

Dynamic Behaviour of Ballasted Railway Track with Special Reference to Transition Zones

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under the supervision of Dr. Sanjay Nimbalkar and
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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, *Piyush Punetha* declare that this thesis, is submitted in fulfilment of the requirements for the award of *Doctor of Philosophy*, in the *School of Civil and Environmental Engineering (Faculty of Engineering and Information Technology, FEIT)* at the University of Technology Sydney.

This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

This document has not been submitted for qualifications at any other academic institution.

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*Dedicated to
My Parents*

मेरे माता-पिता
को समर्पित

ABSTRACT

The poor performance of transition zones in railway tracks has long been a subject of concern for the rail infrastructure managers. These zones are the discontinuities along a railway line that are highly susceptible to differential settlement due to an abrupt variation in the support conditions over a short span. Consequently, these regions require frequent maintenance to ensure adequate levels of passenger safety and comfort. The rapid deterioration of track geometry in these zones is primarily ascribed to limited understanding of the underlying mechanism and scarcity of adequate tools to assess the severity of the potential issue. Therefore, a comprehensive evaluation of their behaviour is paramount to improve the design and ensure adequate service quality. With this objective, a novel methodology is developed which can predict the dynamic behaviour of the transition zones under train-induced repeated loading and assess the suitability of different countermeasures in improving the track performance.

To this end, an integrated approach is first developed by combining track loading, resiliency, and settlement models to evaluate the transient and irrecoverable response of the substructure layers of a standard ballasted railway track. The track substructure layers (ballast, subballast, and subgrade) in this model are simulated as an array of lumped masses that are connected by elastic springs and viscous dampers. The irrecoverable response of the track is evaluated using the empirical settlement models for substructure layers. The accuracy of the method is validated by comparing the predicted results against the field investigation data reported in the literature. Subsequently, the practical applicability of the aforementioned method under different traffic loading and soil conditions is improved by replacing the empirical approach with a mechanistic approach, in which, plastic slider elements are employed to predict the inelastic deformation in the substructure layers. To validate the approach, the predicted results are compared with the in-situ measurements reported in the literature. A good agreement between the predicted results and the field data verified the accuracy of the novel geotechnical rheological track model. A parametric investigation is conducted which highlights the significant influence of axle load, train speed, and granular layer thickness, on the accumulated settlement in the track layers.

The novel geotechnical rheological track model is then applied to an open-track bridge transition by incorporating the inhomogeneous support conditions associated with the critical zone and the adequacy of different countermeasures to mitigate the differential track settlements is examined. The approach is successfully validated with published field data and predictions from the finite element (FE) analysis. The results revealed that an increase in axle load exacerbates the track geometry degradation problem. The results also show that the performance of transition zones with weak subgrade can be improved by increasing the granular layer thickness. Interpretation of the predicted differential settlement for different countermeasures exemplified the practical significance of the proposed methodology.

Subsequently, the influence of principal stress rotation (PSR) experienced by the soil elements during a train passage is incorporated in the geotechnical rheological model. The results revealed that PSR causes significant cumulative deformation in the substructure layers, and disregarding it in the analysis leads to inaccurate predictions. Finally, the adequacy of using three-dimensional (3D) cellular geoinclusions to improve the performance of critical zones is investigated using the proposed methodology and FE analyses. A novel semi-empirical model is first developed to evaluate the magnitude of improvement provided by these inclusions under the 3D stress state. The proposed model is successfully validated against the experimental data. This model is then incorporated in the geotechnical rheological model and the effectiveness of 3D geoinclusions in improving the performance of an open track-bridge transition is investigated. The results show that the geoinclusions significantly reduce the magnitude of differential settlement and therefore, have a huge potential to be used in the transition zones to improve track performance.

The essential contribution of this thesis is that it provides reliable, practical, and adaptable techniques to assist the practising railway engineers in analysing the performance of various sections of ballasted railway tracks, identifying the most effective method to improve the track performance, planning the maintenance operations, and improving the design. The developed techniques are available in the form of MATLAB codes, which can readily be converted into an application that can be used by railway engineers. Nonetheless, the outcomes of this study have huge potential to influence the real-world

design implications of track transition zones. The approaches developed in this study are original, simple yet elegant, and can enhance, if not fully replace, present complex track modelling procedures for anticipating the behaviour of critical zones and adopting appropriate mitigation strategies.

