rectangular and cylindrical coordinate systems. 32
3.3 The audio SPL (dB re 20 μPa) as a function of the propagating distance at 1 kHz calculated with different methods: (a) on the radiation axis (0°), and (b) in the direction of the angle 10°. The parameters in Table 3.1 are used. 35
3.4 Normalized sound pressure along the radiation axis. Solid line, calculated using the closed-form solution given by Eq. (3.32). Dashed line, calculated using the GBE method, where the transducer radius $a = 0.1\,\mathrm{m}$, and the GBE coefficients are chosen from [117]. 36
3.5 The experimental setup. 40
3.6 Audio SPL (dB re 20 μPa) along the z axis at different audio frequencies: (a) 315 Hz, (b) 500 Hz, (c) 800 Hz, and (d) 1 kHz. Solid line, baffled model given by Eq. (3.17); dashed line, non-baffled model given by Eq. (3.46); triangle, measurement. 41
3.7 Audio SPL (dB re 20 μPa) along the z axis obtained using the non-baffled model at 1/3 octave center frequencies from 160 Hz to 1.6 kHz when the surface SPL of ultrasound is 125 dB. Red circle, $z = -2\,\mathrm{m}$; Blue square, $z = -1\,\mathrm{m}$; green triangle, $z = -0.5\,\mathrm{m}$; purple diamond, $z = -0.25\,\mathrm{m}$; orange pentagon, $z = -0.1\,\mathrm{m}$. 42
3.8 Audio SPL (dB re 20 μPa) predicated by the proposed non-baffled model at (a) 315 Hz and (b) 800 Hz; and the measurements at (c) 315 Hz and (d) 800 Hz. The PAL is placed at the origin and the radiation surface is on the plane $z = 0$, denoted by the dashed line. 42

4.1 Sketch of the radiation from a baffled circular rigid piston source. 46

4.2 Numerical calculation of the integral $\mathcal{J}_{2n}^{1}(0, ka)$ using four methods when (a) $ka = 10$ and $n = 0$; (b) $ka = 300$ and $n = 10$; (c) $ka = 900$ and $n = 10$; and (d) using the GHF series at $n = 10$. In (a), (b), and (c): red circle, proposed method; blue square, Gauss-Legendre quadrature; green triangle, Bessel expansion; purple diamond, GHF series. In (d): red circle, $ka = 300$; blue square, $ka = 900$; dashed line, overflow. The function log10 represents the common logarithm with the base of 10. 51

4.3 Calculated normalized sound pressure on the radiation axis (a) and the directivity index (b) when $ka = 350 + 0.09\mathrm{i}$ for a baffled circular radiator with the uniform (rigid piston) and quadratic (simply supported disk) velocity profiles. Red circle, rigid piston, proposed method; blue sqaure, rigid piston, exact solution; green triangle, simply supported, proposed method; purple diamond, simply supported, exact solution. 53

4.4 Convergence of the audio SPL (dB re 20 μPa) as a function of the truncated term N at several typical field points when $y = 0$, where the parameters in Table 4.2 are used. 58

4.5 The audio SPL (dB re 20 μPa) as a function of the propagating distance in different directions at 1 kHz, where the circles are that obtained by using the direct method. Solid line, $\theta = 0°$; dashed line, $\theta = 10°$; dash dotted line, $\theta = 20°$. 58

4.6 Audio SPL (dB re 20 μPa) at 1 kHz as a function of the propagating distance on the radiation axis, where the transducer radius is 0.05 m 59

4.7 The level of normalized ultrasound pressure and Lagrangian density at the average ultrasound frequency 40 kHz. 60

4.8 The audio SPL difference calculated with the Kuznetsov and Westervelt equations as a function of the propagating distance on the radiation axis (a) with different transducer radii when the average ultrasound frequency is 40 kHz, and (b) at different average ultrasound frequencies when the transducer radius is 0.05 m, where the audio frequency is 1 kHz. 61

