# TIE-BREAKING AND EFFICIENCY IN THE LABORATORY SCHOOL CHOICE†

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Abstract. In school choice problems with weak priorities, the deferred acceptance (DA) mechanism may produce inefficient stable matchings due to tie-breaking. The stable-improvement-cycles (SIC) and choice-augmented deferred acceptance (CADA) mechanisms were proposed to remedy inefficiencies but they are manipulable. In a simple environment, we theoretically and experimentally analyze students' strategic behavior when DA, SIC, and CADA are implemented. We show that obtaining the efficiency gain relative to DA crucially depends on whether students report their preferences truthfully in SIC and whether they play a particular equilibrium strategy in CADA. Our laboratory experiment reveals that (i) non-negligible degrees of untruthful reporting are observed but they are not a major drawback for practical efficiency improvements of the mechanisms we consider; (ii) SIC achieves gains from trade whenever they exist, both on and off the equilibrium paths; and (iii) the additional layer of equilibrium coordination required by CADA makes it harder for CADA to fully produce the promised welfare advantage relative to DA. These findings are robust to various environments.

## 1. Introduction

The school choice problem is a pair of a preference profile of students and a priority profile of schools. Each student is to be matched with at most one school, and the capacity constraint of a school determines the number of students it may admit. The school choice mechanism is "a systematic way of selecting a matching for a given school choice problem" (Kesten, 2010). In a seminal paper, Abdulkadiroğlu and Sönmez (2003) approach

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the school choice problem from a mechanism-design perspective, noting that the mechanisms used in practice have shortcomings. They propose two well-known mechanisms as alternatives: the (student-proposing) deferred acceptance (DA) mechanism based on Gale and Shapley (1962) and the top-trading-cycles (TTC) mechanism based on Shapley and Scarf (1974). Both DA and TTC have several desirable properties, most notably achieving fairness (stability) and Pareto efficiency, respectively. Since the pioneering work of Abdulkadiroğlu and Sönmez (2003), DA has taken an early lead in the recent reform of school choice systems in several major cities in the US, including Boston, New York City, and San Francisco, among others (Abdulkadiroğlu, Pathak and Roth, 2009; Abdulkadiroğlu, Pathak, Roth and Sönmez, 2005). Experimental data (e.g., Chen and Sönmez, 2006) have also played a crucial role in policy-makers' decision making and provided useful guidelines for transition.

There are three major reasons why DA has been so successful in real-life school choice reform. First, DA outcomes are "stable". That is, they respect student priorities and are envy-free in the following sense. If a student $i$ prefers a school $s$ over his current assignment, then no other student with a lower priority at school $s$ than $i$ can be assigned to school $s$. Second, DA is strategy-proof, i.e., reporting true preferences is a weakly dominant strategy for each student. While strategy-proofness is often promoted as an incentive property, it also connects with fairness concerns in the context of school choice because a non-strategy-proof mechanism creates a disparity in opportunities between those who can skillfully game the system and those who cannot. Thus, strategy-proofness "levels the playing field" for all (Pathak and Sönmez, 2008). Third, DA selects the best possible outcome among all stable matches: the matching produced by DA Pareto dominates any other stable matching (for this reason, DA is also known as the student-optimal stable mechanism).

One critical condition that guarantees DA's promising performance is that schools' priorities are strict, i.e., at each school, no two students are given the same priority. In practice, however, school priorities not only admit ties but are very coarse. For example, there are only four priority classes in Boston. At each school, the top priority goes to the students who have a sibling attending that school and who live in the school's walk zone; the second priority goes to those who only have a sibling attending that school; the third priority goes to those who only live in the school's walk zone. Then, students in the same priority group are further ordered by a random lottery (Erdil and Ergin, 2008; Kesten, 2010). However, random tie-breaking makes DA inefficient. That is, DA may produce a matching that is Pareto-dominated (according to students' preferences) by another type of stable matching. In effect, tie-breaking constructs *artificial* strict priorities, and the matching obtained by applying DA to them is not guaranteed to be efficient among the matchings that are stable with respect to the original priorities with ties (although it is efficient among the matchings that are stable with respect to the constructed strict priorities). Still worse, the efficiency loss is significant in practice. Analyzing the data from the NYC school district, Abdulkadiroğlu et al. (2009) report that, on average, over 700 students could have been matched with more preferred schools without hurting others.

Two approaches have been taken to correct the efficiency loss in the presence of weak priorities (i.e., priorities admitting ties). The first approach, represented by the stable-improvement-cycles (SIC) mechanism of Erdil and Ergin (2008), relies only on *ordinal* preference rankings to pursue Pareto improvement.[1] On the other hand, the second approach, encapsulated in the choice-augmented deferred acceptance (CADA) mechanism of Abdulkadiroğlu et al. (2015), enriches students' message space by allowing each to submit a "target school", a school at which he wishes to be considered favorably when ties in priorities at that school are to be broken. The latter can be viewed as a device that transmits *cardinal* preference intensity, as the target school submission can be affected by the cardinal utilities derived from different schools. In Section 6 on related literature, we provide details on these mechanisms and explain how they mitigate the inefficiency problem due to random tie-breaking.

Unfortunately, both SIC and CADA lack strategy-proofness. That is, students have an incentive to misrepresent their preferences when participating in these mechanisms.[2] Because students' truth-telling behavior comes into question, the efficiency improvement achieved by SIC and CADA is no longer guaranteed in practice. This calls for a theoretical and experimental study that reveals students' manipulation at work and the extent of deviation from truth-telling.

Our objective is to experimentally investigate efficiency properties of the three mechanisms—DA, SIC, and CADA—when they are subject to participants' strategic behavior while participating in these mechanisms. The first step toward this goal lies in designing an environment that satisfies a number of conditions. First, an ideal environment should be rich enough to demonstrate key features of the three mechanisms and generate disparate matching outcomes for them. In particular, priorities should have ties, and DA should result in an inefficient matching with a positive probability. Second, the environment should be simple enough to permit experiments. Third, theoretical analysis

[1]Other mechanisms proposed in the literature to improve the efficiency properties of DA that belong to the first classification include EADAM (Kesten, 2010) and SEADAM (Tang and Yu, 2014). In our environment, EADAM and SEADAM are strategically equivalent to SIC (due to the fact that there is at most one Pareto improving cycle in our environment); thus, we do not explicitly consider them in our theoretical and experimental investigation.

[2]The lack of strategy-proofness is not specific to SIC and CADA. Indeed, in an environment with strict priorities, Kesten (2010) shows that there is no Pareto-efficient and strategy-proof mechanism that Pareto-dominates DA. Abdulkadiroğlu et al. (2009) further prove that, in an environment with weak priorities, there is no strategy-proof mechanism that Pareto-dominates DA.

of the environment should yield sharp predictions such that the experimental hypotheses are clear and easy to test. Finally, the magnitude of potential efficiency gain should be the same for CADA and SIC so that the comparison between the two mechanisms will be fair.

The simplest environment subject to the above considerations involves three students and three schools with one seat each. Each school gives the top priority to one (distinct) student and the bottom priority to the other two students. As we show in Online Appendix, there are only three preference profiles (up to permutations) of students for which the three mechanisms exhibit potentially varying performances in light of efficiency and manipulability. We test two of them in experiments; see Section 3 for a justification of our choice.

Our laboratory experiments rely on a geographical implementation of the three-student-three-school environment. We presented a map of an island that consists of three school districts indicated by different colors. In each district, there is one school and one student. Schools allocate priorities to students based on whether a student lives in the same district (i.e., has the same color). The cardinal preferences of a student depend on the distance between a school and his location. This implementation is not only natural but also serves as an easier environment for experimental subjects to comprehend and remember all the important components of the school choice problem.

In our experiments, we consider three mechanisms, DA, CADA, and SIC, in four environments. Our **baseline (B)** environment implements the first (called "Profile 1" in Section 3) of the three preference profiles amenable to experimental analysis. The three additional environments are considered to validate our experimental findings. The **replica (R)** environment takes the first preference profile with a variation that each district has two identical students; thus, it implements a six-student-three-school environment. The **cardinal (C)** environment also takes the same preference profile but cardinal utilities students attach to schools are different. This cardinal environment is designed to give CADA its best shot because CADA is meant to provide a channel through which not only ordinal but also cardinal preferences affect tie-breaking. The **new (N)** environment implements a new preference profile (called "Profile 2" in Section 3), which is more favorable to CADA and less so to SIC in terms of the possibility of realizing efficiency gains. The replica, cardinal, and new environments mainly serve the purpose of a robustness check for our results from the baseline environment to a larger economy, a cardinal preference amendment, and an ordinal preference change, respectively.

Our experimental data from the baseline environment presents a clear picture. First, the data reveals that inefficiency created by random tie-breaking in DA was substantial, with more than 45% of the matching outcomes being inefficient. Second, targeting choices made by the vast majority of subjects participating in CADA are consistent with our equilibrium analysis. As a result, CADA attained almost no efficiency gain relative to DA. In particular, the dominated-strategy equilibrium of CADA that Pareto improves upon the DA outcome hardly emerged in the experiment. Targeting in CADA can be viewed as a coordination device among subjects and the device is more likely to be effective in a small-sized environment. It is quite surprising that the coordination did not come into play even in the three-student experiment. Third, subjects participating in SIC reported preferences largely in line with what the Nash equilibria in undominated strategies predicted. Thus, SIC sufficiently improved upon the DA outcome. The cardinal environment implemented a slightly different set of cardinal utilities in order to help subjects coordinate on a Pareto-efficient but "dominated-strategy" Nash equilibrium in CADA. However, it did not help reduce the disparity in efficiency performances of SIC and CADA. Further, the disparity became more pronounced in the replica environment with six students.

Our data from the new environment deepens our understanding of the main source of the observed disparate efficiency performances of the three mechanisms and thus addresses a concern about external validity of our main findings. Even though CADA's unique equilibrium outcome in this environment predicts that CADA has a full efficiency advantage relative to DA, the advantage was not fully achieved in the laboratory. Uniqueness of the equilibrium outcome does not imply that everyone successfully uses targeting to coordinate on a particular Pareto efficient equilibrium. The additional layer of coordination (equilibrium targeting) required by CADA makes the mechanism more vulnerable to strategic uncertainty. Multiplicity of equilibria embedded in the first preference profile is a fundamental source of strategic uncertainty, but uniqueness of the Nash equilibrium outcome embedded in the new environment does not completely get rid of the issue. In case of SIC, the stringent equilibrium requirement of untruthful reporting for some student indeed leads to a substantial deviation from the equilibrium reporting in the new environment. However, the observed deviation did not undermine the efficiency advantage of SIC relative to DA because stable improvement cycles are implemented not only on the equilibrium path but also off the equilibrium path. That is, SIC achieves gains from trade whenever they exist, both on and off the equilibrium path.

To summarize, we theoretically and experimentally investigate efficiency properties of DA, SIC, and CADA in a simple setup where these algorithms may perform differently. We see our paper as a first step toward understanding students' equilibrium behavior in the practically relevant case of weak priorities. Our results highlight that efficiency gains based on non-strategy-proof mechanisms may or may not materialize in practice.[3]

[3] Afacan et al. (2022) is another recent experimental study that investigates stability and efficiency properties of a non-strategy-proof mechanism in the school choice environment.

In other words, although different non-strategy-proof mechanisms could result in full efficiency with respect to "reported preferences," they may result in different efficiency performances with respect to "true preferences." More broadly, we demonstrate that whether a mechanism succeeds in practice hinges on two key aspects: (i) theoretical properties of a mechanism under study (such as undominated strategies and multiplicity of equilibria) and (ii) empirical data obtained in a carefully designed laboratory experiment.

The paper is organized as follows. In Section 2, we introduce the model. In Section 3, we provide our theoretical results. In Section 4, we introduce our experimental design. In Section 5, we provide our experimental results. In Section 6, we discuss the related literature. Section 7 concludes. Appendix A provides proofs of the theoretical results. Appendix B provides experimental instructions for one of the treatments. Appendix C provides nonparametric tests results. Finally, Online Appendix contains additional results and omitted proofs.

## 2. The Model

The school choice problem consists of students and schools that need to match with each other. Let $\boldsymbol{N} \equiv \{1, \cdots, n\}$ be a finite set of students and $\boldsymbol{A} \equiv \{a_1, \cdots, a_{|A|}\}$ a finite set of schools ($|N|, |A| \geq 2$). Each student $i \in N$ has a linear preference relation $\boldsymbol{R_i}$ over $A$.[4] Let $\boldsymbol{P_i}$ and $\boldsymbol{I_i}$ be the strict preference and indifference relations associated with $R_i$, respectively. Let $\boldsymbol{\mathcal{R}}$ be the set of all linear preference relations over $A$. Let $\boldsymbol{R} \equiv (R_i)_{i \in N} \in \mathcal{R}^N$ be a preference profile. Each school $a \in A$ has a complete and transitive priority relation $\boldsymbol{\succsim_a}$ over $N$ and a capacity of $\boldsymbol{q_a} \in \mathbb{N}$ seats. Say that $\succsim_a$ is **weak** if it admits a tie (i.e., for some distinct $i, j \in N$, $i \sim_a j$); it is **strict** otherwise. When $q_a \geq n$, school $a$ is effectively unlimited in capacity and can thus be interpreted as the **null school**, the option of being unassigned. Let $\succsim \equiv (\succsim_a)_{a \in A}$ be a priority profile and $\boldsymbol{q} \equiv (q_a)_{a \in A}$ a capacity profile. The **school choice problem** is then described by a list $(N, A, R, \succsim, q)$, but since we keep $(N, A)$ fixed throughout, we treat $\boldsymbol{\pi} \equiv (R, \succsim, q)$ as a **school choice problem** or simply a **problem**. Let $\boldsymbol{\Pi}$ be the set of all problems.

Let $\boldsymbol{\Delta(A)}$ be the set of all lotteries (probability distributions) over schools. For a given problem $\pi \equiv (R, \succsim, q) \in \Pi$, a (probabilistic) **assignment** is a profile of lotteries $x \equiv (x_i)_{i \in N} \in [\Delta(A)]^N$, where (i) for each $i \in N$, $x_i \equiv (x_{ia})_{a \in A} \in \Delta(A)$, called **student *i*'s (assignment) lottery**, with $\boldsymbol{x_{ia}}$ representing the probability of student $i$ being assigned to school $a$; and (ii) for each $a \in A$, $\sum_{i \in N} x_{ia} \leq q_a$. We refer to (i) and (ii) as the **feasibility conditions**. Let $\boldsymbol{X}$ be the set of all feasible assignments. An assignment is **deterministic** if all of its entries are $0$ or $1$. For deterministic assignments, we informally

[4] A preference relation is linear if it is complete, transitive, and anti-symmetric.

write $x = \begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$ for the assignment that allocates, with probability 1, schools $a$, $b$, and $c$ to students 1, 2, and 3, respectively; similarly, if $x_{ia} = 1$, we write $x_i = a$. It is well known that each non-deterministic assignment can be expressed as a convex combination of (i.e., a lottery over) deterministic assignments (Birkhoff, 1946; Von Neumann, 1953; Budish et al., 2013). A (student assignment) **mechanism** is a mapping $\boldsymbol{\varphi : \Pi \to X}$ that associates with each problem $\pi \equiv (R, \succsim, q) \in \Pi$ an assignment $\varphi(\pi) \equiv (\varphi_{ia}(\pi))_{i \in N, a \in A} \in X$.

In real-life school choice problems, the input information for schools, $(\succsim, q)$, is publicly known and assignment mechanisms rely on students' reports of their preferences. Further, most mechanisms are ordinal in the sense that they determine assignments based solely on ordinal preferences (preference rankings). In principle, however, a mechanism may solicit additional information from students that can guide its choice of an assignment. We capture the latter in messages. Let $\boldsymbol{M}$ be the common message space for all students and for each $i \in N$, let $\boldsymbol{m_i} \in M$ be **student $\boldsymbol{i}$'s message** (in addition to preferences). The message space can include cardinal utilities for schools, or more simply, "target schools" at which student $i$ wishes favorable treatment over others in some prescribed circumstances. When messages in $M$ are allowed, a **problem** is a list $\boldsymbol{(R, m, \succsim, q)}$, where $m \equiv (m_i)_{i \in N} \in M^N$ is a message profile and a **mechanism** associates an assignment with each problem.

When school priorities are weak, ties are often broken randomly and students receive nondegenerate lotteries as assignments, which cannot be evaluated by ordinal preference relations. We assume that students are indeed endowed with von Neumann-Morgenstern (vNM) utility functions. For each student $i \in N$, let $\boldsymbol{u_i : \Delta(A)} \to \mathbb{R}$ be student $i$'s vNM utility function over lotteries (over schools). Given an assignment $x \in X$, student $i$ attaches a utility of $u_i(x_i)$ to assignment $x$. Since $u_i$ is of expected utility form, we identify it with a profile $(u_i(a))_{a \in A}$ of cardinal utilities (where $a$ is the degenerate lottery giving $a$ for sure). Say that $\boldsymbol{R_i}$ **is consistent with** $\boldsymbol{u_i}$, or alternatively, $\boldsymbol{u_i}$ **induces** $\boldsymbol{R_i}$ if the following holds: for all $a, b \in A$, $a \, R_i \, b$ if and only if $u_i(a) \geq u_i(b)$. Let $\boldsymbol{u} \equiv (u_i)_{i \in N}$ be a utility profile. Since utilities contain finer information on preferences than preference relations do, we write $\boldsymbol{(u, \succsim, q)}$ and $\boldsymbol{(u, m, \succsim, q)}$ (if additional messages are allowed) for the problems where students have preference relations induced by $u$.

In the context of school choice and matching at large, two concepts play an important role, as we test the desirability of an assignment or a mechanism. The first is stability. Say that a deterministic assignment $x \in X$ is **stable for** $\boldsymbol{(R, \succsim, q)}$ if (i) it is non-wasteful (i.e., for each $i \in N$ and each $a \in A$, $\sum_{j \in N} x_{ja} < q_a$ implies $x_i \, R_i \, a$)[5]; and (ii) it does not violate any student's priority (i.e., there do not exist $i, j \in N$ such that $x_j \, P_i \, x_i$ and $i \succ_{x_j} j$). A

[5]Non-wastefulness implies individual rationality: for each $i \in N$ and each null school $a \in A$, $x_i \, R_i \, a$.

probabilistic assignment is **stable for $(\boldsymbol{R}, \succsim, \boldsymbol{q})$** if it can be written as a convex combination of stable deterministic assignments.[6] The second test is efficiency. Given a problem $(u, \succsim, q)$ with vNM utilities and assignments $x, y \in X$, say that **$\boldsymbol{x}$ Pareto-dominates $\boldsymbol{y}$ for $(\boldsymbol{u}, \succsim, \boldsymbol{q})$** if for each $i \in N$, $u_i(x_i) \geq u_i(y_i)$, with a strict inequality for at least one $i$. An assignment is (Pareto) **efficient for $(\boldsymbol{u}, \succsim, \boldsymbol{q})$** if it is not Pareto-dominated by any other assignment.[7]

### 2.1. Three Mechanisms.

We focus on three mechanisms: the deferred acceptance mechanism and two variants thereof.

