Real-time Optimal Control of River Basin Networks ⋆

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Abstract: River basins are key components of water supply grids. River basin operators must handle a complex set of objectives including runoff storage, flood control, supply for consumptive use, hydroelectric power generation, silting management, and maintenance of river basin ecology. At present, operators rely on a combination of simulation and optimization tools to help them make operational decisions. The complexity associated with this approach makes it suitable for long term planning but not daily or hourly operation. The consequence is that between longer-term optimized operation points, river basins are largely operated in open loop. This leads to operational inefficiencies most notably wasted water and poor ecological outcomes. This paper proposes a systematic approach using optimal control based on simple low order models for the real-time operation of entire river basin networks.

Keywords: River basin, Optimal control, LQR control, Model predictive control.

1. INTRODUCTION

River basins are key components of water supply grids. However, they are largely operated in open-loop mode. One reason is the difficulty associated with the development of suitable models. Traditionally, river basin modeling efforts have focused on process-based methodologies that are potentially very accurate but not amenable to the design of feedback controllers. For control purposes river basin operators often rely on simulation-optimization and/or rule-based approaches. This method may work well for long-term planning intervals (e.g. months) but is impractical for real-term operations (e.g. hours). This limitation results in suboptimal river flows and releases from water storages. A systematic approach to real-time river operation is needed. Feedback control offers one solution, and is the subject of this paper.

Real-time river basin operation is typical of large-scale control problems that have the following characteristics (Papageorgiou, 1984):

(1) A network structure with some kind of flow along links connecting storage units;
(2) Flow is to be routed from specific sources to designated destinations;
(3) Flow is subject to capacity constraints;
(4) There are time-varying demand variables at the source, along the network and at the destination;
(5) The links and storage units are characterized by transport lags; and,
(6) A communication network with limited bandwidth is used to transmit network state.

The general control problem is to specify control inputs to influence the flow in the network so as to minimize a performance criterion subject to capacity constraints and time-varying loads. River basin real-time operational objectives include in-stream flow rates and water levels in storages.

Modeling and optimization of water resources systems has a rich history (Labadie, 2004). The Saint-Venant equations (Chow, 1988) are the basis for the mathematical modeling of open water channels. These are hyperbolic partial differential equations making them difficult to use in feedback controller design. The study in Weyer (2002) has focused on the use of decentralized PI control, and Litrico and Fromion (2006) used H_∞ control. The studies in Negenborn et al. (2009) explored the use of model predictive control. An alternative to the Saint-Venant equations is to exploit grey-box or data-based models derived using system identification experiments by Jakeman and Horngerger (1993), Young (1998). The key advantage of these models is that feedback controllers are easier to design (Mareels et al., 2005).

Most of the studies cited above focus on modeling and control of irrigation canal networks and short river reaches. Combined simulation-optimization methods are commonly used to plan and operate river basin networks (Bridgart and Bethune, 2009). This paper builds on previous work

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in open canals to develop a framework for real-time river basin operation based on optimal control theory. The River Murray system in Australia is used as a case study.

The River Murray system (MDBC, 2006a) drains a catchment region which covers the south east corner of the Australian continent and extends over 1,060,000 km^2. The total length of the main river channel is 3,780 km and the mean discharge is 0.4 ML/sec. The system is largely fed by precipitation and snow-melt in the Australian Alps. The main consumptive demands are irrigation districts and rural populations and one major metropolitan demand site in Adelaide, South Australia. The River Murray is permanently navigable to a distance of 970 kilometers from the mouth due to a series of locks and weirs.

The River Murray is operated in three modes (MDBC, 2006b): (1) Supplying mode; (2) Storing mode; and (3) Spilling mode. It is possible for different reaches of the river to be in different modes. Supplying mode occurs during the irrigation season. The flow in the river is set to meet demands with little excess. Storing mode generally occurs when the flows in the river are in excess of that required to meet diversions, water supply, and minimum flow requirements; but which are confined within the channel. Spilling mode occurs when flow exceeds the river's channel capacity at a point as a result of runoff generated by heavy rain. This operation can be quite complex as the flow varies as tributaries join the main stream.