4.9 The audio SPL difference calculated with the Kuznetsov and Westervelt equations as a function of the propagating distance on the radiation axis at different audio frequencies (transducer radius is 0.05 m and the average ultrasound frequency is 40 kHz). 63

4.10 The audio SPL difference calculated with the Westervelt equation and the inverse-law property as a function of the propagating distance on the radiation axis (a) with different transducer radii when the average ultrasound frequency is 40 kHz and the audio frequency is 1 kHz, (b) at different average ultrasound frequencies when the transducer radius is 0.05 m and the audio frequency is 1 kHz, and (c) at different audio frequencies when the transducer radius is 0.05 m and the average ultrasound frequency is 40 kHz, where SPL = 1 dB for the dashed lines. 64

4.11 Comparison of the continuous and discrete velocity profile for the ultrasound: (a) normalized amplitude distribution, and (b) normalized phase distribution. . . 71

4.12 Audio SPL (dB re 20 μPa) generated by a conventional PAL with a uniform profile and a size of 0.08 m at (a) 500 Hz, (b) 1 kHz, (c) 2 kHz, and (d) 4 kHz. . 72
4.13 Audio SPL (dB re 20 μPa) generated by a conventional PAL with a uniform profile and a size of 0.08 m calculated using the convolution method and the cylindrical expansion at (a) 500 Hz, (b) 1 kHz, (c) 2 kHz, and (d) 4 kHz. Red hollow square, convolution model; blue hollow circle, cylindrical expansion at 4 m; orange triangle, cylindrical expansion in the far field; green solid square, convolution method from [105]; purple solid circle, measurement from [105]. . . . 73
4.14 Audio SPL (dB re 20 μPa) at 4 kHz generated by a steerable PAL with a steering angle of 70° (denoted by dashed lines), a size of 0.08 m, and a (a) continuous, or (b) discrete profile with a PAL unit size of 0.01 m. 74
4.15 Audio SPL (dB re 20 μPa) at 4 kHz at different angles generated by a steerable PAL with a steering angle of 70°, a size of 0.08 m, and a discrete profile with a PAL unit size of 0.01 m. Red hollow square, convolution model; blue hollow circle, cylindrical expansion at 4 m; green solid square, convolution model from [105]; purple solid circle, measurement from [105]. 75
4.16 Audio SPL (dB re 20 μPa) at 4 kHz generated by a steerable PAL generating dual beams at 70° and 110° (denoted by dashed lines), where the size of the phased array PAL is 0.1 m, the size of PAL units is 0.01 m, and their center separation is 0.0125 m. 76
4.17 Audio SPL (dB re 20 μPa) at 4 kHz at different angles generated by a steerable PAL generating dual beams at 70° and 110°, where the size of the phased array PAL is 0.1 m, the size of PAL units is 0.01 m, and their center separation is 0.0125 m. Red hollow square, convolution model; blue hollow circle, cylindrical expansion at 4 m; green solid square, convolution model from [105]; Purple solid circle, measurement from [105]. 76

5.1 A PAL radiating sound (a) in free field or (b) to an infinitely large reflecting surface with an incident angle θ. 81
5.2 Audio SPL (dB re 20 μPa) at 1 kHz generated by: (a) the original PAL in free field; (b) the image PAL with respect to the reflecting surface; and (c) the PAL near a rigid reflecting surface. 83
5.3 Audio SPL (dB re 20 μPa) at 1 kHz where (a), (b), and (c) are the incident, reflected, and total sound radiated by a piston source, respectively, and (d), (c), and (f) are the incident, reflected, and total sound radiated by a 5-channel end-fire array, respectively. 84
5.4 Audio SPL (dB re 20 μPa) of four components generated by the PAL at 30° incidence near a rigid reflecting surface with a distance of 1 m at 1 kHz: (a) and (b) the audio sound generated by the incident ultrasound and its reflection, respectively; (c) and (d) the audio sound generated by the reflected ultrasound and its reflection, respectively. 84

5.5 Audio SPL (dB re 20 μPa) generated by the original PAL and the image PAL, and the total sound fields with different distance between the PAL and the reflecting surface. (a-c) the distance is 2 m, and (d-f) the distance is 4 m. 85