*The deferred acceptance mechanism.* Originating in Gale and Shapley (1962), the deferred acceptance mechanism has been widely used for two-sided matching problems, such as marriage problems and college admission problems, when both sides have strict preferences. In school choice problems with weak priorities, we may first eliminate ties in priorities and then apply the traditional DA. Given $a \in A$ and school $a$'s priority relation $\succsim_a$ over students, let $\boldsymbol{T}(\succsim_{\boldsymbol{a}})$ be the set of strict priority relations obtained by breaking ties in $\succsim_a$; i.e., $\succsim'_a \in T(\succsim_a)$ if and only if for all $i, j \in N$, $i \succ_a j$ implies $i \succ'_a j$. We call the priority relations in $T(\succsim_a)$ the **tie-breakers for $\succsim_{\boldsymbol{a}}$**. For a profile $\succsim \equiv (\succsim_a)_{a \in A}$ of priority relations, define $\boldsymbol{T}(\succsim) \equiv \prod_{a \in A} T(\succsim_a)$. To define the **deferred acceptance (DA) mechanism**, we apply the following algorithm to each problem $(R, \succsim, q)$.

**Step 1:** Choose a tie-breaker profile $\succsim' \in T(\succsim)$ uniformly at random and consider the problem $(R, \succsim', q)$ with strict priorities.

**Step 2:** Each student $i \in N$ applies to his most preferred school according to $R_i$. For each $a \in A$, among those students who apply to school $a$, school $a$ tentatively accepts up to $q_a$ students with the highest $\succsim'_a$-priority and rejects the rest.

**Step $\boldsymbol{t}$ $(\boldsymbol{t \geq 3})$:** Each student $i \in N$ who was rejected in Step $t-1$ applies to his next most preferred school according to $R_i$. For each $a \in A$, among those students who were tentatively accepted to school $a$ in Step $t-1$ or who apply to school $a$ in this step, school $a$ tentatively accepts up to $q_a$ students with the highest $\succsim'_a$-priority and rejects the rest.

The algorithm terminates after a step in which no additional student is rejected, and the tentative assignment from the last step is finalized. Since the ties in $\succsim$ are broken randomly in Step 1, the finalized assignment is random, and DA selects that assignment. Let us denote it by $DA(R, \succsim, q)$.

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[6]Kesten and Ünver (2015) propose three notions of stability that extend the traditional notion taking into account ties in school priorities and the probabilistic nature of assignments. The stability notion we adopt here is ex post stability in Kesten and Ünver (2015).

[7]A similar concept can be defined for problems $(R, \succsim, q)$ that have only ordinal information on preferences.

*The stable-improvement-cycles mechanism.* An important drawback of DA is that ties may sometimes be broken in a manner that causes inefficiency of the resulting assignment. Erdil and Ergin (2008) show that whenever a stable assignment, including the DA assignment, is Pareto-dominated, we can allow a subset of students to swap assigned schools among themselves, which Pareto-improves upon the welfare of those students. We next define a mechanism exploiting this idea.

Fix a problem $(R, \succsim, q)$ and a deterministic assignment $x$. For each $(i, a) \in N \times A$, say that **student $i$ desires school $a$ at $x$** if $a \, P_i \, x_i$. Let $D(a)$ be the set of highest $\succsim_a$-priority students among those who desire school $a$ at $x$. A sequence of distinct students $i_1, \cdots, i_k$ constitutes a **stable improvement cycle** (with respect to $(R, \succsim, q)$ and $x$) if for each $h \in \{1, \cdots, k\}$, (i) $x_{i_h}$ is not the null school; (ii) student $i_h$ desires school $x_{i_{h+1}}$ (modulo $k$); and (iii) $i_h \in D(x_{i_{h+1}})$. If students $i_1, \cdots, i_k$ form a stable improvement cycle, there is a simple way of Pareto-improving the given assignment $x$: for each $h \in \{1, \cdots, k\}$, student $i_h$ is now assigned to school $x_{i_{h+1}}$ (modulo $k$), while the other students' assignments are not affected. We refer to the latter process as **trading within the cycle**. To define the **stable-improvement-cycles (SIC) mechanism**, we apply the following algorithm to each problem $(R, \succsim, q)$.

**Step 1:** Choose a tie-breaker profile $\succsim' \in T(\succsim)$ uniformly at random and apply DA to the problem $(R, \succsim', q)$ with strict priorities. Let $x^1 \equiv DA(R, \succsim', q)$.

**Step $t$ ($t \geq 2$):** Using assignment $x^{t-1}$ from Step $t-1$, check if there is a stable improvement cycle with respect to $(R, \succsim', q)$ and $x^{t-1}$. If there is, choose any one cycle uniformly at random and allow the students forming the cycle to trade within the cycle. Otherwise, stop.

The algorithm terminates after a step in which no stable improvement cycle exists. Whenever there is a multiplicity of stable improvement cycles or tie-breakers, we choose one of them with equal probabilities and independently across such instances. Therefore, the final assignment is random. SIC selects the latter assignment. Denote it by $SIC(R, \succsim, q)$.

*The choice-augmented deferred acceptance mechanism.* The choice-augmented deferred acceptance (CADA) mechanism allows students to transmit an additional message as well as preference rankings. The additional message is one's "target school" and serves as an avenue through which students indicate their preference intensities. Let $A$ be the common message space (i.e., $M = A$). For each $i \in N$, denote student $i$'s target school by $t_i \in A$. Letting $t \equiv (t_i)_{i \in N} \in A^N$, and the problem is a list $(R, t, \succsim, q)$.

Given $a \in A$, school $a$'s priority relation $\succsim_a$ over students, and a target profile $t \in A^N$, let $\boldsymbol{T(\succsim_a, t)}$ be the set of strict priority relations obtained by breaking ties in $\succsim_a$ as follows: $\succsim'_a \in T(\succsim_a, t)$ if and only if for all $i, j \in N$, (i) $i \succ_a j$ implies $i \succ'_a j$; and (ii) $i \sim_a j$ and $t_i = a \neq t_j$ imply $i \succ'_a j$. We call the priority relations in $T(\succsim_a, t)$ the **target-respecting tie-breakers for** $\boldsymbol{\succsim_a}$. For a profile $\succsim \equiv (\succsim_a)_{a \in A}$ of priority relations, define $\boldsymbol{T(\succsim, t)} \equiv \prod_{a \in A} T(\succsim_a, t)$. [8]

For each problem $(R, t, \succsim, q)$, we apply the following algorithm to obtain $CADA(R, t, \succsim, q)$.

**Step 1:** Choose a target-respecting tie-breaker profile $\succsim' \in T(\succsim, t)$ uniformly at random and consider the problem $(R, \succsim', q)$ with strict priorities.

**Step 2:** Apply DA to problem $(R, \succsim', q)$.

Since the choice of a target-respecting tie-breaker profile in Step 1 is random, so is the assignment obtained after Step 2. CADA selects the latter assignment. Let us denote it by $CADA(R, t, \succsim, q)$.

### 2.2. Revelation Games.

A mechanism relies on the information submitted by students and students may strategically choose what to communicate to the mechanism. Suppose that a mechanism $\varphi$ permits preference relations in $\mathcal{R}$ and messages in $M$ to be reported. Given a problem $(u, \succsim, q)$, $\varphi$ induces a **(revelation) game** $(u, \succsim, q, \mathcal{R} \times M, \varphi)$, defined as follows:

(i) for each $i \in N$, student $i$ chooses a strategy $(R_i, m_i) \in \mathcal{R} \times M$ ($R_i$ need not be consistent with $u_i$, nor is $m_i$ required to be the top school for $R_i$);

(ii) the outcome of the game is an assignment $\varphi(R, m, \succsim, q)$; and

(iii) the payoff to each student $i \in N$ is $u_i(\varphi_i(R, m, \succsim, q))$.

We refer to $(u, \succsim, q, \mathcal{R} \times M, \varphi)$ as the $\boldsymbol{\varphi}$ **(revelation) game** (e.g., DA game and CADA game).

## 3. An Experimental Environment and Theoretical Analysis

Our main objective is to experimentally investigate performances of the three mechanisms in Section 2.1 in terms of efficiency and manipulability. Therefore, a suitable environment should allow the three mechanisms to potentially diverge in behavior and be tractable enough to be tested in experiments.

For the reasons discussed in Section 1, we are led to a simple case involving three students and three schools. To minimize the effect of different priority standings on students' strategic behavior, we construct a symmetric environment in which each student has the

[8] We assume "single tie-breaking" in our analysis. See Abdulkadiroğlu et al. (2015) for a discussion of single versus multiple tie-breaking rules.

top priority at one school and the bottom priority at the other two schools. Given these school priorities, we can show that there exist only three preference profiles (up to permutations) of students for which DA produces an inefficient matching with a positive probability; see Online Appendix for details. For each student $i \in \{1, 2, 3\}$, denoting $i$'s preferences over schools $a$, $b$, and $c$ by $R_i$ and assuming that students 1, 2, and 3, respectively, have the top priority at schools $a$, $b$, and $c$, the three preference profiles are (below boldfaced symbols indicate schools at which students have the top priority):

| Profile 1 | | | Profile 2 | | | Profile 3 | | |
|---|---|---|---|---|---|---|---|---|
| $R_1$ | $R_2$ | $R_3$ | $R_1$ | $R_2$ | $R_3$ | $R_1$ | $R_2$ | $R_3$ |
| $b$ | $c$ | $b$ | $c$ | $c$ | $b$ | $c$ | $c$ | $b$ |
| $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ | $b$ | $a$ | $\boldsymbol{c}$ | $b$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ |
| $c$ | $a$ | $a$ | $\boldsymbol{a}$ | $\boldsymbol{b}$ | $a$ | $\boldsymbol{a}$ | $a$ | $a$ |

Our design of an experimental environment is guided by theoretical analysis of these profiles. For the three profiles, we characterize the set of Nash equilibria in undominated strategies for the preference revelation game when each of DA, SIC, and CADA is in place. These results, which we summarize now, substantiate our choice of experimental setups.

First, consider the environment induced by Profile 1.[9] For DA, the unique Nash equilibrium in undominated strategies is that all students report their preferences truthfully, yielding an inefficient stable matching 50% of the time. For CADA, there is a Nash equilibrium that results in an efficient stable matching with probability one. However, this equilibrium involves a dominated targeting choice by student 1. The prediction from the unique Nash equilibrium in undominated strategies in CADA, in fact, entails all students reporting their true preferences and targeting their respective top schools. For SIC, (at least) two students report preference truthfully in all Nash equilibria in undominated strategies, and an efficient stable matching obtains with probability one. These results[10] suggest that Profile 1 can serve as a good environment where we can compare efficiency performances of the three mechanisms. The DA matching lies quite far away from the Pareto frontier, which gives SIC and CADA much room in which to diverge from DA. The efficiency gain that SIC and CADA achieve relative to DA depends crucially on whether students choose undominated strategies in preference reporting and targeting, which can be tested in experiments. Finally, Profile 1 is also compelling from the perspective of strategic calculation in the lab. When preferences are given by Profile 1, equilibrium strategies are neither trivial nor prohibitively complicated for experiment subjects.

Next, when students' preferences are given by Profile 2, theoretical prediction is quite different. For DA, the unique Nash equilibrium in undominated strategies produces an

[9]Profile 1 is also the main example Erdil and Ergin (2008) considered to illustrate the inefficiency of DA.

[10]These results are given in Propositions 1, 2, 3, and 4 in Section 3.

inefficient assignment with probability $\frac{1}{4}$. CADA replaces this inefficient assignment by an efficient one. Efficiency of CADA in equilibrium follows regardless of whether students' targeting choices are dominated or not. On the other hand, SIC also chooses an efficient matching for sure but it requires student 2 to misrepresent his preferences in equilibrium.[11] Therefore, relative to Profile 1, Profile 2 provides an environment that is more favorable to CADA and less so to SIC in order for the efficiency improvement (over DA) to materialize. This means that we can use Profile 2 to check robustness of our experimental results from Profile 1.

Finally, in Profile 3, school $a$ is unanimously bottom-ranked, which makes the associated school choice problem a rather trivial one of allocating only two schools ($b$ and $c$) to three students. Our theoretical analysis of Profile 3, which is relegated to Online Appendix, shows that the matching outcome for DA is inefficient with probability $\frac{3}{4}$. CADA Pareto improves upon DA but remains inefficient, with probability $\frac{1}{2}$. SIC uniquely yields an efficient assignment, further improving upon CADA. In terms of efficiency, CADA can perform just as well as SIC only if a student's equilibrium targeting choice is dominated. Overall, these results indicate that the environment induced by Profile 3 is very similar to that induced by Profile 1 and we do not conduct experiments on Profile 3.

On these grounds, we take Profile 1 as the main experimental environment and Profile 2 as an external validity test for our results from Profile 1.

Now to state theoretical results more formally, let $N = \{1, 2, 3\}$ and $A = \{a, b, c\}$, with $q_a = q_b = q_c = 1$. Denote Profiles 1 and 2 above by $R^*$ and $R^{**}$, respectively. Consider a problem $(u, \succsim, q)$, where $u$ is any utility profile consistent with $R^*$ or $R^{**}$[12] and the school priorities $\succsim$ are as follows:

| $\succsim_a$ | $\succsim_b$ | $\succsim_c$ |
|---|---|---|
| $1$ | $2$ | $3$ |
| $2, 3$ | $1, 3$ | $1, 2$ |

For each $\varphi \in \{DA, CADA, SIC\}$, we analyze the revelation game $(u, \succsim, q, \mathcal{R} \times M, \varphi)$. As a solution concept, we adopt the Nash equilibrium in undominated strategies, where the dominance notion is weak. We apply weak dominance to refine the Nash equilibria (but we do not iteratively eliminate weakly dominated strategies). To simplify the notation, we write, e.g., $bac$ for a preference relation $R_i \in \mathcal{R}$ with $b\, P_i\, a\, P_i\, c$.

### 3.1. Analysis of Profile 1.

Throughout this subsection, we assume that students have a profile $u$ of vNM utility functions consistent with Profile 1, $R^*$. For each of the three mechanisms we consider,

[11] These results are given in Propositions 5, 6, 7, and 8 in Section 3.

[12] Our theoretical results in this section are based only on ordinal preferences. Specific values of vNM indices required for experiments will be introduced in the next section.

assignments are affected by how ties in priorities are broken. In light of this, it is useful to note that when true preferences are reported, augmenting DA with a *deterministic* tie-breaking rule results in one of only two assignments

$$\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix},$$

with the second Pareto-dominating the first. The first obtains if the tie $1 \sim_b 3$ at school $b$ is broken in favor of student $1$; the second obtains otherwise.

It is well known that truth-telling is a weakly dominant strategy for the DA game with strict priorities. The DA game with weak priorities is a convex combination of DA games with strict priorities, whose strict priorities are consistent with the given weak priorities. Thus, truth-telling is a weakly dominant strategy for the DA game with weak priorities.

**Proposition 1.** *Let $u$ be a utility profile consistent with $R^*$. The DA game $(u, \succsim, q, \mathcal{R}, DA)$ has a unique Nash equilibrium in undominated strategies, which is truth-telling and yields* $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ *as an assignment.*

This result is very well-known, so that proof of Proposition 1 is skipped.

Next, we turn to the CADA game. While a typical strategy in the CADA game has two dimensions, targeting affects only how ties are broken. Therefore, as far as preference reports are concerned, it is a weakly dominant strategy for each student to submit his true preference rankings. Moreover, each student should target his top school. For instance, consider student $1$. He has the highest priority at school $a$ and least prefers school $c$. Thus, targeting $b$ weakly dominates targeting every other school. A similar argument shows that students 2 and 3 should target $c$ and $b$, respectively. In sum, we have the following Proposition (where the proof is skipped since it is straightforward).

**Proposition 2.** *Let $u$ be a utility profile consistent with $R^*$. The CADA game $(u, \succsim, q, \mathcal{R} \times A, CADA)$ has a unique Nash equilibrium in undominated strategies, where each student reports his preferences truthfully and targets his top school, yielding* $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ *as an assignment.*

Relative to DA, CADA has the advantage of eliciting preference intensities through target choices. However, in our experimental environment, it can implement welfare improvements over the DA assignment only when student 1 plays a weakly dominated strategy of targeting his "safety school" (school $a$, which already gives him the highest priority) or bottom-ranked school. In stating this observation, we restrict attention to Nash equilibria that involve truthful preference reporting since the latter is a weakly dominant strategy in both the DA and CADA games.

**Proposition 3.** *Let $u$ be a utility profile consistent with $R^*$ and consider the CADA game $(u, \succsim, q, \mathcal{R} \times A, CADA)$. In each Nash equilibrium with truthful preference reporting, the associated targeting profile is $(t_1, t_2, b)$, where $t_1, t_2 \in \{a, b, c\}$. Thus, the resulting equilibrium assignment is $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ if $t_1 = b$ and $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ if $t_1 \in \{a, c\}$.*

The proof of Proposition 3 is relegated to Appendix A.

The above equilibrium analysis for the DA and CADA games is simple because the two games can be viewed as convex combinations of DA games, each of which has strict priorities. This is not true for the SIC game and the truthful reporting of preferences is no longer a weakly dominant strategy. In fact, a weakly dominant strategy does not exist, and weak dominance allows us to rule out only some of the strategies. Nevertheless, the Nash equilibria in undominated strategies all produce the same outcome, as our next result shows.

**Proposition 4.** *Let $u$ be a utility profile consistent with $R^*$. For the SIC game $(u, \succsim, q, \mathcal{R}, SIC)$, each Nash equilibrium in undominated strategies has students 2 and 3 reporting their preferences truthfully and yields $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ as an assignment.*[13]

The proof of Proposition 4 is relegated to Appendix A.

To summarize equilibrium outcomes from the three revelation games, in the DA and CADA games, Nash equilibria in undominated strategies yield $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$. That $\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$ is chosen with a positive probability is an efficiency loss for DA and CADA and CADA can circumvent it only if student 1 plays a weakly dominated strategy. On the other hand, the SIC game produces $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$, an efficient assignment with probability 1.

### 3.2. Analysis of Profile 2.

In this subsection, we consider revelation games where students' ordinal preferences are given by Profile 2, $R^{**}$. First, analysis of the DA game is similar to that in Section 3 and truth-telling is a unique Nash equilibrium in undominated strategies (hence the proof of the following proposition is skipped).

**Proposition 5.** *Let $u$ be a utility profile consistent with $R^{**}$. The DA game $(u, \succsim, q, \mathcal{R}, DA)$ has a unique Nash equilibrium in undominated strategies, which is truth-telling and yields $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ as an assignment.*

[13] In the equilibrium, student 1 may or may not report truthfully.

Of the the three deterministic assignments DA chooses with a positive probability in equilibrium, $\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$ is inefficient. That is, DA produces an inefficient assignment with probability $\frac{1}{4}$.

Next is the CADA game. When students have preferences in Profile 2, the equilibrium targeting behavior is less obvious than in the case of Profile 1. Student 1, for instance, has the highest priority at his bottom-ranked school and undominatedness only requires targeting one of his top two schools. Nevertheless, the Nash equilibrium in undominated strategies is unique, as shown below.

**Proposition 6.** *Let $u$ be a utility profile consistent with $R^{**}$. The CADA game $(R, \succsim, q, \mathcal{R} \times A, CADA)$ has a unique Nash equilibrium in undominated strategies, where each student reports his preferences truthfully and targets his top school, yielding* $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$.

The proof of Proposition 6 is relegated to Appendix A.

The two deterministic assignments that arise in the CADA game are both efficient. Further, in relation to the DA outcome, CADA shifts probability $\frac{1}{4}$ from an inefficient assignment $\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$ to an efficient assignment $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$. However, the CADA outcome does not Pareto improve upon the DA outcome; student 1 is worse off in the CADA game than in the DA game.

In our analysis of Profile 1, dropping the requirement of strategies being undominated in targeting expanded the set of Nash equilibrium outcomes and if student 1 targets his safety school or bottom-ranked school, CADA yields a Pareto improvement upon the DA assignment. This, however, is no longer true for Profile 2. In fact, the equilibrium targeting choices remain the same as in Proposition 6. The proof of Proposition 7 is relegated to Appendix A.