This paper proposes a systematic approach using optimal control based on simple low order models for the real-time operation of entire river basin networks. It is the preliminary version of Evans et al. (2011).

2. MODELS

A schematic of the River Murray System is illustrated in Figure 1. Following the methodology in Papageorgiou (1984) the river system is subdivided into a sub-networks with storage capabilities. Links connecting the sub-networks are treated as pure delays. In this sense flow rates leaving a sub-network (or storage element) are control variables, whereas the volumes (or water levels) in the storage elements are the state variables. In-stream flow rates further downstream from storage outlets can also be considered as state variables.

2.1 Physical models

Storage models River basin storages are modeled using the continuity equation

$$\dot{V}(t) = \sum_{n\in I} q_{in,n}(t) - \sum_{m\in O} u_m(t). \qquad (1)$$

where V is the storage volume, $q_{in,n}(t)$ and $u_m(t)$ are inflow rate and outflow rates respectively, and I and O denote the set of all inflows and outflows, respectively. The inflow is a measurement some distance upstream of the storage. The outflow is defined as a control variable. This approach was originally proposed in Winn and Moore (1973) and later in Weyer (2001) where the control components are defined in terms of flows.

In river basin operations, storage volume is generally *inferred* from water level measured at the downstream end

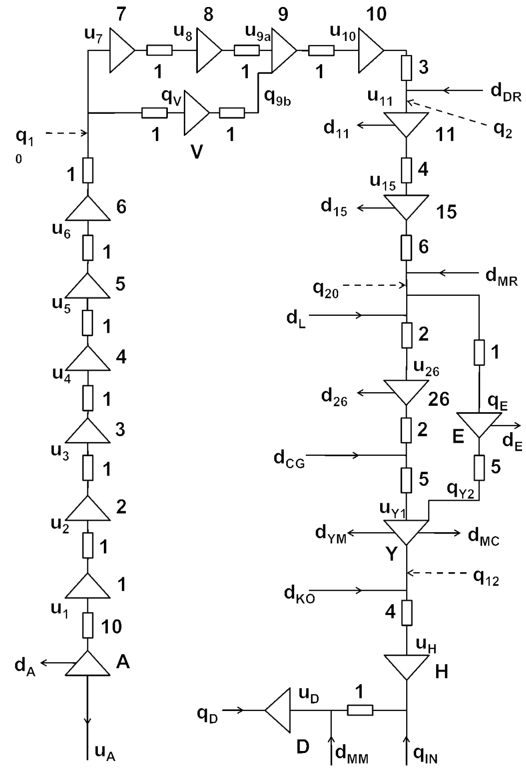


Fig. 1. River Murray schematic

of a storage element, close to the outflow control point. The function relating storage volume and water level depends on the storage element's geometry. Assuming only a single outflow structure is present, the following model for water level in a storage element can be used

$$\dot{y}(t) = \alpha(y)\left(\sum_{n\in I} q_{in,n}(t) - \sum_{m\in O} u_m(t)\right) \qquad (2)$$

where the function $\alpha(y)$ is related to the storage element's geometry. Generally it will be non-linear, for example when the storage is deep and has sloping sides. In this paper, this value is assumed constant. Without this simplification, the system can still be described by a set of linearized models by selecting several operating points over the range of set-points. Gain scheduling is a popular method used for designing controllers for such systems (Shamma and Athans, 1990). Letting T_s denote the sample interval, and using a first order approximation for $\dot{y}$, the discrete-time model for water level is given by

$$y[k+1] = y[k] + T_s\alpha\left(\sum_{n\in I} q_{in,n}[k] - \sum_{m\in O} u_m[k]\right). \qquad (3)$$

