Track Geomechanics for Future Railways: Use of Artificial Inclusions

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Abstract. Given that current ballasted tracks cannot support faster and heavier Australian heavy freight trains, the need to develop innovative and sustainable ballasted tracks for transport infrastructure is crucial. This paper reviews and discusses the use artificial inclusions such as recycled rubber mats, end-of-life tires, and geogrids to stabilize ballasted rail tracks overlying soft formation soil. It also presents a novel solution for increasing the stability and resiliency of track structure by energy-absorbing recycled rubber tires. This study confirms that a capping layer confined by tires will actively reduce ballast breakage within the track substructure. Numerical simulations employing discrete element method-DEM is also carried out to study the micro-mechanical aspects of ballast aggregates and the interaction between the particles and inclusions. This study shows that waste rubber products and geosynthetics will eliminate the need for a capping layer in certain terrain and help to decrease the thickness of the ballast layer. The outcomes of this study will lead to a better understanding of the performance of ballast tracks reinforced with artificial inclusions, and also help to improve the design and cost effectiveness of ballasted tracks, with a view to enhance passenger comfort and safety.

Keywords: Track Geomechanics, Artificial Inclusions, Numerical Modeling.

1 Introduction

Australian railways offer an efficient mode of transportation for freight and passengers across all the States. However, the dynamic impact loads induced by heavy haul trains cause breakage, lateral spreading and settlements of ballast which leads to rail-tie misalignment and decreased load bearing capacity, all of which compromises safety and entails frequent track maintenance [1, 2]. For these reasons conventional ballasted rail tracks should be enhanced so they can deal with the increased demand for heavier freight trains.

Artificial inclusions such as geo-composites (e.g. geogrids) and rubber-energy absorbing inclusions (rubber mats/pads, granulated rubber and recycled tire cell) produced from recycled tires are becoming more popular worldwide because they eliminate the hard interfaces between ballast aggregates and concrete sleepers, or the underlying formation soils, and allow the aggregates to bed into the relatively softer pads; this increases contacts of surface area of the ballast and reduces ballast stresses [3-8]. Moreover, some of these artificial inclusions can be obtained through recycled materials such as waste tires, which in a way reduces the stockpiling of waste materials and the cost by using natural aggregates.

However, since the number of studies that analyze how well synthetic inclusions actually improved performance of tracks is still limited, this paper reviews current research done at the University of Wollongong Australia (UOW) using large-scale test facilities and computational modelling to evaluate the ability of artificial inclusions (i.e. under ballast mats, tire cells, granular waste mixtures and geogrids) to eliminate ballast degradation and associated deformation.

2 Large-scale Laboratory Testing

2.1 Use of Under Ballast Mat for Improved Performance of Ballast

Track process simulation testing apparatus (TPSTA) was used to study the effect that USP has on the load-deformation of ballast. The testing chamber of this TPSTA is based on the unit cell of a standard gauge track (800mm × 600mm × 600mm). It has four moveable walls connected by hydraulic jacks placed on ball bearings and hinges (Fig. 1a), so the walls can be fixed or move laterally. The cyclic loading frequency can be applied up to 30 Hz and the axial load can simulate a train up to 40-tonne axle load. There are pressure plates and settlement pegs (Figs. 1b, 1c) in the TPSTA placed on top of the ballast layer to measure the stress and ballast deformation during testing. A concrete sleeper with rail is placed on top the test specimen to simulate the field condition. Around the concrete sleeper is filled with shoulder ballast. Further details of the instrumentation and the placement of track substructure layers, including a plan view and cross-section of the TPSTA, can be found elsewhere [9, 10].