**Proposition 7.** *Let $u$ be a utility profile consistent with $R^{**}$ and consider the CADA game $(u, \succsim, q, \mathcal{R} \times A, CADA)$. In each Nash equilibrium with truthful preference reporting, each student targets his top school, yielding* $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$.

Finally, we turn to the SIC game. The SIC game too has a unique matching outcome but unlike the DA and CADA games, it choose one deterministic assignment with probability 1. Further, the latter assignment is Pareto efficient.

**Proposition 8.** *Let $u$ be a utility profile consistent with $R^{**}$. For the SIC game $(u, \succsim, q, \mathcal{R}, SIC)$, each Nash equilibrium in undominated strategies has student 2 misrepresenting his preferences*

($i.e.$, *reporting* $cba$) *and student 3 reporting his preferences truthfully and yields* $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ *as an assignment.*[14]

The proof of Proposition 8 is relegated to Appendix A.

Let us now compare equilibrium outcomes from the three revelation games. The DA game produces $\frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$. Of the three deterministic assignments constituting the latter, $\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$ is inefficient and CADA restores efficiency by selecting the other two. SIC produces an efficient assignment $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ for sure. Nevertheless, equilibrium outcomes from the three mechanisms cannot be ranked in the Pareto sense.

## 4. Experimental Design

### 4.1. Experimental Design and Procedure.

Our experimental design is presented in Table 1, which involves 12 treatments in total. The experimental game is adopted from the environment with three students and three schools described in Section 3.

| Environment | | Mechanism | | |
|---|---|---|---|---|
| | | DA | CADA | SIC |
| **B**aseline | | DA-B | CADA-B | SIC-B |
| Robustness | **R**eplica | DA-R | CADA-R | SIC-R |
| | **C**ardinal | DA-C | CADA-C | SIC-C |
| | **N**ew | DA-N | CADA-N | SIC-N |

TABLE 1. Experimental Treatments

We consider three mechanisms, DA, CADA, and SIC, in four environments. Our **baseline** (**B**) environment implements Profile 1 using the utility values in the left panel of Table 2 below.[15] The three additional environments are considered to validate our experimental findings. The **replica** (**R**) environment also takes the same Profile 1 but involves six students, with two copies of each preference type. The **cardinal** (**C**) environment takes Profile 1 but with amended cardinal utility values, as in the middle panel of Table 2. The only

[14]In the equilibrium, student 1 may or may not report truthfully.

[15]One utility value is multiplied by HKD 40 ($\approx$ USD 5) in the laboratory.

difference from the baseline environment is that the utility value that student 1 receives from her safe/priority school is increased to 3. This cardinal value adjustment is meant to give CADA its best shot because CADA is meant to provide a channel (via targeting) through which something other than ordinal preferences affects tie-breaking.[16] The **new**

| | Baseline and Replica | | | Cardinal | | | New | | |
|---|---|---|---|---|---|---|---|---|---|
| | $u_1(\cdot)$ | $u_2(\cdot)$ | $u_3(\cdot)$ | $u_1(\cdot)$ | $u_2(\cdot)$ | $u_3(\cdot)$ | $u_1(\cdot)$ | $u_2(\cdot)$ | $u_3(\cdot)$ |
| $a$ | 1 | 0 | 0 | **3** | 0 | 0 | 1 | 2 | 0 |
| $b$ | 4 | 2 | 4 | 4 | 2 | 4 | 2 | 1 | 4 |
| $c$ | 0 | 3 | 1 | 0 | 3 | 1 | 3 | 3 | 2 |

TABLE 2. Utility Values

**(N)** environment implements Profile 2 using the utility values in the right panel of Table 2. In this environment, theory predicts that CADA, without the multiple-equilibrium issue, has a unique equilibrium outcome always yielding an efficient matching while SIC's efficiency improvement relative to DA comes from the equilibrium requirement that student 2 submits her preference ranking as $c$-$b$-$a$ *untruthfully*. Thus, the new environment provides us with an experimental setting that is more favorable to CADA and less favorable to SIC to materialize the efficiency gain (relative to DA) that theory promises. The replica, cardinal, and new environments mainly serve the purpose of a robustness check for our results from the baseline environment to a larger economy, cardinal preference amendment, and ordinal preference change, respectively. In what follows, we take Treatment CADA-B to illustrate the experimental procedure. All other treatments share a similar general procedure. The full version of the experimental instructions for CADA-B can be found in Appendix B. Two sample instructions for DA and SIC are also available in Online Appendix.

Throughout the experiments, we used Students B, R, and G to refer to students 1, 2, and 3 and Schools BLUE, RED, and GREEN to refer to schools $a$, $b$, and $c$, respectively. The names of the schools and students are chosen to help experimental subjects comprehend the experimental environment including the school priority profile. When describing our experimental environment, procedure, and results, we will use the notation consistent with the experimental instructions.

**General Environment and Procedure.** We considered three students who live on an island for which a map is presented in Figure 1 below. The island consisted of three administrative districts: the BLUE zone, RED zone, and GREEN zone. One student lived in each

[16]In particular, it is necessary for student 1 to target her safe school to achieve any efficiency improvement in CADA relative to DA. Thus, we presume that increasing the cardinal utility value student 1 receives from her safe school would help achieve efficiency improvement in CADA.

zone. There were three schools on the island: one in the BLUE zone, one in the RED zone, and one in the GREEN zone. Each school had only one seat. We called the student and school in the BLUE / RED / GREEN zone Student B / R / G and School BLUE / RED / GREEN, respectively. In each round, the three participants in each group were *randomly* assigned to the role of Student B / R / G. The role assigned to each subject was revealed at the beginning of each round.

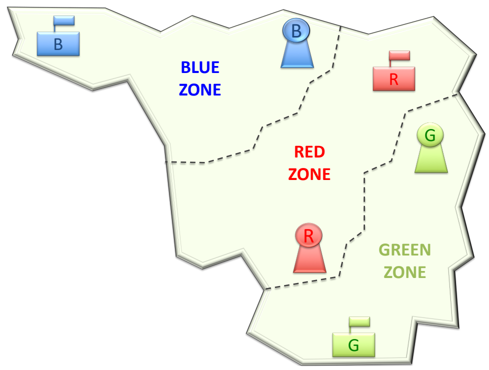



FIGURE 1. Island Map

At the beginning of each round, each subject needed to submit 1) the list of his preference rankings and 2) the target school to the central admission office. To make admission decisions, the admission office used

a. the submitted preference rankings and target schools from all students in each group and
b. each school's priority information.

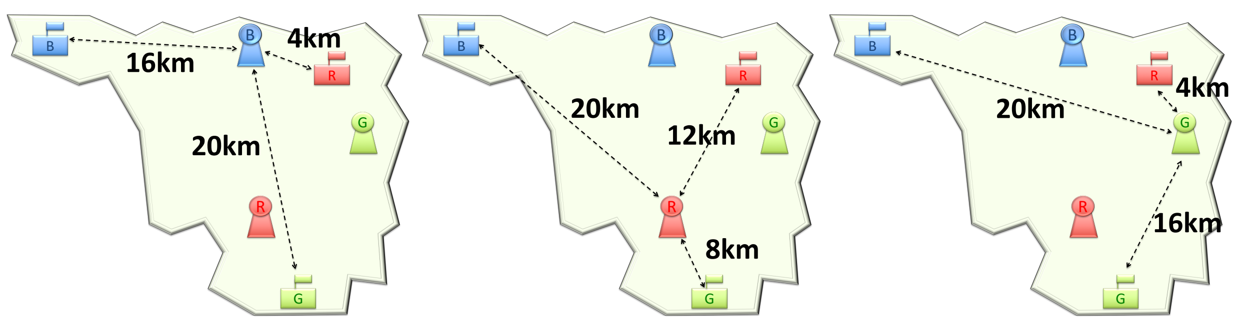
Each school gave a *same-district priority* to the student who lived in the same district. For those who did not live in the same district, each school gave a *target priority* to any student who targeted the school. The same-district priority was more important than and preceded the target priority. The admission procedure was as follows:

(1) Each student's application was sent to the school of his top choice.
(2) If a school received only one application, it tentatively kept the student.
(3) If a school received more than one application, then it determined which student to retain based on the student's priority. Among the students who applied to the school, it chose the student with the highest priority (if there were several highest-priority students, it randomly chose one of them).
(4) Whenever a student's application was rejected at a school, his application was sent to the next highest school on his submitted list.
(5) Whenever a school received new applications, these applications were considered together with the retained application for that school. Among the retained and new applications, each school chose one based on priority. Again, it chose the

student with the highest priority or randomly chose one of the highest-priority students if there were several such students.

(6) The allocation was finalized when no more applications could be rejected. Each student was admitted by the school that held his application at the end of the process.

The payoff of each student in HKD was $200 - 10 \times$ the distance in kilometers between the student and the school that admitted him, where the distance was presented as follows:




Student B Student R Student G

We conducted four sessions for each treatment and thus had 48 sessions in total. Each session had 12-18 participants. The random-matching protocol and a between-subjects design were used. Our experiment was conducted in English using the z-Tree (Fischbacher, 2007) and oTree (Chen et al., 2016a) at the Hong Kong University of Science and Technology (HKUST). A total of 639 subjects who had no prior experience in our experiment were recruited from the graduate and undergraduate populations of the university.[17]

After all three subjects in each group submitted their preference rankings and target schools, the submitted rankings and the target schools were revealed to everyone in the group. At the end of each round, the computer provided each subject with the following end-of-each-round feedback: 1) which school a subject was finally admitted to, 2) payoff,

[17]We conducted 4 sessions (72 subjects for 2 DA-B, 1 CADA-B, and 1 SIC-B sessions) via the face-to-face laboratory mode with z-Tree in November 2019 and 20 sessions (255 subjects for 2 DA-B, 3 CADA-B, 3 SIC-B, and all 12 sessions with the replica environment) via the real-time online mode with oTree in February 2021. 24 more sessions (312 subjects for the cardinal and new environments) were conducted via the real-time online mode with oTree in October and November 2021. The real-time online mode was based on Zoom, where the instructions were provided and all questions were handled by the experimenter via Zoom. Turning on their videos during the entire course of the experiment was strictly required for subjects (i.e., any subject who did not agree to turn on their video could be dismissed at the beginning of the experiment, and one such case occurred). The aggregate-level data from these two types of experiments share all qualitative features, which made us feel comfortable merging the data. Comparing the data from these two forms of experiments, albeit interesting, goes beyond the main research questions of the paper, and thus, we did not pursue it. Online Appendix presents the data from the lab experiments only.

3) the submitted preference rankings and target schools from all students in each group, 4) which school other students were admitted to, and 5) their payoffs. All treatments shared the same end-of-each-round feedback.[18]

The computer randomly selected 1 round out of the 10 rounds to calculate the payment. The total payment in HKD was the payoff each subject earned in the selected round plus an HKD 40 show-up fee. Subjects on average earned HKD 140 (≈ USD 18) by participating in a session that lasted 65 minutes. The final earnings were paid in cash for all experiments we conducted in the laboratory, and for all sessions we conducted using Zoom and oTree (the real-time online mode), they were paid electronically via the HKUST Autopay System to the bank account each participant provided to the Student Information System (SIS).

Here are the reported preference rankings (You are Student B)

| | Student R | Student G | Student B (You) |
|---|---|---|---|
| First choice | BLUE | BLUE | RED |
| Second choice | RED | GREEN | BLUE |
| Third choice | GREEN | RED | GREEN |
| Target School | GREEN | BLUE | RED |

Please click the PROCEED button to start the admission process

PROCEED

FIGURE 2. CADA-B: Submitted Preference Rankings and Target Schools

**Feedback Procedure.** The admission process was presented with *full transparency* such that every subject was able to see which school tentatively kept or rejected his application. The step-by-step feedback procedure we adopted is to ensure that experimental subjects fully understand the school choice environment and how the mechanism works without having a potential concern of the experimental demand effect, a typical concern attached to excessively long experimental instructions with multiple quiz questions. Given the large stake size of the school choice problem in reality, students and parents have strong incentives to spend time to understand the underlying mechanism and carefully choose their preferences. In contrast, the stake size is relatively small and the decision time is short in the lab. Thus it is crucial to create an environment in the lab in which experimental subjects have no doubt about how the mechanism works. For more details, consider the example in Figure 2 above. The submitted rankings and target schools are (RED-BLUE-GREEN,

[18]More feedback about how the admission process occurred was provided after all subjects in each group submitted their preference rankings (and target schools); we will further elaborate on this feedback later.

RED), (BLUE-RED-GREEN, GREEN), and (BLUE-GREEN-RED, BLUE) for students B, R, and G, respectively.

When the admission process began, the students' applications were sent to their top choices. As a result, Students R and G applied to School BLUE, and Student B applied to School RED, while no one applied to School GREEN. Given that neither Student R nor G had the same-district priority at School BLUE while Student G targeted School BLUE, Student G was tentatively admitted by School BLUE. Student B was, without any competitor, tentatively admitted by School RED. This outcome is illustrated by the screenshot in the top-left panel of Figure 3, where the school names highlighted in boldface or de-highlighted by grey imply tentative acceptance and rejection, respectively.

Proceed to step 2

Submitted Preference Rankings. (You are Student B)

| | Student R | Student G | Student B (You) |
|---|---|---|---|
| First choice | BLUE | **BLUE** | **RED** |
| Second choice | RED | GREEN | BLUE |
| Third choice | GREEN | RED | GREEN |
| Target School | GREEN | BLUE | RED |

Below is the result from STEP 1 :

| Step 1 | School RED | School GREEN | School BLUE |
|---|---|---|---|
| Student(s) Applied | B, | None | R, G, |
| Student Tentatively Accepted | B, | None | G, |

(a) Step 1

Proceed to step 3

Submitted Preference Rankings. (You are Student B)

| | Student R | Student G | Student B (You) |
|---|---|---|---|
| First choice | BLUE | **BLUE** | RED |
| Second choice | **RED** | GREEN | BLUE |
| Third choice | GREEN | RED | GREEN |
| Target School | GREEN | BLUE | RED |

Below is the result from STEP 2 :

| Step 2 | School RED | School GREEN | School BLUE |
|---|---|---|---|
| Student(s) Applied | R, B, | None | G, |
| Student Tentatively Accepted | R, | None | G, |

(b) Step 2

Proceed to step 4

Submitted Preference Rankings. (You are Student B)

| | Student R | Student G | Student B (You) |
|---|---|---|---|
| First choice | BLUE | BLUE | RED |
| Second choice | **RED** | GREEN | **BLUE** |
| Third choice | GREEN | RED | GREEN |
| Target School | GREEN | BLUE | RED |

Below is the result from STEP 3 :

| Step 3 | School RED | School GREEN | School BLUE |
|---|---|---|---|
| Student(s) Applied | R, | None | B, G, |
| Student Tentatively Accepted | R, | None | B, |

(c) Step 3

Submitted Preference Rankings. (You are Student B)

| | Student R | Student G | Student B (You) |
|---|---|---|---|
| First choice | BLUE | BLUE | RED |
| Second choice | **RED** | **GREEN** | **BLUE** |
| Third choice | GREEN | RED | GREEN |
| Target School | GREEN | BLUE | RED |

Below is the result from STEP 4 :

All students are placed in one of the schools.

| Step 4 | School RED | School GREEN | School BLUE |
|---|---|---|---|
| Student(s) Applied | R, | G, | B, |
| Student Tentatively Accepted | R, | G, | B, |

(d) Final Step

FIGURE 3. CADA-B: Feedback about the Admission Process

Student R, who was rejected in the previous step, then applied to School RED, which was his second choice, and competed with Student B. Due to the same-district priority that Student R had with School RED, Student R was tentatively accepted, while Student B was rejected by School RED. This outcome is illustrated in the screenshot in the top-right panel of Figure 3.

Student B, who was rejected in the previous step, then applied to School BLUE, which was his second choice, and competed with Student G. Due to the same-district priority that Student B had with School BLUE, Student B was tentatively accepted, while Student G was rejected by School BLUE. This outcome is illustrated in the screenshot in the bottom-left panel of Figure 3.

Student G, who was rejected in the previous step, then applied to School GREEN, which was his second choice. Given that he was the only applicant, Student G was tentatively accepted by School GREEN. No more applications could be rejected, and thus, the allocation was finalized. This outcome is illustrated in the screenshot in the bottom-right panel of Figure 3.

**4.2. Experimental Hypotheses.**

In this subsection, we present three sets of testable hypotheses driven by the theoretical predictions in Section 3. The first set of hypotheses concerns manipulability of the three mechanisms. They thus focus on comparing reporting strategies (including targeting). The second set of hypotheses further compares the three mechanisms in terms of assignment outcomes. Our last set of hypotheses discusses efficiency properties of the three mechanisms.[19] For each set of hypotheses, we first present a hypothesis that jointly applies to the first three (baseline, replica, and cardinal) environments, given that they share the same ordinal preferences profile. We then present a hypothesis that applies to the new environment with theoretical predictions that are qualitatively different from those for the first three environments. All statements that belong to each hypothesis applies to the environment(s) specified at the top of the hypothesis.

The undominated-strategy Nash equilibria for Profile 1 characterized in Propositions 1, 2, and 4 predict that Students R and G always report their preferences truthfully regardless of the underlying mechanisms. In the case of Student B, truthful reporting is also predicted in DA and CADA. However, Student B may not report truthfully in SIC. Regarding the choice of target schools in CADA, the undominated-strategy Nash equilibria predict that every student targets his top choice. Summarizing these results, our first hypothesis is as follows:

**Hypothesis 1** (Reporting and Targeting Strategies in the **B**, **R**, and **C** Environments)**.**

(a) In every treatment, Students R and G truthfully report their preferences.

[19] In spite of the tight theoretical connections among the assignment outcome, efficiency properties, and payoffs, it is still meaningful to scrutinize the experimental data in different aspects because observed behavior in the lab may deviate substantially from the theoretical predictions.

(b) The rate of truthful reporting for Student B is significantly lower in Treatment SIC than in Treatment DA and in Treatment CADA.
(c) In Treatment CADA, all students choose their top choice as their target school.

The undominated-strategy Nash equilibria for Profile 2 are provided by Propositions 5, 6, and 8. In this environment, Student G always reports her preference truthfully regardless of the underlying mechanisms. In the case of Students B and R, truthful reporting is also predicted in DA and CADA. In SIC, however, it is predicted that Student R reports *untruthfully* while Student B may or may not report truthfully. Regarding the choice of target schools in CADA, the undominated-strategy Nash equilibria predict that every student targets his top choice. The following hypothesis summarizes them:

**Hypothesis N1** (Reporting and Targeting Strategies in the **N** Environment)**.**

(a) In every treatment, Student G truthfully report their preferences.
(b) The rate of truthful reporting for Student R is significantly lower in Treatment SIC than in Treatment DA and in Treatment CADA.
(c) In Treatment CADA, all students choose their top choice as their target school.

For notational simplicity, let $(a, a', a'')$ denote the assignment outcome $\begin{pmatrix} B & R & G \\ a & a' & a'' \end{pmatrix}$ for $a, a', a'' \in \{$BLUE, RED, GREEN$\}$. Observe that the predictions from the undominated-strategy Nash equilibria for Profile 1 result in outcome equivalence of DA and CADA. However, the assignment outcome predicted in SIC is distinct from DA and CADA in that the outcome (BLUE, GREEN, RED) is predicted with certainty. We thus have our second hypothesis as follows:

**Hypothesis 2** (Matching Outcomes in the **B**, **R**, and **C** Environments)**.**

(a) The proportion of (BLUE, GREEN, RED) in Treatment SIC is significantly larger than that in Treatment DA and in Treatment CADA.
(b) The proportions of (BLUE, RED, GREEN) in Treatments DA and CADA are not different from each other. Similarly, the proportions of (BLUE, GREEN, RED) in the two treatments are not different from each other.

