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Dynamic Behaviour of Ballasted Railway Track with Special Reference to Transition Zones

by Piyush Punetha

Thesis submitted in fulfilment of the requirements for
the degree of

Doctor of Philosophy

under the supervision of Dr. Sanjay Nimbalkar and
Prof. Hadi Khabbaz

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, *Piyush Punetha* declare that this thesis, is submitted in fulfilment of the requirements for the award of *Doctor of Philosophy*, in the *School of Civil and Environmental Engineering (Faculty of Engineering and Information Technology, FEIT)* at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

This research is supported by the Australian Government Research Training Program.

Signature: Production Note: Signature removed prior to publication.

Date: 10/01/2022

Dedicated to

My Parents

मेरे माता-पिता

को समर्पित

ABSTRACT

The poor performance of transition zones in railway tracks has long been a subject of concern for the rail infrastructure managers. These zones are the discontinuities along a railway line that are highly susceptible to differential settlement due to an abrupt variation in the support conditions over a short span. Consequently, these regions require frequent maintenance to ensure adequate levels of passenger safety and comfort. The rapid deterioration of track geometry in these zones is primarily ascribed to limited understanding of the underlying mechanism and scarcity of adequate tools to assess the severity of the potential issue. Therefore, a comprehensive evaluation of their behaviour is paramount to improve the design and ensure adequate service quality. With this objective, a novel methodology is developed which can predict the dynamic behaviour of the transition zones under train-induced repeated loading and assess the suitability of different countermeasures in improving the track performance.

To this end, an integrated approach is first developed by combining track loading, resiliency, and settlement models to evaluate the transient and irrecoverable response of the substructure layers of a standard ballasted railway track. The track substructure layers (ballast, subballast, and subgrade) in this model are simulated as an array of lumped masses that are connected by elastic springs and viscous dampers. The irrecoverable response of the track is evaluated using the empirical settlement models for substructure layers. The accuracy of the method is validated by comparing the predicted results against the field investigation data reported in the literature. Subsequently, the practical applicability of the aforementioned method under different traffic loading and soil conditions is improved by replacing the empirical approach with a mechanistic approach, in which, plastic slider elements are employed to predict the inelastic deformation in the substructure layers. To validate the approach, the predicted results are compared with the in-situ measurements reported in the literature. A good agreement between the predicted results and the field data verified the accuracy of the novel geotechnical rheological track model. A parametric investigation is conducted which highlights the significant influence of axle load, train speed, and granular layer thickness, on the accumulated settlement in the track layers.

The novel geotechnical rheological track model is then applied to an open-track bridge transition by incorporating the inhomogeneous support conditions associated with the critical zone and the adequacy of different countermeasures to mitigate the differential track settlements is examined. The approach is successfully validated with published field data and predictions from the finite element (FE) analysis. The results revealed that an increase in axle load exacerbates the track geometry degradation problem. The results also show that the performance of transition zones with weak subgrade can be improved by increasing the granular layer thickness. Interpretation of the predicted differential settlement for different countermeasures exemplified the practical significance of the proposed methodology.

Subsequently, the influence of principal stress rotation (PSR) experienced by the soil elements during a train passage is incorporated in the geotechnical rheological model. The results revealed that PSR causes significant cumulative deformation in the substructure layers, and disregarding it in the analysis leads to inaccurate predictions. Finally, the adequacy of using three-dimensional (3D) cellular geoinclusions to improve the performance of critical zones is investigated using the proposed methodology and FE analyses. A novel semi-empirical model is first developed to evaluate the magnitude of improvement provided by these inclusions under the 3D stress state. The proposed model is successfully validated against the experimental data. This model is then incorporated in the geotechnical rheological model and the effectiveness of 3D geoinclusions in improving the performance of an open track-bridge transition is investigated. The results show that the geoinclusions significantly reduce the magnitude of differential settlement and therefore, have a huge potential to be used in the transition zones to improve track performance.