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LIST OF NOTATIONS

Latin Symbols:

- A_b Equivalent area of ballast at a particular depth (m^2)
- A_b^S Equivalent shear area of ballast (m^2)
- A_d Maximum normal operating cant deficiency angle (rad)
- A_f Fourier amplitude
- A_g Equivalent area of subgrade at a particular depth (m^2)
- A_g^S Equivalent shear area of subgrade (m^2)
- A_s Equivalent area of subballast at a particular depth (m^2)
- A_s^S Equivalent shear area of subballast (m^2)
- A_y Angle of lateral ramp discontinuity (rad)
- a, a' Equivalent radius of sleeper-ballast contact area in softer and stiffer side (m)
- a^* Empirical coefficient
- a' Material parameter for subgrade
- a_h Cyclic hardening parameter
- a_o, b_o Locomotive and track maintenance factors
- a_t Number of wheels considered
- b Intermediate principal stress ratio
- b^*, b' Empirical coefficients
- b_g Material parameter for subgrade
- b_{ps} Intermediate principal stress ratio at plane-strain condition
- b_{sl} Sleeper width (m)
- b_t Track width (m)
- C Coefficient in Japanese standard
- c Cohesion (N/m^2)
- c_b, c_s, c_g Viscous damping coefficients for ballast, subballast and subgrade (Ns/m)
- c_b^r Viscous damping coefficient for ballast in the stiffer side (Ns/m)
- c_b^S, c_s^S, c_g^S Shear damping coefficients for ballast, subballast and subgrade (Ns/m)
- $c_b^{S,r}$ Shear damping coefficient for ballast in the stiffer side (Ns/m)
- D Dilatancy
- D_g Diameter of geoinclusion opening (m)

D_p	Plastic dilatancy
D_r	Dilatancy rate
D_w	Train wheel diameter (m)
d_s	Diameter of soil specimen in triaxial test (m)
$dF_{b,n}^r$	Force increment acting on ballast layer in the stiffer side (N)
$dF_{g,n}, dF_{s,n}, dF_{b,n}$	Force increment acting on subgrade, subballast and ballast (N)
$d\hat{p}$	Hydrostatic stress increment in characteristic stress space
dp_i	Image mean effective stress increment (N/m ²)
$d\hat{q}$	Deviatoric stress increment in characteristic stress space
dt	Time step (s)
E	Young's modulus of infill (N/m ²)
E_b, E_s, E_g	Resilient modulus of ballast, subballast and subgrade (N/m ²)
E_b^r	Resilient modulus of ballast in the stiffer side (N/m ²)
E_i	Initial Young's modulus (N/m ²)
E_m	Young's modulus of geoinclusion (N/m ²)
E_R	Resilient modulus (N/m ²)
E_r	Young's modulus of rail (N/m ²)
E_{sec}	Secant Young's modulus (N/m ²)
e, e_0	Current and initial void ratio
e_c	Void ratio on critical state line at the current mean effective stress
F	Frequency (Hz)
F_{wr}	Wheel-rail contact force (N)
f_c, f_r, f_t	Current, reference and transitional surfaces
f_g, f_s, f_b	Yield surface for subgrade, subballast and ballast
G	Wheel-rail contact constant (m/N ^{2/3})
g	Acceleration due to gravity (m/s ²)
g_p	Potential function
g_t	Centre-to-centre distance between the rails (m)
H	Hardening parameter in Nor-sand model
H_L	Lateral load (N)
H_m	Influence height of planar geosynthetic (m)

H_{mean}	Mean lateral load (N)
H_w	Crosswind force (N)
h	Vertical distance from rail top to centre of gravity of train (m)
h_b, h_s, h_g	Ballast, subballast and subgrade thickness (m)
h_b^r	Ballast thickness in the stiffer side (m)
h_{bL}, h_{sL}, h_{gL}	Overlap height in ballast, subballast and subgrade along longitudinal direction (m)
h_{bt}, h_{st}, h_{gt}	Overlap height in ballast, subballast and subgrade along transverse direction (m)
h_d	Cant or superelevation deficiency (m)
h_{eb}, h_{es}	Equivalent thickness of ballast and subballast layers (m)
h_{gl}	Granular layer thickness (m)
h_i	Thickness of i^{th} substructure layer (m)
h_{se}	Superelevation (m)
I_r	Moment of inertia of rail (m^4)
i_1, i_2	Empirical parameters for calculating dynamic amplification factor
K	Coefficient representing the internal friction
K_1, K_2, K_3, K_4	Fitting parameters that depend on type and physical state of the soil
K_j	Track stiffness at joint (N/m)
K_y	Effective lateral rail stiffness per wheel (N/m)
k	Track modulus (N/m^2)
k^r	Track modulus for stiffer side of the transition (N/m^2)
k^*, k_1^*, k_2^*, k_3^*	Empirical parameters
k_0	Lateral earth pressure coefficient
k_1, k_2, k_3, k_4	Empirical parameters
$k_1^b, k_2^b, k_3^b, k_4^b$	Empirical parameters for calculating ballast deformation
$k_1^s, k_2^s, k_3^s, k_4^s$	Empirical parameters for calculating subballast deformation
k_b, k_s, k_g	Stiffness of ballast, subballast and subgrade (N/m)
k_b^r	Stiffness of ballast in the stiffer side (N/m)
k_b^s, k_s^s, k_g^s	Shear stiffness of ballast, subballast and subgrade (N/m)
$k_b^{s,r}$	Shear stiffness of ballast in the stiffer side (N/m)