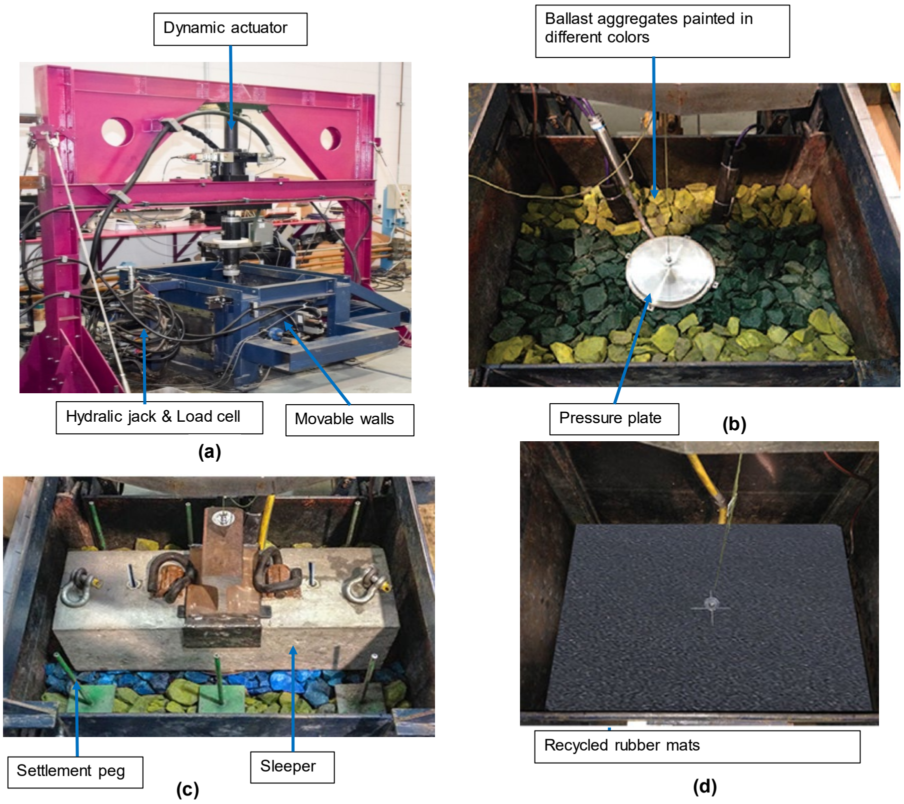
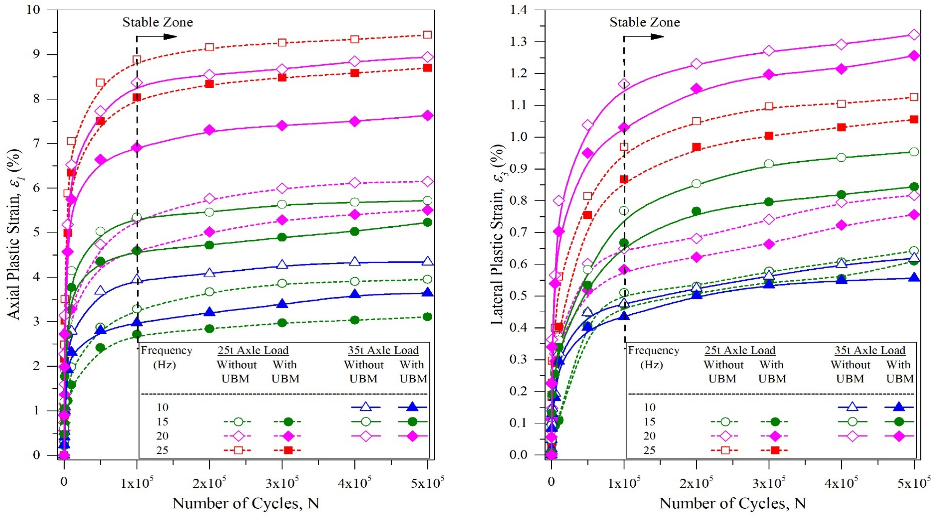


Fig. 1. TPSTA: (a) 3D view; (b) Ballast aggregates and pressure plate; (c) Settlement pegs; and (d) Recycled rubber mat

The ballast is ordinary igneous rock aggregates collected from a quarry in Wollongong and crushed to comply with Australian standard AS:2758.7: (2015). The capping layer (sub-ballast) is a mixture of sand and gravel and the soft subgrade is a mixture of sandy-clay soil. There is a 10mm thick under-ballast mats (UBM) made from recycled rubber tires to be placed beneath the ballast layer (Fig. 1d) to investigate its performance under cyclic loading. These tests took place under 25 and 35-tonne axle loads at varied frequencies (f= 15-25 Hz), during which the vertical and lateral displacement of the ballast assemblies were monitored at given time intervals [4]. The ballast aggregates were sieved after each test to determine the amount of breakage.

Figure 2 shows the accumulated vertical and lateral deformations and the corresponding strain of ballast (with/without UBM). Under a given axle load and frequency f, the UBM reduced the vertical and lateral strain of ballast by up to 21% and 12%, respectively. Moreover, the overall ballast degradation (breakage) had decreased by 35%, and there was an almost 19% reduction in vertical stress at the base of the ballast-concrete interface.



(a) (b)

Fig. 2. Measured laboratory test data: (a) axial strain, ε_1 versus load cycles (N); and (b) lateral strain, ε_3 v.s N (*modified after Navaratnarajah et al. [4] with permission from ASCE*)

2.2 Use of Scrap Rubber Tires for Capping Confinement

Scrap tires have been trialed at the UOW to confine the capping (sub-ballast) layer because they provide additional cellular confinement to the infilled-aggregates and also increase the strength, stiffness of the railway track. Indraratna et al. [11] proposed using rubber tires as a base for the capping layer where one side-wall has been cut and the tires were infilled with aggregates. A geotextile was placed between tires and the subgrade as a separator. Previous studies revealed that from an engineering perspective, a tire cell has three main effects: (i) extra confinement provided by the tire will increase the stiffness; (ii) provide more uniform stress distribution onto the subgrade; (iii) the tires will also increase the damping of the track and its ability to attenuate dynamic forces. The TPSTA used to carry out cyclic tests of recycled tires and confine the capping layer is presented in Figure 3. The cyclic loading frequency is 15 Hz and the maximum vertical pressure is 385 kPa to simulate the track field condition of a heavy haul with 40-tonne axle load running at 115 km/h. The cyclic loading test was continuing until 500,000 cycles were achieved.