In the case of Profile 2, the undominated-strategy Nash equilibria predict that the three mechanisms produce all distinct outcomes. The assignment outcome predicted in DA leads to the inefficient matching (RED, BLUE, GREEN) 25% of the time, while none of CADA and SIC leads to an inefficient matching. CADA is predicted to result in two efficient matchings (BLUE, GREEN, RED) and (GREEN, BLUE, RED) with equal probabilities while in SIC (BLUE, GREEN, RED) is predicted with certainty. We thus have the following hypothesis:

**Hypothesis N2** (Matching Outcomes in the **N** Environment)**.**

(a) The proportion of (RED, BLUE, GREEN) in Treatment DA is significantly larger than that in Treatment CADA and in Treatment SIC.
(b) The proportions of (BLUE, GREEN, RED) in Treatment SIC is significantly larger than that in Treatment DA and in Treatment CADA.

Our final set of hypothesis is about welfare properties of the three mechanisms. We start from the three environments induced by Profile 1. On one hand, the distinct outcome predicted by the undominated-strategy Nash equilibria in SIC implies that SIC exhibits a *welfare advantage* over DA. On the other hand, the outcome equivalence of DA and CADA implies that CADA achieves no such advantage over DA. These two observations are highlighted in the following hypothesis.

**Hypothesis 3** (Welfare in the **B**, **R**, and **C** Environments)**.**

(a) There is no difference in the average earnings of each student between Treatments DA and CADA.
(b) The average earnings of Student R and Student G are higher in Treatment SIC than in Treatment DA.

In the new environment, the welfare ranking is not straightforward due to the fact that there are two efficient stable matchings in this environment. Only DA leads to the inefficient matching with a positive probability, but that inefficient matching is Pareto dominated by one of the two efficient matchings. Both CADA and SIC increase the likelihood of the efficient matching that does *not* Pareto dominate the inefficient matching but not the likelihood of the other efficient matching relative to DA. As a result, the average earnings of Students R and G are predicted to be higher in CADA and SIC than in DA, but the ranking is reversed for Student B.

**Hypothesis N3** (Welfare in the **N** Environment)**.**

(a) The average earning of Student B is higher in Treatment DA than in each of Treatments CADA and SIC.
(b) The average earnings of Students R and G are lower in Treatment DA than in each of Treatments CADA and SIC.

It is worth mentioning that all hypotheses above are established based on the undominated-strategy Nash equilibrium predictions. This fact has a crucial implication for evaluating the performance of CADA, particularly in terms of manipulability and efficiency properties because there is a Nash equilibrium in weakly dominated strategies in CADA that achieves a welfare advantage over DA.

## 5. Experimental Results

We report our experimental findings in this section. We first report results from the first three (**B**, **R**, and **C**) environments in Section 5.1 that share the same ordinal preferences profile and next present results from the new (**N**) environment in Section 5.2. Only very mild learning was observed among subjects, but when reporting the results we use the last 5 rounds of data for statistical tests (unless stated otherwise) to focus on the behavior after convergence.[20] Tables 5, 6, 7, and 8 in Appendix C report the results ($p$-values) from all non-parametric tests in this section.[21] For robustness checks of the results from the non-parametric tests, we also conduct regression analyses using the last 5 rounds individual-level observations and the results are reported in Appendix D. Tables 9-12 report regression results on matching outcomes, Tables 13-16 report regression results on preference reporting, Tables 17-20 report regression results on targeting, and Table 21-24 report regression results on earnings.[22] All results are qualitatively consistent with those obtained by the non-parametric tests regardless of the regression models used.

### 5.1. Baseline, Replica, and Cardinal Environments.

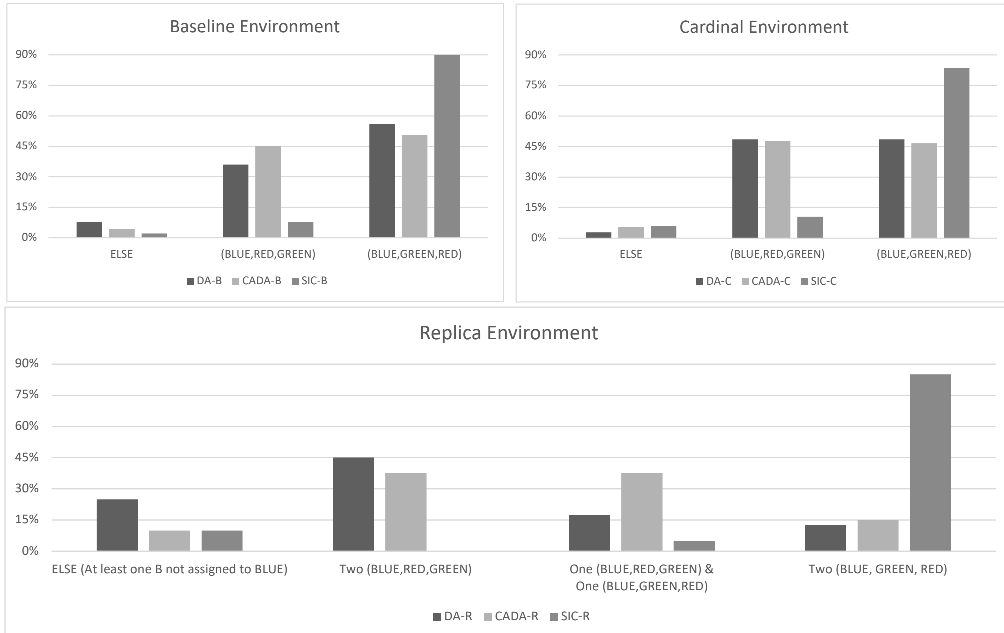
**5.1.1.** *Outcomes.* Figure 4 compares the matching outcomes obtained in the three treatments with the baseline environment (the top-left panel), the cardinal environment (the top-right panel) and the replica environment (the bottom panel), aggregated over the last 5 rounds for each treatment. There are two stable matchings in this environment, (BLUE, RED, GREEN) and (BLUE, GREEN, RED) and only the second matching is Pareto efficient.

We begin with the baseline environment. Two observations are apparent. First, the outcomes achieved in both Treatments DA-B and CADA-B are qualitatively consistent with the prediction of the half-half mixture between (BLUE, RED, GREEN) and (BLUE, GREEN, RED). As a result, the proportions of (BLUE, RED, GREEN) across Treatments DA-B and CADA-B are only marginally different from each other (36% vs. 45%) and the proportions of (BLUE, GREEN, RED) are not significantly different from each other (56% vs. 51%). Second, the vast majority (90%) of the outcomes achieved in Treatment SIC-B are (BLUE, GREEN, RED), significantly and substantially larger than the proportions achieved in the other two treatments (90% vs. 56% or 51%).

[20]All nonparametric tests conducted in the paper are based on the session-level data aggregated over the last 5 rounds. The qualitative features of the data do not depend on whether we use the first or last 5 rounds of data or all 10 rounds of data. Online Appendix reports time trend data.

[21]When we describe the results in this section, "marginal", "significant", and "insignificant" refer to the case in which the $p$-value from the non-parametric test is between $0.05$ and $0.1$, strictly below $0.05$, and strictly above $0.1$, respectively.

[22]In each of Tables 9-20, we report results from six types of regressions: linear, logit, and probit regressions, each with and without session-level clustering.




Note: Among the two stable matchings, (BLUE, RED, GREEN) is Pareto dominated by (BLUE, GREEN, RED).

FIGURE 4. Matching Outcome

The matching outcome obtained in the cardinal environment is essentially the same as that in the baseline environment. The outcomes achieved in both Treatments DA-C and CADA-C are qualitatively consistent with the prediction of the half-half mixture between (BLUE, RED, GREEN) and (BLUE, GREEN, RED). The proportions of (BLUE, RED, GREEN) across Treatments DA-C and CADA-C are not significantly different from each other (49% vs. 48%) and the same is true for the proportions of (BLUE, GREEN, RED) (49% vs. 47%). The vast majority (84%) of the outcomes achieved in Treatment SIC-C are (BLUE, GREEN, RED) and this proportion is significantly and substantially larger than the proportions achieved in the other two treatments (84% vs. 49% or 47%).

Now we move on to the replica environment. The first "ELSE" column of the bottom panel in Figure 4 reports the proportion of matching outcomes in which at least one Student B is allocated to a school other than BLUE.[23] The second and fourth columns present the proportions of groups with two (BLUE, RED, GREEN) matchings and with two (BLUE, GREEN, RED) matchings, respectively. Thus, they correspond to the second and third columns of the other panels in Figure 4, respectively. The third column reports the proportion of groups with one (BLUE, RED, GREEN) and one (BLUE, GREEN,

[23] In the experiments with the replica environment, there are six students in each group, and thus, the matching outcome must be a sixtuple. When reporting the matching outcomes in the bottom panel of Figure 4, however, we divide a group of six students into two subgroups of (B,R,G) whenever there is an objective way for us to do the division. By doing so, we can straightforwardly compare the matching outcomes from different environments. The first column of the bottom panel in Figure 4 presents the cases in which there is no objective way for us to report the outcome separately.

RED) matching. Two results are worth highlighting. First, the proportion of the outcomes with one (BLUE, RED, GREEN) and one (BLUE, GREEN, RED) is substantially higher in CADA-R than in DA-R (37.5% vs. 17%), although the difference is only marginally significant (one-sided test), while those of two (BLUE, RED, GREEN) outcomes across CADA-R and DA-R are not different from each other (37.5% vs. 45%). This result indicates that CADA may perform slightly better than DA in obtaining an efficient stable matching in the replica environment. Second, the proportion of two (BLUE, GREEN, RED) outcomes is significantly and substantially higher in Treatment SIC-R than in the other two treatments (85% vs. 13% or 15%). Confirming Hypotheses 2(a) and 2(b), we thus have the following result.

*Observation* 1 (Matching Outcomes in the **B**, **R**, and **C** Environments). In the **B** and **C** environments, the proportions of (BLUE, RED, GREEN) across Treatments DA and CADA are only marginally or insignificantly different, and those of (BLUE, GREEN, RED) are not significantly different from each other. In the **R** environment, the proportion of (BLUE, GREEN, RED) in Treatment CADA is higher than that in Treatment DA. In all three environments, the vast majority of the matching outcomes in Treatment SIC are the efficient matching (BLUE, GREEN, RED).

This result implies that the efficiency advantage of SIC relative to DA is materilzed in the lab which is robust to the larger economy with six students and the cardinal preference change. The efficiency advantage of CADA predicted by the dominated-strategy (targeting) Nash equilibrium was not realized in the lab regardless of whether the cardinal preferences are modified to help subjects to coordinate on the particular Nash equilibrium. CADA improved efficiency of DA incrementally in the larger economy although the efficiency gain is not as prominent as that provided by SIC.

**5.1.2.** *Strategies and manipulability.* To understand the main source of the observed differences in the matching outcomes across treatments, we now analyze strategy-level data.

Table 3 shows the rate of truthful reporting for each student type in each treatment of the baseline, cardinal, and replica environments. Again, we begin with the baseline environment. A few observations emerge immediately. First, the rate of truthful reporting does not substantially vary across treatments for Student R (89% vs. 84% vs. 94%) and for Student G (89% vs. 97% vs. 96%). Moreover, they are reasonably close to 100%, even though the difference from 100% is marginally significant for a few cases. Similarly, the rate of truthful reporting for Student B does not vary across treatments (55% vs. 74% vs. 64%), either. However, it deviates significantly and substantially from 100% regardless of the treatment.

| Environment | | Baseline | Cardinal | Replica |
|---|---|---|---|---|
| Student B | DA | 55% | 84% | 84% |
| | CADA | 74% | 79% | 80% |
| | SIC | 64% | 79% | 60% |
| Student R | DA | 89% | 90% | 91% |
| | CADA | 84% | 92% | 93% |
| | SIC | 94% | 87% | 94% |
| Student G | DA | 89% | 94% | 91% |
| | CADA | 97% | 94% | 95% |
| | SIC | 96% | 92% | 99% |

TABLE 3. Truthful Reporting (%)

| Environment | | Baseline | Cardinal | Replica |
|---|---|---|---|---|
| Student B | Round 1-5 | 91% | 92% | 96% |
| | Round 6-10 | 92% | 94% | 95% |
| | All | 91% | 93% | 96% |
| Student R | Round 1-5 | 91% | 94% | 96% |
| | Round 6-10 | 93% | 99% | 95% |
| | All | 92% | 97% | 96% |
| Student G | Round 1-5 | 97% | 96% | 96% |
| | Round 6-10 | 98% | 98% | 99% |
| | All | 97% | 97% | 97% |

TABLE 4. Undominated Targeting (%)

The cardinal environment again leads to the behavior qualitatively consistent with what we observed from the baseline environment with only one exception of the higher rates of truthful reporting from Student B in all three treatments. The rate of truthful reporting does not substantially vary across treatments for Student R (90% vs. 92% vs. 87%) and for Student G (94% vs. 94% vs. 92%). They are reasonably close to 100%. The rate of truthful reporting for Student B does not vary across treatments (84% vs. 79% vs. 79%), either. It still deviates significantly and substantially from 100% regardless of the treatment, but the difference is not as substantial as the baseline.

One noticeable difference of the replica environment from the other two environments is that the rate of truthful reporting for Student B is significantly higher in DA-R than in SIC-R (84% vs. 60%). A similar observation holds for CADA-R and SIC-R although the difference is insignificant (80% vs. 60%). For Student R, the rate of truthful reporting is lowest in DA-R and highest in SIC-R, but the differences are neither substantial nor significant (91% vs. 93% vs. 94%). The same observation is true for Students G, with more substantial and significant differences (91% vs. 95% vs. 99%).

These observations are qualitatively consistent with Hypothesis 1(a), but we have to partially reject Hypothesis 1(b).

*Observation* 2 (Preference Reporting in the **B**, **R**, and **C** Environments). In all treatments across all environments for all students types, truthful reporting is a modal behavior. In the **B** and **C** environments, for a given student type, the truth-telling rates have no difference across treatments. The truth-telling rate for Student B is lower than 100% with marginal statistical significance in all treatments. In the **R** environment, the rates of truthful reporting for Students R and G have no or marginal differences across treatments. However, the rates of truthful reporting for Student B are substantially lower in Treatment SIC-R than in the other two treatments.

We now move our attention to the targeting choices in Treatments CADA. Table 4 presents the rate of undominated targeting for each student type in Treatment CADA for the three environments. Clearly, the undominated strategy provides a strong behavioral guideline to people in the lab. The rates of undominated targeting were observed to be 90% even in the first 5 rounds and it became slightly higher in later rounds. This reveals that learning did not help our experimental subjects to coordinate on the "dominated-strategy" Nash equilibrium that produces a Pareto efficient matching. Neither the changes in utility values induced by the cardinal environment nor the larger economy provided by the replica environment helped. Thus, confirming our Hypothesis 1(c), our next result is as follows.

*Observation* 3 (Targeting Behavior in the **B**, **R**, and **C** Environments). In all three environments, a vast majority of students in Treatment CADA target their top schools regardless of the student types.

The results above reveal that the outcomes from Treatments CADA can be easily rationalized by simply combining truthful reporting and undominated targeting strategies. This observation speaks to the multiplicity of equilibrium outcomes for the CADA game, as noted in Proposition 3. Our data vividly demonstrate that the winning prediction comes from the combination of truthful reporting and undominated targeting, not from the combination of truthful reporting and dominated targeting.

**5.1.3.** *Efficiency and welfare.* Where does the apparent efficiency gain SIC achieved over DA come from? To address this question, we look at the transition matrices—the top-left panel for the baseline environment, the top-right panel for the cardinal environment, and the bottom panel for the replica environment—presented in Figure 5 and see how swaps took place in the last 5 rounds of Treatments SIC.

Given the similarity between the transition matrices for the baseline and cardinal environments, we describe the results for these two environments together. Before swapping, the average matching outcome was approximately the 55-45 (baseline) or 40-60 (cardinal) mixture between (BLUE, RED, GREEN) and (BLUE, GREEN, RED). However, the vast majority of (BLUE, RED, GREEN) outcomes were transformed into (BLUE, GREEN, RED) via implementation of stable improvement cycles. The social cost of misreporting individual preferences in SIC is captured by the 7 and 9 cases (the mid-sized bubble at the center of each of the two top panels of Figure 5) in which no stable improvement cycle was found and 2 and 5 cases (the small bubbles at the bottom-left corner) in which the pre-swap outcome is unstable. They are absolutely non-negligible but not substantial in their magnitudes, either.

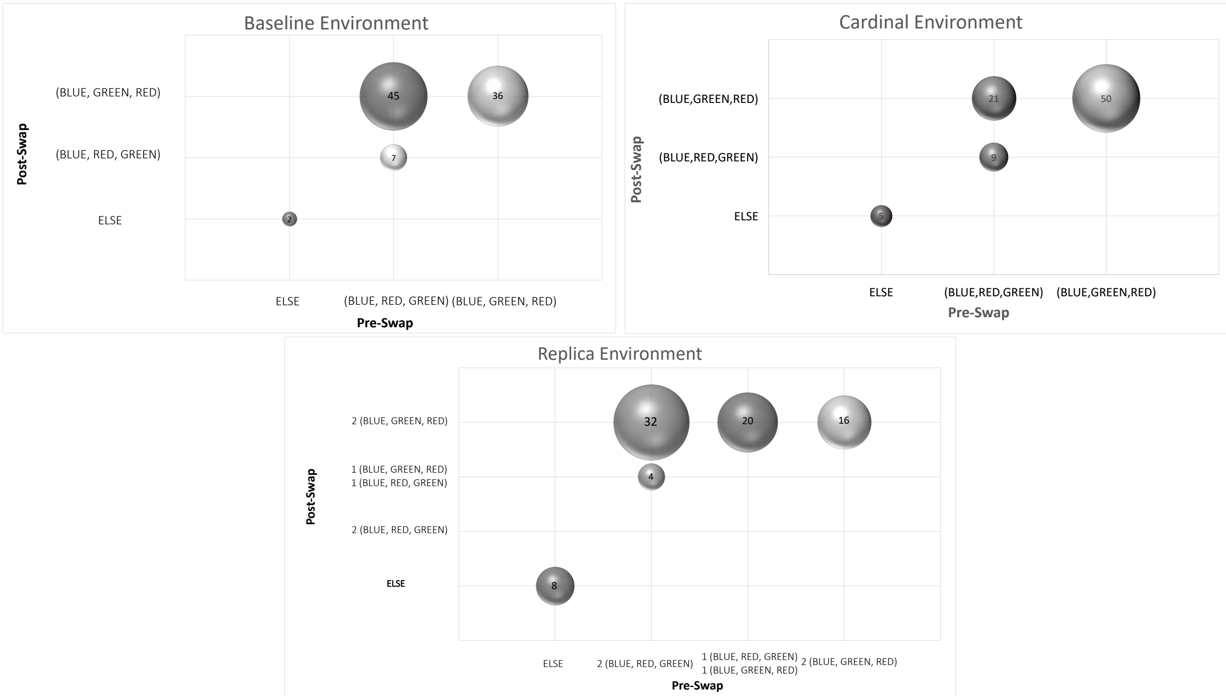



FIGURE 5. Transition of Matching Outcomes in Treatments SIC

The result from the replica environment is not different. First, 90% (32 out of 36) of the pre-swapping matching with two (BLUE, RED, GREEN) outcomes became two (BLUE, GREEN, RED) matching outcomes, and the rest (10%) became one (BLUE, RED, GREEN) and one (BLUE, GREEN, RED) after swapping. Second, 100% of the pre-swapping matching with outcomes of one (BLUE, RED, GREEN) and one (BLUE, GREEN, RED) turned into two (BLUE, GREEN, RED) matching outcomes after swapping. The social cost of misreporting individual preferences in SIC-R is captured by the 4 cases (the smallest bubble in the bottom panel of Figure 10) in which only a partial improvement was implemented and 8 cases (the bubble at the bottom-left corner in the bottom panel of Figure 10) in which the pre-swap outcome was unstable.