River reach models The Saint-Venant equations are a good starting point for modeling river reaches. It has been shown in Litrico and Georges (1999) (see also Weyer (2001)) that under relatively mild assumptions, the Saint-Venant equations can be linearlized about a reference flow rate resulting in the following river reach dynamics

$$\dot{q}_{out,i} + \frac{1}{K} q_{out,i} = q_{in,i}(t - \tau_i) \qquad (4)$$

where τ_i is the input delay, and K is the time constant. It is important to note the parameters in (4) vary with the reference flow rate. The above first-order system takes into account the transport delay, in-stream storage phenomena and the dispersion of the flow (or wave attenuation) as it moves downstream. In this study the river reach model is simplified to a transport delay. As for storages described above, without this simplification, river reaches can still be described by a set linearized models by selecting a several operating points over the range of set-points. Once again, gain scheduling can be applied. With the above simplification, in discrete-time notation we have

$$q_{out_i}[k+1] = q_{in_i}[k - \lceil \tau_i / T_s \rceil]. \qquad (5)$$

2.2 State-space representation

We define a set of states that are deviations from setpoints, for example $x_{i,0}[k] = y_i[k] - y_i^*[k]$, where $y_i^*[k]$ is the setpoint at node i. The same applies to flow, but with the following notation $x_{i,0}[k] = q_i[k] - q_i^*[k]$. To deal with time delays associated with flows within the network, we introduce the states $x_{j,i}[k] = u_j[k-i]$ which implies that whenever the term $u_j[k-\tau]$ appears in the model equations we introduce the following auxiliary state equations

$$\begin{aligned} x_{j,1}[k+1] &= u_j[k] \\ x_{j,i+1}[k+1] &= x_{j,i}[k], i = 1, \ldots, \tau - 1 \end{aligned}$$

and the substitute $x_{j,\tau}[k]$ in all model equations where $u_j[k-\tau]$ appears (Weyer, 2003).

For the network in Fig. 1, the state equations for the first two storages and the last storage take on the form

$$\begin{aligned} x_{D,0}[k+1] &= x_{D,0}[k] + a_D(q_D[k] - u_D[k]) + v_{sp,D} \\ x_{D,1}[k+1] &= u_D[k] \\ x_{H,0}[k+1] &= x_{H,0}[k] + a_H(x_{D,1}[k] + d_{MM}[k-1] \\ &\quad + q_{IN}(k) - u_H[k]) + v_{sp,H} \\ x_{H,1}[k+1] &= u_H[k] \\ x_{H,i+1}[k+1] &= x_{H,i}[k], i = 1, \ldots, 3 \\ &\vdots \\ x_{A,0}[k+1] &= x_{A,0}[k] + a_A(x_{1,10}[k] - u_A[k] - d_A[k]) \\ &\quad + v_{sp,A} \\ x_{A,1}[k+1] &= u_A[k] \end{aligned}$$

The disturbances q_D, d_{MM}, q_{IN} and d_A are inflows or offtakes while disturbances $v_{sp,A}$, $v_{sp,D}$, $v_{sp,H}$ represent water level or flow setpoints. All these disturbances are known in advance and their effects can be minimized through feedforward. Note that while the last control input $u_A[k]$ does not feed any storage within the network, it is still necessary to introduce the state $x_{A,1}[k]$ if we are to control flow and gate movement of storage A.

Using the above set of equations as an example, the state space equation for the entire network can be generated and takes on the form

$$\tilde{\mathbf{x}}[k+1] = A\tilde{\mathbf{x}}[k] + B\mathbf{u}[k] + (A-I)\mathbf{x}_{SP} + B\mathbf{u}_{SP} + \mathbf{w}[k] \qquad (6)$$

where $\tilde{\mathbf{x}}[k] = [x_{D,0}[k], x_{D,1}[k], x_{H,0}[k], x_{H,1}[k], \ldots, x_{H,4}[k], \ldots, x_{A,0}[k], x_{A,1}[k]]^T$, $\mathbf{u}[k] = [\, u_D[k],\ u_H[k],\ \ldots,\ u_A[k]\,]^T$, $\mathbf{x}_{SP}$ and $\mathbf{u}_{SP}$ are the set setpoint vectors for state $\tilde{\mathbf{x}}(k)$ and control input $\mathbf{u}(k)$, respectively. Note that all state variables are accessible and so there is no need for an observer or estimator.