5.6 Audio SPL (dB re 20 μPa) generated by the original PAL and its image, and the total sound field with different sound absorption coefficient of the reflecting surface. (a-c) are for ultrasound absorption coefficient of 0.5; and (d-f) are for ultrasound absorption coefficient of 0.9. 85

5.7 Audio SPL (dB re 20 μPa) generated by the PAL, its image and the total sound field at 30° incidence near a rigid reflecting surface with $D = 1\,\mathrm{m}$. (a-c) are for the ultrasound frequencies of 100 kHz and 101 kHz, (d-f) are for the ultrasound frequencies of 200 kHz and 201 kHz. 86

5.8 Sketch of the experimental setup when a PAL radiates toward ground with and without a cotton sheet. 87

5.9 Photos of the experimental setups when different loudspeakers radiate toward the ground without the cotton sheet: (a) the PAL, (b) the traditional omnidirectional loudspeaker, and (c) the horn loudspeaker, and with the cotton sheet: (d) the PAL, (e) the traditional omnidirectional loudspeaker, and (f) the horn loudspeaker. 87

5.10 Measured audio SPL (dB re 20 μPa) at 1 kHz generated by different loudspeakers at 30° incidence without the cotton sheet for: (a) the PAL, (b) the traditional omnidirectional loudspeaker and (c) a horn loudspeaker, and with the cotton sheet on the ground for: (d) the PAL, (e) the traditional omnidirectional loudspeaker, and (f) a horn loudspeaker. 89

5.11 Sketch for a PAL near a thin partition. 90

5.12 Audio SPL (dB re 20 μPa) at $f_\mathrm{a} = 1\,\mathrm{kHz}$ with a 50 μm thick aluminum partition and a source at $z_\mathrm{s} = -1\,\mathrm{m}$ without the partition for the: (a) PAL, (b) point monopole, (c) end-fire array, and with the partition for the: (d) PAL, (e) point monopole, (f) end-fire array. The lower ultrasound frequency $f_2 = 60\,\mathrm{kHz}$. 96

5.13 Insertion loss of sound radiated by different sources through an aluminum partition with a thickness of 50 μm. 96

5.14 Audio SPL (dB re 20 μPa) at $f_\mathrm{a} = 1\,\mathrm{kHz}$ with a 1 mm thick polyester fibre blanket partition and a source at $z_\mathrm{P} = -1\,\mathrm{m}$: (a) for a PAL; (b) for a point monopole; (c) an end-fire array. 97

5.15 Insertion loss of sound radiated by different sources through a polyester fiber blanket partition with a thickness of 1 mm. 97

5.16 Sketch of the experimental setups; a photo of (b) a 60-channel microphone array; (c) the PAL and the aluminum foil; (d) the traditional loudspeaker (point monopole) and the aluminum foil; (e) the PAL and the cotton sheet; (f) the horn loudspeaker (traditional directional source) and the cotton sheet. 98

5.17 The model of the horn loudspeaker used in Virtual.Lab Acoustics. 100

5.18 Audio SPL (dB re 20 μPa) at 1 kHz with different sound sources at $z = -1\,\mathrm{m}$ and $x = 0$ above the ground with a height of 1.9 m in the simulations: (a) the PAL, (b) the traditional loudspeaker, and (c) the horn loudspeaker, and in the experiments: (d) the PAL, (e) the traditional loudspeaker, and (f) the horn loudspeaker. 100

5.19 Experimental results of the audio SPL (dB 20 re μPa) at 1 kHz with different sound sources at $z = -1\,\mathrm{m}$ and $x = 0$ above the ground with a height of 1.9 m with a sheet of aluminum foil for: (a) the PAL, (b) the traditional loudspeaker, and (c) the horn loudspeaker, and with a cotton sheet for: (d) the PAL, (e) the traditional loudspeaker, and (f) the horn loudspeaker. 101