The theoretical efficiency gain of SIC is translated into actual gains in welfare. As is clear in Figure 6 and consistent with the theoretical prediction, there is no payoff gain for Student B in Treatment SIC or CADA relative to Treatment DA in all environments. However, the average earnings for Students R and G are significantly and substantially higher in Treatment SIC than in Treatment DA in any environment. In contrast, the average earnings for Students R and G in Treatment CADA are not different at all from those in Treatment DA. Confirming Hypotheses 3(a) and 3(b), we thus have our next result as follows.

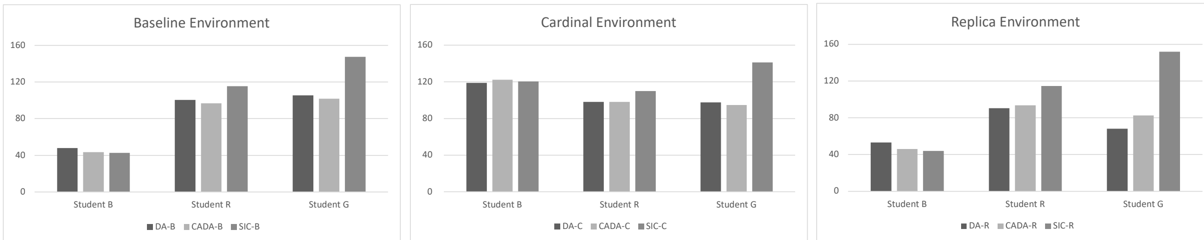


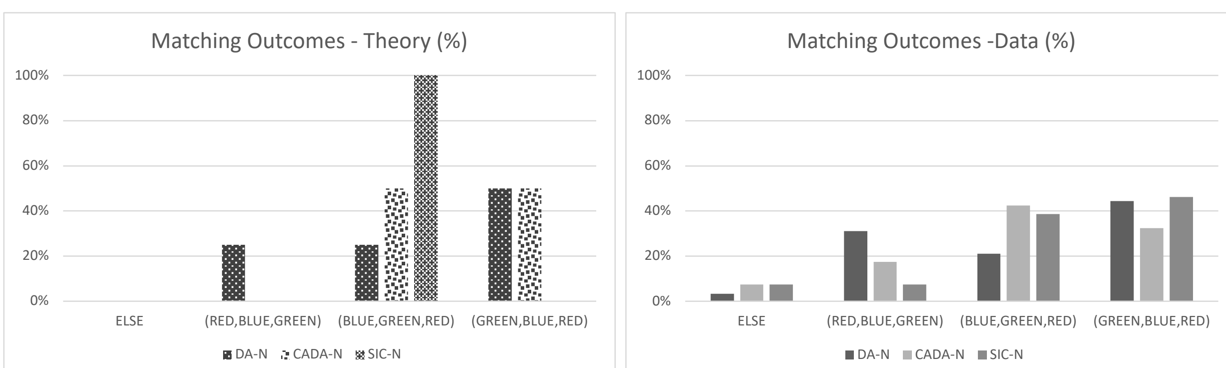

FIGURE 6. Earnings

*Observation* 4 (Earnings in the **B**, **R**, and **C** Environments). There is no difference in the earnings of Student B across treatments. However, for the earnings of Students R and G, the following rankings are observed:

$$\text{Earnings (SIC)} > \text{Earnings (DA)} \approx \text{Earnings (CADA)}.$$

### 5.2. New Environment.

This section presents the data from the new (**N**) environment. As detailed in Subsection 3.2, the theory predicts that CADA has a unique equilibrium outcome always yielding an efficient matching while SIC's efficiency improvement relative to DA comes from the equilibrium requirement that student 2 untruthfully reports GREEN-RED-BLUE. By investigating the data from this environment, we aim at understanding the following: (i) what prevents CADA from achieving its theoretical efficiency advantage, and (ii) what enables SIC to successfully translate its theoretical efficiency advantage into empirical advantage, and eventually address a potential concern of external validity of our findings in the previous subsection.




Note: Among the three stable matchings, both (BLUE, GREEN, RED) and (GREEN, BLUE, RED) are efficient while (RED, BLUE, GREEN) is Pareto dominated by (GREEN, BLUE, RED).

FIGURE 7. Matching Outcomes: Theory (Left) and Data (Right)

**5.2.1.** *Outcomes.* Figure 7 presents the matching outcomes predicted by theory (left) and those obtained by the data (right) from the three treatments aggregated across the last 5 rounds of all sessions for the new environment. It demonstrates a substantial discrepancy, especially for the SIC treatment. In this environment, there are three stable matchings. Among them, both (BLUE, GREEN, RED) and (GREEN, BLUE, RED) are efficient while (RED, BLUE, GREEN) is Pareto dominated by (GREEN, BLUE, RED). There is no Pareto ranking between (BLUE, GREEN, RED) and (RED, BLUE, GREEN).

Three results are worth highlighting. First, consistent with theory, the proportion of (RED, BLUE, GREEN) is substantially higher in DA than in CADA and in SIC (31% vs. 18%/8%) although the difference is significant only in the case of SIC. Second, consistent with theory, the proportion of (BLUE, GREEN, RED) is substantially higher in CADA than in DA (43% vs. 21%) with a marginal statistical significance, providing evidence for the efficiency advantage of CADA relative to DA. The same observation holds for SIC (39% vs. 21%) but the difference is insignificant. Inconsistent with theory, however, CADA still results in 18% of the outcome with the inefficient matching outcome of (RED, BLUE, GREEN). Third, inconsistent with theory, the proportion of (BLUE, GREEN, RED) is *not* statistically different between Treatment SIC and Treatment CADA (39% vs. 43%). Unexpectedly, SIC leads to another efficient matching (GREEN, BLUE, RED) 46% of the time. Partially confirming Hypotheses N2(a) but rejecting N2(b), we have the following result.

*Observation* 5 (Matching Outcome in the **N** Environment). The proportion of the inefficient matching (RED, BLUE, GREEN) is lower in Treatments CADA and SIC than in Treatment DA. The proportion of efficient matching (BLUE, GREEN, RED) is higher in Treatments CADA and SIC than in Treatment DA. In Treatment SIC, the vast majority of outcomes obtained is one of the two efficient matchings, (BLUE, GREEN, RED) and (GREEN, BLUE, RED).

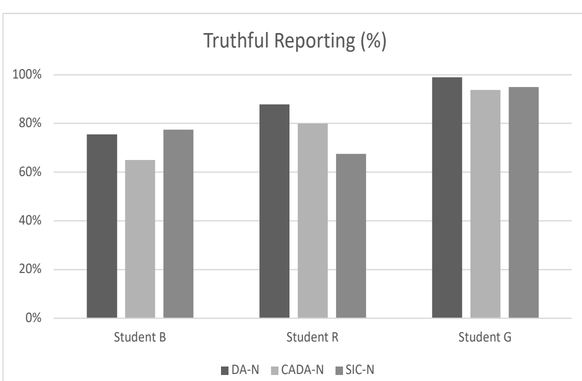



FIGURE 8. Truthful Reporting

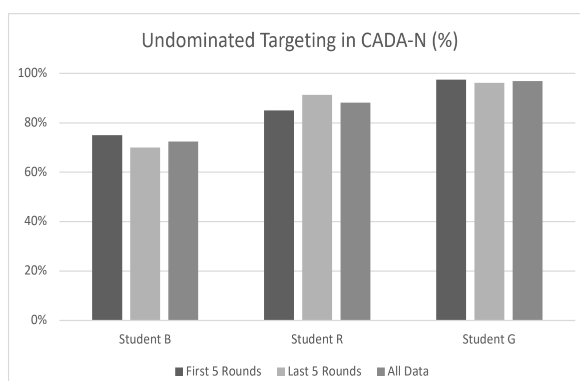



FIGURE 9. Targeting in CADA-N

**5.2.2.** *Strategies and manipulability.* The matching outcomes discussed above reveal two substantial discrepancies between the theoretical prediction and the experimental data.

First, CADA still leaves room for further efficiency improvement despite the fact that theory promises to achieve the full efficiency gain. Second, SIC leads to a 50-50 mixture between two efficient matchings, instead of generating (BLUE, GREEN, RED) with certainty. Where do these discrepancies come from? To answer the question, first let us look at the rate of truthful reporting in Figure 8. Observe that the truth-telling rate in CADA is only 65% for Student B and 80% for Student R. Furthermore, as Figure 9 shows, the rate of undominated targeting for Student B in CADA is only around 70%. As a result, we have to reject Hypothesis N1(c). Combining these two observations, it is evident that a substantial proportion of subjects in CADA failed to coordinate on the unique undominated-strategy Nash equilibrium, which explains why CADA was not able to fully achieve the predicted efficiency advantage relative to DA.

Regarding the outcomes obtained in SIC that deviated largely from the unique equilibrium prediction, recall that the Nash equilibrium in Proposition 8 requires that Student R *untruthfully* reports GREEN-RED-BLUE. In contrast, Figure 8 shows that still 68% of Student R truthfully reported. This value is not statistically different from the rates of truthful reporting in the other two treatments and thus we reject Hypothesis N1(b). It turns out that only 21% of the time, Student R submits the particular untruthful rankings required by theory which explains the observed discrepancy. However, it is still puzzling how SIC resulted in the other efficient matching equally frequently and we shall address this issue in the next subsection.

*Observation* 6 (Truthful Reporting and Targeting Behavior in the **N** Environment). Truthful preference reporting is a modal behavior although the rate of truthful reporting for Students B and R are substantially below 100% in all treatments. In CADA, undominated targeting is a modal behavior for all students, but a non-negligible proportion of Student B made a dominated targeting.

This result highlights the fact that the additional layer of coordination (equilibrium targeting) required by CADA makes the mechanism more vulnerable to strategic uncertainty. Multiplicity of equilibrium embedded in Profile 1 is a fundamental source of strategic uncertainty as we have seen in the previous section, but the uniqueness of Nash equilibrium outcome embedded in the current **N** environment (from Profile 2) does not completely get rid of the issue.

**5.2.3.** *Efficiency and welfare.* Figure 10 presents the transition matrix for the matching outcomes aggregated across the last 5 rounds of all sessions of Treatment SIC-N. There are two occasions that implementing stable improvement cycles provides an efficiency gain. First, there are 15 cases in which the (unstable) matching (BLUE, RED, GREEN) was obtained before swapping but 67% of them were transformed into the efficient matching

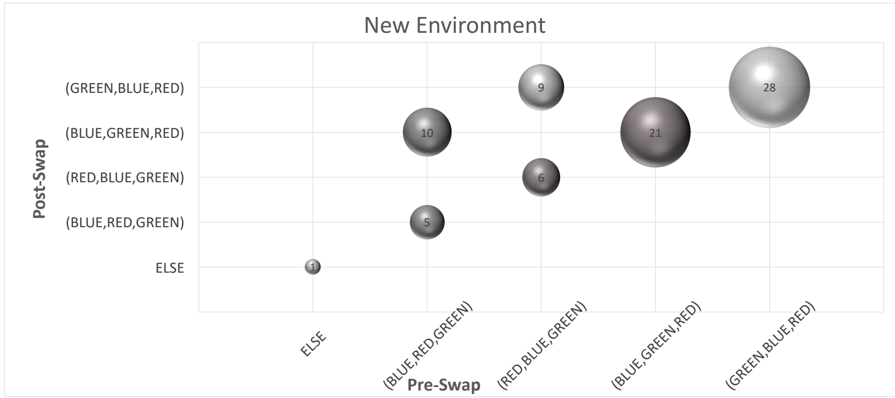



FIGURE 10. Transition of Matching Outcomes

(BLUE, GREEN, RED). And this is exactly what theory predicts to happen on the equilibrium path. Second, there are another 15 cases in which inefficient stable matching (RED, BLUE, GREEN) was obtained before swapping but 60% of them were transformed into the efficient matching (GREEN, BLUE, RED). The latter case of efficiency improvement is not predicted by theory and it explains why SIC frequently produces (GREEN, BLUE, RED). When non-negligible proportions of Students B and R deviated from the equilibrium play in SIC, the stable improvement cycles were still implemented no matter whether it was on or off the equilibrium path, resulting in efficient matchings for the majority of the observations in SIC.

In spite of the fact that the behavior observed in Treatment SIC-N deviated substantially from the theoretical predictions, implementation of stable improvement cycles on and off the equilibrium path allows SIC to achieve the efficiency gain relative to DA. Figure 11 shows that this efficiency gain is translated into welfare gain for Students R and G, albeit with a small magnitude. Consistent with theory, the average earning of Student B is higher in Treatment DA than in CADA and SIC although the difference is statistically significant only for CADA. For both Student R and G, the average earnings in Treatments CADA and SIC are higher than in Treatment DA. The difference are either insignificant or only marginally significant, with one exception that Student G's average earning in Treatment SIC is significantly higher than that in Treatment DA. Given that the differences are not statistically significant, we cannot confirm Hypotheses N3(a) and N3(b).

*Observation* 7 (Earnings in the **N** Environment). The average earning of Student B is higher in Treatment DA than in SIC. The average earning of Student G is higher in Treatment SIC than in Treatment DA.

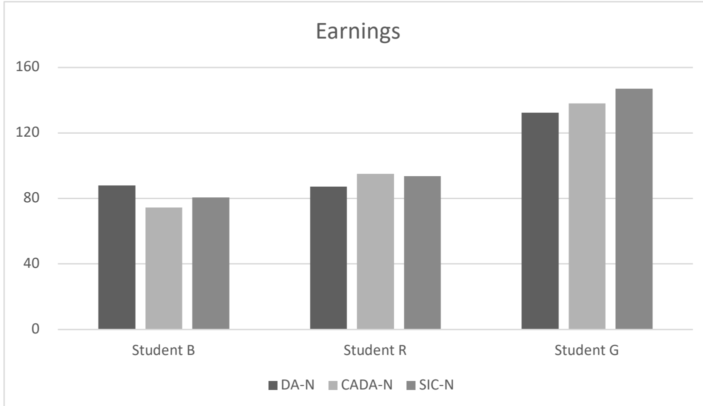



FIGURE 11. Earnings

## 6. Literature Review

In this section, we discuss the relevant theoretical and experimental studies in the matching literature.

Erdil and Ergin (2008) is a seminal paper that focuses on the efficiency loss due to random tie-breaking when priorities are not strict. They show that if DA (combined with some tie-breaking) selects an inefficient stable matching, then a cyclic trade of assignments among a group of students can Pareto-improve DA matching without sacrificing stability. They call such cycles that enable trades "stable improvement cycles." Students in a stable improvement cycle are chosen in a way that when a student "desires" another student's school (in the sense that he prefers that school to his own assignment), no other student has a higher priority at that school and desires it. The stable-improvement-cycles (SIC) mechanism by Erdil and Ergin (2008) first applies DA to a given school choice problem and corrects for inefficiency due to tie-breaking by implementing stable improvement cycles.

Abdulkadiroğlu et al. (2015)'s study is also motivated mainly by the prevalence of coarse priorities. They propose the choice-augmented DA (CADA) mechanism, which allows one more message, namely, one's target school, to break ties. When a school has weak priorities, they reorder students within a priority class by favoring those who target that school. CADA applies DA to a modified school choice problem where school priorities are broken in this manner. Targeting can be viewed as a medium through which to communicate cardinal preference intensities, and CADA outperforms DA in terms of efficiency.

In a seminal paper, Kesten (2010) introduces the "Efficiency Adjusted Deferred Acceptance Mechanism" (EADAM), which was revisited via the notion of "underdemanded schools" in "Simplified EADAM" (SEADAM) by Tang and Yu (2014). More recently, Dur et al. (2019) consider all "consent-proof mechanisms" (i.e. mechanisms where a consenting student is never hurt by her decision), and show that EADAM is the unique constrained efficient mechanism that is consent-proof.

The main focus of earlier experimental studies on the school choice problem has been to compare three theoretically and practically influential school choice mechanisms—the Boston, the DA, and the top-trading-cycles (TTC) mechanisms—from the perspectives of efficiency and manipulability. Chen and Sönmez (2006) are the first to consider the three mechanisms in the laboratory environment. They find that a significant degree of preference manipulation occurs among participants in the Boston mechanism, leading to an efficiency loss. In their environment, DA outperforms both TTC and the Boston mechanism.[24] On the other hand, in the experiments by Pais and Pintér (2008), TTC emerges as the most desirable among the above three mechanisms in terms of efficiency and truth-telling rates (in particular, manipulation is more pervasive under the Boston mechanism than under the other two mechanisms). Pais and Pintér (2008) also study the interplay between students' strategic behavior and the amount of information assumed on the part of students. Students tend to report their preferences more truthfully when they have little information on the game they are playing.

More recently, researchers have paid more attention to the stability of mechanisms. Chen and Kesten (2017, 2019) theoretically and experimentally compared the Boston mechanism, its Chinese parallel, and DA and show that moving from one extreme to the other resulted in systematic changes in manipulability, stability, and efficiency properties. In the laboratory, participants are more likely to report their preferences truthfully in DA than in the Chinese parallel mechanism while they are least likely to do so in the Boston mechanism. Furthermore, although DA is significantly more stable than the Chinese parallel mechanism (which is more stable than the Boston mechanism), efficiency comparisons vary across environments. Chen et al. (2016b) revisit the experimental environment of Chen and Sönmez (2006) and show that TTC outperforms DA and the Boston mechanism, whereas DA is more stable than the others. Providing more information on others' payoffs to participants improves the efficiency performance of the Boston mechanism and TTC but not that of DA. Calsamiglia et al. (2010) consider the case in which students are only allowed to submit a list containing a limited number of schools, and show that introducing the constraint has large negative effects not only on the manipulability but also on efficiency and stability of the mechanisms.

Chen et al. (2018) experimentally compare the large-market performance of DA and the Boston mechanism. They show that as the market size grows from 4 to 40, participant

[24]However, Calsamiglia et al. (2011) show that the efficiency performances of DA and TTC in Chen and Sönmez (2006) are comparable when the number of recombinations in the recombinant estimation technique is sufficiently increased.

truth-telling increases under DA but decreases under the Boston mechanism. However, the inefficiency of matching outcomes for both mechanisms worsens. In a very recent paper, in a strict priorities environment, Cerrone et al. (2021) experimentally compare DA with two variants of EADAM. Their main result is that efficiency and truthtelling rates are substantially higher under EADAM than under DA, with more efficiency gains obtained when priority waiver is enforced. Since the literature on experimental school choice is vast, we would like to refer the readers to Pan (Section 3.3, 2020) and Hakimov and Kübler (Section 3, 2020) for comprehensive surveys of the recent experimental school choice literature.