3. CONTROL OBJECTIVES

Controlling a river basin network involves regulating a selected set of states around their set-points based on the operational mode of interest. For example in storage mode a river operator maintains water levels in storages at specified levels while allowing flows to take on values necessary to maintain those levels. On the other hand, in supply mode a river operator maintains constant flow rates in-stream while allowing storage levels to take on values necessary to maintain those flows. This section summarizes the main operational objectives for the River Murray (MDBC, 2006a):

(1) Meet water demands for both consumptive use and environmental flows, expressed as a flow rate (set-point regulation);
(2) Keep storage water levels close to a reference level (also set-point regulation);
(3) Reject disturbances caused by urban and irrigation withdrawals and rainfall-runoff;
(4) Minimize control effort by minimizing gate movement; and,
(5) Maintain rate of rise and rate of fall within bounds to avoid river bank slumping.

The remainder of this paper develops two alternative optimal control frameworks to achieve these objectives.

4. CONTROLLER DESIGN

4.1 Centralized LQR controller

In this section, the River Murray control problem is formulated as a finite horizon LQR problem incorporating feedforward of forecast disturbances.

For this puepose, the state-space model in Section 2.2 is considered. For a set of selected states it is possible to achieve zero steady-state error in the presence of disturbances by including the integral of these setpoint errors in the controller criterion function. Let $\mathbf{x}_{\text{int}}[k]$ be the vector of integrated setpoint errors. If E is a matrix that maps the river network state vector $\tilde{\mathbf{x}}[k]$ to the integrated error state vector $\mathbf{x}_{\text{int}}[k]$, then the state equation for the integrated error state vector takes on the form

$$\mathbf{x}_{\text{int}}[k+1] = \mathbf{x}_{\text{int}}[k] + TE\tilde{\mathbf{x}}[k]) \qquad (7)$$

where T is the sampling interval. A new augmented state vector $\mathbf{x}[k] = [\, \tilde{\mathbf{x}}^T[k]\ \mathbf{x}_{\text{int}}^T[k]\,]^T$ and a new state equation obtained from (6) and (7) therefore takes on the form

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] + \mathbf{d}[k]. \qquad (8)$$

Here $\mathbf{A} = \begin{bmatrix} A & 0 \\ E & I \end{bmatrix}, \mathbf{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \mathbf{d}[k] = \begin{bmatrix} (A-I)\mathbf{x}_{SP} + B\mathbf{u}_{SP} + \mathbf{w}[k] \\ 0 \end{bmatrix}$.

The quadratic cost function to be minimized is

$$J_K = \frac{1}{2}\mathbf{x}[K]^T\mathbf{S}\mathbf{x}[K] + \frac{1}{2}\sum_{k=0}^{K-1} \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{u}[k] \end{bmatrix}^T \begin{bmatrix} \mathbf{Q} & \mathbf{N} \\ \mathbf{N}^T & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{u}[k] \end{bmatrix}, \qquad (9)$$

where $\mathbf{R} > 0$ and $\begin{bmatrix} \mathbf{Q} & \mathbf{N} \\ \mathbf{N}^T & \mathbf{R} \end{bmatrix} \geq 0$.

From objective 4, the physical input effort is the gate movement and so terms of the form $(u_j[k] - u_j[k-1])^2$ should be penalized, where $j = 1, 2, ..., N_D$ is the storage index. When expressed in terms of state variables and inputs, these terms take on the form $(u_j[k]-x_{j,1}[k])^2$. We note that these terms include both inputs and states and so we get a non-zero $\mathbf{N}$ matrix in the cost function. The weight matrices therefore take on the form $\begin{bmatrix} \mathbf{Q}_1 & \mathbf{N}_1 \\ \mathbf{N}_1^T & \mathbf{R}_1 \end{bmatrix} = \sum_{i=1}^{N_D} s_i s_i^T$, where the vectors $s_i, i = 1,2,3,...,N_D$ have a η in the position corresponding to the $u_j(k)$ and $-\eta$ in the position corresponding to $x_{j1}(k)$ and zero elsewhere.

