Shakedown response of recycled rubber-granular waste mixtures under cyclic loading

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Abstract:

Determining the long-term deformation behaviour of granular materials under repeated (cyclic) loads is imperative for ensuring acceptable longevity of railway tracks. One of most relevant characteristics of granular materials under repeated loading and unloading is their ability achieve a relatively stable state (shakedown) after being subjected to initial compression. This study reports the shakedown response of rubber-granular waste mixtures based on a series of cyclic triaxial tests conducted at different loading conditions. The combined influence of the rubber content, confining stress, and cyclic loading amplitude on the permanent deformation of the granular matrix is analysed in relation to plastic shakedown and also identifying the occurrence of plastic creep. The results show that the permanent axial deformations reach a state of plastic shakedown at a lower rubber content, while a higher rubber content leads to enhanced plastic creep, which exacerbates at higher loading amplitudes. A unified method of estimating the shakedown limit is proposed herein to estimate the appropriate cyclic loading amplitude corresponding to the rubber content and confining stress. The proposed method is validated with two independent sets of drained cyclic triaxial test data on coal wash-rubber crumb mixtures in contrast to standard rail ballast.

Keywords: Shakedown, rubber crumbs, permanent deformations, granular materials

1. Introduction:

The growing number of scrap rubber tyres has become a concern to accumulating areas of landfill, leaving unsightly stockpiles occupying otherwise usable land, also causing significant environmental pollution over time. As a sustainable solution, numerous studies have proposed the adoption of recycled rubber in various forms in earthquake-resistant foundations and liquefaction-resistant backfills (Li et al., 2016, Mashiri et al., 2017, Tsang, 2008, Senetakis et al., 2012). Recently, Indraratna et al. (2018) proposed an alternative for capping layer in track infrastructure by using rubber crumbs (RC) blended with granular by-products from coal and steel industries, namely, steel furnace slag (SFS) and coal wash (CW). Under cyclic loading conditions appropriate for typical heavy haul train passages, Qi et al. (2019) found that the presence of rubber crumbs in a granular matrix would increase the permanent axial and volumetric strains (compression), while enhancing the damping properties of the mixture in relation to the hysteretic loading-unloading response. However, they also found that when the rubber content in the mixture exceeded 10%, the mixture showed marginal benefits, and at significantly higher rubber content approaching 20%, the blended matrix suffered from significantly reduced resilient modulus and unacceptable compression that will not be appropriate as a track substructure material.

Granular materials in railway infrastructure are subjected to thousands of loading cycles during their lifetime. During initial loading cycles, attributed to particle re-arrangement, the permanent deformation of the granular matrix increases rapidly, causing cyclic densification (Sun et al., 2015). Subsequently, after being subjected to a certain number of loading cycles, the material reaches a shakedown state that is usually characterised by the permanent strains reaching a plateau with insignificant accumulated strain thereafter. However, at significantly higher stress levels, the permanent deformations may not become stable but continue to increase, thereby exceeding the maximum allowable deformations within the designed lifespan of a capping layer. This post-densification response has been studied by other researchers (Sharp and Booker, 1984, Werkmeister et al., 2001, Alnedawi et al., 2019, Xiao et al., 2018) to analyse the settlement of unbound pavement materials under cyclic loading. Based on these past studies, three distinct deformation zones can be identified: (i) plastic shakedown, (ii) plastic creep and (iii) incremental collapse, as shown in Figure 1(a). The critical cyclic stress limit associated with the onset of unstable creep deformations and incremental collapse can be termed as 'shakedown limit' and 'creep limit', respectively. Further, the incremental axial strain response with the accumulated axial strain (Figure 1b) was also quantified, and different criteria have been proposed based on the incremental strain in the range of 3000-5000 loading cycles to define the apparent shakedown zones (Werkmeister et al., 2001, Gu et al., 2017).

Various studies have used this concept of cyclic shakedown to analyse the permanent deformations of bounded granular materials such as railway ballast (Xiao et al., 2017, Sun et al., 2015). They reported that the effective confining stress on the material as well as the cyclic loading amplitude and frequency had affected the shakedown response. However, the estimation of the shakedown limit for granular materials with varying confining stress and cyclic loading amplitude has not yet been studied in detail. In addition, it is most beneficial to quantify how the rubber content can influence the shakedown behaviour of rubber-granular waste mixtures, especially when the addition of rubber to a mixture of harder and highly angular aggregates is expected to cause a reduction in its overall shear strength. In this regard, this paper provides an insightful study into the role of rubber inclusion on the permanent deformation in distinctly different shakedown regions. More importantly, a unified method of estimating the shakedown limit is proposed, considering the peak static shear strength under various cyclic loading conditions and material compositions.