The essential contribution of this thesis is that it provides reliable, practical, and adaptable techniques to assist the practising railway engineers in analysing the performance of various sections of ballasted railway tracks, identifying the most effective method to improve the track performance, planning the maintenance operations, and improving the design. The developed techniques are available in the form of MATLAB codes, which can readily be converted into an application that can be used by railway engineers. Nonetheless, the outcomes of this study have huge potential to influence the real-world design implications of track transition zones. The approaches developed in this study are original, simple yet elegant, and can enhance, if not fully replace, present complex track modelling procedures for anticipating the behaviour of critical zones and adopting appropriate mitigation strategies.

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6. **Punetha, P.**, Nimbalkar, S. & Khabbaz, H. 2020, 'Evaluation of additional confinement for three-dimensional geoinclusions under general stress state', *Canadian Geotechnical Journal*, vol. 57, no. 3, pp. 453-461. **[IF: 3.725** (2020); Citations: **16** (Google scholar)**]** [link]

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9. **Punetha, P.** and Nimbalkar, S. 2021, ‘Performance improvement of ballasted railway tracks for high-speed rail operations’**,** *Challenges and Innovations in Geomechanics*, Springer [Citations: **2** (Google scholar)]. [link]

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Sakdirat Kaewunruen	Manuscript witing: review and editing	

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Latin Symbols:

A_{b}	Equivalent area of ballast at a particular depth (m^2)
A_{b}^{s}	Equivalent shear area of ballast (m^2)
A_{d}	Maximum normal operating cant deficiency angle (rad)
A_{f}	Fourier amplitude
A_{g}	Equivalent area of subgrade at a particular depth (m^2)
A_{g}^{s}	Equivalent shear area of subgrade (m^2)
A_{s}	Equivalent area of subballast at a particular depth (m^2)
A_{s}^{s}	Equivalent shear area of subballast (m^2)
A_{y}	Angle of lateral ramp discontinuity (rad)
a, a^r	Equivalent radius of sleeper-ballast contact area in softer and stiffer side (m)
a^*	Empirical coefficient
a'	Material parameter for subgrade
a_{h}	Cyclic hardening parameter
a_{o}, b_{o}	Locomotive and track maintenance factors
a_{t}	Number of wheels considered
b	Intermediate principal stress ratio
b^*, b'	Empirical coefficients
b_{g}	Material parameter for subgrade
b_{ps}	Intermediate principal stress ratio at plane-strain condition
b_{sl}	Sleeper width (m)
b_{t}	Track width (m)
C	Coefficient in Japanese standard
c	Cohesion (N/m^2)
c_{b}, c_{s}, c_{g}	Viscous damping coefficients for ballast, subballast and subgrade (Ns/m)
c_{b}^{r}	Viscous damping coefficient for ballast in the stiffer side (Ns/m)
$c_{\mathrm{b}}^{s}, c_{\mathrm{s}}^{s}, c_{\mathrm{g}}^{s}$	Shear damping coefficients for ballast, subballast and subgrade (Ns/m)
$c_{\mathrm{b}}^{s,r}$	Shear damping coefficient for ballast in the stiffer side (Ns/m)
D	Dilatancy
D_{g}	Diameter of geoinclusion opening (m)