From control objectives 1, 2 and 4, the terms of interest are the level setpoint errors $x_{j,0}[k]$, the setpoint errors $x_{j,1}[k]$ of delayed gate outflows, the integrated setpoint errors $x_{j,\text{int}}[k]$, and the control input or gate flow errors $u_j[k]$. We denote the weights associated with the setpoint errors by γ_j, the weights associated with the delayed setpoint errors of gate outflows by ξ_j, the weights associated with the integrated setpoint errors by ζ_j, and the weights associated with gate flow errors by σ_j for $j = 1,...,N_D$. We therefore construct a matrix $\begin{bmatrix} \mathbf{Q}_2 & \mathbf{N}_2 \\ \mathbf{N}_2^T & \mathbf{R}_2 \end{bmatrix}$ with the weights γ_i, ξ_i, and ζ_i, and $\sigma_j, i = 1,...,N_D$ in the appropriate places.

Hence the physical control problem can be formulated in an LQ framework, with the matrices $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{N}$ given by $\mathbf{Q} = \mathbf{Q}_1 + \mathbf{Q}_2$, $\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2$, $\mathbf{N} = \mathbf{N}_1 + \mathbf{N}_2$. The control objective 3 is achieved through feedforward control while the remaining objectives can be achieved through the use of constraints. This is a basic description of the composition of the $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{N}$ matrices and assumes that each storage has a single control gate. Extension to include storages with multiple gates is straightforward.

For a given disturbance trajectory $\mathbf{w}[k], k = 1,\dots,K-1$ we seek a control $\mathbf{u}(k)$ that minimizes (9) subject to (8) with initial condition $\mathbf{x}[0] = \mathbf{x}_0$. Such an LQ controller that employs feedforward for disturbance rejection can be obtained by first formulating the discrete-time Hamiltonian and then applying the *maximum principle* (Sage and White, 1977). When the resulting difference equations are solved we obtain

$$\mathbf{u}[k] = -\mathbf{K}\mathbf{x}[k] - \mathbf{K}_d\mathbf{p}[k+1]$$

where $\mathbf{K} = (\mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}(\mathbf{B}^T\mathbf{P}\mathbf{A} + \mathbf{N}^T)$ and $\mathbf{K}_d = \mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T$ are the feedback and feedforward gain matrices respectively (Marinaki and Papageorgiou, 2005; Sage and White, 1977). The matrix $\mathbf{P}$ is the positive definite solution to the steady-state Riccati equation

$$\mathbf{P} = \mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{Q} - (\mathbf{A}^T\mathbf{P}\mathbf{B} + \mathbf{N})(\mathbf{B}^T\mathbf{P}\mathbf{B} + \mathbf{R})^{-1}(\mathbf{B}^T\mathbf{S}\mathbf{A} + \mathbf{N}^T)$$

and the feedforward signal is

$$\mathbf{p}[k] = \mathbf{P}\mathbf{d}[k-1] + \mathbf{A}^T(\mathbf{I} - \mathbf{P}\mathbf{B}\mathbf{K}_d)\mathbf{p}[k+1]. \qquad (10)$$

The vector $\mathbf{p}[k]$ is calculated by backward integration of equation (10) starting from $\mathbf{p}[K+1] = 0$.