5.20 Theoretical prediction audio SPL (dB re 20 μPa) corresponding to Figs. 5.19(a) and (d), i.e. the sound fields at 1 kHz with the PAL at $z = -1\,\mathrm{m}$ and $x = 0$ with (a) a sheet of aluminum foil and (b) a cotton sheet. 102

5.21 Sketch of a circular PAL near a sphere. 103

5.22 Audio SPL (dB re 20 μPa) at different zenith angles with the distance of d to the center of the sphere generated by a PAL: (a) $d = 0.1\,\mathrm{m}$ at 500 Hz, (b) $d = 1.0\,\mathrm{m}$ at 500 Hz, (c) $d = 0.1\,\mathrm{m}$ at 1 kHz, (d) $d = 1.0\,\mathrm{m}$ at 1 kHz, (e) $d = 0.1\,\mathrm{m}$ at 2 kHz, and (f) $d = 1.0\,\mathrm{m}$ at 2 kHz. Solid line, Kuznetsov equation; dashed line, Westervelt equation. 106

5.23 Audio SPL (dB re 20 μPa) generated by a PAL without the sphere for (a), (c), and (e) and with the sphere for (b), (d), and (f) at 500 Hz for (a) and (b), 1 kHz for (c) and (d), and 2 kHz for (e) and (f). 107

5.24 Audio SPL (dB re 20 μPa) generated by a traditional loudspeaker without the sphere for (a), (c), and (e) and with the sphere for (b), (d), and (f) at 500 Hz for (a) and (b), 1 kHz for (c) and (d), and 2 kHz for (e) and (f). 109

5.25 Audio SPL (dB re 20 μPa) at different zenith angles with a distance of 1.0 m to the center of the sphere generated by a PAL for (a), (c), and (e) and a traditional loudspeaker for (b), (d), and (f) with and without the sphere at 500 Hz for (a) and (b), 1 kHz for (c) and (d), and 2 kHz for (e) and (f). Solid line, with sphere; dashed line, without sphere. 110

5.26 Audio SPL (dB re 20 μPa) at the zenith angle $\theta = 135°$ and the radius of 1.0 m generated by (a) a PAL and (b) a traditional loudspeaker from 100 Hz to 4 kHz. Solid line, with sphere; dashed line, without sphere. 111

5.27 Front view of the experiment setup. 111

5.28 Photos of the experiment setup: (a) a PAL; (b) a solid wooden sphere; (c) a 60-channel microphone array; (d) the measurement system. 112

5.29 Audio SPL (dB re 20 μPa) generated by a PAL: (a) without the sphere at 1 kHz; (b) with the sphere at 1 kHz; (c) without the sphere at 2 kHz; and (d) with the sphere at 2 kHz. 113

5.30 Measured audio SPL (dB re 20 μPa) at the microphone located at $x = 0.7\,\mathrm{m}$ and $z = -0.7\,\mathrm{m}$ generated by a PAL at the frequencies from 315 Hz to 4 kHz: (a) the SPL with and without the sphere; solid line with sphere; dashed line, without sphere; (b) the SPL increment by simulations and experiments; solid line, experiment; dashed line, simulation. 113