k_c	Ratio of circumferential strain to the radial strain in the geocell
k_p	Stiffness of rail pad (N/m)
k_p^r	Rail pad stiffness in the stiffer side (N/m)
$k_{\sigma,2}, k_{\sigma,3}$	Normalised additional confinement
L	Characteristic length (m)
l_c	Distance between rail top and centre of gravity of train (m)
l_e	Effective length of sleeper (m)
l_g	Gauge width (m)
l_{sl}	Sleeper length (m)
l_w	Distance between centre of rails and the resultant wind force (m)
M	Critical stress ratio
\hat{M}	Critical stress ratio in characteristic stress space
M_i	Critical stress ratio corresponding to image state
M_{itc}	Critical stress ratio corresponding to the image state for triaxial compression
M_m	Mobilised modulus of geocell (N/m)
M_p	Peak stress ratio
M_t	Tensile stiffness of the geocell (N/m)
M_{tc}	Critical stress ratio under triaxial compression
M_u	Effective lateral unsprung mass per axle (kg)
M_y	Effective lateral rail mass per wheel (kg)
m, n	Empirical coefficients
m^*	Material parameter for subgrade
m_b, m_s, m_g	Vibrating mass of ballast, subballast and subgrade (kg)
m_b^r	Vibrating mass of ballast in the stiffer side (kg)
N	Number of load cycles
N_d	Number of days
N_{lim}	Number of load cycles required to reach stable zone
N_{limit}	Number of load repetitions required to reach the resilient state
N_v	Volumetric coupling coefficient
P_a	Atmospheric pressure (N/m ²)

- p Mean effective stress (N/m²)
- p_i Image mean effective stress (N/m²)
- p_{ic}, p_{im} Hardening parameters
- \hat{p}_{xg} Intersection of potential function with \hat{p} axis
- $\hat{p}_{xt}, \hat{p}_{xc}, \hat{p}_{xr}$ Intersection of transitional, current and reference surfaces with \hat{p} axis
- Q, Q_a Static wheel and axle load (N)
- Q_d Design wheel load (N)
- Q_{dy} Dynamic component of load (N)
- Q_{qs} Quasi-static wheel load (N)
- $Q_{r,n}$ Vertical rail seat load at n^{th} sleeper (N)
- Q_t Tensile load (N)
- Q_{tv} Total vertical wheel load (N)
- q Deviatoric stress (N/m²)
- \hat{q} and \hat{p} Deviatoric and hydrostatic stress invariants in the characteristic stress space
- R Parameter that controls the magnitude of plastic volumetric strain increment
- R_c Radius of curvature of track (m)
- R_i Parameter that controls the magnitude of plastic strain accumulation
- R_s Stress ratio
- R_w Nominal radius of the wheel (m)
- r Spacing ratio
- S Sleeper spacing (m)
- $s_{1\alpha}, s_{2\alpha}$ Constitutive parameters to account for the effects of principal stress rotation
- s_b, s_s Settlement of ballast and subballast (m)
- s_g, s_{gl} Settlement of subgrade and granular layers (m)
- s_{ij} Deviatoric stress tensor
- s_t Settlement of track substructure (m)
- s_v^r Vertical resilient deformation (m)
- T Cumulative tonnage (kg)
- T_n^s Average shear stress vector
- T_x, T_y Tensile stresses in planar geosynthetic along x and y directions (N/m)