4.2 MPC controller design

Model predictive control (MPC) (Maciejowski, 2002; Mayne et al., 2000) is one of the leading advanced control technologies in the process industries. The most attractive feature of MPC is the ability to accommodate complex performance objectives, dynamic systems and constraints in a unified framework. Similar to the process industries, the dynamics of water systems are relatively slow. Also, during control design, physical limitations and managing water level and flow within certain bounds need to be considered. MPC is a suitable controller design strategy for the current problem. Applications of MPC to water systems can be found in van Overloop (2006); Negenborn et al. (2009); Blanco et al. (2008).

While a deviation model is adopted in Section 2.2, in this section, for the purpose of MPC design, a slightly different non-deviation model is used, as shown below,

$$\begin{aligned} \mathbf{x}_m[k+1] &= A\mathbf{x}_m[k] + B\mathbf{u}_m[k] + w[k], \\ \mathbf{z}_m[k] &= C\mathbf{x}_m[k]. \end{aligned} \qquad (11)$$

Here $\mathbf{x}_m = \tilde{\mathbf{x}} + \mathbf{x}_{SP}$, $\mathbf{u}_m = \mathbf{u} + \mathbf{u}_{SP}$ where $\tilde{\mathbf{x}}, \mathbf{x}_{SP}, \mathbf{u}, \mathbf{u}_{SP}$ are defined in equation (8). The main control objectives are set-point tracking, minimizing energy consumption and disturbance rejection. As stated before, disturbance rejection can be achieved through feedforward. Denoting H_p prediction horizon and H_u control horizon, set-point tracking and minimizing energy consumption can be achieved by minimizing the following quadratic objective function at time k

$$J[k] = \sum_{i=1}^{H_p} \|\mathbf{z}_m[k+i|k] - \mathbf{r}[k+i|k]\|^2_{Q[i]} + \sum_{i=0}^{H_u-1} \|\Delta\mathbf{u}_m[k+i|k]\|^2_{R[i]}, \qquad (12)$$

subject to (11), where $\mathbf{r}$ is a filtered version of set-point signals, $\Delta\mathbf{u}_m[i] := \mathbf{u}_m[i] - \mathbf{u}_m[i-1]$.

As introduced in Section 3, river operation is subject to many constraints. The constraints considered here comprise of hard constraints and soft constraints. Hard constraints are those which cannot be violated, for example, positive water level and flow rate, velocity of gate movement, $\mathbf{x} > 0$, $\mathbf{u} > 0$, $|\Delta\mathbf{u}_m| < \epsilon_u$. Soft constraints can be violated, but only a very short period, to prevent the overall optimization problem infeasible, for example, the upper and lower bounds on outputs, $|\mathbf{z}_m| < \epsilon_z$. Now, a typical optimization problem at time k can be formulated as follows,

minimize J[k], subject to (11) and underlying constraints, (13)

where $J[k]$ is defined in equation (12). Once the optimal solution $\{\Delta\mathbf{u}[k|k], \Delta\mathbf{u}[k+1|k], \cdots, \Delta\mathbf{u}[k+H_u-1|k]\}$ to the optimization problem in (13) is obtained, only the first one $\Delta\mathbf{u}[k|k]$ is used to calculate the control action at time k, $\mathbf{u}[k] = \Delta\mathbf{u}[k|k] + \mathbf{u}[k-1]$.

5. EXPERIMENTAL RESULTS

The following section outlines simulation results obtained using BasinCad, a computer aided design software tool for simulating river basin networks. Figures 2, 3(a), and 3(b) show storage water level, flow rate transients and rate of change of flow rate for different coefficients in weighting matrix $\mathbf{R}$. For this example the relevant elements of $\mathbf{Q}$ were set to one. Using this figure, one can select the appropriate weighting coefficients based on actuator constraints. A unit step input disturbance with amplitude of 6GL/day was introduced at q_D. Using Figure 2 and assuming that 250ML/day is the maximum permissible rate of change of flow rate, we select $\sigma = 10^{-6}$ for the results that follow.