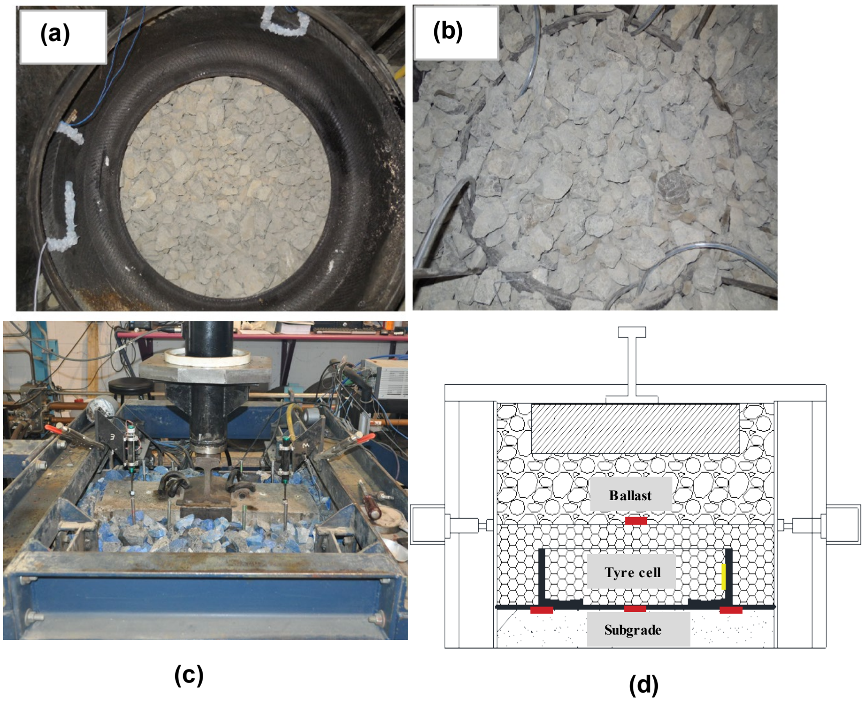


Fig. 3. (a) Recycled tires before installing the strain gauges, (b) tires filled with capping aggregate, (c) TPSTA test set up, (d) schematic showing the placement of tire in capping

Measured test results show that having a tire in the capping layer vastly improved lateral confinement of infills, thus mitigating the lateral spreading and degradation of ballast, while improving the damping properties of the system (Fig. 4a). It also reduced the track modulus (Fig. 4b), which may be of some benefit for sections of track over rigid foundations such as transition zones and concrete bridge decks where vertical stiffness should be reduced to avoid abrupt changes in stiffness along railway embankments.

Finite element modeling (FEM) was carried out to study the beneficial effect that recycled tires have on track embankments, as shown in Figure 5. Details of the FEM model, including the loading and boundary conditions and the model parameters can be found in Indraratna et al. [11]. The simulations showed that with waste tires, the stress imparted onto the subgrade was almost 12% less than where there are no tires (Fig. 6). This means that a capping layer stabilized with rubber tires reduces the stress transferred onto the ballast and subgrade layers, mitigates ballast breakage and particle displacements, and enhances damping of the system.

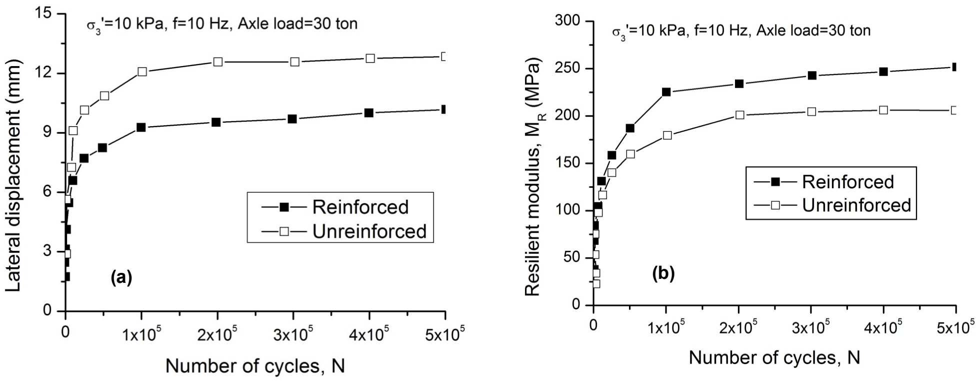


Fig. 4. (a) Measured lateral displacement; (b) measured resilient modulus (*modified after Indraratna et al. [11]*)