t	Time instant (s)
t_g	Thickness of geoinclusion (m)
t_u	Factor that depends on the upper confidence limit
V	Train speed (m/s)
V_m	Maximum normal operating speed (m/s)
W_u	Unsprung weight at one wheel (N)
W_{vd}	Vertical deformation (m)
W_{wr}	Deformation at the wheel-rail contact point (m)
w	Vertical track deflection (m)
w_b	Vertical displacement at ballast top (m)
w_{bm}	Mean value of track displacement in stiffer zone (m)
w_t	Vertical track displacement (m)
x	Distance along longitudinal direction (m)
x_n^j	Distance of n^{th} sleeper from j^{th} wheel (m)
Z	Plastic softening parameter
z	Depth (m)
$z_{b,n}, \dot{z}_{b,n}, \ddot{z}_{b,n}$	Displacement, velocity and acceleration of ballast below n^{th} sleeper
$z_{b,n}^p, \dot{z}_{b,n}^p$	Plastic displacement and velocity of ballast below n^{th} sleeper
$z_{s,n}, \dot{z}_{s,n}, \ddot{z}_{s,n}$	Displacement, velocity and acceleration of subballast below n^{th} sleeper
$z_{s,n}^p, \dot{z}_{s,n}^p$	Plastic displacement and velocity of subballast below n^{th} sleeper
$z_{g,n}, \dot{z}_{g,n}, \ddot{z}_{g,n}$	Displacement, velocity and acceleration of subgrade below n^{th} sleeper
$z_{g,n}^p, \dot{z}_{g,n}^p$	Plastic displacement and velocity of subgrade below n^{th} sleeper
$z_g^{ve}, z_s^{ve}, z_b^{ve}$	Viscoelastic displacement in subgrade, subballast and ballast (m)

Greek Symbols:

α	Stress distribution angle for ballast ($^\circ$)
α^*	Empirical parameter
α'	Coefficient relating track irregularities, train suspension and speed
α''	Stress distribution angle for ballast in the stiffer side ($^\circ$)
α_0	Reference stress distribution angle in ballast ($^\circ$)
α_m	Bonding coefficient for planar geosynthetic

- α_v Angle between major principal stress direction and vertical ($^\circ$)
- β Stress distribution angle for subballast ($^\circ$)
- β' Coefficient accounting for the movement of train along a curve
- β_0 Reference stress distribution angle in subballast ($^\circ$)
- Γ Critical void ratio at $p = 1$ kPa
- $\hat{\Gamma}, \hat{N}$ Void ratio of critical state line and normal compression line at $\hat{p}=1$ kPa
- γ Stress distribution angle for subgrade ($^\circ$)
- γ' Coefficient that depends on speed and design of train, and track condition
- $\Delta\sigma$ Increase in stress (N/m^2)
- $\Delta\sigma'_2, \Delta\sigma'_3$ Effective additional confining stress in the direction of σ'_2 and σ'_3 (N/m^2)
- $\Delta\sigma_3$ Additional confining stress in the direction of σ_3 (N/m^2)
- $\Delta\sigma_x, \Delta\sigma_y$ Additional confining pressure along x and y directions (N/m^2)
- $d\varepsilon_{ij}^p, \varepsilon_v^p$ Plastic strain increment and cumulative plastic volumetric strain
- $d\varepsilon_v, d\varepsilon_q$ Volumetric and deviatoric strain increments
- $d\varepsilon_v^p, d\varepsilon_q^p$ Plastic volumetric and deviatoric strain increments
- $d\varepsilon_z^p$ Plastic strain increment in vertical direction
- δ_{ij} Kronecker delta
- δ_t Factor that depends on the track condition
- $\varepsilon_0/\varepsilon_r, \rho_p, \beta_p$ Fitting parameters
- $\varepsilon_1, \varepsilon_2, \varepsilon_3$ Major, intermediate and minor principal strains
- $\varepsilon_1^e, \varepsilon_2^e, \varepsilon_3^e$ Resilient components of major, intermediate and minor principal strains
- $\varepsilon_1^p, \varepsilon_2^p, \varepsilon_3^p$ Plastic components of major, intermediate and minor principal strains
- $\varepsilon_{1,1}^p$ Plastic axial strain after the first load cycle
- ε_a Axial strain
- ε_a^p Accumulated plastic axial strain
- $\varepsilon_b^p, \varepsilon_s^p, \varepsilon_g^p$ Cumulative plastic strain in ballast, subballast and subgrade
- ε_c Circumferential strain
- ε_q Deviatoric strain
- ε_r Radial strain
- ε_t Tensile strain