6.1 Schematic diagram of the experiment setup. 118
6.2 A photo of the experiment setup in the semi-anechoic room (left); and a photo of the LDV error sensing system (right). 118
6.3 SPL (dB re 20 μPa) measured by the left ear simulator of the HATS with and without ANC, where the secondary source was (a) a traditional loudspeaker and (b) a PAL, at a distance of 1 m from the error sensing point. 119
6.4 SPL (dB re 20 μPa) at point #2 when the secondary source was (a) a traditional loudspeaker and (b) a PAL; and at point #7 when the secondary source was (c) a traditional loudspeaker and (d) a PAL. The distance between the secondary source and the error point was 1 m. 119
6.5 Overall noise reductions from 1 kHz to 6 kHz (a) at the left ear of the HATS, and at the evaluation points, where the secondary source was (b) a traditional loudspeaker and (c) a PAL. 120
6.6 Sketch of an ANC system where the primary and secondary sources are located on the xOy plane. 121
6.7 Sketch of an ANC system where the primary and secondary sources are located in three-dimensional space. 122
6.8 Audio SPL (dB re 20 μPa) at 1 kHz (a) for the primary noise comes from the direction $\varphi_{\mathrm{p}} = 22.5°$, (b) under the optimal control with 8 PALs, and (c) under the optimal control with 8 point monopoles. 124
6.9 The quiet zone size and energy gain as a function of the primary source azimuthal angles at 1 kHz for different numbers of secondary sources: (a) and (b) the quiet zone size created by PALs and point monopoles, respectively; (c) and (d) the energy gain caused by PALs and point monopoles, respectively. Red circles, $N_{\mathrm{s}} = 1$; blue squares, $N_{\mathrm{s}} = 4$; green triangles, $N_{\mathrm{s}} = 8$; purple diamonds, $N_{\mathrm{s}} = 16$; dashed line, $\lambda/10$. 125
6.10 Audio SPL (dB re 20 μPa) at 1 kHz (a) generated by 8 point monopoles randomly located on the xOy plane, (b) under the optimal control with 8 PALs, and (c) under the optimal control with 8 point monopoles. 126
6.11 For random 2D primary sound fields under the optimal control with the 2D secondary sources, (a), (b), and (c) the quiet zone size when the secondary source number is 4, 8, and 16, respectively; (d), (e), and (f) the energy gain when the secondary source number is $N_{\mathrm{s}} = 4, 8$, and 16, respectively, where the value and error bar are the mean value and standard deviation of 100 random trials, and λ is the wavelength. Red circles, PAL; blue squares, monopole; dashed lines, 0.75λ, 1.5λ, and 3λ for (a), (b), and (c), respectively. 127

6.12 For random 2D primary sound fields under the optimal control with 2D secondary sources at 1 kHz and 2 kHz: (a) the quiet zone size as a function of secondary source number; (b) the energy gain as a function of secondary source number. Red circles, PAL at 1 kHz; blue squares, monopole at 1 kHz; green triangles, PAL at 2 kHz; purple diamonds, monopole at 2 kHz. 127

6.13 For random 3D primary sound fields under the optimal control with eight 2D secondary sources in the plane xOy, (a) the quiet zone size as a function of frequency; (b) the energy gain as a function of frequency. Red circles, PAL; blue squares, monopole; dashed line, 0.75λ. 128

6.14 For random 3D primary sound fields controlled by 2D secondary sources at 1 kHz and 2 kHz: (a) the quiet zone size as a function of secondary source number; (b) the energy gain as a function of secondary source number. Red circles, PAL at 1 kHz; blue squares, monopole at 1 kHz; green triangles, PAL at 2 kHz; purple diamonds, monopole at 2 kHz; dashed line, $0.095\lambda N_\mathrm{s}$. 129

6.15 For random 3D primary sound fields under the optimal control with the 3D secondary sources, (a) the quiet zone size as a function of frequency; (b) the energy gain as a function of frequency. Red circles, PAL when $N_\mathrm{s} = 8$; blue squares, monopole when $N_\mathrm{s} = 8$; green triangles, PAL when $N_\mathrm{s} = 20$; purple diamonds, monopole when $N_\mathrm{s} = 20$; dashed line, λ; dotted line, 2.2λ. 129

6.16 For random 3D primary sound fields under the optimal control with the 3D secondary sources, (a) the quiet zone size as a function of secondary source number; (b) the energy gain as a function of secondary source number. Red circles, PAL at 1 kHz; blue squares, monopole at 1 kHz; green triangles, PAL at 2 kHz; purple diamonds, monopole at 2 kHz; dashed line, $0.55\lambda\sqrt{N_\mathrm{s}}$. 130

6.17 Top view of the experiment setup in a full anechoic room. 131

6.18 Photo of the experiment setup in a full anechoic room. 131

6.19 Experimental measurements and predictions of SPL (dB re 20 μPa) for 2 and 4 PALs as secondary sources. Red circles, experimental measurements for 4 PALs; Blue squares, experimental measurements for 2 PALs; dashed line, predictions 0.75λ; dash dotted line, predictions 0.38λ. 133