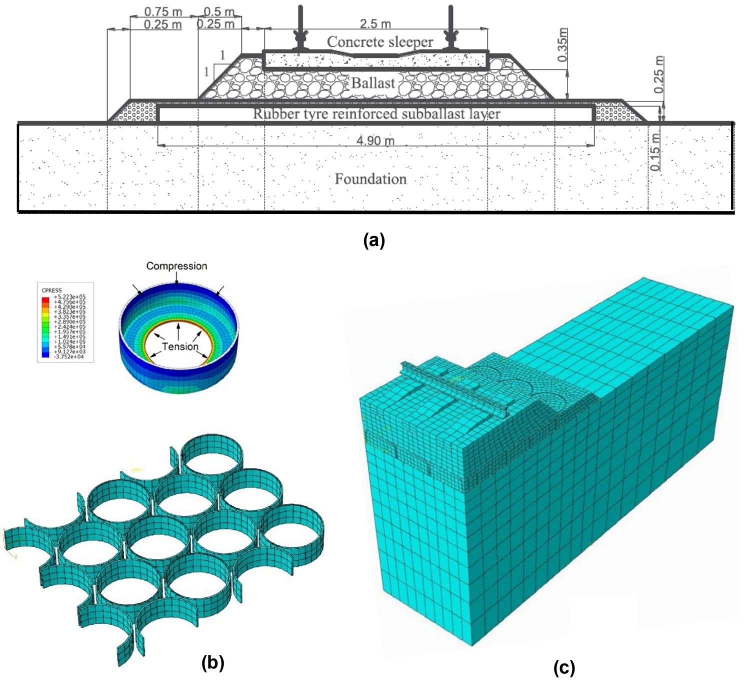
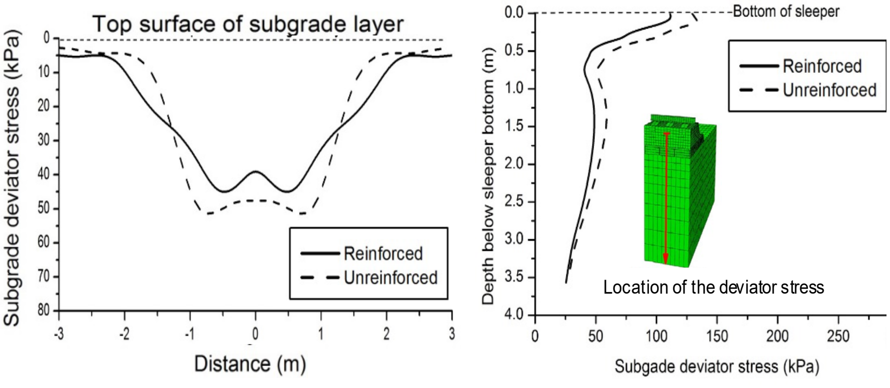


Fig. 5. (a) Track geometry with recycled rubber tire, (b) FEM mesh for recycled rubber tires, and (c) Simulated track section with tires (*modified after Indraratna et al. [11]*)



(a)

(b)

Fig. 6. Role of recycled rubber tires on: (a) distribution of subgrade stress, and (b) distribution of stresses along the depth (*modified after Indraratna et al. [11]*)

2.3 A Synthetic Energy Absorbing Layer for Railway Subballast

It has been reported that recycled rubber crumbs (RC) have higher damping properties and higher energy absorbing properties than natural aggregates [7, 12-17]. To extend the use of mining waste such as steel furnace slag (SFS) and coal wash (CW) in dynamic loading projects such as rail tracks, Indraratna et al. [15] incorporated RC into blends of SFS and CW to develop a synthetic energy absorbing layer (SEAL) for railway subballast. The optimal waste mixture for subballast has been optimized as SFS:CW=7:3 with 10% RC mixing by weight (i.e. SEAL10). A series of laboratory tests proved that adding 10% of RC into a mixture of SFS and CW will, (i) reduce the swelling potential of SFS and the particle breakage of CW, and (ii) increase the ductility and energy absorbing property of these waste mixtures [12-18]. Qi et al. [17] also proposed an energy absorbing concept and found that installing SEAL10 under a rail track will help to reduce ballast degradation and minimise track deformation as more energy is absorbed by the SEAL rather than the other layers. To verify this concept, large-scale cubical triaxial tests using the TPSTA at the University of Wollongong have been carried out to examine the deformation and ballast degradation of a track specimen with and without SEAL10.

Figure 7(a) is a schematic of the large-scale test specimen with a 150mm thick SEAL between a 200mm thick layer of ballast and a 100mm thick layer of structural fill, and a concrete sleeper with a rail on top of the test specimen. The area surrounding the concrete sleeper is filled with shoulder ballast (in red). Figure 7b-c shows the procedure for preparing the test specimen. To check ballast breakage, the ballast directly under the sleeper is painted white and is sieved again after each test. The cyclic loading test is carried out at a frequency f=15 Hz and the vertical stress under the sleeper is 230 kPa to simulate a train with a 25-tonne axle load running at 115 km/h. The effective confining pressure is 15 kPa to simulate the field conditions. Each test is completed after reaching 500,000 cycles.