2. Shakedown response of rubber-granular waste mixtures:

In this study, to analyse the shakedown response of rubber-granular waste mixtures (SFS-CW -RC), permanent axial strain data from the cyclic drained triaxial tests conducted by Qi et al. (2019) was adopted, where the rubber contents (R_b) range from 0% to 40% by weight? plus three levels of confining stress (σ_3'=10, 40, 70 kPa). A granular mix with SFS:CW=7:3 is considered, which is the optimal ratio to ensure a sufficient shear strength, acceptable permeability and insignificant swelling effects of steel slag to be used as subballast? (Indraratna et al., 2018). The cyclic loading amplitude for these tests is applied with a constant cyclic stress ratio (CSR, given in Eq.1) of 0.8.

$$CSR = \frac{q_{cyc}}{2\sigma_3'} \tag{1}$$

The variation of incremental permanent axial strains with accumulated axial strains for varying rubber content at different confining stresses is shown in Figure 2. Isometric lines are drawn on the plots to denote 1st, 1000th, 10000th and 50000th loading cycles. As observed in Figure 2a, at low confining stresses ($\sigma_3' = 10\ kPa$), mixtures with R_b=0%-40% experience cyclic densification within 100 loading cycles and they reach a stable state of plastic shakedown quickly after about 500 cycles. Although a higher rubber content caused an increase in the accumulated strain after N=50000, the shakedown response was not altered by the rubber content. However, at higher confining stresses of 40 kPa and 70 kPa (Figure 2b and 2c), the densification period increases to N=500 for mixtures with lower levels of R_b (i.e. 0 and 10%) and N=1000 for higher R_b (20%,30% and 40%). After the cyclic densification stage, while mixtures with lower R_b experienced plastic shakedown, those mixtures with R_b=20%-40% experienced irrecoverable strains in the plastic creep region, as shown in Figures 2b and 2c. For $R_b > 10\%$, the incremental strain rate decreases gradually within the range of 1000-50000 loading cycles, with no stabilisation observed in relation to the accumulated axial strains. Also, the accumulated axial strain at N=50000 increases by approximately 1.5 times when the rubber content (R_b) increases from 10% to 20% for both σ_3'=40 and 70 kPa, indicating a distinct boundary between the plastic shakedown and plastic creep stage. This change in shakedown response at higher rubber content can be attributed to the reduced interaction between the rigid SFS and CW aggregates; thus the behaviour of the mixture is dominated by the compressible rubber crumbs.

3. Shakedown limit:

To better understand the influence of applied cyclic loading amplitude on the shakedown response, a normalised cyclic stress ratio (Ψ) is computed by taking the ratio of cyclic loading amplitude to the peak shear strength (static), as given in Eq. 2.

$$\Psi = \frac{q_{cyc}}{q_{peak}} \tag{2}$$

For CSR=0.8, Figure 3 shows the variation of peak static shear strength (q_{peak}) of the recycled rubber-granular waste mixture and the corresponding Ψ values for test specimens with different rubber contents. It can be seen that the peak shear strength of the material decreases with the increase in R_b for all the three confining stresses, and the value of Ψ increases with higher rubber contents. The influence of Ψ on the permanent axial deformations (ϵ_a) after the cyclic densification stage at three loading intervals (N=1000, 10000 and 50000 cycles) for all cyclic loading cases is shown in Figure 4. At relatively low values of Ψ (< 0.23), the permanent axial deformations for all three stages become closer together, irrespective of the rubber content and confining stress. As the accumulated deformation does not change for N = 1000, 10000 and 50000, the loading cases in this region fall within the plastic shakedown range as shown in Fig 3. In contrast, when $\Psi > 0.23$, a clear scatter of the accumulated strains at all three stages is observed with plastic strains unable to attain a stable shakedown state even after 50000 loading cycles, so this part of the plot in Figure 4 is termed as the plastic creep region. The bounding value of Ψ at which the change in response occurs can be considered as the 'shakedown limit' (Ψ_{sh}=0.23) for these rubber-mixed granular mixtures. It is to be noted that, even though CSR is kept constant (0.8) for all confining stresses and rubber contents, the corresponding Ψ values at relatively low confining stress (10 kPa) lie within the shakedown limit, and the material reaches a plastic shakedown state even with 40% rubber crumbs as observed in Figure 2(a). In contrast, at higher confining stresses (70 kPa), the magnitude of Ψ slightly exceeds the shakedown limit at R_b=10%, and plastic creep is observed as shown in Figure 2(c). Besides, the separation between the three loading intervals (N=1000,10000,50000) increases with an increase in Ψ, signifying the combined influence of cyclic loading amplitude, confining stress and rubber content on the instability of these recycled rubber-granular waste mixtures.