B.1 Attenuation coefficient at 1 atm and 293.15 K (20°C) are shown as a function of frequency at different relative humidity (RH). 146

B.2 Absorption distance of PALs at 1 atm and 293.15 K (20°C) are shown as a function of frequency at different relative humidity. 147

List of Tables

2.1 Commercial ultrasonic emitters. 15
2.2 Commercial products of the PAL. 17

3.1 The parameters used in the simulations. 34

4.1 Comparison of the calculation time when the GHF is calculated by the built-in function “hypergeom” of MATLAB R2018a (based on a personal computer with a 2.5 GHz CPU) and the proposed method. 52
4.2 The parameters used for validating the proposed method in Sec. 4.2 57
4.3 Comparison of the calculation time of the SWE method and the direct integration method. 59
4.4 The first maxima of the ultrasound pressure amplitude and the transition distances from the near field to the Westervelt far field for several sets of parameters. 62
4.5 The transition distance from the Westervelt far field to the inverse-law far field for the parameters. 64

List of Abbreviations

Abbreviation	**Full**
2D	Two-dimensional
3D	Three-dimensional
ANB	Active Noise Barrier
ANC	Active Noise Control
BEM	Boundary Element Method
CWE	Cylindrical Wave Expansion
CPU	Central Processing Unit
DFW	Difference Frequency Wave
DSB	Double Sideband
DSP	Digital Signal Processor
FDTD	Finite-Difference Time-Domain
FEM	Finite Element Method
FFT	Fast Fourier Transform
FPGA	Field Programmable Gate Array
FxLMS	Filtered-x Least Mean Square
GBE	Gaussian Beam Expansion
GHF	Generalized Hypergeometric Function
IL	Insertion Loss
KZK	Khokhlov-Zabolotskaya-Kuznetsov
LDV	Laser Doppler Vibrometer
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor
NR	Noise Reduction
PAA	Parametric Acoustic Array
PAL	Parametric Array Loudspeaker
PMUT	Piezoelectric Micromachined Ultrasonic Transducer
PNC	Passive Noise Control
PWE	Plane Wave Expansion
RH	Relative Humidity
SSB	Single Siddeband
SWE	Spherical Wave Expansion
SPL	Sound Pressure Level
SRT	Square Root
VSB	Virtual Sound Barrier

List of Symbols

Symbol	Description
a	radius of the circular transducer or PAL
A_n	GBE coefficients
B_n	GBE coefficients
$B_n(\cdot)$	in Appendix A: a Bessel function $J_n(\cdot)$ or a Hankel function $H_n(\cdot)$
$\mathrm{d}^3\mathbf{r}$	the volume element $\mathrm{d}x\mathrm{d}y\mathrm{d}z$
$\mathcal{D}(\vartheta, k)$	the directivity at the angle of ϑ and the wavenumber of k
$\exp(x)$	exponential function
f_1, f_2	higher and lower ultrasonic frequencies, respectively. $f_1 > f_2$
f_a	audio frequency
f_u	average ultrasonic frequency
$g(\mathbf{r}_1, \mathbf{r}_2, k)$	the Green's function in a free field between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ at a wavenumber of k
$g_{2\mathrm{D}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k)$	the 2D Green's function in a free field between the points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ at a wavenumber of k
$G(\mathbf{r}_1, \mathbf{r}_2, k)$	the Green's function in an arbitrary acoustic environment between the points $\mathbf{r}_1$ and $\mathbf{r}_2$ at a wavenumber of k
$\mathrm{j}_n(z)$	spherical Bessel function of the first kind with an argument of z of order n
$J_n(z)$	Bessel function of the first kind with an argument of z of order n
$\mathcal{J}$	cost function for the ANC system
k_1, k_2	the wavenumber of ultrasound at higher and lower ultrasonic frequencies, respectively
k_a	the wavenumber of audio sound at the frequency of f_a
$\mathrm{h}_n(z)$	spherical Hankel function of the first kind with an argument of z of order n
$H_n(z)$	Hankel function of the first kind with an argument of z of order n
$\mathbf{H}_n(z)$	Struve function with an argument of z of order n
i	imaginary unit
i	1 or 2 indexing the ultrasound when it is used as the subscript
$\mathbf{I}$	the identity matrix
l	Index distinguishing different modes
$\mathscr{L}$	Lagrangian density
m	Index distinguishing different modes
n	Index distinguishing different modes
N_e	the number of error sensors in ANC systems
N_p	the number of primary sources in ANC systems
N_s	the number of secondary sources in ANC systems
O	orgin of coordinate systems
$p(\mathbf{r}, k)$	the sound pressure field at the field point $\mathbf{r}$ and the wavenumber k
P	ambient pressure