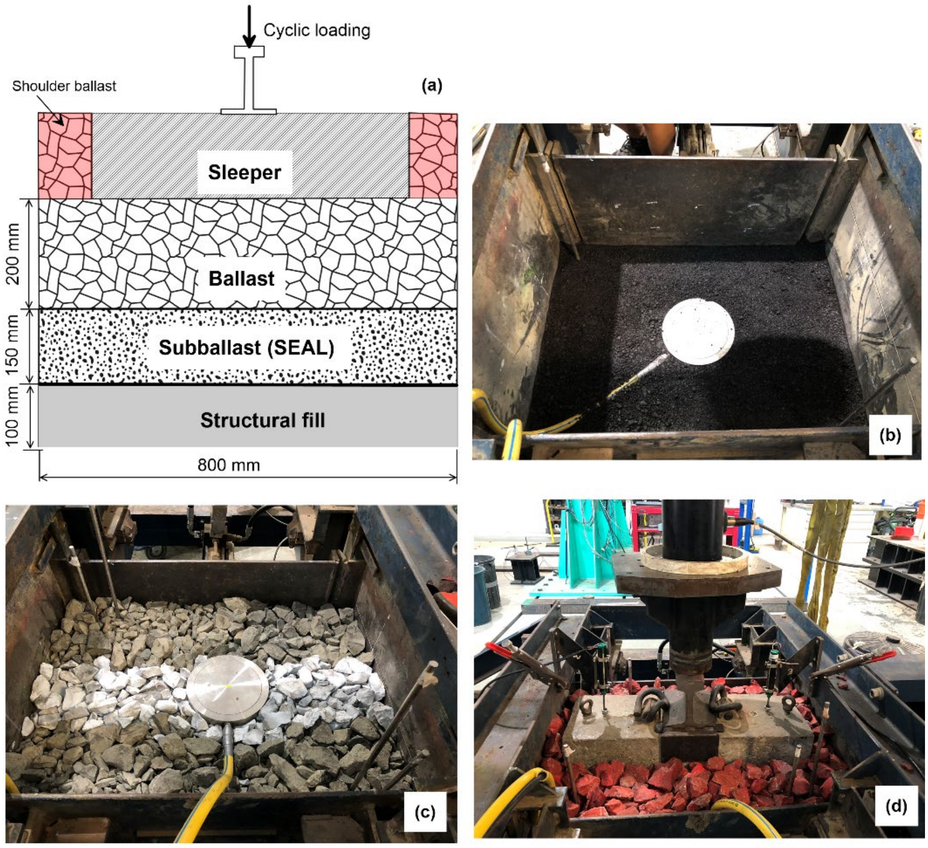


Fig. 7. (a) Schematic showing the track specimen incorporating SEAL, (b) installing the SEAL, (c) compacted ballast on top of SEAL, and (d) installing the shoulder ballast and the concrete sleeper

The vertical and lateral deformation of the track specimen with SEAL10 as it varies with the loading cycles are shown in Figure 8. To compare its performance with a traditional track, the results from previous tests [4, 9] of traditional track materials under the same loading conditions are also shown in Figure 8. Note that the settlement and lateral deformation (dilation) accumulate rapidly before 50,000 cycles due to initial densification and particle rearrangement, and then the rate of accumulation gradually decreases towards the end of the test. When these results are compared with traditional materials, the test specimen with SEAL10 shows a promising performance with less settlement and lateral deformation.

Ballast breakage is evaluated using the ballast breakage index (BBI) initially proposed by Indraratna et al. [19] by comparing the particle size distribution curve of ballast before and after each test:

$$BBI=A/(A+B) \tag{1}$$

where A denotes the area enclosed by the particle size distribution curves of ballast before and after the test, and B denotes the area enclosed by the initial particle size distribution curve and the arbitrary boundary of maximum breakage. A detailed definition of BBI is shown in Figure 9 (a). Figure 9 (b) shows the BBI of the test specimen with and without SEAL10, where the BBI of the track specimen with traditional track materials is around 0.6 while the one with SEAL10 is 0.31; these figures show that incorporating SEAL10 will significantly reduce ballast degradation. These results further verify the energy absorbing concept proposed by Qi et al. [17].