The dependence of accumulated axial strain with Ψ can be mathematically expressed using a linear relationship as given in Eq. 3 and plotted in Figure 4.

$$\epsilon_a = \alpha(\Psi - \Psi_{sh}) + \epsilon_{sh} \tag{3}$$

where, α is the ratio of permanent axial strain increments to the increment of Ψ, Ψ_{sh} is the shakedown limit and ϵ_{sh} is the accumulated axial strain at the corresponding shakedown limit. For all the rubber-granular waste mixtures, Ψ_{sh} is found to be 0.23, with ϵ_{sh}= 1.3%. It can be seen from Figure 4 that $\alpha = 6.2$ is unique for all N in the shakedown region, while α increases from 17.75 at N=1000 cycles to 22.4 at N=50000 cycles. It is also observed that the regression line in the shakedown region intercepts the x-axis at $\Psi = 0.02$, and this can be considered as the cyclic stress level below which the material experiences very small deformations. Further, this behaviour is validated for the waste mixture having $R_b = 10\%$ tested with a lower CSR of 0.4, and the permanent axial strain data are shown in Figure 4. As observed, the CSR of 0.4 corresponds to lower Ψ values in the plastic shakedown region ($\Psi < 0.23$), and the permanent axial strains reach the plastic shakedown state at around N=1000 cycles, and the observed trend of data matches the proposed relationship well.

4. Validation of the proposed method:

This approach is also applied to two independent sets of permanent axial strain data from drained cyclic triaxial tests on coal wash (CW) and rubber crumb (RC) mixtures (Tawk et al., 2021) and typical Australian railway ballast (Lackenby et al., 2007), tested under cyclic triaxial conditions at different confining stresses and cyclic stress amplitudes (Figures 5a and b). As explained by Tawk et al., (2021) for CW+RC mixtures (Figure 5a), N=5000 is considered as the completion of the initial compaction phase. It is observed that the shakedown limit (Ψ_{sh}=0.24) of the mixture can be uniquely identified irrespective of the confining stress and CSR, and the permanent axial strains follow a linear relationship (r^2~0.9). Also, the mixtures with low rubber contents ($R_b = 0\%, 5\%$) subjected to relatively low values of CSR (0.8) and confining stress (25 kPa) are found to lie in the shakedown region, while mixtures with higher rubber content (R_b=10%, 15%) fall within the region of plastic creep for higher CSR (2.0) and higher confining stress (50 kPa). Further, the permanent axial strain response of ballast without any rubber (Figure 5b), also follows a linear relationship with Ψ (r^2~0.88); the distinction between plastic shakedown and creep regions is observed at a much higher value of Ψ_{sh}=0.8. The elevated shakedown limit indicates that the material can resist higher cyclic load amplitudes before showing a plastic creep behaviour compared to the rubber-granular waste mixtures. Further, the shakedown curve is found to pass through the origin, which can be attributed to the angular ballast aggregates ($D_{50} = 35\,mm$) when compared to less angular rubber-granular waste mixtures ($D_{50} = 1.8\,mm$), forcing the angular corners to break even for low cyclic loading amplitudes, thus resulting in permanent deformation. These observations indicate without doubt that the parameter Ψ that incorporates the effects of both confining stress and rubber content, can be used as a useful parameter to define the shakedown limit of granular materials.

4.1 Limitations of this study:

The observations made in this study are limited to a constant loading frequency (5 Hz) and the effects of varying loading frequency on the shakedown limit considering the influence of rebound of rubber crumbs during different unloading-reloading periods has not been investigated and reported in this paper. The authors intend to combine the regions of shakedown and plastic creep of the current study with such rubber rebounding model at a later stage, but not within the scope of this paper.