$q(\mathbf{r})$	the virtual source denstiy at point $\mathbf{r}$ for audio sound generated by a PAL
$\mathbf{r}$	field point position; vector from origin to point with coordinates (x, y, z)
$\mathbf{r}_\mathrm{s}$	position on the transducer surface
$r_{\mathrm{s},<}$	$\min(r, r_\mathrm{s})$
$r_{\mathrm{s},>}$	$\max(r, r_\mathrm{s})$
$r_{\mathrm{v},<}$	$\min(r, r_\mathrm{v})$
$r_{\mathrm{v},>}$	$\max(r, r_\mathrm{v})$
$\mathcal{R}$	the radial component for audio sound generated by a PAL
$\mathscr{R}$	Rayleigh distance; pressure-amplitdue reflection coefficient
S	the radiation surface of a planar source
t	time
$\mathscr{T}$	pressure amplitude transmission coefficient
$\mathbf{v}(\mathbf{r}, k)$	the velocity field (also known as acoustic particle velocity vector) at the field point $\mathbf{r}$ and the wavenumber k
v_x, v_y, v_z	the components of the velocity field $\mathbf{v}$ in x, y, and z directions, respectively
v_r, v_θ, v_φ	the components of the velocity field $\mathbf{v}$ in radial r, zenithal θ, and azimuathal φ directions, respectively
v_ρ	the component of the velocity field $\mathbf{v}$ in polar radial ρ direction
V	the volume of the virtual audio sound source
(x, y, z)	rectangular (also known as Cartesian) coordinates

Greek letters

Symbol	**Description**
α	pure-tone sound absorption coefficient for atmospheric absorption, describing amplitude decay with distance, Np/m
δ	the sound diffusivity parameter
δ_{mn}	Kronecker delta function; the value is 0 if $m \neq n$, and 1 if $m = n$
ϵ	error function
ε_n	Neumann factor
θ	zenith (also known as polar) angle in spherical coordinates
$\Gamma(\cdot)$	Gamma function
ρ	the polar radius in cylindrical coordinates (ρ, φ, z)
ρ_0	linear ambient density of air
$\tilde{\rho}$	the fluid density
$\boldsymbol{\rho}$	the transverse coordinate vector (x, y)
τ	retarded time
$\Phi(\mathbf{r}, k)$	the velocity potential field at the field point $\mathbf{r}$ and the wavenumber k
ω	angular frequency
ω_1, ω_2	the angular frequency of ultrasound
ω_a	the angular frequency of audio sound

Other symbols

Symbol	Description
$\boldsymbol{\nabla}$	Gradient operator
$\boldsymbol{\nabla}^2$	Laplace operator
$\boldsymbol{\nabla}_\perp^2$	the transverse Laplace operator
$n!$	factorial of n
$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$	Wigner $3j$ symbol

Constants

Symbol	Description
$c_0 = 343\,\mathrm{m/s}$	linear speed of sound
$\rho_0 = 1.21\,\mathrm{kg/s^3}$	linear ambient density of air
$\beta = 1.2$	the nonlinear coefficient in air
$\pi = 3.1415926$	Archimedes's constant