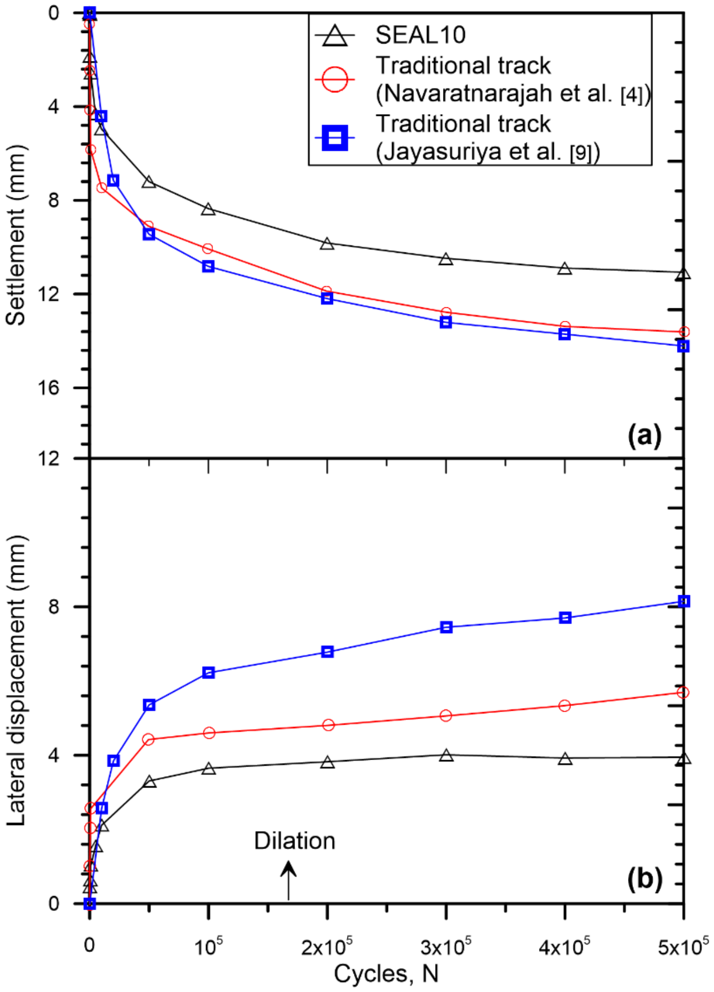


Fig. 8. Settlement and lateral deformation of the large-scale cubical triaxial test with and without SEAL (*modified after Qi and Indraratna [20]*)

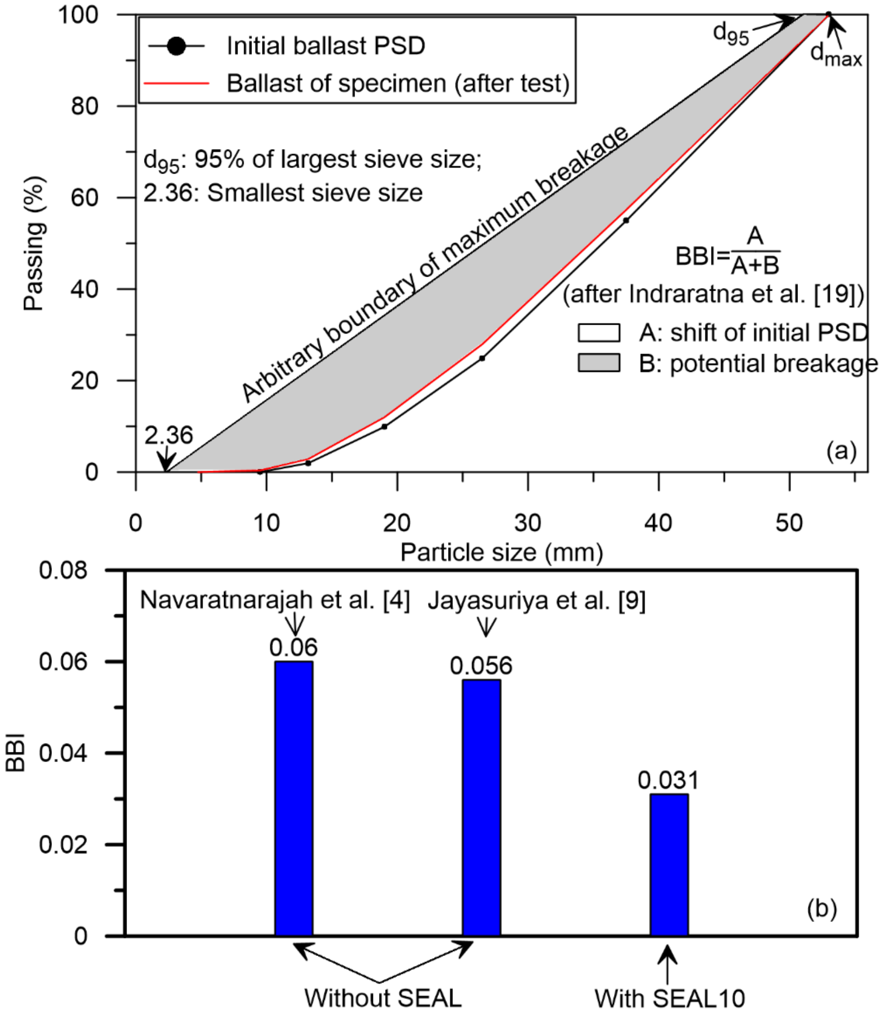


Fig. 9. (a) Definition of the ballast breakage index (BBI, modified after Indraratna et al [19]) and (b) BBI of the large-scale cubical triaxial test with and without SEAL (*modified after Qi and Indraratna [20]*)