- $\varepsilon_x, \varepsilon_y$ Strain along x and y directions
- $\varepsilon_x^m, \varepsilon_y^m$ Strains in geosynthetic in x and y directions
- ε_z^p Cumulative plastic strain in vertical direction
- η Stress ratio
- $\hat{\eta}$ Stress ratio in characteristic stress space
- η_v Factor that depends on the speed of vehicle
- θ Bulk stress (N/m²)
- $\theta_1 + \theta_2$ Total dip angle of the rail joint (rad)
- J Tensorial invariant
- Λ_s, Λ_g Scalars
- λ, κ Slope of critical state line and swelling line in e - $\ln p$ space
- ν_b, ν_s, ν_g Poisson's ratio of ballast, subballast and subgrade
- ν_b^r Poisson's ratio of ballast in the stiffer side
- ν_i Poisson's ratio of infill
- ν_m Poisson's ratio of geoinclusion material
- ζ, A Dimensionless material parameters
- ρ_b, ρ_s, ρ_g Density of ballast, subballast and subgrade (kg/m³)
- ρ_b^r Density of ballast in the stiffer side (kg/m³)
- $\sigma_1, \sigma_2, \sigma_3$ Major, intermediate and minor principal stresses (N/m²)
- σ_{bb}^r Vertical stress at the bottom of substructure layer in the stiffer side (N/m²)
- $\sigma_{C,2}, \sigma_{C,3}$ Circumferential stresses in the direction of σ'_2 and σ'_3 (N/m²)
- σ_c Confining pressure in triaxial tests (N/m²)
- σ'_c Effective confining pressure (N/m²)
- σ_{cyc} Cyclic deviator stress (N/m²)
- σ_d Deviator stress (N/m²)
- σ'_d Effective deviatoric stress (N/m²)
- σ_{di} Deviator stress at which slope of E_R versus σ_d curve changes (N/m²)
- σ_g Compressive strength of the soil (N/m²)
- σ_{ij} Stress tensor
- $\hat{\sigma}_{ij}$ Characteristic stress tensor

σ_j	Principal stress (N/m ²)
σ_{oct}	Octahedral normal stress (N/m ²)
σ_{ref}	Reference stress (N/m ²)
$\sigma_{\text{sb}}, \sigma_{\text{bs}}, \sigma_{\text{sg}}, \sigma_{\text{go}}$	Vertical stresses at the sleeper-ballast, ballast-subballast, subballast-subgrade interfaces and bottom of subgrade layer (N/m ²)
σ_v	Vertical stress (N/m ²)
$\sigma'_x, \sigma'_y, \sigma'_z$	Effective stresses along x, y and z directions (N/m ²)
τ_{cyc}	Cyclic shear stress amplitude (N/m ²)
τ_{oct}	Octahedral shear stress (N/m ²)
ϕ	Friction angle (°)
ϕ_c, ϕ_e	Critical state friction angles under triaxial compression and extension (°)
ϕ_d	Dynamic amplification factor
ϕ'_m	Mobilised friction angle (°)
χ_i, χ_{tc}	Dilatancy parameter corresponding to image state and triaxial compression
ψ	State parameter
ψ_d	Dilation angle (°)
ψ_i	Image state parameter
ψ_m	Mobilised dilation angle (°)

Abbreviations:

2D	Two-dimensional
2.5D	Two and a half dimensional
3D	Three-dimensional
ACR	Additional confinement ratio
AREA	American Railway Engineering Association
ARTC	Australian Rail Track Corporation
BEM	Boundary element method
BoEF	Beam on elastic foundation
CBM	Cement bound mixture
CBR	California Bearing Ratio
CG	Coir geotextile

CSL Critical state line
CSSR Cyclic shear stress ratio
CVSR Cyclic vertical stress ratio
DEM Discrete element method
FDM Finite difference method
FEM Finite element method
GB Geocell reinforced ballast
GG Geocell reinforced subgrade
GS Geocell reinforced subballast
HDPE High-density polyethylene
HMA Hot-mix asphalt
LVDT Linear variable displacement transformer
MDD Multi-depth deflectometers
MGT Million gross tonnes
MSD Mass-spring-dashpot
NCL Normal compression line
ORE Office for Research and Experiments
PE Polyethylene
PP Polypropylene
PSR Principal stress rotation
RAP Recycled asphalt pavement
SS Silica sand
UGM Unbound granular material
UR Unreinforced
US United States
WMATA Washington Metropolitan Area Transit Authority