5. Conclusions:

A novel method to estimate the plastic shakedown limit of rubber blended granular mixtures is proposed. Based on this study, the following conclusions can be drawn:

1. An increase in the rubber content reduced the capability of SFS+CW+RC mixtures, reaching a state of plastic shakedown within 50000 loading cycles at CSR = 0.8 and confining pressures of 40 and 70 kPa. The permanent deformations for mixtures after the initial compression phase showed a plastic shakedown behaviour at lower rubber contents ($R_b \leq 10$ %), and the mixtures with higher rubber contents (≥ 20 %) showed plastic creep response, while a constant increase in accumulated axial strains was observed even after N=50,000.
2. A linear relationship was observed between the normalised cyclic stress ratio and accumulated axial strains, with the shakedown limit for SFS+CW+RC mixtures found to be at Ψ_{sh}=0.23. A similar relationship was observed for CW+RC mixtures (i.e. no steel furnace slag) with Ψ_{sh}=0.24, whereas, for coarse granular materials such as pure ballast (no rubber), the shakedown limit increases to 0.8. This indicates that the presence of rubber crumbs would influence the granular mixtures to exhibit a plastic creep response at a relatively lower cyclic stress amplitude, even though the loading conditions are kept the same.
3. In practice, this proposed approach would help determine the allowable cyclic loading amplitudes that can be applied on granular layers in rail tracks to minimise deformations exhibiting plastic creep.

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List of notations:

CSR	Cyclic stress ratio
q_{cyc}	Cyclic deviatoric stress amplitude
q_{peak}	Peak shear strength
R_b	Rubber content
σ_3'	Effective confining stress
Ψ	Normalised cyclic stress ratio
Ψ_{sh}	Shakedown limit
ϵ_a	Permanent axial strain
ϵ_{sh}	Permanent axial strain at the shakedown limit

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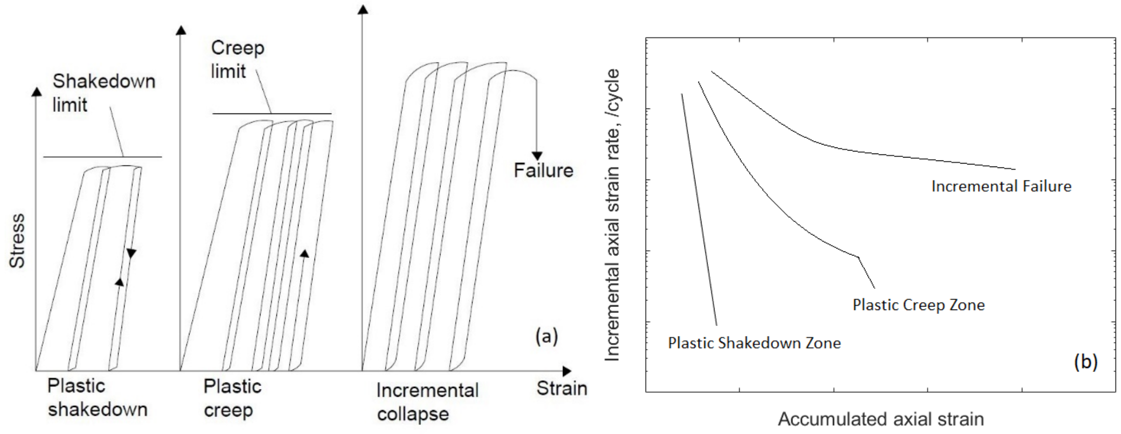