D_{p}	Plastic dilatancy
D_{r}	Dilatancy rate
D_{w}	Train wheel diameter (m)
d_{s}	Diameter of soil specimen in triaxial test (m)
$dF_{\mathrm{b,n}}^{r}$	Force increment acting on ballast layer in the stiffer side (N)
$dF_{\mathrm{g,n}}$, $dF_{\mathrm{s,n}}$, $dF_{\mathrm{b,n}}$	Force increment acting on subgrade, subballast and ballast (N)
$d\hat{p}$	Hydrostatic stress increment in characteristic stress space
dp_{i}	Image mean effective stress increment (N/m^2)
$d\hat{q}$	Deviatoric stress increment in characteristic stress space
dt	Time step (s)
E	Young's modulus of infill (N/m^2)
E_{b}, E_{s}, E_{g}	Resilient modulus of ballast, subballast and subgrade (N/m^2)
E_{b}^{r}	Resilient modulus of ballast in the stiffer side (N/m^2)
E_{i}	Initial Young's modulus (N/m^2)
E_{m}	Young's modulus of geoinclusion (N/m^2)
E_{R}	Resilient modulus (N/m^2)
E_{r}	Young's modulus of rail (N/m^2)
E_{sec}	Secant Young's modulus (N/m^2)
e, e_0	Current and initial void ratio
e_{c}	Void ratio on critical state line at the current mean effective stress
F	Frequency (Hz)
F_{wr}	Wheel-rail contact force (N)
f_{c}, f_{r}, f_{t}	Current, reference and transitional surfaces
f_{g}, f_{s}, f_{b}	Yield surface for subgrade, subballast and ballast
G	Wheel-rail contact constant (m/N$^{2/3}$)
g	Acceleration due to gravity (m/s^2)
g_{p}	Potential function
g_{t}	Centre-to-centre distance between the rails (m)
H	Hardening parameter in Nor-sand model
H_{L}	Lateral load (N)
H_{m}	Influence height of planar geosynthetic (m)

H_{mean}	Mean lateral load (N)
H_{w}	Crosswind force (N)
h	Vertical distance from rail top to centre of gravity of train (m)
h_{b}, h_{s}, h_{g}	Ballast, subballast and subgrade thickness (m)
h_{b}^{r}	Ballast thickness in the stiffer side (m)
h_{bL}, h_{sL}, h_{gL}	Overlap height in ballast, subballast and subgrade along longitudinal direction (m)
h_{bt}, h_{st}, h_{gt}	Overlap height in ballast, subballast and subgrade along transverse direction (m)
h_{d}	Cant or superelevation deficiency (m)
h_{eb}, h_{es}	Equivalent thickness of ballast and subballast layers (m)
h_{gl}	Granular layer thickness (m)
h_{i}	Thickness of i^{th} substructure layer (m)
h_{se}	Superelevation (m)
I_{r}	Moment of inertia of rail (m^4)
i_1, i_2	Empirical parameters for calculating dynamic amplification factor
K	Coefficient representing the internal friction
K_1, K_2, K_3, K_4	Fitting parameters that depend on type and physical state of the soil
K_{j}	Track stiffness at joint (N/m)
K_{y}	Effective lateral rail stiffness per wheel (N/m)
k	Track modulus ($\mathrm{N/m^2}$)
k^{r}	Track modulus for stiffer side of the transition ($\mathrm{N/m^2}$)
k^{*}, k_1^{*}, k_2^{*}, k_3^{*}	Empirical parameters
k_0	Lateral earth pressure coefficient
k_1, k_2, k_3, k_4	Empirical parameters
$k_1^{b}, k_2^{b}, k_3^{b}, k_4^{b}$	Empirical parameters for calculating ballast deformation
$k_1^{s}, k_2^{s}, k_3^{s}, k_4^{s}$	Empirical parameters for calculating subballast deformation
k_{b}, k_{s}, k_{g}	Stiffness of ballast, subballast and subgrade (N/m)
k_{b}^{r}	Stiffness of ballast in the stiffer side (N/m)
$k_{\mathrm{b}}^{s}, k_{\mathrm{s}}^{s}, k_{\mathrm{g}}^{s}$	Shear stiffness of ballast, subballast and subgrade (N/m)
$k_{\mathrm{b}}^{s,r}$	Shear stiffness of ballast in the stiffer side (N/m)

