UNIVERSITY OF TECHNOLOGY SYDNEY
Faculty of Engineering and Information Technology

Analysis of Uncertainty for Dynamic Pricing: Models, On-demand Attractors, and Artificial Chaos

by

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A Thesis Submitted in Fulfillment of the Requirements for the Degree

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Certificate of Original Authorship

I, Shuixiu Lu, declare that this thesis is submitted in fulfilment of the requirements for the award of Doctor of Philosophy in the Faculty of Engineering and Information Technology at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Abstract

Dynamic pricing is a pricing strategy that adapts and optimizes prices based on information about demand. The optimization models interactions between price and demand. Uncertain demand poses a challenge in the modeling. In addressing the uncertainty, stochastic demand is widely assumed. From dynamical systems' perspective, nonlinear interactions between variables yield a rational route that can exhibit uncertainty. However, uncertain demand because of nonlinear interactions remains to be elucidated.

This thesis analyzes uncertain demand from theoretical and empirical perspectives. A theoretical model addresses a hypothetically rational route to uncertain demand. The rational route has discontinuities in demand functions and optimizations. By a bifurcation analysis, the theoretical impacts of discontinuous interactions are investigated. A reconstruction of real-life on-demand attractor addresses a data-driven identification of uncertain demand. Recurrence-based attractor reconstruction is proposed and applied on empirical data from RideAustin, a company providing ride share service in the city of Austin, Texas, the United States. Recurrence plots and Pareto optimality are applied to find optimal embedding and time delay dimensions. The ones under which recurrence plots yield optimal recurrence quantification measures, the determinism and the trapping time, are chosen for an attractor reconstruction.

Border collision bifurcations are observed from the theoretical mode, justifying dynamic pricing from dynamical systems' perspective. A period-7 limit cycle is reconstructed from empirical data. Results suggest that nonlinear interactions could cause uncertain demand of which a rational route is a constituent part. The findings emphasize data-driven modeling of uncertain demand. For optimal revenue, demand dynamics should be identified.

Finally, uncertainty in deterministic chaos or dynamic pricing is increasingly analyzed by machine learning methods. However, for an artificial system, a system that employs machine learning methods for mimicking deterministic chaos, the role of initial conditions remains unclear. This thesis analyzes the sensitive dependence of an artificial system on initial conditions. Nonlinear time series analysis is introduced to study machine behavior, the behavior of an artificial system under varying initial conditions. We observe that machine behaviors coincide chaotic trajectories, however, alter original basins. Garbled symbolic dynamics is observed, further indicating that a coincidence of a single chaotic trajectory could mislead conclusions. The results highlight that when machine learning meets complex dynamics, an artificial system should be performed under varying initial conditions, instead of a single chaotic trajectory. Machine behaviors would help showing and comparing the sensitive dependence on initial conditions between a mimicked chaotic and an artificial systems.

List of Publications

Related to the Thesis :

Journal Papers

J-1. **Lu S**, Oberst S, Zhang G, Luo Z (2019). "Bifurcation analysis of dynamic pricing processes with nonlinear external reference effects", *Communications in Nonlinear Science and Numerical Simulation*, 79:104929. Doi: 10.1016/j.cnsns.2019.104929.

Conference Papers

C-1. **Lu S**, Oberst S, Zhang G, Luo Z. "Comparing complex dynamics using machine learning-reconstructed attracting sets", *Colloquium on "Irregular Engineering Oscillations and Signal Processing".* Hamburg, Germany, Sep., 2018. Doi: 10.13140/RG.2.2.27988.96647.

C-2. **Lu S**, Oberst S, Zhang G, Luo Z. "Period adding bifurcations in dynamic pricing processes", *2019 IEEE Conference on Computational Intelligence for Financial Engineering & Economics (CIFEr).* Shenzhen, China, 2019, pp. 1-6. Doi: 10.1109/CIFEr.2019.8759118.

C-3. **Lu S**, Oberst S, Zhang G, Luo Z. "Capturing determinism buried in surrogate time series of dynamic pricing processes: applying transition networks on order pattern plots", 8^{th} *International Symposium on Recurrence Plots*. Zhenjiang, China, Aug. 2019.

Book Chapter

B-1. **Lu S**, Oberst S, Zhang G, Luo Z. "Novel Order Patterns Recurrence Plot-Based Quantification Measures to Unveil Deterministic Dynamics from Stochastic Processes", *Theory and Applications of Time Series Analysis*. 2019, pp 57–70. editor: Valenzuela, Olga, Rojas, Fernando, Pomares, Héctor, and Rojas, Ignacio. Springer International Publishing (Cham). Doi: 10.1007/978-3-030-26036-1_5.

Poster

P-1. **Lu S**, Luo Z, Zhang G, Oberst S. "Order pattern recurrence plots: unveiling determinism buried in noise", University of Technology Sydney, Faculty of Engineering and IT Research Showcase. Sydney, Australia. Jun., 2018. Doi: 10.13140/RG.2.2.32482.02249.

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