3 Discrete Element Modeling for Ballast

While the discrete element method (DEM) developed by Cundall and Strack [21] has been widely used to study the mechanical behavior of granular aggregates, it has also been used to simulate ballast grains as it can capture the discrete nature which contains arbitrarily shaped discrete particles [22-28]. The DEM approach can simulate ballast particles of various sizes and shapes by bonding many spheres to represent the actual shape and angularity of ballast (Fig. 10a). A biaxial geogrid with a 40 mm × 40 mm aperture was modeled by connecting a number of small spheres, similar to the geogrids tested in the laboratory (Fig.10b). The DEM for a large-scale direct shear test of geogrid-reinforced fresh and fouled ballast is shown in Figs.10c-d. Micro-mechanical parameters used to simulate ballast, geogrid, and coal-fouling material to obtain these parameters can be found in Ngo et al. [24]. DEM simulations for large-scale direct shear box tests were implemented for fresh balalst and 40%-VCI-fouled ballast (with/without geogrid).

Figure 11 shows a comparison of the predicted shear stress-strain with those measured in the laboratory (with/without geogrid). Note that this simulation agrees quite well with the laboratory data from Indraratna *et al.* [29]. The strain softening and volumetric dilatancy of ballast shown in all the simulations indicates that the higher σ_n the greater the shear strength and the smaller the dilation. This can be explained by the interlocking effects that occur between the ballast particles and geogrid [30-31].

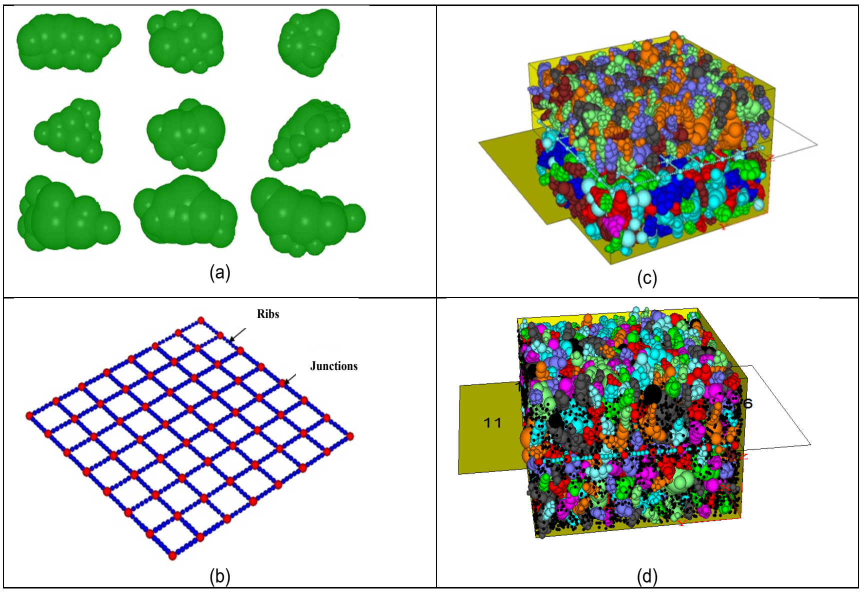


Fig. 10. Simulated large-scale direct shear test for geogrid-reinforced ballast (*Ngo et al. [24]*)

4 Selected Case Studies for Research Impact

New ballast gradation for Australian rail tracks:

Other than adopting a subtle modification of British Standards, the standards and specifications for ballast in Australia have not changed for many decades. Ground conditions in Australia are completely different from other parts the world which is why there is no international ballast design which considers the rate and extent of particle breakage. Particle breakage caused by the passage of trains has been considered in the development of new ballast gradations AS:2758.7 [32], which is why the 60-Graded gradation has been adopted by Sydney Trains (previously, RailCorp) for the design of new tracks. This new gradation has enhanced the capability for track upgrades, while allowing greater loads to be carried at increased speeds by utilizing existing infrastructure with minimum maintenance costs.