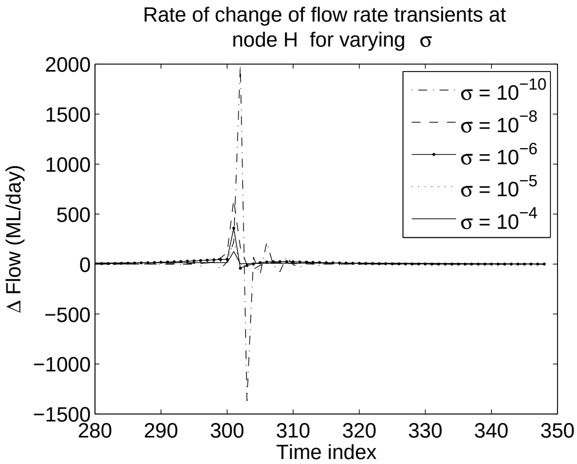
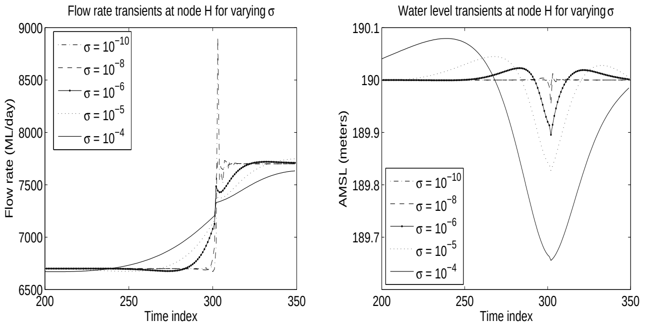


Fig. 2. Rate of change of flow-rate transients at storage H in response to disturbance flow q_D.

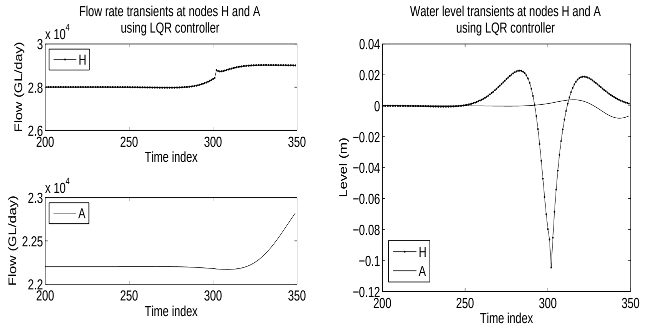


(a) Flow rate transients (b) Water level transients

Fig. 3. Water level and flow transients at storage H in response to disturbance flow q_D.

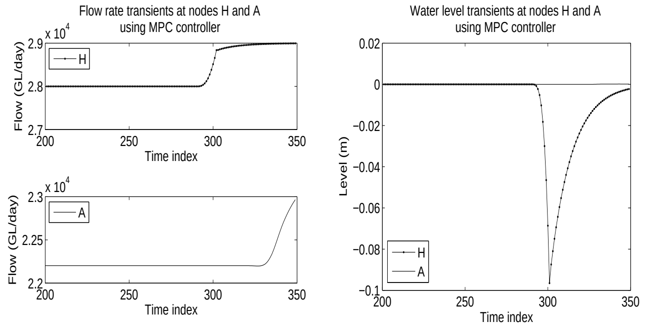
Figure 4 and 5 illustrate the effect of upstream disturbances and compares the performance of the LQR and MPC controller schemes. In this example a step change at q_D of 1 GL/day is introduced at time index 300. Figures 4(a) and 4(b) illustrate the corresponding water level and flow rate transients at nodes H and A for the LQR controller. Figures 5(a) and 5(b) correspond to MPC controller. The results clearly indicate firstly the pre-release of water to accommodate the disturbance inflow. This is apparent from the rise in water levels and flow rates before time index 300. Secondly, the integral action inherent in this system smoothes out the transients as we move further downstream.