Some studies indicate the possibility of non-strategy-proof mechanisms outperforming strategy-proof mechanisms. They are relevant to our paper since we also consider non-strategy-proof mechanisms. Klijn et al. (2019) and Bó and Hakimov (2020a) establish theoretical and experimental findings in the context of dynamic implementation of DA algorithms, favoring dynamic implementations to the static ones. In a similar vein, Bó and Hakimov (2020b) study "pick-an-object" (PAO) mechanisms that can dynamically implement DA and TTC. Their experimental results establish better performance of PAO mechanisms and obviously strategy-proof mechanisms as compared to their direct (static) counterparts.

To our knowledge, only two papers investigate experimental, empirical, or simulation aspects of random tie-breaking accompanying DA. However, their focus differs from ours and is to compare two common tie-breaking rules: one in which all schools break ties according to a single lottery (single-tie-breaking) and the other in which each school uses a separate independent lottery (multiple-tie-breaking). More specifically, Schmelzer (2016) compares the two tie-breaking rules in a laboratory environment and show that a significant fraction of individuals prefer multiple- to single-tie-breaking because of fairness concerns. Ashlagi and Nikzad (2020) conduct a simulation exercise with the NYC school choice data and show that in a market with a surplus of seats, single-tie-breaking is less equitable but more efficient.

## 7. Conclusion

In a simple school choice environment, we theoretically and experimentally investigated efficiency properties of three important mechanisms in the literature. To our knowledge, our paper is the first to attempt an equilibrium analysis in the practically relevant case of weak priorities and random tie-breaking. We provided several valuable insights into whether and how one can achieve efficiency gains in practice by employing non-strategy-proof mechanisms. First, our laboratory data vividly demonstrated that the efficiency loss due to random tie-breaking was substantial. Indeed, our results confirmed that "the extent of the inefficiencies that can arise in the student-optimal stable matching is a matter of real practical importance" (Reny, 2021). On the one hand, SIC showed a significant welfare advantage relative to DA, and preference reporting under SIC observed in the lab was largely consistent with the predictions of our equilibrium analysis. On the other hand, CADA attained no or marginal efficiency gain. In the other three environments involving more students, amended cardinal utility values, or different ordinal preferences, the gap in efficiency persisted (even widened in some case) and brought strategic issues surrounding SIC and CADA to light.

If a contribution of experimental studies lies in providing data that "speak to theorists" (Roth, 1995), then our paper delivers this contribution. We obtained clear (empirical) efficiency rankings over DA, SIC, and CADA. Participants' strategic behavior we documented in the lab was in line with the implications we established theoretically. More importantly, we provided experimental data from a simple environment that is easily understandable to various parties in real-world school choice programs, including school board officials, city councilors, state legislators, and parents. Our setup and results are useful to 1) explain the workings of different mechanisms, 2) demonstrate inefficiencies caused by weak priorities and tie-breaking, and 3) convince policy-makers and stakeholders to reconsider the existing assignment mechanisms based on DA. In this vein, our paper "whispers into the ears of princes" (Roth, 1995).

An essential remaining task is to externally validate our findings more broadly. In this paper, we chose a simple setup with three schools and three (or six) students to exploit its analytical tractability. Our results do not indicate that efficiency gains of SIC (with respect to DA) will be always greater than that of CADA in all environments; rather it theoretically and experimentally establishes that different non-strategy-proof mechanisms that result in full efficiency with respect to "reported preferences" may demonstrate different efficiency performances with respect to "true preferences." Therefore, the next step, we believe, is not to carry our results directly to the field. Instead, additional experiments of a similar kind should be conducted. They may shift the focus to other competing issues, e.g., when and how students' preference misrepresentation serves to affect actual workings of SIC and CADA in a positive or negative manner. Experiments of a larger scale will render findings more solid. Such further work will not only dispel concerns about external validity of our study but also reduce the social cost of implementing inefficient school choice programs.

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## Appendix A. Proofs of Theoretical Results

**Proof of Proposition 3.** Consider a strategy profile in which students' preferences are truthful. Targeting choices affect only how the relevant tie, $1 \sim_b 3$, is broken. With true preferences reported, each case of tie-breaking yields the following assignments: $\begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$ if $1 \succ_b 3$; and $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ if $3 \succ_b 1$. This can be used to find each student's best response in targeting.

For each $i \in N$, let $t_i$ be student $i$'s targeting choice and $BR_i : A^2 \to A$ be $i$'s best response in targeting. Note that student 2's targeting does not affect how the tie $1 \sim_b 3$ is broken and that student 1 gets $a$ for sure regardless of how $1 \sim_b 3$ is broken and of what school he targets. Thus, for each $(t_2, t_3) \in A^2$, $BR_1(t_2, t_3) = \{a, b, c\}$. Also, for each $(t_1, t_3) \in A^2$, $BR_2(t_1, t_3) = \{a, b, c\}$. Finally, for each $(t_1, t_2) \in A^2$, $BR_3(t_1, t_2) = \{b\}$. With these best responses, it is simple to see that (combined with true preferences) $\{(t_1, t_2, b) : t_1, t_2 \in \{a, b, c\}\}$ constitutes equilibria, yielding assignments given in the proposition. □

**Proof of Proposition 4.** First, we show that in the SIC game $(u, \succsim, q, \mathcal{R}, SIC)$, $abc$, $acb$, and $cab$ are weakly dominated for student 1. To show that $bac$ weakly dominates each $s_1 \in \{abc, acb\}$, note that if student 1 plays $s_1$, then since he is top-ranking his safety school $a$, he is never involved in a stable improvement cycle and he always obtains $a$ with probability 1. By contrast, if student 1 plays $bac$, then he always obtains a mixture of $b$ and $a$ and sometimes obtains $b$ (if his opponents play appropriately). Thus, $bac$ weakly dominates $s_1$. Next, to show that $bac$ weakly dominates $cab$, observe that $bac$ always gives student 1 a mixture of $b$ and $a$. If student 1 plays $cab$, then he always obtains a mixture of $a$ and $c$, and sometimes $c$ (if his opponents play appropriately). Thus, $bac$ weakly dominates $cab$.

A similar argument shows that for student 2, $bac$, $bca$, and $abc$ are weakly dominated (by $cba$); and that for student 3, $cab$, $cba$, and $acb$ are weakly dominated (by $bca$).

Now we prove the proposition. Let $s \equiv (s_1, s_2, s_3)$ be an undominated Nash equilibrium. By the claim proved above, $s_1 \in \{bac, bca, cba\}$, $s_2 \in \{acb, cab, cba\}$, and $s_3 \in \{abc, bac, bca\}$. When students' choices are confined to these strategies, one can use an oTree simulation to obtain SIC assignments (the SIC game is reduced to a $3 \times 3 \times 3$ game now).[25] It is then simple to show that Nash equilibria of the reduced game are $\{(s'_1, cba, bca) : s'_1 \in \{bac, bca, cba\}\}$ (as long as the utility profile $u$ is consistent with $R^*$).

[25] See Online Appendix for details.

Since $s$ belongs to the latter set and since the strategy profiles in the set all produce the same assignment $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$, the proposition follows. □

**Proof of Proposition 6.** Since targeting only affects how ties in priorities are broken in DA, reporting true preferences is still part of a weakly dominant strategy in the CADA game. Concerning targeting, let $t \equiv (t_1, t_2, t_3)$ be a targeting profile. Clearly, for student 3, $t_3 = b$ is uniquely undominated. For students 1 and 2, undominatedness requires $t_1 \in \{b, c\}$ and $t_2 \in \{a, c\}$. It is simple to find CADA outcomes when students report $R^{**}$ and one of these targeting choices. If $t = (b, a, b)$, then $CADA(R, t, \succsim, q) = \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$; if $t = (b, c, b)$, then $CADA(R, t, \succsim, q) = \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$; if $t = (c, a, b)$, then $CADA(R, t, \succsim, q) = \begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix}$; and if $t = (c, c, b)$, then $CADA(R, t, \succsim, q) = \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$. Now it follows that a unique Nash equilibrium in undominated strategies should have $t = (c, c, b)$. □

**Proof of Proposition 7.** Consider a strategy profile in which students' preferences are truthful. Targeting choices affect only how the relevant ties, $1 \sim_c 2$ and $1 \sim_b 3$, are broken. With true preferences reported, each case of tie-breaking yields the following assignments: $(*)$ $\begin{pmatrix} 1 & 2 & 3 \\ c & a & b \end{pmatrix}$ if $1 \succ_c 2$; $\begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$ if $2 \succ_c 1$ and $1 \succ_b 3$; and $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$ if $2 \succ_c 1$ and $3 \succ_b 1$. This can be used to find each student's best response in targeting.

For each $i \in N$, let $t_i$ be student $i$'s targeting choice and $BR_i : A^2 \to A$ be $i$'s best response in targeting. First, for student 1, for each $(t_2, t_3) \in A^2$, $BR_1(t_2, t_3)$ is given by the following table (each cell contains the value of $BR_1(t_2, t_3)$).[26]

| | | $t_3$: $a$ | $b$ | $c$ |
|---|---|---|---|---|
| | $a$ | $\{c\}$ | $\{c\}$ | $\{c\}$ |
| $t_2$ | $b$ | $\{c\}$ | $\{c\}$ | $\{c\}$ |
| | $c$ | subset of $\{b, c\}$ | $\{c\}$ | subset of $\{b, c\}$ |

Next, student 2's best response $BR_2(t_1, t_3)$ in targeting is given by the following table.

[26]The exact value of $BR_1(c, a)$ depends on student 1's vNM utilities but however they are specified, the targeting part of an equilibrium with truthful preference reporting remains the same as in Proposition 7. A similar comment applies to the value of $BR_1(c, c)$.

| | | $t_3$ | | |
|---|---|---|---|---|
| | | $a$ | $b$ | $c$ |
| | $a$ | $\{c\}$ | $\{c\}$ | $\{c\}$ |
| $t_1$ | $b$ | $\{a,b,c\}$ | $\{c\}$ | $\{a,b,c\}$ |
| | $c$ | $\{c\}$ | $\{c\}$ | $\{c\}$ |

Also, student 3's best response $BR_3(t_1,t_2)$ in targeting is given by the following table.

| | | $t_2$ | | |
|---|---|---|---|---|
| | | $a$ | $b$ | $c$ |
| | $a$ | $\{b\}$ | $\{b\}$ | $\{b\}$ |
| $t_1$ | $b$ | $\{b\}$ | $\{b\}$ | $\{b\}$ |
| | $c$ | $\{a,b,c\}$ | $\{a,b,c\}$ | $\{b\}$ |

With these best responses, we now show that (combined with true preferences) only $(c,c,b)$ constitutes an equilibrium. Let $(t_1,t_2,t_3)$ be the targeting part of an equilibrium with truthful preference reporting.

First, to show that $t_1=c$, suppose not. Since $BR_1$ never contains $a$ in its values, $t_1 \neq c$ implies $t_1=b$. By $BR_3$, regardless of the value of $t_2$, $BR_3(t_1,t_2)=\{b\}$, i.e., $t_3=b$. By $BR_2$, $BR_2(t_1,t_3)=\{c\}$, so that $t_2=c$. However, $BR_1(t_2,t_3)=\{c\}$, which does not contain $t_1=b$, a contradiction.

Next, to show that $t_2=c$, suppose not. Since $t_2\neq c$, for each $t_3\in A$, $BR_1(t_2,t_3)=\{c\}$, so that $t_1=c$. By $BR_2$ then, $BR_2(t_1,t_3)=\{c\}$, so that $t_2=c$, a contradiction.

Now given that $t_1=t_2=c$, $BR_3(t_1,t_2)=\{b\}$, so that $t_2=b$. In sum, $(t_1,t_2,t_3)=(c,c,b)$. Finally, it is simple to see that $(c,c,b)$ is indeed the targeting part of an equilibrium with truthful preference reporting. □

**Proof of Proposition 8.** First, by an argument similar to that in the proof of Proposition 4, we can show the following: for student 1, *cba* weakly dominates *abc* and *acb*; for student 2, *cab* weakly dominates *bac* and *bca*; and for student 3, *bca* weakly dominates *cab*, *cba*, and *acb*.

Let $s\equiv(s_1,s_2,s_3)$ be an undominated Nash equilibrium. Then $s_1\in\{bac,bca,cab,cba\}$, $s_2\in\{abc,acb,cab,cba\}$, and $s_3\in\{abc,bac,bca\}$. When students' choices are confined to these strategies, one can use an oTree simulation to obtain SIC assignments (the SIC game is reduced to a $4\times4\times3$ game now).[27] It is then simple to show that Nash equilibria of the reduced game are $\{(s_1',cba,bca): s_1'\in\{bac,bca,cab,cba\}\}$ (as long as the utility profile

[27]See Online Appendix for details.

$u$ is consistent with $R^{**}$). Since $s$ belongs to the latter set and since the strategy profiles in the set all produce the same assignment $\begin{pmatrix} 1 & 2 & 3 \\ a & c & b \end{pmatrix}$, the proposition follows. □

## Appendix B. Experimental Instructions: Treatment CADA-B

Welcome to this experiment. Please read these instructions carefully. This experiment studies the interaction of decisions made by three individuals. In the following one and a half hours or so, you will participate in 10 rounds of decision making. The payment you will receive from this experiment will depend on the decisions you make. The amount you earn will be paid **electronically via the HKUST Autopay System to the bank account you provide to the Student Information System (SIS)**. The auto-payment will be arranged by the Finance Office of HKUST, which takes about two weeks or more.

In each round, you will be randomly matched with two other participants to form a group of three. Your group will be formed randomly and independently in each round. You will not be told the identity of the participants you are matched with, nor will those participants be told your identity even after the end of the experiment.

**Overview.** The experiment is about three students who are trying to enter a school. The three participants in the same group represent students competing for school seats.

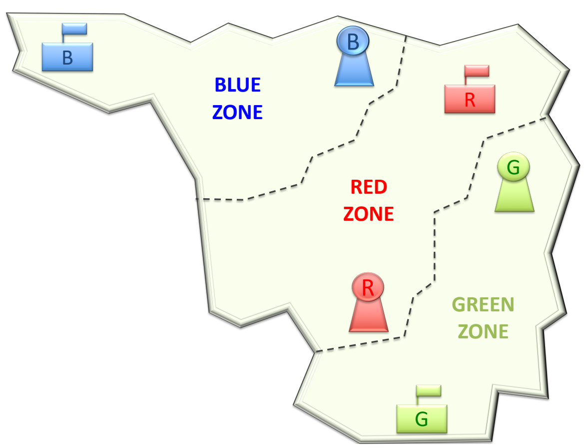



FIGURE 12. Island - BLUE / RED / GREEN Zones

The three students live in an island whose map is presented in Figure 12 above. The island consists of three administrative districts – BLUE zone, RED zone, and GREEN zone. There is one student who lives in each zone. There are three schools in the island, one in the BLUE zone, one in the RED zone, and one in the GREEN zone. Each school has only one seat. For the rest of the instruction, we shall call the student and the school in **B**LUE / **R**ED / **G**REEN zone Student B / R / G and School BLUE / RED / GREEN.

In each round, the three participants in your group will be randomly assigned to the role of Student B / R / G. You will be informed about your role at the beginning of each round.

Your payoff depends on to which school you are admitted. In order to get an admission to any school, you have to participate in the centralized allocation mechanism described below.

**Admission Process via Central Admission Office**

At the beginning of each round, you will be informed whether you are Student B / R / G. You will then submit 1) the list of your **preference rankings** and 2) your **target school** to the central admission office. To make admission decisions, the admission office will use

a. the submitted preference rankings and target schools from all students in your group, and
b. each school's priority information.

Each school gives a *same-district priority* to the student who lives in the same district. For those who do not live in the same district, each school gives a *target priority* to any student who targets the school. **The same-district priority is more important than and precedes the target priority.** For example, School BLUE gives Student B the same-district priority (first priority). School BLUE gives Student R the target priority (second priority) if he is the only one among Students R and G to target School BLUE. School BLUE treats Students R and G equally (as if they are without any priority) if none or both of them target School BLUE. The admission procedure is as follows:

(1) Each student's application is sent to the school of his/her top choice.
(2) If a school receives only one application, it tentatively keeps the student.
(3) If a school receives more than one application, then it determines which student to retain based on the **priority**. Among the students who applied to the school, it chooses the student with the highest priority. If there are several highest-priority students, it randomly chooses one of them.
(4) Whenever an application is rejected at a school, his/her application is sent to the next highest school on his/her submitted list.
(5) Whenever a school receives new applications, these applications are considered together with the retained application for that school. Among the retained and new applications, each school chooses one based on the priority.
(6) The allocation is finalized when no more applications can be rejected. Each student is **admitted** by the school that holds his/her application at the end of the process.

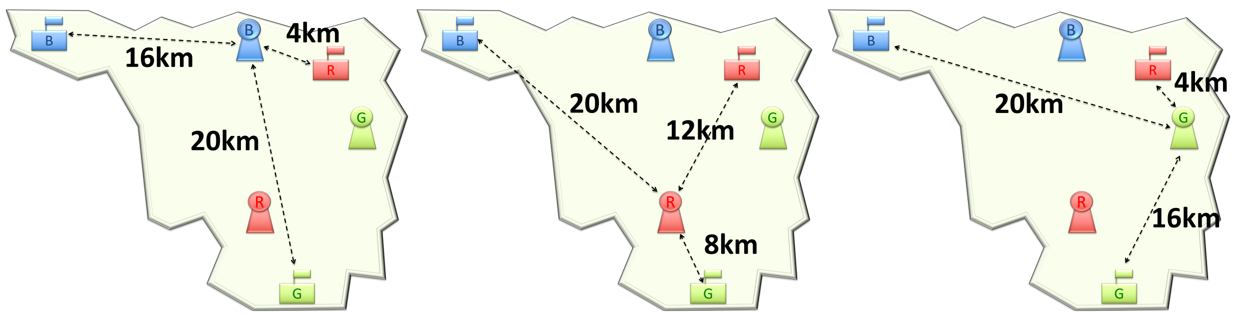
Note that the only thing each student needs to do is to submit his/her preference rankings and the target school. (If you do not submit your preference ranking within 2 minutes, a randomly generated preference ranking will be automatically submitted.) All the steps described above take place in the admission system automatically, without any further inputs from the students. The entire admission process (i.e., who applies to and who targets which school and who are tentatively accepted or rejected by each school) in each round will be presented to you via your computer screen in a transparent way.

**Payoffs**

Your payoff depends on the distance (km) between your location and the school you are admitted to. Your payoff will be higher if you are admitted by the school that is **closer** to your location. Precisely,

**Your Payoff (in HKD) = 200 − 10 × [The distance (in km) between you and the school that admits you]**

where the distance is presented as follows:




Student B Student R Student G

For example,

- Student B receives the payoff of $(200 - 160) = 40$ if he is admitted by School BLUE.
- Student R receives the payoff of $(200 - 80) = 120$ if she is admitted by School GREEN.
- Student G receives the payoff of $(200 - 40) = 160$ if he is admitted by School RED.

**Information Feedback**

After you and two other subjects in your group submit their preference rankings and target schools, the submitted rankings and the target schools will be revealed to everyone in your group. The admission process will be presented with full transparency such that you can see which school tentatively keeps or rejects your application. At the end of each round, the computer will provide you with some feedback, including 1) which school you are finally admitted to, 2) your payoff, 3) the submitted preference rankings and target schools from all students in your group, 4) which school other students are admitted to, and 5) their payoffs.

**Your Payment**

The computer will randomly select 1 round out of the 10 rounds to calculate your payment. So it is in your best interest to take each round equally seriously. Your total payment in HKD will be the payoff you earned in the selected round plus a HKD 40 show-up fee.