k_c	Ratio of circumferential strain to the radial strain in the geocell
k_p	Stiffness of rail pad (N/m)
k_p^r	Rail pad stiffness in the stiffer side (N/m)
$k_{\sigma,2}$, $k_{\sigma,3}$	Normalised additional confinement
L	Characteristic length (m)
l_c	Distance between rail top and centre of gravity of train (m)
l_e	Effective length of sleeper (m)
l_g	Gauge width (m)
l_sl	Sleeper length (m)
l_w	Distance between centre of rails and the resultant wind force (m)
M	Critical stress ratio
$\widehat{M}$	Critical stress ratio in characteristic stress space
M_i	Critical stress ratio corresponding to image state
M_itc	Critical stress ratio corresponding to the image state for triaxial compression
M_m	Mobilised modulus of geocell (N/m)
M_p	Peak stress ratio
M_t	Tensile stiffness of the geocell (N/m)
M_tc	Critical stress ratio under triaxial compression
M_u	Effective lateral unsprung mass per axle (kg)
M_y	Effective lateral rail mass per wheel (kg)
m, n	Empirical coefficients
m^*	Material parameter for subgrade
m_b, m_s, m_g	Vibrating mass of ballast, subballast and subgrade (kg)
m_b^r	Vibrating mass of ballast in the stiffer side (kg)
N	Number of load cycles
N_d	Number of days
N_lim	Number of load cycles required to reach stable zone
N_limit	Number of load repetitions required to reach the resilient state
N_v	Volumetric coupling coefficient
P_a	Atmospheric pressure (N/m^2)

p	Mean effective stress (N/m^2)
p_i	Image mean effective stress (N/m^2)
p_ic, p_im	Hardening parameters
$\hat{p}_\mathrm{xg}$	Intersection of potential function with $\hat{p}$ axis
$\hat{p}_\mathrm{xt}$, $\hat{p}_\mathrm{xc}$, $\hat{p}_\mathrm{xr}$	Intersection of transitional, current and reference surfaces with $\hat{p}$ axis
Q, Q_a	Static wheel and axle load (N)
Q_d	Design wheel load (N)
Q_dy	Dynamic component of load (N)
Q_qs	Quasi-static wheel load (N)
$Q_\mathrm{r,n}$	Vertical rail seat load at n^th sleeper (N)
Q_t	Tensile load (N)
Q_tv	Total vertical wheel load (N)
q	Deviatoric stress (N/m^2)
$\hat{q}$ and $\hat{p}$	Deviatoric and hydrostatic stress invariants in the characteristic stress space
R	Parameter that controls the magnitude of plastic volumetric strain increment
R_c	Radius of curvature of track (m)
R_i	Parameter that controls the magnitude of plastic strain accumulation
R_s	Stress ratio
R_w	Nominal radius of the wheel (m)
r	Spacing ratio
S	Sleeper spacing (m)
$s_{1\alpha}$, $s_{2\alpha}$	Constitutive parameters to account for the effects of principal stress rotation
s_b, s_s	Settlement of ballast and subballast (m)
s_g, s_gl	Settlement of subgrade and granular layers (m)
$\boldsymbol{s}_\mathrm{ij}$	Deviatoric stress tensor
s_t	Settlement of track substructure (m)
s_v^{r}	Vertical resilient deformation (m)
T	Cumulative tonnage (kg)
$\boldsymbol{T}_\mathrm{n}^{s}$	Average shear stress vector
T_x, T_y	Tensile stresses in planar geosynthetic along x and y directions (N/m)

t	Time instant (s)
t_g	Thickness of geoinclusion (m)
t_u	Factor that depends on the upper confidence limit
V	Train speed (m/s)
V_m	Maximum normal operating speed (m/s)
W_u	Unsprung weight at one wheel (N)
W_vd	Vertical deformation (m)
W_wr	Deformation at the wheel-rail contact point (m)
w	Vertical track deflection (m)
w_b	Vertical displacement at ballast top (m)
w_bm	Mean value of track displacement in stiffer zone (m)
w_t	Vertical track displacement (m)
x	Distance along longitudinal direction (m)
x_n^{j}	Distance of nth sleeper from jth wheel (m)
Z	Plastic softening parameter
z	Depth (m)
$z_\mathrm{b,n}, \dot{z}_\mathrm{b,n}, \ddot{z}_\mathrm{b,n}$	Displacement, velocity and acceleration of ballast below nth sleeper
$z_\mathrm{b,n}^{p}, \dot{z}_\mathrm{b,n}^{p}$	Plastic displacement and velocity of ballast below nth sleeper
$z_\mathrm{s,n}, \dot{z}_\mathrm{s,n}, \ddot{z}_\mathrm{s,n}$	Displacement, velocity and acceleration of subballast below nth sleeper
$z_\mathrm{s,n}^{p}, \dot{z}_\mathrm{s,n}^{p}$	Plastic displacement and velocity of subballast below nth sleeper
$z_\mathrm{g,n}, \dot{z}_\mathrm{g,n}, \ddot{z}_\mathrm{g,n}$	Displacement, velocity and acceleration of subgrade below nth sleeper
$z_\mathrm{g,n}^{p}, \dot{z}_\mathrm{g,n}^{p}$	Plastic displacement and velocity of subgrade below nth sleeper
$z_\mathrm{g}^{ve}, z_\mathrm{s}^{ve}, z_\mathrm{b}^{ve}$	Viscoelastic displacement in subgrade, subballast and ballast (m)