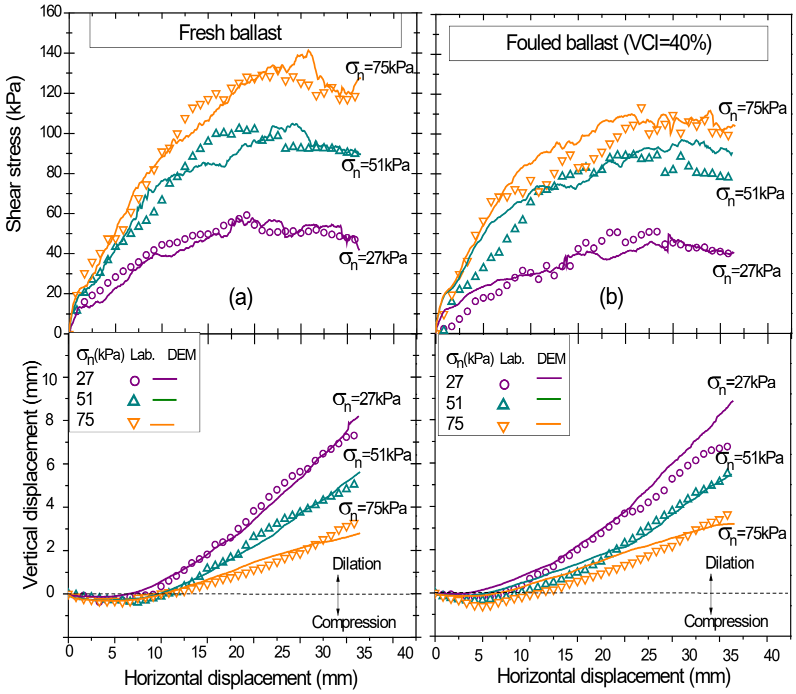


Fig. 11. Comparisons between DEM and laboratory test results: (a) fresh ballast, and (b) 40%VCI-fouled fouled ballast (*Ngo et al. [24]*)

The use of geosynthetics with recycled ballast:
The use of degraded ballast (i.e. recycled ballast from the spoil tips of Sydney Trains) strengthened with artificial plastic grids increased the internal stability by reducing the movement of aggregates. By using this technique to move from theory to practice, the novel track built at Bulli (north of Wollongong) via a joint collaboration between RailCorp and the University of Wollongong (UOW) has resulted in a resilient track that will bear greater train loads with less settlement; this track has been tested and monitored over a period of two year periods. This method has proved to be an excellent solution for the need to quarry fresh ballast, while proactively contributing to reducing environmental degradation and saving the Australian rail industry millions of dollars.

New design software:
Practicing engineers still prefer to develop simplified software that will provide solutions that are accurate enough for ballasted track design rather than sophisticated numerical modeling. The salient research outcomes that have evolved at the CGRE (Centre for Geomechanics and Railway Engineering), UOW over past two decades has resulted in a software: *Supplementary Methods of Analysis for Rail Track* (SMART) based on MATLAB subroutines, as the main user interface shown in Figure 12. The SMART is unique and more comprehensive for preliminary track design and analysis because it is expected to revolutionize the current state-of-the-art of track design.

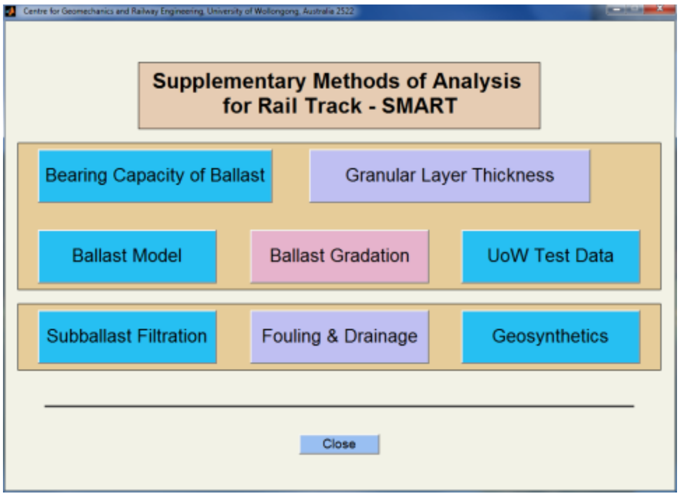


Fig. 12. A snapshot of SMART design software developed at UOW

5 Conclusions

This paper has reviewed recent studies carried out at the University of Wollongong Australia to examine the ability of artificial inclusions to minimize ballast deformation and associated breakage induced by heavy haul freight trains. The following conclusions can be drawn:

- The laboratory tests showed that the inclusion of under ballast mat (UBM) reduced the permanent deformation of ballast and decreased particle breakage by up to 50% because their energy-absorbing properties reduced the dynamic amplification of train loads.

- An approach of confining the capping layer with recycled rubber tires for increased stability and resiliency of track substructure was tested using large-scale TPSTA tests under cyclic loading conditions. FEM simulations on the use of tires to provide additional confinement to ballasted tracks were also implemented. The laboratory and FEM results proved the role of recycled rubber tire by reducing the stress transmitted to the subgrade that could effectively decrease the required ballast thickness.

- The performance of a track specimen fitted with the proposed synthetic energy absorbing layer (SEAL10) was investigated through the large-scale cubical triaxial test. The results show that SEAL10 has reduced the vertical and lateral deformation of the track, as well as reducing the ballast degradation by almost 50% more than the test specimen with traditional materials.

- Laboratory tests and DEM simulations on ballast reinforced by geogrids proved that the geogrid reinforcement plays a substantial role in increasing the shear strength and controlling the permanent deformations of the ballast. This is attributed to the aggregate-geogrid interlock mechanism, which contributes to improved stability of the granular assembly leading to a more uniform distribution of the internal loads.

- There is no doubt that rubber tires improved the geogrid-ballast interaction and increased the confinement of the capping layer, and moreover, the use of rubber products, particularly on harder subgrades resulted in improved resilient tracks with enhanced longevity and stability with significant implications for the ongoing cost of track maintenance.

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