**Example and Practice**

To ensure your understanding of the instructions, we will provide you with an example through the computer screen. After the example, you will participate in a practice round. The practice round is part of the instructions and is not relevant to your payment. Its objective is to get you familiar with the computer interface and the flow of the decisions in each round. Once the practice round is over, the computer will tell you "The official rounds begin now!"

**Completion of the Experiment**

After the 10th round, the experiment will be over. You will be instructed to fill in the receipt for your payment. The amount you earn will be paid electronically via the HKUST Autopay System to the bank account you provide to the Student Information System (SIS). The auto-payment will be arranged by the Finance Office of HKUST.

## Appendix C. Non-parametric Tests Results

| Env. | Test | Two-sided? | Null Hypothesis | Treatment Averages | $p$-values |
|---|---|---|---|---|---|
| Baseline | MW | ✓ | The proportion of (BLUE, RED, GREEN) is the same across DA-B and CADA-B. | 36.0% vs. 45.3% | 0.0796 |
| | MW | ✓ | The proportion of (BLUE, GREEN, RED) is the same across DA-B and CADA-B. | 56.0% vs. 50.5% | 0.4568 |
| | MW | ✓ | The proportion of (BLUE, GREEN, RED) is the same across SIC-B and DA-B. | 90% vs. 56% | 0.0284 |
| | MW | ✓ | The proportion of (BLUE, GREEN, RED) is the same across SIC-B and CADA-B. | 90.0% vs. 50.5% | 0.0286 |
| Cardinal | MW | ✓ | The proportion of (BLUE,RED,GREEN) across DA-C and CADA-C is the same. | 48.6% vs. 47.8% | 0.8845 |
| | MW | ✓ | The proportion of (BLUE,GREEN,RED) across DA-C and CADA-C is the same. | 48.6% vs. 46.7% | 0.8824 |
| | MW | ✓ | The proportion of (BLUE,GREEN,RED) across SIC-C and DA-C is the same. | 83.5% vs. 48.6% | 0.0294 |
| | MW | ✓ | The proportion of (BLUE,GREEN,RED) across SIC-C and CADA-C is the same. | 83.5% vs. 46.7% | 0.0284 |
| Replica | MW | One-sided | The proportion of the outcome with one (BLUE, RED, GREEN) and one (BLUE, GREEN, RED) is higher in CADA-R than in DA-R. | 37.5% vs. 17.5% | 0.0671 |
| | MW | ✓ | The proportion of the outcome with two (BLUE,RED,GREEN) across DA-R and CADA-R is the same. | 45.0% vs.37.5% | 1 |
| | MW | ✓ | The proportion of the outcome with two (BLUE,GREEN,RED) across SIC-R and DA-R is the same. | 85.0% vs. 12.5% | 0.027 |
| | MW | ✓ | The proportion of the outcome with two (BLUE,GREEN,RED) across SIC-R and CADA-R is the same. | 85% vs. 15% | 0.0247 |
| New | KW | N/A | The proportion of (RED,BLUE,GREEN) across DA-N, CADA-N, and SIC-N is the same. | 31.1% vs. 17.5% vs. 7.5% | 0.0218 |
| | MW | ✓ | The proportion of (RED,BLUE,GREEN) across DA-N and CADA-N is the same. | 31.1% vs. 17.5% | 0.0796 |
| | MW | ✓ | The proportion of (RED,BLUE,GREEN) across DA-N and SIC-N is the same. | 31.1% vs. 7.5% | 0.0286 |
| | MW | ✓ | The proportion of (RED,BLUE,GREEN) across CADA-N and SIC-N is the same. | 17.5% vs. 7.5% | 0.1441 |
| | KW | N/A | The proportion of (BLUE,GREEN,RED) across DA-N, CADA-N, and SIC-N is the same. | 21.1% vs. 42.5% vs. 38.8% | 0.1201 |
| | MW | ✓ | The proportion of (BLUE,GREEN,RED) across DA-N and CADA-N is the same. | 21.1% vs. 42.5% | 0.0796 |
| | MW | ✓ | The proportion of (BLUE,GREEN,RED) across DA-N and SIC-N is the same. | 21.1% vs. 38.8% | 0.1832 |
| | MW | ✓ | The proportion of (BLUE,GREEN,RED) across CADA-N and SIC-N is the same. | 42.5% vs. 38.8% | 0.6489 |
| | KW | N/A | The proportion of (GREEN,BLUE,RED) across DA-N, CADA-N, and SIC-N is the same. | 44.4% vs. 32.5% vs. 46.2% | 0.1201 |
| | MW | ✓ | The proportion of (GREEN,BLUE,RED) across DA-N and CADA-N is the same. | 44.4% vs. 32.5% | 0.1832 |
| | MW | ✓ | The proportion of (GREEN,BLUE,RED) across DA-N and SIC-N is the same. | 44.4% vs. 46.2% | 0.559 |
| | MW | ✓ | The proportion of (GREEN,BLUE,RED) across CADA-N and SIC-N is the same. | 32.5% vs. 46.2% | 0.0396 |

■ MW, Wilc, and KW refer to the Mann-Whitney U (rank-sum) test, one-sample Wilcoxon (signed rank) test, and Kruskal-Wallis test, respectively.
■ $p$-values reported as a range inside square brackets jointly provides the range (minimum and maximum) of $p$-values from three pair-wise tests.
■ The test statistic of the KW test is approximated by Chi-square distribution, which is asymmetric. From the perspective of the test statistic, it can be considered as one-sided; from the perspective of the statement of null-hypothesis, essentially we are testing whether there is a significant difference in distribution but have no clue on whose median is larger/or who stochastically dominates whom, so it might be considered as two-sided. Hence, we leave it as N/A.

Table 5. Non-parametric Tests for Assignment Outcomes

| Env. | Test | Two-sided? | Null Hypothesis | Treatment Averages | $p$-values |
|---|---|---|---|---|---|
| Baseline | MW | ✓ | For Student R, the rate of truthful reporting is the same across treatments. | 89.0% vs. 84.2% vs. 94.4% | [0.559, 1] |
| | KW | N/A | | | 0.7346 |
| | MW | ✓ | For Student G, the rate of truthful reporting is the same across treatments. | 89.0% vs. 96.8% vs. 95.6% | [0.2396, 0.3719] |
| | KW | N/A | | | 0.2347 |
| | MW | ✓ | For Student B, the rate of truthful reporting is the same across treatments. | 55.0% vs. 73.7% vs. 64.4% | [0.1038, 0.3836] |
| | KW | N/A | | | 0.1382 |
| | Wilc | One-sided | For Student R, the rate of truthful reporting is lower than 100% | 100% vs. 89.0%/84.2%/94.4% | [0.0625, 0.1855] |
| | Wilc | One-sided | For Student G, the rate of truthful reporting is lower than 100% | 100%vs. 89.0%/96.8%/95.6% | [0.0868, 0.0907] |
| | Wilc | One-sided | For Student B, the rate of truthful reporting is lower than 100% | 100% vs. 55.0%/73.7%/64.4% | [0.0445, 0.0625] |
| Cardinal | MW | ✓ | For Student R, the rate of truthful reporting is the same across treatments. | 90.0% vs.92.2% vs. 87.1% | [0.1832, 1] |
| | KW | N/A | | | 0.5053 |
| | MW | ✓ | For Student G, the rate of truthful reporting is the same across treatments. | 94.3% vs.94.4% vs. 91.8% | [[0.6489, 0.8839]] |
| | KW | N/A | | | 0.8331 |
| | MW | ✓ | For Student B, the rate of truthful reporting is the same across treatments. | 84.3% vs.78.9% vs. 78.8% | [0.2975, 1] |
| | KW | N/A | | | 0.4282 |
| | Wilc | One-sided | For Student R, the rate of truthful reporting is lower than 100% | 100% vs. 90.0%/92.2%/87.1% | [0.0488, 0.1855] |
| | Wilc | One-sided | For Student G, the rate of truthful reporting is lower than 100% | 100% vs. 94.3%/94.4%/91.8% | [0.0907, 0.1729] |
| | Wilc | One-sided | For Student B, the rate of truthful reporting is lower than 100% | 100% vs. 84.3%/78.9%/78.8% | [0.0488, 0.0625] |
| Replica | MW | ✓ | For Student R, the rate of truthful reporting is the same across treatments. | 91.2% vs. 92.5% vs. 93.8% | [0.7568, 1] |
| | KW | N/A | | | 0.9307 |
| | MW | ✓ | For Student G, the rate of truthful reporting is the same across treatments. | 91.2% vs. 95.0% vs. 98.8% | [0.0485, 0.3529] |
| | KW | N/A | | | 0.0724 |
| | MW | ✓ | For Student B, the rate of truthful reporting is the same across treatments. | 83.8% vs. 80.0% vs. 60.0% | [0.0408, 1] |
| | KW | N/A | | | 0.0995 |
| | MW | One-sided | For Student B, the rate of truthful reporting is higher in DA-R than in SIC-R. | 83.8% vs. 60.0% | 0.0204 |
| | MW | One-sided | For Student B, the rate of truthful reporting is higher in CADA-R than in SIC-R. | 80.0% vs. 60.0% | 0.0956 |
| | Wilc | One-sided | For Student R, the rate of truthful reporting is lower than 100% | 100% vs. 91.2%/92.5%/93.8% | [0.0445, 0.0868] |
| | Wilc | One-sided | For Student G, the rate of truthful reporting is lower than 100% | 100% vs. 91.2%/95.0%/98.8% | [0.0488, 0.5] |
| | Wilc | One-sided | For Student B, the rate of truthful reporting is lower than 100% | 100% vs. 83.8%/80.0%/60.0% | [0.0488, 0.0625] |
| New | MW | ✓ | For Student R, the rate of truthful reporting is the same across treatments. | 87.8% vs.80.0% vs. 67.5% | [0.1913, 0.6573] |
| | KW | N/A | | | 0.2973 |
| | MW | ✓ | For Student G, the rate of truthful reporting is the same across treatments. | 98.9% vs.93.8% vs. 95.0% | [0.4047, 0.8744] |
| | KW | N/A | | | 0.5059 |
| | MW | ✓ | For Student B, the rate of truthful reporting is the same across treatments. | 75.6% vs.65.0% vs. 77.5% | [0.1913, 0.4568] |
| | KW | N/A | | | 0.2682 |
| | Wilc | One-sided | For Student R, the rate of truthful reporting is lower than 100% | 100% vs. 87.8%/80.0%/67.5% | [0.0488, 0.0625] |
| | Wilc | One-sided | For Student G, the rate of truthful reporting is lower than 100% | 100% vs. 98.9%/93.8%/95.0% | [0.1729, 0.5] |
| | Wilc | One-sided | For Student B, the rate of truthful reporting is lower than 100% | 100% vs. 75.6%/65.0%/77.5% | [0.0473, 0.0625] |

■ MW, Wilc, and KW refer to the Mann-Whitney U (rank-sum) test, one-sample Wilcoxon (signed rank) test, and Kruskal-Wallis test, respectively.
■ $p$-values reported as a range inside square brackets jointly provides the range (minimum and maximum) of $p$-values from three pair-wise tests.
■ The test statistic of the KW test is approximated by Chi-square distribution, which is asymmetric. From the perspective of the test statistic, it can be considered as one-sided; from the perspective of the statement of null-hypothesis, essentially we are testing whether there is a significant difference in distribution but have no clue on whose median is larger/or who stochastically dominates whom, so it might be considered as two-sided. Hence, we leave it as N/A.

TABLE 6. Non-parametric Tests for Preference Reporting

| Env. | Test | Two-sided? | Null Hypothesis | Treatment Averages | $p$-values |
|---|---|---|---|---|---|
| Baseline | Wilc | One-sided | For Student R, the rate of undominated targeting is lower than 100%. | 100% vs. 92.6% | 0.0868 |
| | Wilc | One-sided | For Student G, the rate of undominated targeting is lower than 100%. | 100% vs. 97.9% | 0.1855 |
| | Wilc | One-sided | For Student B, the rate of undominated targeting is lower than 100%. | 100% vs. 91.6% | 0.0907 |
| Cardinal | Wilc | One-sided | For Student R, the rate of undominated targeting is lower than 100%. | 100% vs. 98.9% | 0.5 |
| | Wilc | One-sided | For Student G, the rate of undominated targeting is lower than 100%. | 100% vs. 97.8% | 0.1729 |
| | Wilc | One-sided | For Student B, the rate of undominated targeting is lower than 100%. | 100% vs. 94.4% | 0.0907 |
| Replica | Wilc | One-sided | For Student R, the rate of undominated targeting is lower than 100%. | 100% vs. 95.0% | 0.1855 |
| | Wilc | One-sided | For Student G, the rate of undominated targeting is lower than 100%. | 100% vs. 98.8% | 0.5 |
| | Wilc | One-sided | For Student B, the rate of undominated targeting is lower than 100%. | 100% vs. 95.0% | 0.5 |
| New | Wilc | One-sided | For Student R, the rate of undominated targeting is lower than 100%. | 100% vs. 91.2% | 0.1855 |
| | Wilc | One-sided | For Student G, the rate of undominated targeting is lower than 100%. | 100% vs. 96.2% | 0.1855 |
| | Wilc | One-sided | For Student B, the rate of undominated targeting is lower than 100%. | 100% vs. 70.0% | 0.0625 |

■ Wilc refers to the one-sample Wilcoxon (signed rank) test.

TABLE 7. Non-parametric Tests for Targeting Behavior in CADA

| Env. | Test | Two-sided? | Null Hypothesis | Treatment Averages | $p$-values |
|---|---|---|---|---|---|
| Baseline | MW | ✓ | Student B's average earning is the same across treatments | 48.00 vs. 43.37 vs. 42.67 | [0.1776, 0.8744] |
| | KW | N/A | | | 0.2016 |
| | MW | One-sided | Student R's average earning is higher in SIC than in DA. | 115.56 vs.100.40 | 0.0147 |
| | MW | ✓ | Student R's average earning is not different between SICM and DA. | 115.56 vs.100.40 | 0.0294 |
| | MW | One-sided | Student G's average earning is higher in SIC than in CADA. | 147.56 vs.101.89 | 0.0143 |
| | MW | ✓ | Student G's average earning is not different between SIC and CADA. | 147.56 vs.101.89 | 0.0286 |
| | MW | ✓ | Student R's average earning is not different between DA and CADA. | 100.40 vs. 96.84 | 1 |
| | MW | ✓ | Student G's average earning is not different between DA and CADA. | 105.60 vs. 101.89 | 0.559 |
| Cardinal | MW | ✓ | Student B's average earning is the same across treatments. | 118.86 vs.122.22 vs. 120.47 | [0.2784, 0.8817] |
| | KW | N/A | | | 0.4848 |
| | MW | One-sided | Student R's average earning is higher in SIC than in DA. | 110.12 vs. 98.29 | 0.021 |
| | MW | ✓ | Student R's average earning is not different between SICM and DA. | 110.12 vs. 98.29 | 0.0421 |
| | MW | One-sided | Student G's average earning is higher in SIC than in CADA. | 141.18 vs. 94.67 | 0.0142 |
| | MW | ✓ | Student G's average earning is not different between SIC and CADA. | 141.18 vs. 94.67 | 0.0284 |
| | MW | ✓ | Student R's average earning is not different between DA and CADA. | 98.29 vs. 98.22 | 0.8845 |
| | MW | ✓ | Student G's average earning is not different between DA and CADA. | 97.71 vs. 94.67 | 0.8824 |
| Replica | MW | ✓ | Student B's average earning is the same across treatments. | 53.00 vs. 46.00 vs. 44.00 | [0.3778, 0.6552] |
| | KW | N/A | | | 0.4808 |
| | MW | One-sided | Student R's average earning is higher in SIC than in DA. | 114.50 vs. 90.50 | 0.0142 |
| | MW | ✓ | Student R's average earning is not different between SIC and DA. | 114.50 vs. 90.50 | 0.0284 |
| | MW | One-sided | Student G's average earning is higher in SIC than in CADA. | 152.00 vs. 82.50 | 0.0147 |
| | MW | ✓ | Student G's average earning is not different between SIC and CADA. | 152.00 vs. 82.50 | 0.0294 |
| | MW | ✓ | Student R's average earning is not different between DA and CADA. | 90.50 vs. 93.50 | 0.8817 |
| | MW | ✓ | Student G's average earning is not different between DA and CADA. | 68.00 vs. 82.50 | 0.5614 |
| New | MW | One-sided | Student B's average earning is higher in DA than in CADA. | 88.00 vs. 74.50 | 0.0407 |
| | MW | ✓ | Student B's average earning is not different between DA and CADA. | 88.00 vs. 74.50 | 0.0814 |
| | MW | One-sided | Student B's average earning is higher in DA than in SIC. | 88.00 vs. 80.50 | 0.0956 |
| | MW | ✓ | Student B's average earning is not different between DA and SIC. | 88.00 vs. 80.50 | 0.1913 |
| | MW | One-sided | Student R's average earning not different between SIC and CADA. in DA. | 93.50 vs. 87.11 | 0.0732 |
| | MW | ✓ | Student R's average earning is not different between DA and SIC. | 87.11 vs. 93.50 | 0.1465 |
| | MW | One-sided | Student G's average earning not different between SIC and CADA. in DA. | 147.00 vs. 132.44 | 0.0204 |
| | MW | ✓ | Student G's average earning is not different between DA and SIC. | 132.44 vs. 147.00 | 0.0408 |
| | MW | One-sided | Student R's average earning is higher in CADA than in DA. | 95.00 vs. 87.11 | 0.054 |
| | MW | ✓ | Student R's average earning is not different between DA and CADA. | 87.11 vs. 95.00 | 0.1081 |
| | MW | One-sided | Student G's average earning is higher in CADA than in DA. | 138.00 vs.132.44 | 0.3306 |
| | MW | ✓ | Student G's average earning is not different between DA and CADA. | 132.44 vs.138.00 | 0.6612 |

■ MW, Wilc, and KW refer to the Mann-Whitney U (rank-sum) test, one-sample Wilcoxon (signed rank) test, and Kruskal-Wallis test, respectively.
■ $p$-values reported as a range inside square brackets jointly provides the range (minimum and maximum) of $p$-values from three pair-wise tests.
■ The test statistic of the KW test is approximated by Chi-square distribution, which is asymmetric. From the perspective of the test statistic, it can be considered as one-sided; from the perspective of the statement of null-hypothesis, essentially we are testing whether there is a significant difference in distribution but have no clue on whose median is larger/or who stochastically dominates whom, so it might be considered as two-sided. Hence, we leave it as N/A.