Greek Symbols:

α	Stress distribution angle for ballast (°)
α^{*}	Empirical parameter
α'	Coefficient relating track irregularities, train suspension and speed
α^{r}	Stress distribution angle for ballast in the stiffer side (°)
α_0	Reference stress distribution angle in ballast (°)
α_m	Bonding coefficient for planar geosynthetic

α_v	Angle between major principal stress direction and vertical (°)
β	Stress distribution angle for subballast (°)
β'	Coefficient accounting for the movement of train along a curve
β_0	Reference stress distribution angle in subballast (°)
Γ	Critical void ratio at p = 1 kPa
$\hat{\Gamma}$, $\hat{N}$	Void ratio of critical state line and normal compression line at $\hat{p}$=1 kPa
γ	Stress distribution angle for subgrade (°)
γ'	Coefficient that depends on speed and design of train, and track condition
$\Delta\sigma$	Increase in stress (N/m^2)
$\Delta\sigma'_2$, $\Delta\sigma'_3$	Effective additional confining stress in the direction of σ'_2 and σ'_3 (N/m^2)
$\Delta\sigma_3$	Additional confining stress in the direction of σ_3 (N/m^2)
$\Delta\sigma_x$, $\Delta\sigma_y$	Additional confining pressure along x and y directions (N/m^2)
$d\boldsymbol{\varepsilon}_{ij}^p$, ε_v^p	Plastic strain increment and cumulative plastic volumetric strain
$d\varepsilon_v$, $d\varepsilon_q$	Volumetric and deviatoric strain increments
$d\varepsilon_v^p$, $d\varepsilon_q^p$	Plastic volumetric and deviatoric strain increments
$d\varepsilon_z^p$	Plastic strain increment in vertical direction
δ_{ij}	Kronecker delta
δ_t	Factor that depends on the track condition
$\varepsilon_0/\varepsilon_r$, ρ_p, β_p	Fitting parameters
ε_1, ε_2, ε_3	Major, intermediate and minor principal strains
ε_1^e, ε_2^e, ε_3^e	Resilient components of major, intermediate and minor principal strains
ε_1^p, ε_2^p, ε_3^p	Plastic components of major, intermediate and minor principal strains
$\varepsilon_{1,1}^p$	Plastic axial strain after the first load cycle
ε_a	Axial strain
ε_a^p	Accumulated plastic axial strain
ε_b^p, ε_s^p, ε_g^p	Cumulative plastic strain in ballast, subballast and subgrade
ε_c	Circumferential strain
ε_q	Deviatoric strain
ε_r	Radial strain
ε_t	Tensile strain