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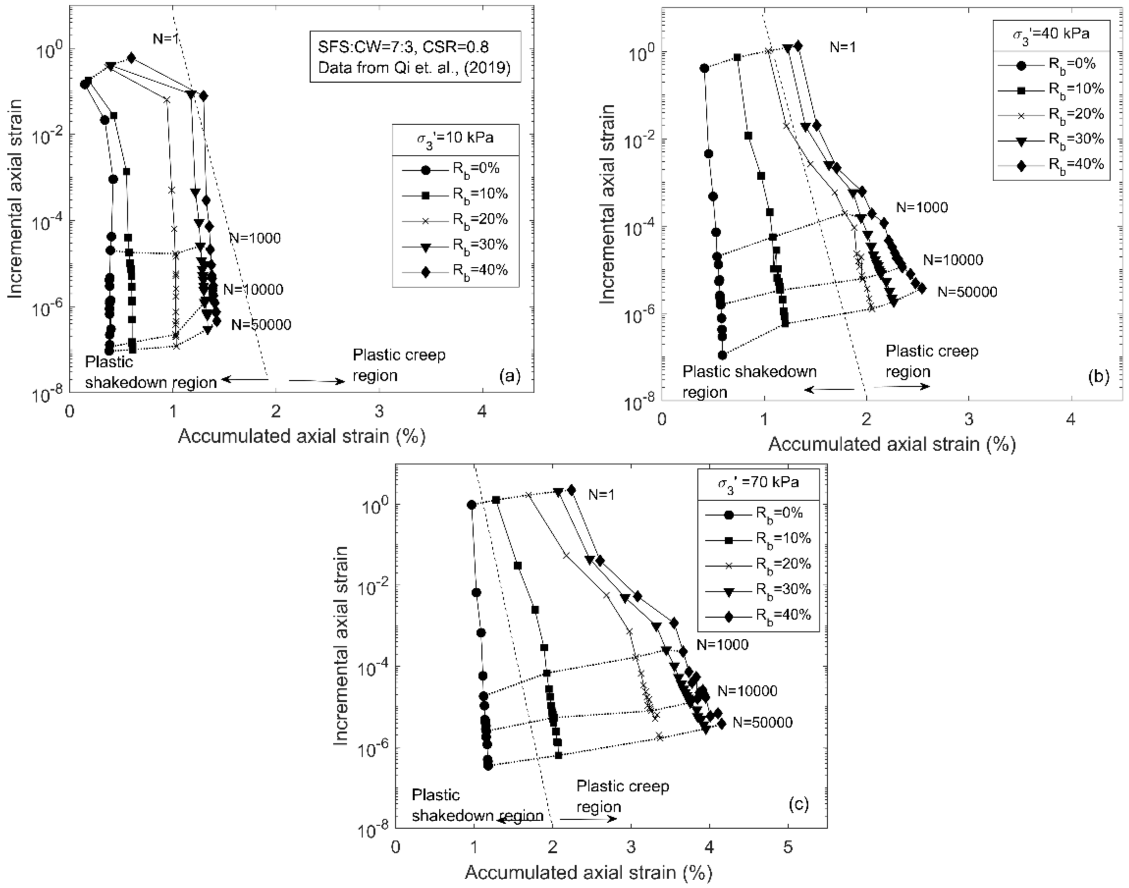

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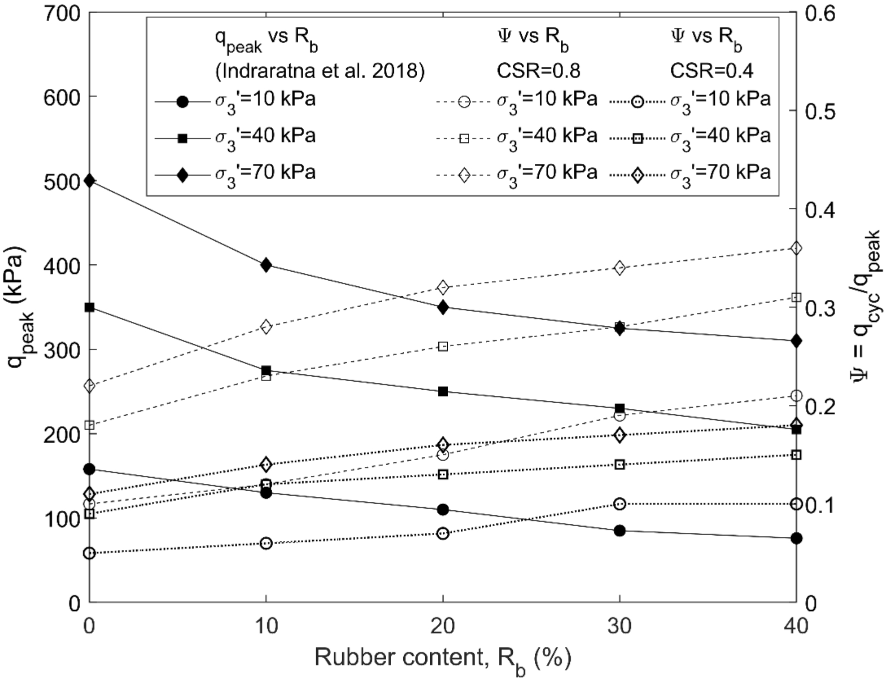