Table 8. Non-parametric Tests for Earnings

## Appendix D. Regression Results

Table 9. Regression results on matching outcomes: Baseline Environment

| | Efficient Outcome | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | −0.0547 | −0.0547 | −0.2201 | −0.2201 | −0.1378 | −0.1378 |
| | (0.0641) | (0.0389) | (0.2876) | (0.1551) | (0.1799) | (0.0972) |
| SIC | 0.3400*** | 0.3400*** | 1.9560*** | 1.9560*** | 1.1310*** | 1.1310*** |
| | (0.0650) | (0.0438) | (0.4050) | (0.4828) | (0.2198) | (0.2477) |
| Constant | 0.5600*** | 0.5600*** | 0.2412 | 0.2412*** | 0.1510 | 0.1510*** |
| | (0.0448) | (0.0042) | (0.2015) | (0.0170) | (0.1259) | (0.0106) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 285 | 285 | 285 | 285 | 285 | 285 |
| $R^2$ | 0.1298 | 0.1298 | | | | |
| Log Likelihood | | | −163.7000 | −163.7000 | −163.7000 | −163.7000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

Table 10. Regression results on matching outcomes: Replica Environment

| | Efficient Outcome | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | 0.0250 | 0.0250 | 0.2113 | 0.2113 | 0.1139 | 0.1139 |
| | (0.0789) | (0.0510) | (0.6517) | (0.4457) | (0.3509) | (0.2383) |
| SIC | 0.7250*** | 0.7250*** | 3.6800*** | 3.6800*** | 2.1870*** | 2.1870*** |
| | (0.0789) | (0.0631) | (0.6517) | (0.5316) | (0.3509) | (0.2861) |
| Constant | 0.1250** | 0.1250*** | −1.9460*** | −1.9460*** | −1.1500*** | −1.1500*** |
| | (0.0558) | (0.0437) | (0.4781) | (0.3959) | (0.2540) | (0.2103) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 120 | 120 | 120 | 120 | 120 | 120 |
| $R^2$ | 0.4818 | 0.4818 | | | | |
| Log Likelihood | | | −48.8900 | −48.8900 | −48.8900 | −48.8900 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 11. Regression results on matching outcomes: Cardinal Environment

| | Efficient Outcome | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | −0.0190 | −0.0190 | −0.0764 | −0.0764 | −0.0478 | −0.0478 |
| | (0.0736) | (0.0925) | (0.3191) | (0.3692) | (0.1999) | (0.2313) |
| SIC | 0.3496*** | 0.3496*** | 1.6810*** | 1.6810*** | 1.0110*** | 1.0110*** |
| | (0.0745) | (0.0839) | (0.3778) | (0.3402) | (0.2209) | (0.2120) |
| Constant | 0.4857*** | 0.4857*** | −0.0572 | −0.0572 | −0.0358 | −0.0358 |
| | (0.0552) | (0.0833) | (0.2391) | (0.3320) | (0.1498) | (0.2080) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 245 | 245 | 245 | 245 | 245 | 245 |
| $R^2$ | 0.1228 | 0.1228 | | | | |
| Log Likelihood | | | −148.7000 | −148.7000 | −148.7000 | −148.7000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 12. Regression results on matching outcomes: New Environment

| | Efficient Outcome | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | 0.0944 | 0.0944 | 0.4551 | 0.4551 | 0.2741 | 0.2741 |
| | (0.0660) | (0.0693) | (0.3404) | (0.3384) | (0.2042) | (0.2029) |
| SIC | 0.1944*** | 0.1944*** | 1.0910*** | 1.0910*** | 0.6361*** | 0.6361*** |
| | (0.0660) | (0.0524) | (0.3837) | (0.2600) | (0.2187) | (0.1543) |
| Constant | 0.6556*** | 0.6556*** | 0.6436*** | 0.6436*** | 0.4004*** | 0.4004*** |
| | (0.0453) | (0.0490) | (0.2218) | (0.2160) | (0.1360) | (0.1324) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 250 | 250 | 250 | 250 | 250 | 250 |
| $R^2$ | 0.0340 | 0.0340 | | | | |
| Log Likelihood | | | −136.8000 | −136.8000 | −136.8000 | −136.8000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

Table 13. Regression results on preference reporting: Baseline Environment

| | Truthful Reporting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | 0.0153 | 0.0153 | 0.1664 | 0.1664 | 0.0856 | 0.0856 |
| | (0.0362) | (0.0509) | (0.3353) | (0.5675) | (0.1724) | (0.2906) |
| SIC | 0.0600 | 0.0600 | 0.8537** | 0.8537 | 0.4183** | 0.4183 |
| | (0.0367) | (0.0370) | (0.4099) | (0.5494) | (0.1966) | (0.2636) |
| StudentB | −0.3400*** | −0.3400*** | −1.8900*** | −1.8900*** | −1.1010*** | −1.1010*** |
| | (0.0437) | (0.0720) | (0.3025) | (0.4587) | (0.1722) | (0.2568) |
| CADA*StudentB | 0.1716*** | 0.1716* | 0.6626 | 0.6626 | 0.4224* | 0.4224 |
| | (0.0626) | (0.0946) | (0.4551) | (0.6139) | (0.2543) | (0.3373) |
| SIC*StudentB | 0.0344 | 0.0344 | −0.4597 | −0.4597 | −0.1736 | −0.1736 |
| | (0.0635) | (0.0918) | (0.5069) | (0.7504) | (0.2698) | (0.3920) |
| Constant | 0.8900*** | 0.8900*** | 2.0910*** | 2.0910*** | 1.2260*** | 1.2260*** |
| | (0.0252) | (0.0299) | (0.2260) | (0.3048) | (0.1177) | (0.1587) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 855 | 855 | 855 | 855 | 855 | 855 |
| $R^2$ | 0.1296 | 0.1296 | | | | |
| Log Likelihood | | | −346.7000 | −346.7000 | −346.7000 | −346.7000 |

*Note:* $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01

Table 14. Regression results on preference reporting: Replica Environment

| | Truthful Reporting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | 0.0250 | 0.0250 | 0.3635 | 0.3635 | 0.1778 | 0.1778 |
| | (0.0352) | (0.0334) | (0.4301) | (0.4747) | (0.2096) | (0.2324) |
| SIC | 0.0500 | 0.0500* | 0.9006* | 0.9006* | 0.4242* | 0.4242* |
| | (0.0352) | (0.0293) | (0.5013) | (0.4602) | (0.2312) | (0.2186) |
| StudentB | −0.0750* | −0.0750 | −0.7048* | −0.7048 | −0.3721* | −0.3721 |
| | (0.0431) | (0.0516) | (0.4125) | (0.4963) | (0.2188) | (0.2595) |
| CADA*StudentB | −0.0625 | −0.0625 | −0.6169 | −0.6169 | −0.3204 | −0.3204 |
| | (0.0610) | (0.1030) | (0.5957) | (0.8621) | (0.3124) | (0.4589) |
| SIC*StudentB | −0.2875*** | −0.2875*** | −2.1350*** | −2.1350*** | −1.1550*** | −1.1550*** |
| | (0.0610) | (0.0852) | (0.6287) | (0.6492) | (0.3190) | (0.3408) |
| Constant | 0.9125*** | 0.9125*** | 2.3450*** | 2.3450*** | 1.3560*** | 1.3560*** |
| | (0.0249) | (0.0270) | (0.2798) | (0.3371) | (0.1405) | (0.1693) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 720 | 720 | 720 | 720 | 720 | 720 |
| $R^2$ | 0.1093 | 0.1093 | | | | |
| Log Likelihood | | | −239.8000 | −239.8000 | −239.8000 | −239.8000 |

*Note:* $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01

Table 15. Regression results on preference reporting: Cardinal Environment

| | Truthful Reporting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | 0.0119 | 0.0119 | 0.1771 | 0.1771 | 0.0863 | 0.0863 |
| | (0.0364) | (0.0389) | (0.4335) | (0.5390) | (0.2115) | (0.2657) |
| SIC | −0.0273 | −0.0273 | −0.3284 | −0.3284 | −0.1660 | −0.1660 |
| | (0.0368) | (0.0412) | (0.4010) | (0.5488) | (0.2017) | (0.2722) |
| StudentB | −0.0786* | −0.0786 | −0.7823* | −0.7823 | −0.4085* | −0.4085 |
| | (0.0472) | (0.0644) | (0.4544) | (0.6546) | (0.2383) | (0.3370) |
| CADA*StudentB | −0.0659 | −0.0659 | −0.5385 | −0.5385 | −0.2900 | −0.2900 |
| | (0.0630) | (0.0851) | (0.6021) | (0.7507) | (0.3156) | (0.3948) |
| SIC*StudentB | −0.0273 | −0.0273 | −0.0369 | −0.0369 | −0.0399 | −0.0399 |
| | (0.0638) | (0.0771) | (0.5824) | (0.7070) | (0.3112) | (0.3693) |
| Constant | 0.9214*** | 0.9214*** | 2.4620*** | 2.4620*** | 1.4150*** | 1.4150*** |
| | (0.0273) | (0.0380) | (0.3141) | (0.5227) | (0.1551) | (0.2580) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 735 | 735 | 735 | 735 | 735 | 735 |
| $R^2$ | 0.0300 | 0.0300 | | | | |
| Log Likelihood | | | −260.8000 | −260.8000 | −260.8000 | −260.8000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

Table 16. Regression results on preference reporting: New Environment

| | Truthful Reporting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| CADA | −0.0646 | −0.0646 | −0.7491** | −0.7491* | −0.3806** | −0.3806* |
| | (0.0403) | (0.0423) | (0.3796) | (0.3998) | (0.1908) | (0.2098) |
| SIC | −0.1208*** | −0.1208*** | −1.1730*** | −1.1730*** | −0.6139*** | −0.6139*** |
| | (0.0403) | (0.0293) | (0.3610) | (0.2531) | (0.1839) | (0.1327) |
| StudentB | −0.1778*** | −0.1778*** | −1.5110*** | −1.5110*** | −0.8090*** | −0.8090*** |
| | (0.0479) | (0.0647) | (0.3866) | (0.4353) | (0.2037) | (0.2411) |
| CADA*StudentB | −0.0410 | −0.0410 | 0.2397 | 0.2397 | 0.0738 | 0.0738 |
| | (0.0699) | (0.1165) | (0.5091) | (0.7349) | (0.2792) | (0.4142) |
| SIC*StudentB | 0.1403** | 0.1403* | 1.2810** | 1.2810*** | 0.6773** | 0.6773** |
| | (0.0699) | (0.0748) | (0.5120) | (0.4955) | (0.2808) | (0.2764) |
| Constant | 0.9333*** | 0.9333*** | 2.6390*** | 2.6390*** | 1.5010*** | 1.5010*** |
| | (0.0277) | (0.0113) | (0.2988) | (0.1813) | (0.1438) | (0.0872) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 750 | 750 | 750 | 750 | 750 | 750 |
| $R^2$ | 0.0511 | 0.0511 | | | | |
| Log Likelihood | | | −328.0000 | −328.0000 | −328.0000 | −328.0000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 17. Regression results on targeting: Baseline Environment

| | Top School Targeting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| StudentR | 0.0105 | 0.0105 | 0.1450 | 0.1450 | 0.0716 | 0.0716 |
| | (0.0343) | (0.0321) | (0.5392) | (0.4599) | (0.2661) | (0.2253) |
| StudentG | 0.0632* | 0.0632* | 1.4530* | 1.4530* | 0.6552* | 0.6552* |
| | (0.0343) | (0.0380) | (0.8045) | (0.7712) | (0.3447) | (0.3435) |
| Constant | 0.9158*** | 0.9158*** | 2.3860*** | 2.3860*** | 1.3770*** | 1.3770*** |
| | (0.0243) | (0.0344) | (0.3695) | (0.4447) | (0.1844) | (0.2219) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 285 | 285 | 285 | 285 | 285 | 285 |
| $R^2$ | 0.0136 | 0.0136 | | | | |
| Log Likelihood | | | −62.1400 | −62.1400 | −62.1400 | −62.1400 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 18. Regression results on targeting: Replica Environment

| | Top School Targeting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| StudentR | 0.0000 | 0.0000 | −0.0000 | −0.0000 | 0.0000 | 0.0000 |
| | (0.0301) | (0.0615) | (0.7255) | (1.2890) | (0.3341) | (0.5938) |
| StudentG | 0.0375 | 0.0375 | 1.4250 | 1.4250 | 0.5965 | 0.5965 |
| | (0.0301) | (0.0557) | (1.1290) | (1.6870) | (0.4507) | (0.7135) |
| Constant | 0.9500*** | 0.9500*** | 2.9440*** | 2.9440*** | 1.6450*** | 1.6450*** |
| | (0.0213) | (0.0502) | (0.5130) | (1.0530) | (0.2363) | (0.4848) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 240 | 240 | 240 | 240 | 240 | 240 |
| $R^2$ | 0.0087 | 0.0087 | | | | |
| Log Likelihood | | | −37.1400 | −37.1400 | −37.1400 | −37.1400 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 19. Regression results on targeting: Cardinal Environment

| | Top School Targeting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| StudentR | 0.0444* | 0.0444 | 1.6550 | 1.6550 | 0.6933 | 0.6933 |
| | (0.0253) | (0.0368) | (1.1060) | (1.4200) | (0.4351) | (0.5938) |
| StudentG | 0.0333 | 0.0333 | 0.9510 | 0.9510** | 0.4167 | 0.4167 |
| | (0.0253) | (0.0210) | (0.8504) | (0.4443) | (0.3641) | (0.7135) |
| Constant | 0.9444*** | 0.9444*** | 2.8330*** | 2.8330*** | 1.5930*** | 1.5930*** |
| | (0.0179) | (0.0295) | (0.4602) | (0.5608) | (0.2153) | (0.4848) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 270 | 270 | 270 | 270 | 270 | 270 |
| $R^2$ | 0.0124 | 0.0124 | | | | |
| Log Likelihood | | | −34.4000 | −34.4000 | −34.4000 | −34.4000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 20. Regression results on targeting: New Environment

| | Top School Targeting | | | | | |
|---|---|---|---|---|---|---|
| | Linear | Linear | Logit | Logit | Probit | Probit |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| StudentR | 0.2125*** | 0.2125** | 1.4970*** | 1.4970 | 0.8319*** | 0.8319 |
| | (0.0524) | (0.1033) | (0.4648) | (0.9889) | (0.2474) | (0.5938) |
| StudentG | 0.2625*** | 0.2625*** | 2.3980*** | 2.3980*** | 1.2560*** | 1.2560* |
| | (0.0524) | (0.0829) | (0.6371) | (0.8995) | (0.2987) | (0.7135) |
| Constant | 0.7000*** | 0.7000*** | 0.8473*** | 0.8473*** | 0.5244*** | 0.5244 |
| | (0.0371) | (0.0648) | (0.2440) | (0.3074) | (0.1474) | (0.4848) |
| Cluster: | None | Session | None | Session | None | Session |
| Observations | 240 | 240 | 240 | 240 | 240 | 240 |
| $R^2$ | 0.1065 | 0.1065 | | | | |
| Log Likelihood | | | −85.4000 | −85.4000 | −85.4000 | −85.4000 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 21. Regression results on earnings: Baseline Environment

| | Earnings | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Student B | Student B | Student R | Student R | Student G | Student G | Pooled | Pooled |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| CADA | −4.6320 | −4.6320 | −3.5580 | −3.5580 | −3.7050 | −3.7050 | −3.9650 | −3.9650** |
| | (3.4960) | (2.9270) | (3.5960) | (3.5920) | (7.8700) | (4.0290) | (4.1260) | (1.9140) |
| SIC | −5.3330 | −5.3330* | 15.1600*** | 15.1600*** | 41.9600*** | 41.9600*** | 17.2600*** | 17.2600*** |
| | (3.5450) | (2.8430) | (3.6470) | (2.4960) | (7.9820) | (5.3620) | (4.1840) | (2.3150) |
| Constant | 48.0000*** | 48.0000*** | 100.4000*** | 100.4000*** | 105.6000*** | 105.6000*** | 84.6700*** | 84.6700*** |
| | (2.4400) | (2.3720) | (2.5100) | (1.0660) | (5.4930) | (0.8953) | (2.8800) | (0.5123) |
| Cluster: | None | Session | None | Session | None | Session | None | Session |
| Observations | 285 | 285 | 285 | 285 | 285 | 285 | 855 | 855 |
| $R^2$ | 0.0096 | 0.0096 | 0.0928 | 0.0928 | 0.1223 | 0.1223 | 0.0321 | 0.0321 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 22. Regression results on earnings: Replica Environment

| | Earnings | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Student B | Student B | Student R | Student R | Student G | Student G | Pooled | Pooled |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| CADA | −7.0000 | −7.0000 | 3.0000 | 3.0000 | 14.5000* | 14.5000 | 3.5000 | 3.5000 |
| | (4.7720) | (6.3580) | (3.9160) | (4.4960) | (7.9900) | (10.5500) | (4.3170) | (6.0760) |
| SIC | −9.0000* | −9.0000 | 24.0000*** | 24.0000*** | 84.0000*** | 84.0000*** | 33.0000*** | 33.0000*** |
| | (4.7720) | (7.3230) | (3.9160) | (4.6170) | (7.9900) | (10.6200) | (4.3170) | (5.8640) |
| Constant | 53.0000*** | 53.0000*** | 90.5000*** | 90.5000*** | 68.0000*** | 68.0000*** | 70.5000*** | 70.5000*** |
| | (3.3750) | (5.9560) | (2.7690) | (4.0880) | (5.6500) | (9.8950) | (3.0530) | (5.8460) |
| Cluster: | None | Session | None | Session | None | Session | None | Session |
| Observations | 240 | 240 | 240 | 240 | 240 | 240 | 720 | 720 |
| $R^2$ | 0.0163 | 0.0163 | 0.1584 | 0.1584 | 0.3477 | 0.3477 | 0.0895 | 0.0895 |

*Note:* *p<0.1; **p<0.05; ***p<0.01

TABLE 23. Regression results on earnings: Cardinal Environment

| | Earnings | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Student B | Student B | Student R | Student R | Student G | Student G | Pooled | Pooled |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| CADA | −7.0000 | −7.0000 | 3.0000 | 3.0000 | 14.5000* | 14.5000 | 0.0847 | 0.0847 |
| | (4.7720) | (6.3580) | (3.9160) | (4.4960) | (7.9900) | (10.5500) | (3.5180) | (5.5580) |
| SIC | −9.0000* | −9.0000 | 24.0000*** | 24.0000*** | 84.0000*** | 84.0000*** | 18.9700*** | 18.9700*** |
| | (4.7720) | (7.3230) | (3.9160) | (4.6170) | (7.9900) | (10.6200) | (3.5630) | (5.1020) |
| Constant | 53.0000*** | 53.0000*** | 90.5000*** | 90.5000*** | 68.0000*** | 68.0000*** | 105.0000*** | 105.0000*** |
| | (3.3750) | (5.9560) | (2.7690) | (4.0880) | (5.6500) | (9.8950) | (2.6390) | (5.0640) |
| Cluster: | None | Session | None | Session | None | Session | None | Session |
| Observations | 240 | 240 | 240 | 240 | 240 | 240 | 735 | 735 |
| $R^2$ | 0.0163 | 0.0163 | 0.1584 | 0.1584 | 0.3477 | 0.3477 | 0.0528 | 0.0528 |

*Note:* $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

TABLE 24. Regression results on earnings: New Environment

| | Earnings | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Student B | Student B | Student R | Student R | Student G | Student G | Pooled | Pooled |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| CADA | −7.0000 | −7.0000 | 3.0000 | 3.0000 | 14.5000* | 14.5000 | 0.0847 | 0.0847 |
| | (4.7720) | (6.3580) | (3.9160) | (4.4960) | (7.9900) | (10.5500) | (3.5180) | (5.5580) |
| SIC | −9.0000* | −9.0000 | 24.0000*** | 24.0000*** | 84.0000*** | 84.0000*** | 18.9700*** | 18.9700*** |
| | (4.7720) | (7.3230) | (3.9160) | (4.6170) | (7.9900) | (10.6200) | (3.5630) | (5.1020) |
| Constant | 53.0000*** | 53.0000*** | 90.5000*** | 90.5000*** | 68.0000*** | 68.0000*** | 105.0000*** | 105.0000*** |
| | (3.3750) | (5.9560) | (2.7690) | (4.0880) | (5.6500) | (9.8950) | (2.6390) | (5.0640) |
| Cluster: | None | Session | None | Session | None | Session | None | Session |
| Observations | 240 | 240 | 240 | 240 | 240 | 240 | 735 | 735 |
| $R^2$ | 0.0163 | 0.0163 | 0.1584 | 0.1584 | 0.3477 | 0.3477 | 0.0528 | 0.0528 |

*Note:* $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$