ε_x, ε_y	Strain along *x* and *y* directions
$\varepsilon_\mathrm{x}^m, \varepsilon_\mathrm{y}^m$	Strains in geosynthetic in *x* and *y* directions
ε_z^p	Cumulative plastic strain in vertical direction
η	Stress ratio
$\hat{\eta}$	Stress ratio in characteristic stress space
η_v	Factor that depends on the speed of vehicle
θ	Bulk stress (N/m^2)
$\theta_1 + \theta_2$	Total dip angle of the rail joint (rad)
$\mathcal{I}$	Tensorial invariant
$\Lambda_\mathrm{s}, \Lambda_\mathrm{g}$	Scalars
λ, κ	Slope of critical state line and swelling line in *e*–ln *p* space
ν_b, ν_s, ν_g	Poisson's ratio of ballast, subballast and subgrade
ν_b^r	Poisson's ratio of ballast in the stiffer side
ν_i	Poisson's ratio of infill
ν_m	Poisson's ratio of geoinclusion material
ξ, A	Dimensionless material parameters
ρ_b, ρ_s, ρ_g	Density of ballast, subballast and subgrade (kg/m^3)
ρ_b^r	Density of ballast in the stiffer side (kg/m^3)
σ_1, σ_2, σ_3	Major, intermediate and minor principal stresses (N/m^2)
σ_bb^r	Vertical stress at the bottom of substructure layer in the stiffer side (N/m^2)
$\sigma_\mathrm{C,2}$, $\sigma_\mathrm{C,3}$	Circumferential stresses in the direction of σ'_2 and σ'_3 (N/m^2)
σ_c	Confining pressure in triaxial tests (N/m^2)
σ'_c	Effective confining pressure (N/m^2)
σ_cyc	Cyclic deviator stress (N/m^2)
σ_d	Deviator stress (N/m^2)
σ'_d	Effective deviatoric stress (N/m^2)
σ_di	Deviator stress at which slope of E_R versus σ_d curve changes (N/m^2)
σ_g	Compressive strength of the soil (N/m^2)
$\boldsymbol{\sigma}_\mathrm{ij}$	Stress tensor
$\hat{\boldsymbol{\sigma}}_\mathrm{ij}$	Characteristic stress tensor

σ_j	Principal stress (N/m^2)
σ_{oct}	Octahedral normal stress (N/m^2)
σ_{ref}	Reference stress (N/m^2)
σ_{sb}, σ_{bs}, σ_{sg}, σ_{go}	Vertical stresses at the sleeper-ballast, ballast-subballast, subballast-subgrade interfaces and bottom of subgrade layer (N/m^2)
σ_v	Vertical stress (N/m^2)
σ'_x, σ'_y, σ'_z	Effective stresses along x, y and z directions (N/m^2)
τ_{cyc}	Cyclic shear stress amplitude (N/m^2)
τ_{oct}	Octahedral shear stress (N/m^2)
φ	Friction angle (°)
φ_c, φ_e	Critical state friction angles under triaxial compression and extension (°)
φ_d	Dynamic amplification factor
φ'_m	Mobilised friction angle (°)
χ_i, χ_{tc}	Dilatancy parameter corresponding to image state and triaxial compression
ψ	State parameter
ψ_d	Dilation angle (°)
ψ_i	Image state parameter
ψ_m	Mobilised dilation angle (°)

Abbreviations:

2D	Two-dimensional
2.5D	Two and a half dimensional
3D	Three-dimensional
ACR	Additional confinement ratio
AREA	American Railway Engineering Association
ARTC	Australian Rail Track Corporation
BEM	Boundary element method
BoEF	Beam on elastic foundation
CBM	Cement bound mixture
CBR	California Bearing Ratio
CG	Coir geotextile

CSL	Critical state line
CSSR	Cyclic shear stress ratio
CVSR	Cyclic vertical stress ratio
DEM	Discrete element method
FDM	Finite difference method
FEM	Finite element method
GB	Geocell reinforced ballast
GG	Geocell reinforced subgrade
GS	Geocell reinforced subballast
HDPE	High-density polyethylene
HMA	Hot-mix asphalt
LVDT	Linear variable displacement transformer
MDD	Multi-depth deflectometers
MGT	Million gross tonnes
MSD	Mass-spring-dashpot
NCL	Normal compression line
ORE	Office for Research and Experiments
PE	Polyethylene
PP	Polypropylene
PSR	Principal stress rotation
RAP	Recycled asphalt pavement
SS	Silica sand
UGM	Unbound granular material
UR	Unreinforced
US	United States
WMATA	Washington Metropolitan Area Transit Authority