An important function for a controller is water pre-release for flood mitigation. This is demonstrated by generating a pulse disturbance of amplitude 10 GL/day over 25 days resulting a total volume of 250GL. The immediate downstream node D has an output flow constraint set at 6GL/day and a maximum water level of 486m. A successful control strategy must incorporate pre-release to overcome the outflow constraint and maximum water level constraint. Figure 6 illustrates the water level and outflow from node D in response to the disturbance indicated by the dashed line. The key point to note is the mandatory pre-release which is evident in Figure 6(a) between time indices 290 and 300.



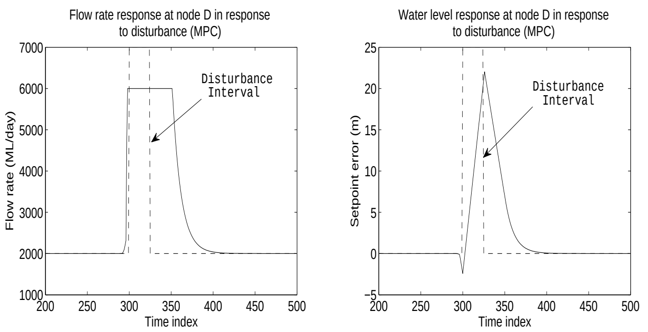
(a) Flow response (b) Level response

Fig. 4. LQR control: Disturbance rejection at storages A and H in response to disturbance flow q_D.



(a) Flow response (b) Level response

Fig. 5. MPC control: Disturbance rejection at storages A and H in response to disturbance flow q_D.



(a) Flow response (b) Level response

Fig. 6. Flood mitigation using MPC.

It is well known that control of flow networks with transport lags require accurate knowledge of time delays. As indicated in the previous discussion, the transport delays in the link element change with flow rate. In this paper we assume that these delays are constant, however this not always the case. In the following example all link delays are increased by one time index. Figure 7 shows flow rate transients using LQR at storages A and H in response to delay mismatch. This can lead to potential instability at downstream nodes as illustrated in Figure 7.

6. CONCLUSIONS AND FURTHER WORK

This paper has introduced a systematic framework of modeling and controlling river basin networks using simple linear models and optimal control principles. Two con-

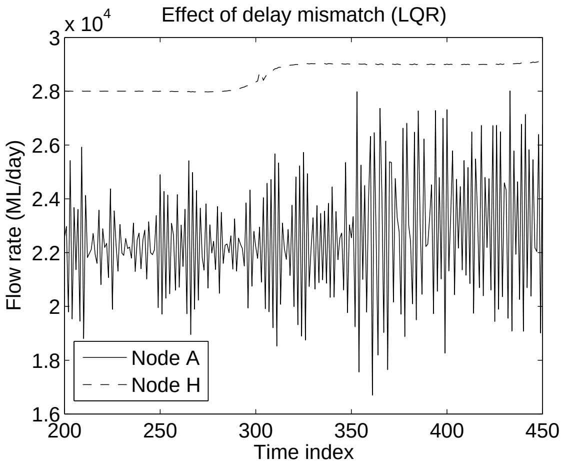


Fig. 7. Flow rate at storages A and H in response to delay mismatch

troller designed have been proposed based on LQR and MPC. This paper has investigated the effects of disturbances, constraints, and sensitivity to transport delay and disturbance estimation. There are three important aspects that require further attention. Firstly, this paper has assumed constant parameter linear models for both links and storage elements. In practice these elements will exhibit non-linear and time-varying characteristics. As mentioned previously, this is typical in river channels where time-delay varies with flow rate and water level changes in storages depend on geometry of the reservoir. Another example of such non-linearities is the rainfall-runoff rate as a function of soil moisture. A potential practical solution to this problem is the use of gain switching mechanisms using a set of linear models that capture the relevant dynamics. This may be compatible with MPC, but perhaps not so with LQR. Secondly, as the sampling rate is increased the dimensionality of the problem may preclude the use of centralized control strategies described here. Future studies will investigate the application of distributed control to address this challenge. A third important extension of this work is to incorporate water quality and groundwater reservoirs in the problem formulation. This poses a significant modeling challenge.

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