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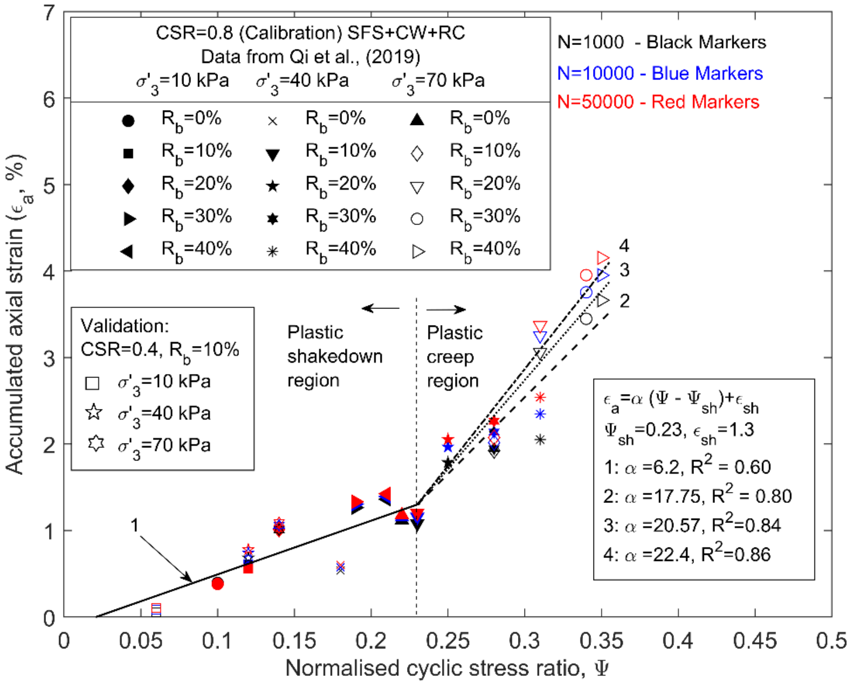

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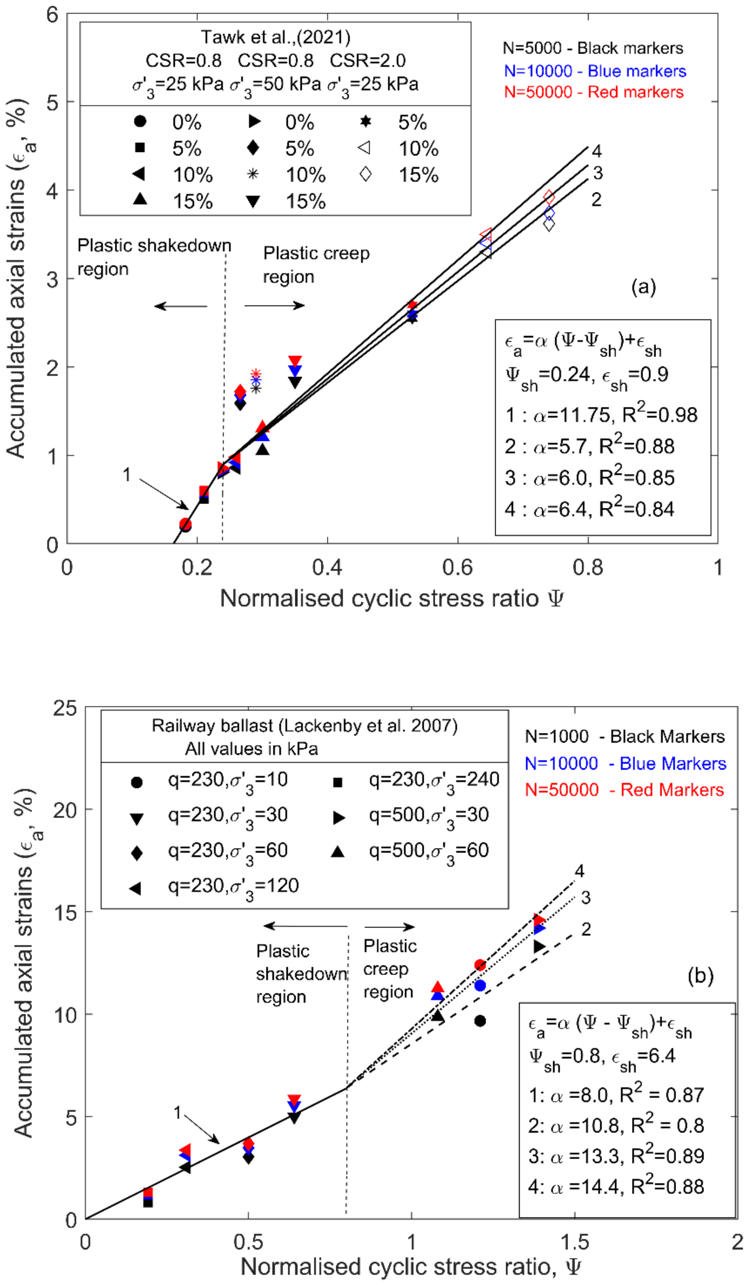

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